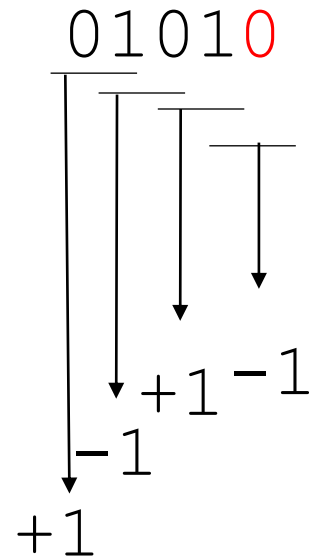


# Booth's Algorithm

- Treats positive and negative multipliers uniformly.
- Rewrites multiplier in terms of sums and differences.
- Convert code according to next bit at right
- $0 \text{ to } 1 \Rightarrow +1$
- $1 \text{ to } 0 \Rightarrow -1$
- Otherwise, 0
- Right of lsb is “nothing”, *i.e.*, equal to 0

# Booth's Algorithm



Steps in obtaining Booth's equivalent of a binary number

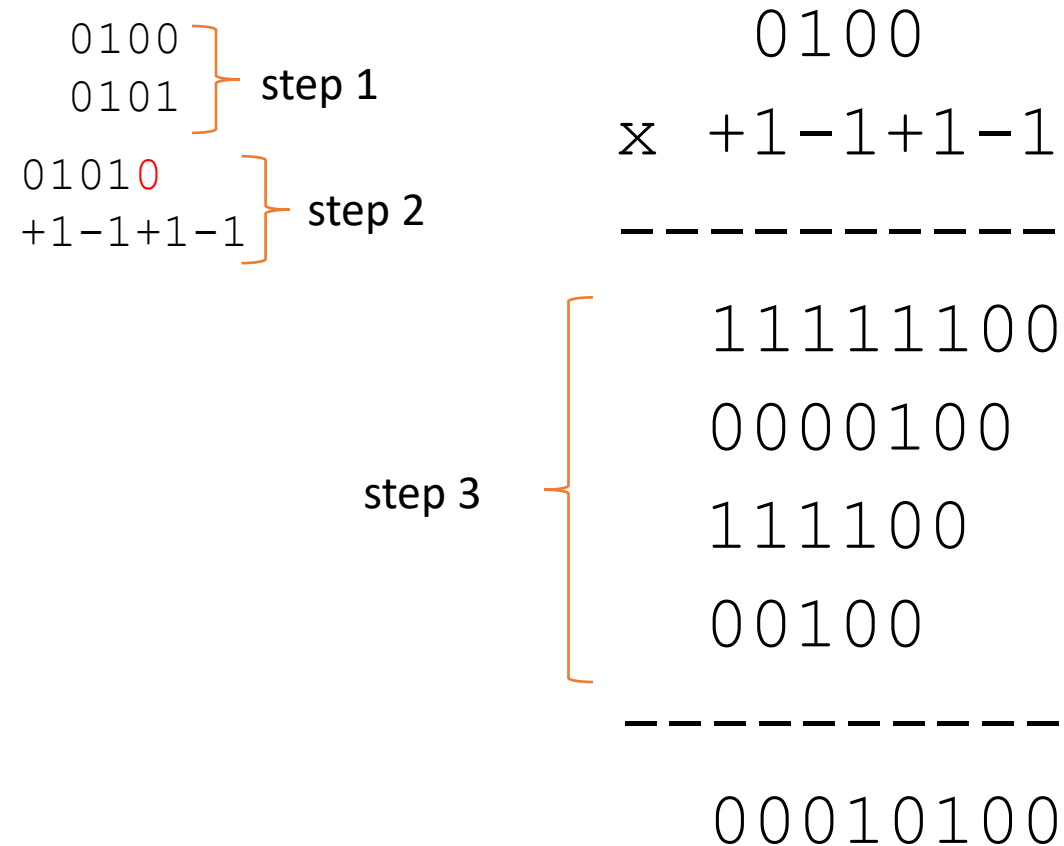
1. append 0 at the LSb side
2. pair 2 bits starting at LSb
3. 00 → 0; 01 → +1; 10 → -1; 11 → 0

