A Test Particle Simulation for the Jovian Magnetospheric Electrons Precipitating into Europa’s Oxygen Atmosphere

SPS 2023 • Appendix

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THIS STUDY IS NOW IN PREPARATION TO SUBMIT TO JGR.

Models

Jupiter's Dipole Field

- The field

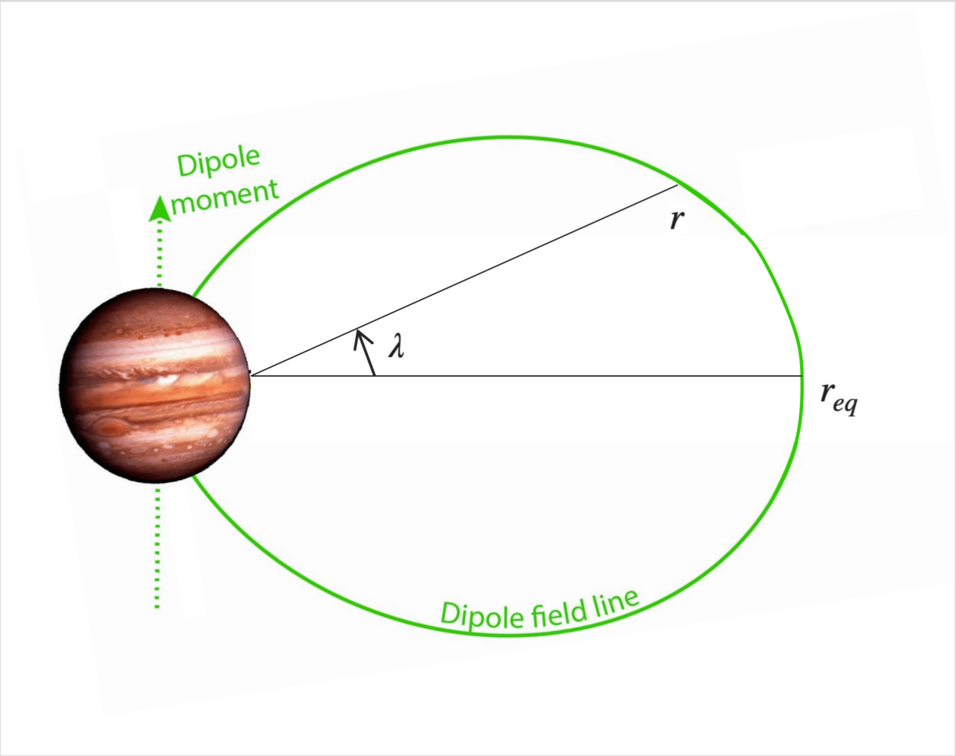
$$B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\left(1 + 3\sin^2\lambda\right)^{\frac{1}{2}}$$

λ: magnetic latitude

$$\vec{B}(r,\theta) = \frac{\mu_0 M}{4\pi}\left(\frac{3Mz\vec{r}}{r^5} - \frac{M}{r^3}\vec{e_z}\right)$$

- Field line equation

$$r = r_{eq}\cos^2\lambda$$



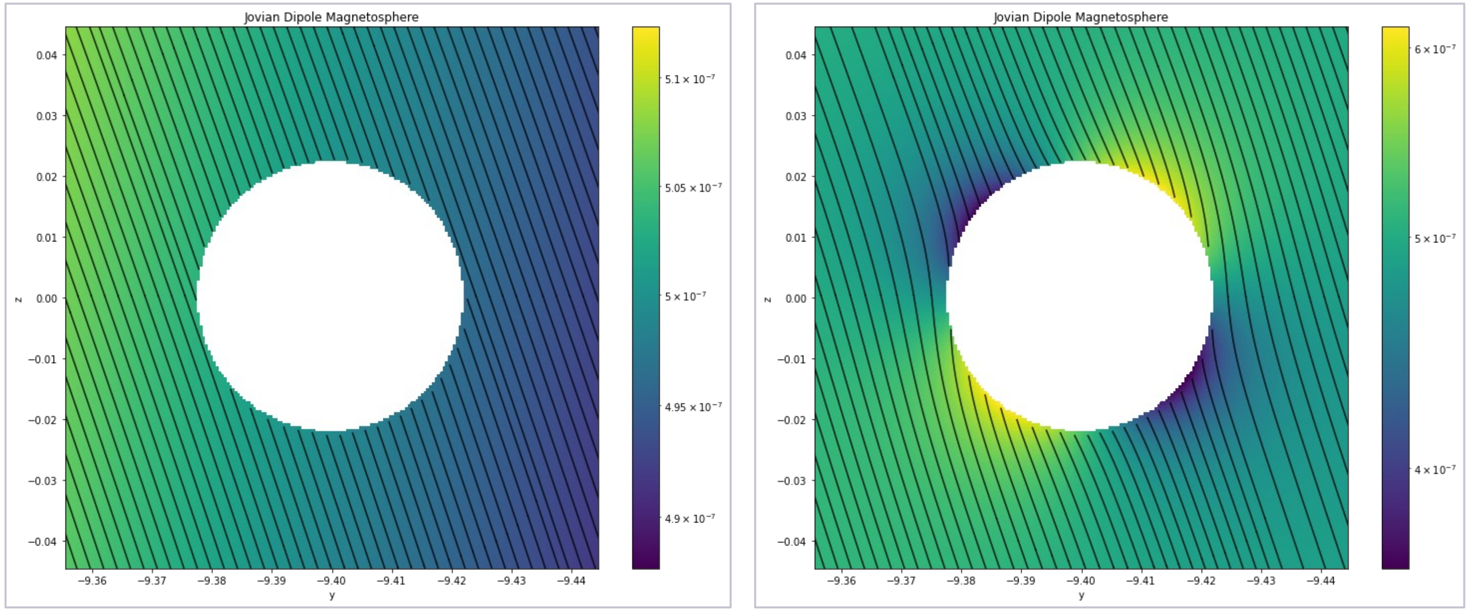
Models

Analytical Model of Static Induced Magnetic Field

- Introduced by Neubauer+1998, Liuzzo+2016, etc.
- Treating the subsurface ocean as a highly conducting medium
  → cancels the primary field at the "magnetic poles"

