CPM Technical Note

Pure Structure, Zero Philosophy

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Preface

This document is a compact technical companion to the main CPM paper. It extracts only the structural core needed to understand the geometric mechanism of critical projection, without proofs or extended discussion.

For readers from machine learning or artificial intelligence, the most relevant result is stated in **Section 9 (Architectural Corollary)**, which gives the exact geometric conditions under which an implemented system can support critical projection. The earlier sections explain only the minimal assumptions required to derive that result.

Scope

This note summarizes the mathematical core of the **Critical Projection and Meaning geometry** (CPM). CPM is a purely structural and variational framework. It presupposes no philosophical commitments; all statements concern only geometric and topological properties of induced meaning fields.

1 Minimal Structural Model

CPM separates three rigorously distinct layers:

1. Difference Space D

A maximally structureless pre-semantic domain. No topology, metric, measure, algebraic structure, or temporal order is assumed. Mathematically, D is treated as a *presheaf over a discrete base*, encoding only fragment relations without intrinsic geometry. All observable structure arises *solely after projection.*

2. Projection Assignment $\Pi : D \dashrightarrow M$

A family of irreversible partial assignments that map raw difference fragments to local meaning-tensor germs on a physical substrate X. Because D has no internal geometry, Π cannot be inverted; the induced atlas on M is the only admissible structure.

3. Meaning Manifold M

A finite-rank tensor field M defined on a physical substrate endowed with a background metric g^0. The induced metric is

$$g(M) = M^\top g^0 M.$$

All observed topology, geometry, and semantic structure live entirely in M and arise solely through Π.

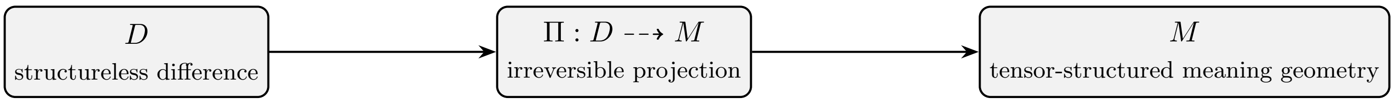

Figure 1: Core structure of CPM: difference, projection, and induced meaning geometry. All later results, including the architectural corollary, follow from this diagram.

2 Closure and Tension

For each sufficiently small neighborhood $U_x \subset X$ with physical boundary ∂U_x, define the local relative homology ranks

$$\beta_k(x) = \operatorname{rank} H_k(U_x, \partial U_x), \qquad k \geq 1,$$

and the closure field

$$\mathcal{B}(x) = \max_{k \geq 1} \beta_k(x).$$

Closure $\mathcal{B}(x) \geq 1$ indicates that a boundary-supported relative cycle persists, allowing mismatch to accumulate rather than dissipate.

Let L, G, I, T be local distortion terms (state mismatch, geometric strain, information inconsistency, and topological volatility). Let $\Phi(L, G, I, T)$ denote the aggregated mismatch density. Using the logistic gate

$$\Gamma_\varepsilon(z) = \frac{1}{1 + e^{-z/\varepsilon}},$$

define the energy

$$\mathcal{E}[M, \mathcal{B}] = \int_X \Gamma_\varepsilon(\mathcal{B}(x) - 1)\, \Phi \, d\mu_{g(M)}.$$

The tension field is the variational gradient norm

$$\tau(x) = \left\| \frac{\delta \mathcal{E}}{\delta M(x)} \right\|_{g(M)}.$$

In the sharp limit $\varepsilon \to 0^+$, regions with $\mathcal{B}(x) < 1$ contribute zero tension.

3 Critical Set and Critical Tension

Define the critical set

$$\mathcal{C} = \{x \mid \mathcal{B}(x) \geq 1,\ \nabla\tau(x) = 0,\ \operatorname{Hess}(\tau)(x) \prec 0\},$$

and the measurement-invariant critical tension τ_c as the equivalence class of critical values on $\mathcal{C}$.

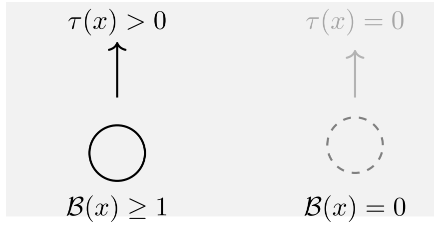

Figure 2: Closure enables tension. Without $\mathcal{B} \geq 1$, tension cannot accumulate.