$$0 * m = 0$$

$$1 * m = m$$

$$-1 * M = 2's \text{ complement}(m)$$

# Booth's Algorithm

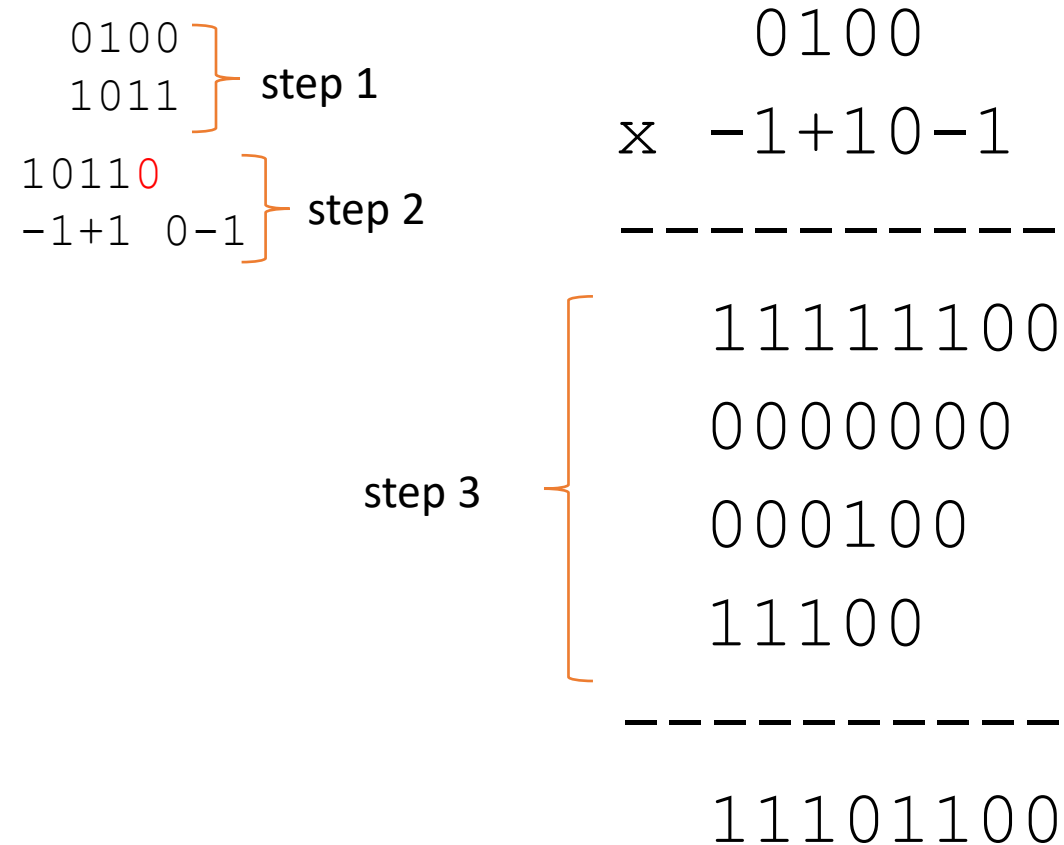


Example: +4 \* +5

Steps:

- 1.) represent both multiplicand (m) and multiplier (n) using 2's complement format
- 2.) convert multiplier to its Booth's equivalent
  - append 0 at the LSb side
  - pair 2 bits starting at LSb
  - 00 → 0; 01 → +1; 10 → -1; 11 → 0
- 3.) Multiply using pencil-and-paper method ignoring extra steps if multiplier is negative

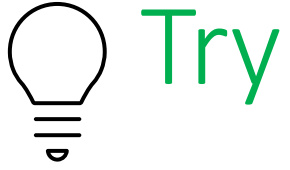
# Booth's Algorithm



Example: +4 \* -5

Steps:

- 1.) represent both multiplicand (m) and multiplier (n) using 2's complement format
- 2.) convert multiplier to its Booth's equivalent
  - append 0 at the LSb side
  - pair 2 bits starting at LSb
  - 00 → 0; 01 → +1; 10 → -1; 11 → 0
- 3.) Multiply using pencil-and-paper method ignoring extra steps if multiplier is negative