$$\mathbf{M}_{\text{ind}} = -\frac{2\pi R_C^3}{\mu_0}\left(B_{x,0}\hat{\mathbf{x}} + B_{y,0}\hat{\mathbf{y}}\right)$$

(Fig)
Left: Magnetic field lines without induced magnetic field.
Right: Induced magnetic field included.



Models

Electrons in the Jovian Dipole Field

- Gyro-motion prorpety
  - Period

$$T_{gyro} = \frac{2\pi m}{eB} = 7.1 \times 10^{-5} \text{ sec}$$

  - Radius

$$R_{gyro} = \frac{mv}{eB}$$

eV	1	10	100	1K	10K	100K	250K
ratio to light speed	1.9×10^{-3}	6.2×10^{-3}	1.9×10^{-2}	6.2×10^{-2}	1.9×10^{-1}	6.2×10^{-1}	9.8×10^{-1}
R_gyro [m]	6.7	21	67	213	674	2133	3373

4th Runge-Kutta method

- Iterative method for approximate solutions of ordinary differential equations, such as equation of motion
- Total accumulated errors on order of $O(h^4)$ (h is the step-size.)

Let an initial value problem be specified as follows: $$\frac{dy}{dt} = f(t, y), \quad y(t_0) = y_0 .$$

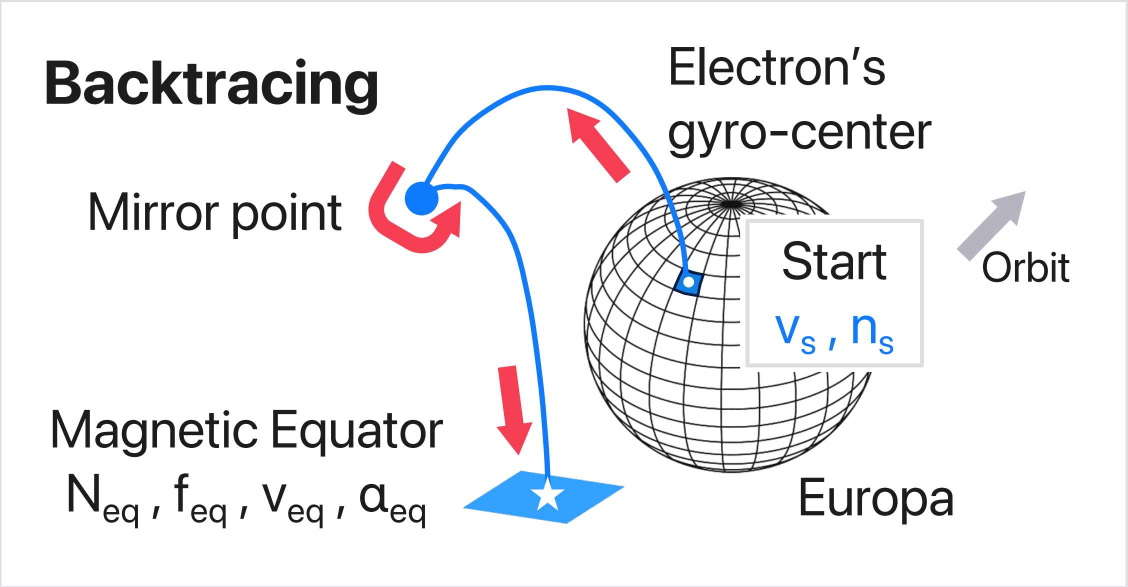
y is an unknown function of time t. Pick a step-size $h > 0$ and for n = 0, 1, 2, ..., define

$$y_{n+1} = y_n + \frac{1}{6} h \left(k_1 + 2k_2 + 2k_3 + k_4\right)$$
$$t_{n+1} = t_n + h$$

$$\begin{aligned} k_1 &= f(t_n, y_n), \\ k_2 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_1}{2}\right), \\ k_3 &= f\left(t_n + \frac{h}{2}, y_n + h\frac{k_2}{2}\right), \\ k_4 &= f\left(t_n + h, y_n + hk_3\right). \end{aligned}$$

Methods

Calculation of precipitation flux (method by Cassidy+2013)



v_s : Electron's velocity vector at the starting point on the surface

n_s : Normal unit vector at the Starting point

n_{eq} : Electron density at the magnetic equator

f_{eq} : Electron velocity distribution at the magnetic equator

v_{eq} : An electron velocity at the magnetic equator

α_{eq} : An electron's pitch angle at the magnetic equator

Electron number flux into a grid labeld as "s"

$$F_s = \int_{v_{eq}} \int_{\alpha_{eq}} (\vec{v_s} \cdot \vec{n_s})\, n_{eq} f_{eq}(v_{eq}, \alpha_{eq}) \mathrm{d}v_{eq} \mathrm{d}\alpha_{eq}$$

