MSCS 446 Numerical Analysis I
Written Assignment 10
Adhere to the Homework Guidelines
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1. (N) Compare Simpson's rule, the trapezoidal rule, Gausian quadrature, and Clenshaw-Curtis quadrature to one another over grid-sizes of $n = 5, 7, 9, \ldots, 23$ for approximating

$$\int_0^1 e^{\cos(\pi x)}\, dx.$$

This integral is a Scipy special function **iv(0,1)** so you will need to load the Scipy special package.

2. (A) Prove that $T_3(x)$ is orthogonal to all polynomials of degree ≤ 2. (Hint: Consider rewriting $ax^2 + bx + c$ in terms of T_0, T_1, and T_2.)

3. (A) Show that $(m, n = 0, 1, 2, \ldots)$

(a) $$\frac{1}{\pi}\int_{-\pi}^{\pi} \cos(mx)\cos(nx)\, dx = \delta_{mn}$$

(b) $$\frac{1}{\pi}\int_{-\pi}^{\pi} \sin(mx)\sin(nx)\, dx = \delta_{mn}$$

(c) $$\frac{1}{\pi}\int_{-\pi}^{\pi} \cos(mx)\sin(nx)\, dx = 0$$

where δ_{mn} denotes the Kronecker delta

$$\delta_{mn} = \begin{cases} 1, & m = n \\ 0, & m \neq n \end{cases}$$

4. (A) Prove that Chebyshev polynomials are orthogonal with respect to the weight function, $w(x) = \dfrac{1}{\sqrt{1-x^2}}$, that is, show

$$\int_{-1}^{1} T_n(x)T_m(x)\frac{1}{\sqrt{1-x^2}}\, dx = \frac{\pi}{2}\delta_{mn}.$$

(You may re-use computations from problem #3.)

5. (N) Plot

$$C(x) = \int_0^x \cos(\frac{\pi}{2}t^2)\,dt$$

on the interval $[0, 4]$ using Gaussian quadrature. Use enough plotting values and quadrature nodes and weights to obtain a figure similar to the one below (the horizontal asymptote $y = \frac{1}{2}$ is included but is not required in your plot).