Try:  $+13 \times -6$  (using Booth's algorithm)

```
  01101
x 11010
-----
```

```
  01101
  11010 } step 1
110100 } step 2
0-1+1-10
```

```
          01101
x 0-1+1-10
-----
0000000000
111110011
00001101
1110011
000000
-----
1110110010
```

## Extended Booth's Algorithm

- Also known as fast multiplication or bit-pair recording
- Bit-Pair Recording – reduces to half the number of summands The number of summands is reduced by pairing multiplier bits

Bit-pair recording:

0 0 0  $\Rightarrow$  0

0 0 1  $\Rightarrow$  +1

0 1 0  $\Rightarrow$  +1

0 1 1  $\Rightarrow$  +2

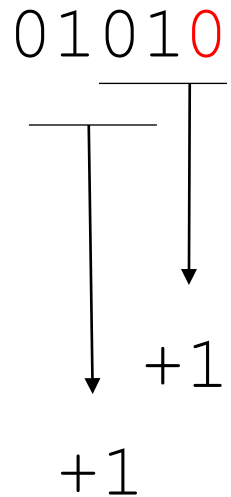
1 0 0  $\Rightarrow$  -2

1 0 1  $\Rightarrow$  -1

1 1 0  $\Rightarrow$  -1

1 1 1  $\Rightarrow$  0

# Extended Booth's Algorithm



Steps in obtaining Booth's equivalent of a binary number

1. append 0 at the LSb side
2. if odd number of bits, sign-extend
3. bit-pair starting at LSb

$$0 * m = 0$$

$$1 * m = m$$

$$-1 * m = 2's \text{ complement}(m)$$

$$+2 * m = m0$$

$$-2 * m = [2's \text{ complement}(m)]0$$

Bit-pair recording:

$$0 \ 0 \ 0 \Rightarrow 0$$

$$0 \ 0 \ 1 \Rightarrow +1$$

$$0 \ 1 \ 0 \Rightarrow +1$$

$$0 \ 1 \ 1 \Rightarrow +2$$

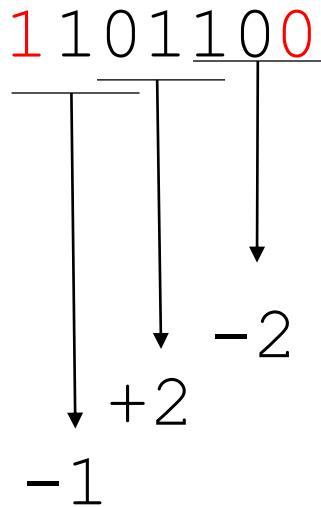
$$1 \ 0 \ 0 \Rightarrow -2$$

$$1 \ 0 \ 1 \Rightarrow -1$$

$$1 \ 1 \ 0 \Rightarrow -1$$

$$1 \ 1 \ 1 \Rightarrow 0$$

# Extended Booth's Algorithm



Steps in obtaining Extended Booth's equivalent of a binary number

1. append 0 at the LSb side
2. if odd number of bits, sign-extend
3. bit-pair starting at LSb

$$0 * m = 0$$

$$1 * m = m$$

$$-1 * m = 2's \text{ complement}(m)$$

$$+2 * m = m0$$

$$-2 * m = [2's \text{ complement}(m)]0$$

Bit-pair recording:

$$0 \ 0 \ 0 \Rightarrow 0$$

$$0 \ 0 \ 1 \Rightarrow +1$$

$$0 \ 1 \ 0 \Rightarrow +1$$

$$0 \ 1 \ 1 \Rightarrow +2$$

$$1 \ 0 \ 0 \Rightarrow -2$$

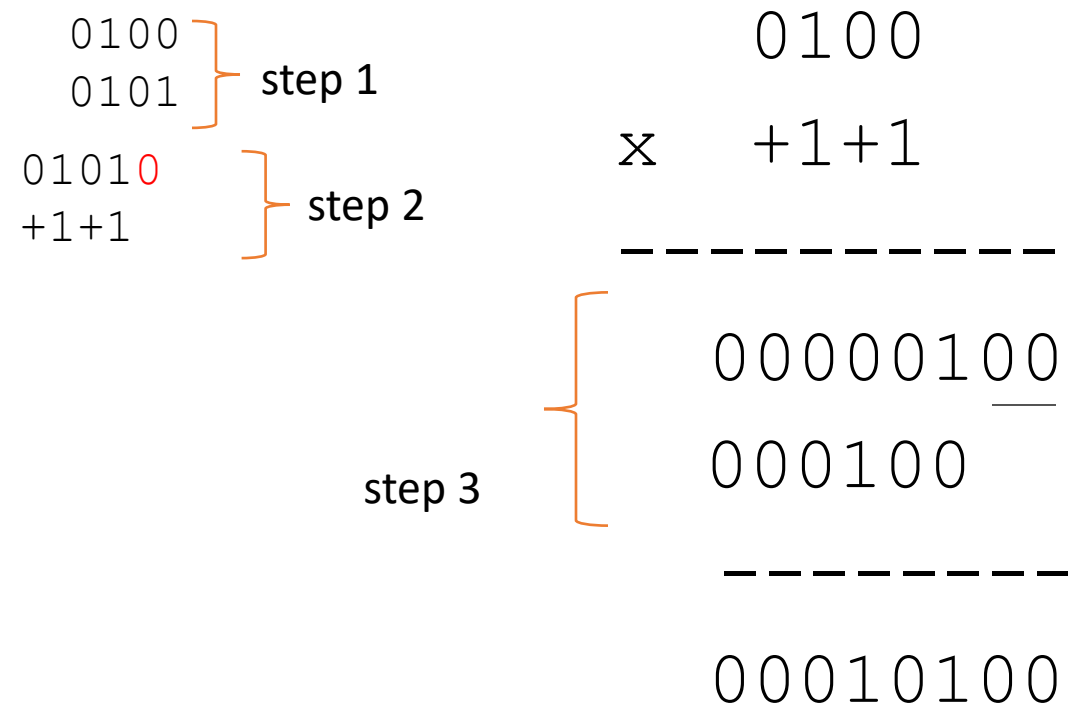
$$1 \ 0 \ 1 \Rightarrow -1$$

$$1 \ 1 \ 0 \Rightarrow -1$$

$$1 \ 1 \ 1 \Rightarrow 0$$



# Extended Booth's Algorithm



Example: +4 \* +5

Steps:

- 1.) represent both multiplicand (m) and multiplier (n) using 2's complement format
- 2.) convert multiplier to Extended Booth's equivalent
- 3.) Multiply using pencil-and-paper method ignoring extra steps if multiplier is negative. Since each bit-pair is equivalent to 2 bits, skip two after the initial intermediate product

# Extended Booth's Algorithm

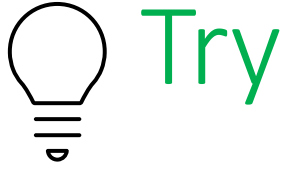
$$\begin{array}{r}
 0100 \\
 1011 \\
 \hline
 10110 \\
 -1-1 \\
 \hline
 \end{array}
 \begin{array}{l}
 \text{step 1} \\
 \text{step 2}
 \end{array}$$
  

$$\begin{array}{r}
 0100 \\
 \times \quad -1-1 \\
 \hline
 11111100 \\
 111100 \\
 \hline
 11101100
 \end{array}
 \begin{array}{l}
 \text{step 3}
 \end{array}$$

Example:  $+4 * -5$

Steps:

- 1.) represent both multiplicand (m) and multiplier (n) using 2's complement format
- 2.) convert multiplier to Extended Booth's equivalent
- 3.) Multiply using pencil-and-paper method ignoring extra steps if multiplier is negative. Since each bit-pair is equivalent to 2 bits, skip two after the initial intermediate product



Try:  $+13 \times -6$  (using Extended Booth's algorithm)

```
  01101
x 11010
-----
```

```
  01101
  11010 } step 1
1110100 } step 2
0-1-2
```

```
      01101
x    0-1-2
-----
1111100110
11110011
000000
-----
1110110010
```