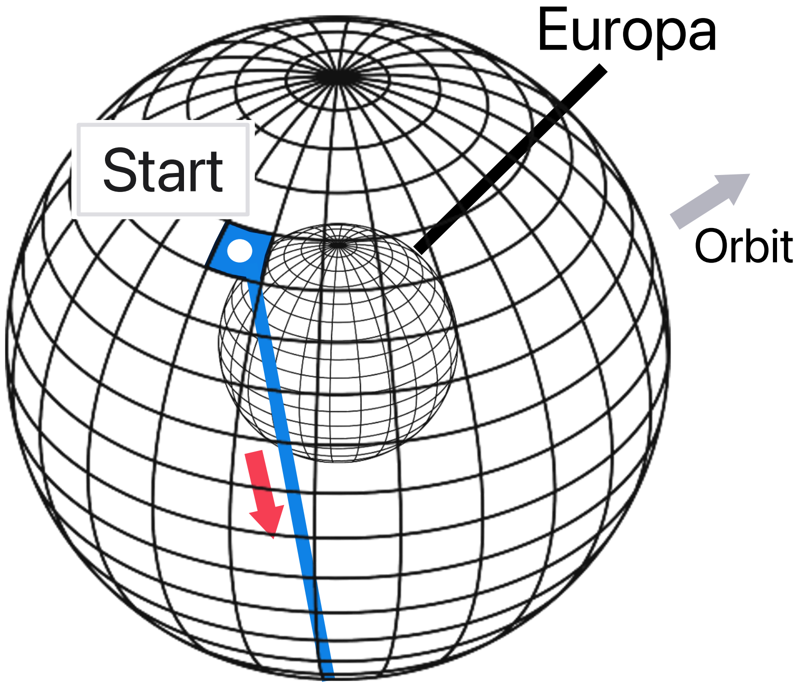
Methods

Calculation of Volume Emission Rate

- Same electric-magnetic field
- Cross section for dissociative excitation of O_2 provided by Kanik+2003
- No other chemical reactions included

- Calculate volume emission rate each time-step
- Integrate VER over line of sight
  → derive brightness in Rayleigh

Forward-tracing



Sphere defined as top of atmosphere

Motion of a particle

Methods

Calculation of Volume Emission Rate

- One single electron impacting a column of O_2 molecules

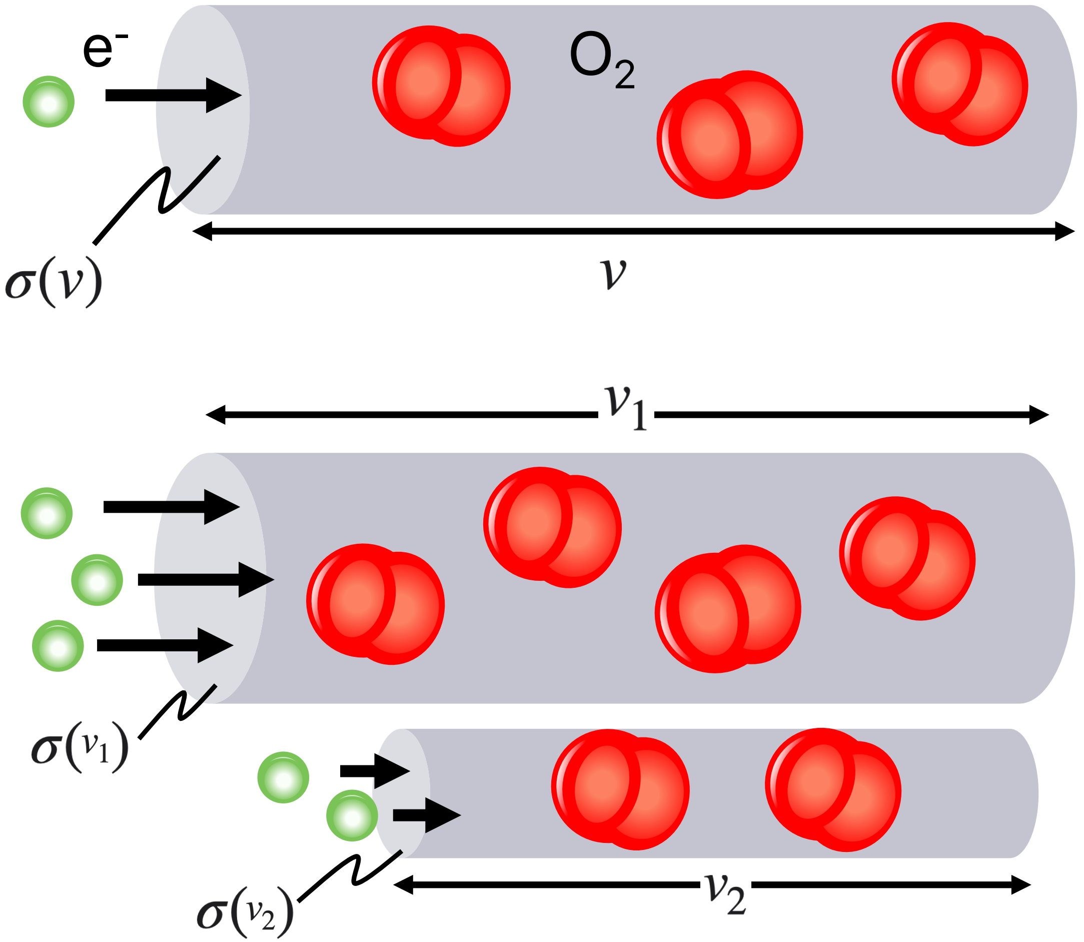
$$R(\vec{r}, v) = \sigma(v) v n_{O_2}(\vec{r})$$ [photons s^{-1}]