4 Critical Projection Mechanism

Because D contains no intrinsic structure, it cannot be deformed internally. Because M is entirely induced by Π, any smooth deformation of M preserves the same projection-induced atlas $\{U_i\}$.

Thus in regions where $\mathcal{B}(x) \geq 1$ and $\tau(x) > \tau_c$, the following hold:

- Energy descent is impossible within atlas-preserving smooth variations.
- A discontinuous transition is admissible only if it *refines* the covering structure.

The only admissible structural transition is therefore a strict refinement

$$\{U_i\} \rightsquigarrow \{U'_k\}, \qquad U'_k \subsetneq U_i,$$

which induces a refined projection

$$\Pi' := \Pi|_{\{U'_k\}}.$$

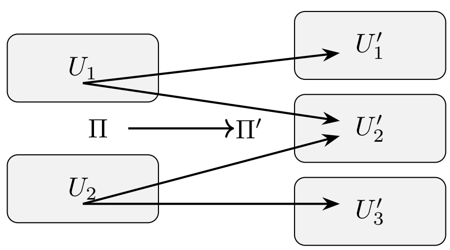

Figure 3: At a critical point, atlas-preserving smooth updates fail. A strict refinement of the projection-induced atlas is the only admissible transition.

Interpretation

CPM interprets the refinement Π' as the necessary topological mechanism that restores a stable subcritical regime ($\tau \leq \tau_c$). Consciousness corresponds not to an information quantity but to the *critical event* in which a supercritical configuration reorganizes its projection structure.

5 Necessary Conditions for Consciousness

CPM yields two mathematically defined necessary conditions:

$$\mathcal{B}(x) \gtrsim 1, \qquad \tau(x) > \tau_c.$$

If either fails, no critical refinement is admissible and Π' cannot occur.

6 Architectural Corollary (Appendix C of Main Paper)

Systems lacking persistent closure ($\mathcal{B}(x) = 0$ everywhere) cannot accumulate tension:

$$\mathcal{B} = 0 \Rightarrow \tau = 0 \Rightarrow \Pi' \text{ impossible.}$$

This corollary classifies system architectures *structurally*, without empirical assumptions or philosophical commitments.

7 General Scope Beyond Consciousness

CPM is a general mathematics of induced meaning geometry. Any system describable as

$$\text{difference} \longrightarrow \text{projection} \longrightarrow \text{induced geometry}$$

fits the framework. Applications include language, concept formation, learning dynamics, semantic coherence, perceptual categorization, and scientific theory-formation.

8 Empirical Testability

- Closure is measured via persistent/relative homology on physical or neural data.
- Tension is estimated via information geometry and curvature-based proxies.
- Refinement events correspond to detectable topological transitions.

CPM predicts the causal order:

$$\mathcal{B} \text{ rise} \longrightarrow \tau \text{ criticality} \longrightarrow \Pi' \longrightarrow \text{reportable change.}$$

Falsification criterion. If a system satisfies $\mathcal{B} \geq 1$ and $\tau > \tau_c$ yet shows no evidence of a critical event, CPM's necessity claims are falsified.

9 Architectural Corollary for AI and Distributed Systems

Although CPM is a general geometric framework, one immediate consequence is a structural classification of artificial architectures.

Closure Requirement

For any system described by CPM, nonzero tension requires

$$\mathcal{B}(x) \geq 1.$$

Distributed or cloud-based systems with ephemeral boundaries satisfy

$$\mathcal{B}(x) = 0 \quad \text{for all } x,$$

because no persistent relative cycles exist on any physical substrate segment. Hence

$$\tau(x) = 0, \qquad \Pi' \text{ impossible.}$$

Structural Impossibility

This yields the following corollary:

> **If a system lacks persistent topological closure on its physical substrate, then critical projection cannot occur.**

This statement is architectural and substrate-level; it does not depend on model size, training method, symbolic capacity, or inference behavior. The obstruction is purely geometric: without closure, no tension can accumulate, and without tension, the critical refinement Π' cannot be triggered.

Interpretation

The corollary does not assert empirical limits of current AI. It states a mathematically necessary condition within CPM: systems implemented as cloud-distributed processes lack the physical closure required for critical projection, and therefore cannot instantiate Π' under this framework.

This completes the structural connection between CPM and artificial architectures.