- Electrons (density n_{e0}) impacting a column of O_2 molecules

$$R'(\vec{r}) = \int_{v=\chi}^{\infty} \sigma(v) v n_{O_2}(\vec{r}) n_{e_0} f(v) \mathrm{d}v$$ [photons cm^{-3} s^{-1}]

Volume emission rate

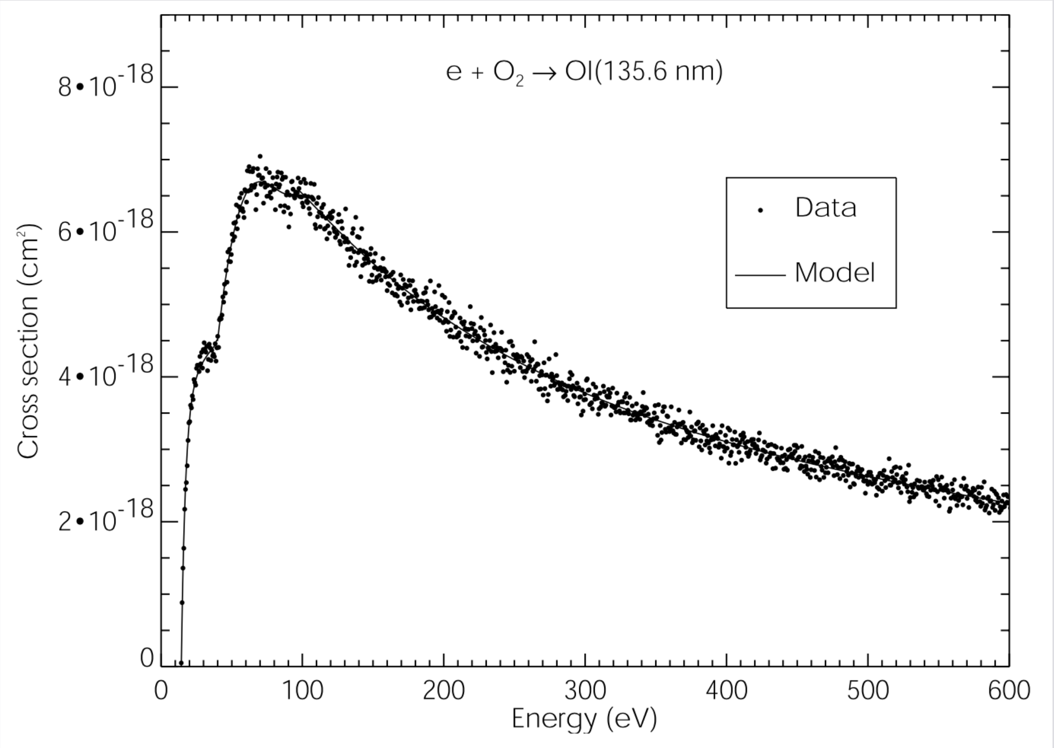
σ(v): absolute cross section of electron-impact dissociative excitation of O2
v: velocity of electron



Methods

Cross Section of O_2 Dissociative Excitation

Laboratory data of cross section for dissociative excitation of O_2 with 135.6 nm emission (Kanik+2003)



$$O(^5S) \rightarrow O(^3P) + h\nu(135.6\ \mathrm{nm})$$

- Kanik+2003 obtained the absolut value of σ(v) from 14.3 eV (threshold) to 600 eV.
- We adopt a linear extrapolation on the higher electron energy (> 600 eV).

Methods

Processing of Model Spectral Images – 1 of 4

- The volume emission rate of 135.6 nm emission due to the dissociative excitation of O_2 is calculated by the following expression:

$$R_{135.6}(\vec{r}) = \int_{v=\chi}^{\infty} \sigma_{O_2}(v) v n_{O_2}(\vec{r}) n_e(\vec{r}) f(v) \mathrm{d}v$$

$R_{135.6}$: 135.6 nm volume emission rate
σ_{O2} : cross section for the dissociative excitation of O_2
n_e : local electron density
f : local electron velocity distribution
n_{O2} : local O_2 density
v : electron's velocity

- Integrating it along a line sight, we obtain the absolute brightness $I_{135.6}$ on a pixel of the detector labeled as (p, q) in the unit of Rayleigh (1 R = 10^6 / 4π photons cm^{-2} sr^{-1} s^{-1}).

$$I_{135.6}(p,q) = \frac{10^{-6}}{4\pi} \int_{l_{(p,q)}} R_{135.6}(\vec{r}) \mathrm{d}l_{(p,q)}$$

$I_{135.6}$ (p, q): 135.6 nm brightness at the (p, q) pixel
$l_{(p,q)}$: line of sight at the (p, q) pixel

Methods

Processing of Model Spectral Images – 2 of 4

- The obtained "raw" brightness is converted into the photon counts N_{count} by the following expression:

$$N_{\text{count}}(p,q) = \frac{10^6}{4\pi} I_{135.6}(p,q)\Omega_{\text{px}} A_{\text{eff}} \tau_{\text{e}}$$

$$A_{\text{eff}} = A_{\text{HST}} T_{\text{G140L}}$$

$I_{135.6}$ (p, q): 135.6 nm brightness at the (p, q) pixel
$l_{(p,\,q)}$: line of sight at the (p, q) pixel
Ω_{px} : solid angle of a single pixel of the STIS/FUV-MAMA calculated from the spatial plate scale of 0.0246 arcsec/pixel
A_{eff} : the effective area of the HST
A_{HST} : the area of the HST primary mirror, 45239 cm^2
τ_e : exposure time, here assumed to be 45 minutes
T_{G140L} : throughput of G140L grating, 4% for 135.6 nm

- Assuming that photon noise of the detector follows the normal distribution, the photon noise of the detector N_{noise} is calculated for the (p, q) pixel:

$$N_{\text{noise}}(p,q) = \sqrt{N_{\text{count}}(p,q)} \times (\text{random value from the normal distribution})$$

Methods

Processing of Model Spectral Images – 3 of 4

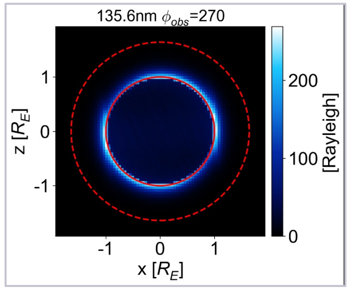
- We use the NumPy random.randn() function to generate random values from the normal distribution.
- N_{noise} is added to N_{count}, and then the summation $N_{count} + N_{noise}$ is converted to the unit of Rayleigh → model spectral image with photon noise for STIS/FUV-MAMA
- If $N_{count} + N_{noise}$ is negative on a pixel, we suppose that the pixel does not detect any photons at the exposure.

Methods

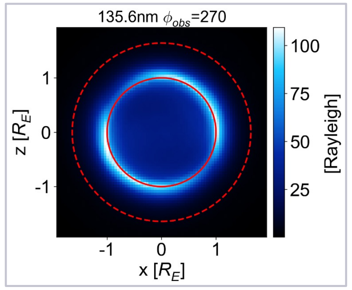
Processing of Model Spectral Images – 4 of 4

- The model spectral images with photon noise will be binned in 3 × 3 pixels.
- Finally, it will be smoothened for six times (Roth+2016):

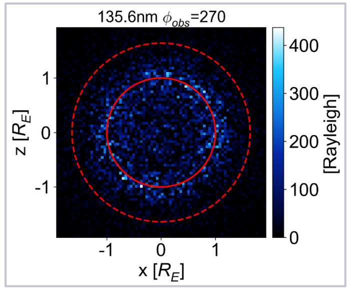
$$f(i_x, i_y) = \frac{1}{8}(4f(i_x, i_y) + f(i_{x-1}, i_y) + f(i_{x+1}, i_y) + f(i_x, i_{y-1}) + f(i_x, i_{y+1})),$$



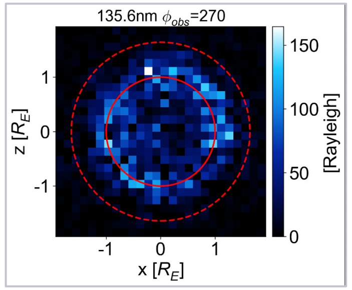
Raw image



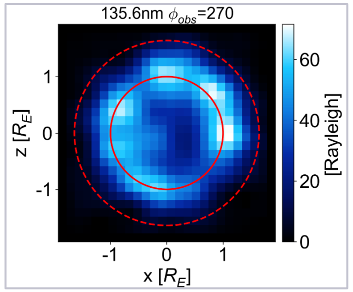
PSF convolved



Noise added



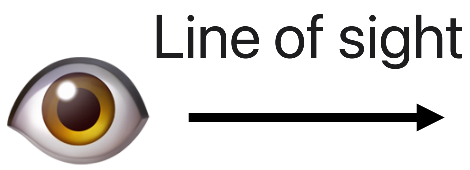
Binned



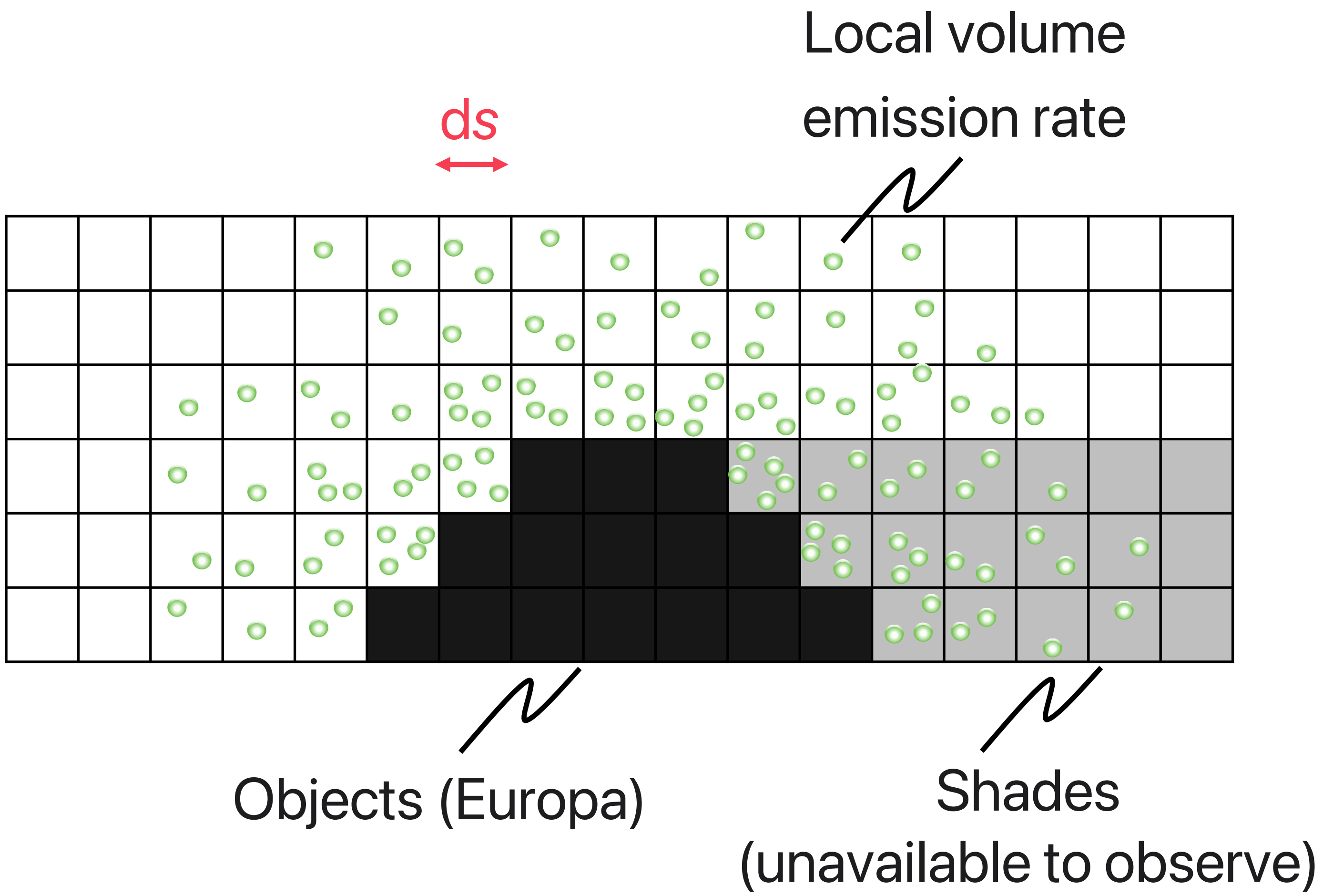
Smoothened

Methods

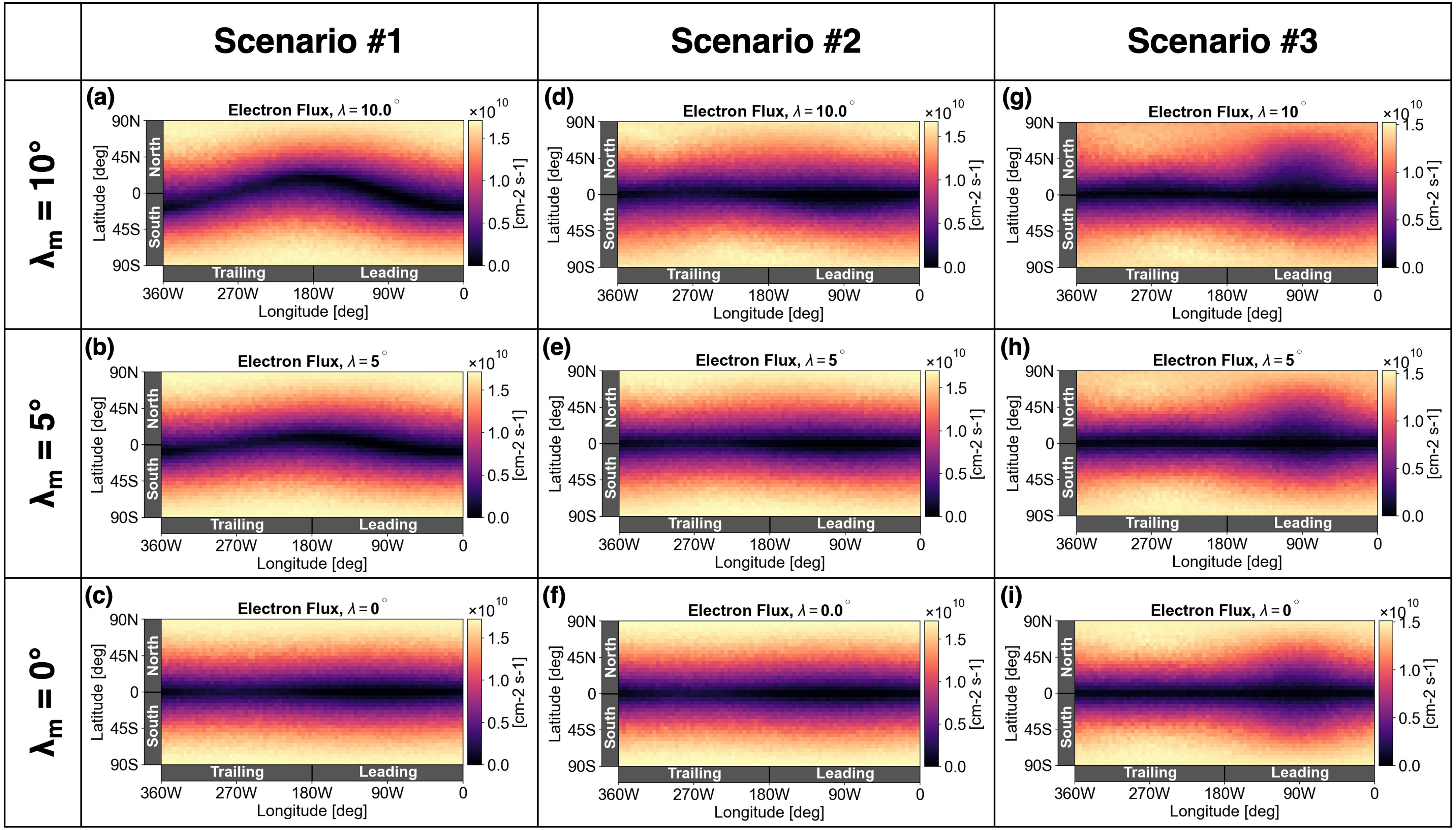
Integration on Line of Sight



$$I = \frac{10^{-6}}{4\pi}\int_0^\infty \gamma n_e n_{O_2}\, \mathrm{d}s \text{ [Rayleigh]}$$

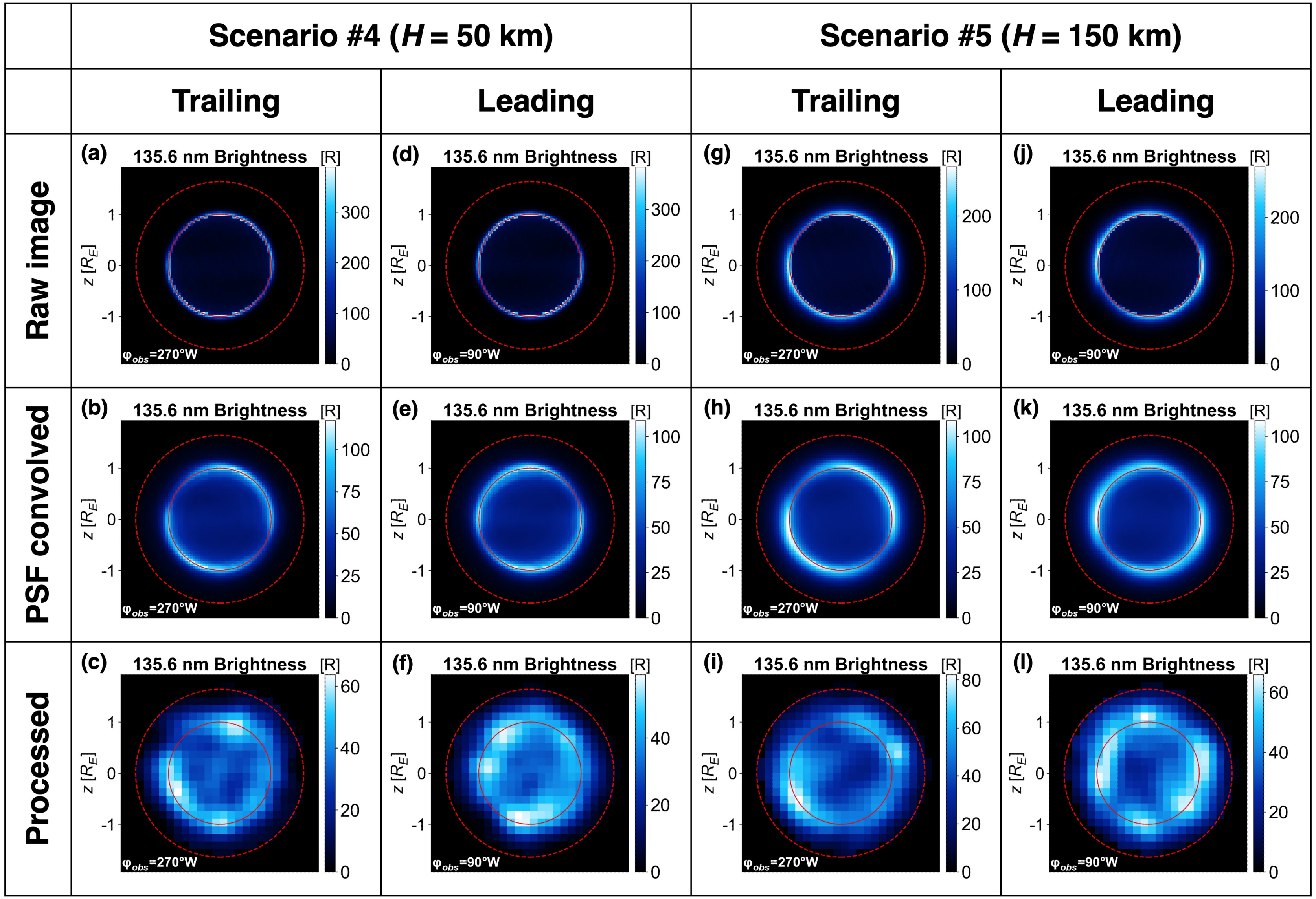


Results

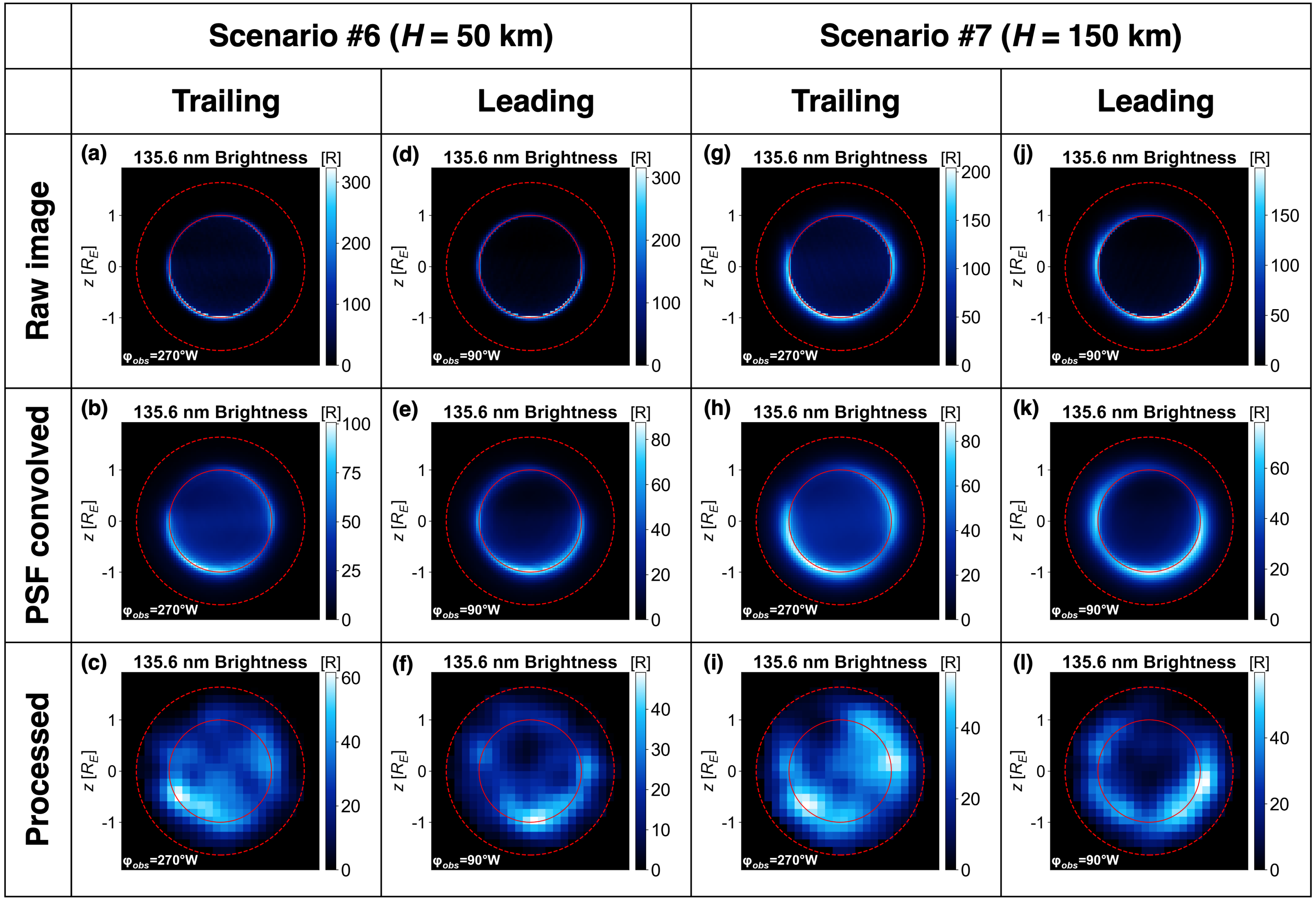


Scenario #1
Scenario #2
Scenario #3
λm = 10°
λm = 5°
λm = 0°
(a) Electron Flux, λ = 10.0°
(b) Electron Flux, λ = 5°
(c) Electron Flux, λ = 0°
(d) Electron Flux, λ = 10.0°
(e) Electron Flux, λ = 5°
(f) Electron Flux, λ = 0.0°
(g) Electron Flux, λ = 10°
(h) Electron Flux, λ = 5°
(i) Electron Flux, λ = 0°
Latitude [deg]
90N
45N
0
45S
90S
North
South
Trailing
Leading
360W
270W
180W
90W
0
Longitude [deg]
×10^10
0.0
0.5
1.0
1.5
[cm-2 s-1]

Results


Scenario #4 (H = 50 km)
Scenario #5 (H = 150 km)
Trailing
Leading
Trailing
Leading
Raw image
PSF convolved
Processed
(a) 135.6 nm Brightness [R]
(d) 135.6 nm Brightness [R]
(g) 135.6 nm Brightness [R]
(j) 135.6 nm Brightness [R]
(b) 135.6 nm Brightness [R]
(e) 135.6 nm Brightness [R]
(h) 135.6 nm Brightness [R]
(k) 135.6 nm Brightness [R]
(c) 135.6 nm Brightness [R]
(f) 135.6 nm Brightness [R]
(i) 135.6 nm Brightness [R]
(l) 135.6 nm Brightness [R]
z [R_E]
φ_obs=270°W
φ_obs=90°W

Results


Scenario #6 (H = 50 km)
Scenario #7 (H = 150 km)
Trailing
Leading
Trailing
Leading
Raw image
PSF convolved
Processed
(a) 135.6 nm Brightness [R]
(d) 135.6 nm Brightness [R]
(g) 135.6 nm Brightness [R]
(j) 135.6 nm Brightness [R]
(b) 135.6 nm Brightness [R]
(e) 135.6 nm Brightness [R]
(h) 135.6 nm Brightness [R]
(k) 135.6 nm Brightness [R]
(c) 135.6 nm Brightness [R]
(f) 135.6 nm Brightness [R]
(i) 135.6 nm Brightness [R]
(l) 135.6 nm Brightness [R]
z [R_E]
φobs=270°W
φobs=90°W

Results

- The limb peak brightness for H = 50 km is smoothened more heavily and becomes lower than for the case with H = 150 km.
- This is because of the full width at half maximum (FWHM) of the PSF, which is ~ 1.5 raw pixels (Branton & Riley, 2021) and equivalent to ~ 113 km on Europa's disk.