MSCS 446 Numerical Analysis I
Written Assignment 8
Adhere to the Homework Guidelines
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1. (N) Compute

$$\int_0^{2\pi} \frac{\cos(2x)}{e^x}\,dx \qquad \text{the exact answer is } \frac{1}{5}(1-e^{-2\pi})$$

using $n = 120$ equispaced points for the

(a) composite trapezoidal rule

(b) composite Simpson's rule (remember that you need 120 subintervals!)

(c) Gaussian Three-Point Rule (this rule is NOT Gaussian quadrature, it is the rule given below)

Which method is most accurate in terms of absolute error?

The **Gaussian Three-Point Rule** is defined

$$\int_a^b f(x)\,dx \approx h\sum_{i=1}^{n/2}\left[\frac{5}{9}f\left(x_{2i-1}-h\sqrt{\frac{3}{5}}\right)+\frac{8}{9}f\left(x_{2i-1}\right)+\frac{5}{9}f\left(x_{2i-1}+h\sqrt{\frac{3}{5}}\right)\right]$$

Notice that this rule only uses the odd-indexed nodes (half as many nodes as the other two methods). Compute it using a dot product.

2. (A) In class we derived Simpson's $\frac{1}{3}$ Rule for 3 points. Complete that calculation from start to finish.

3. (A) Determine the degree of precision of the quadrature rule.

$$\int_{-1}^{1} f(x)\,dx \approx \frac{1}{4}\left[3f(-2/3)+2f(0)+3f(2/3)\right]$$

4. (A) How large must n be if the composite trapezoidal rule is being used to estimate $\displaystyle\int_0^{\pi}\sin(x)\,dx$ with error $\leq 10^{-12}$? Will the estimate be too big or too small (justify using Calculus)? Repeat this exercise using the composite Simpson's rule.

5. (A) Construct a rule of the form (determine the α, β, and γ)

$$\int_{-1}^{1} f(x)\,dx \approx \alpha f\left(-\frac{1}{2}\right)+\beta f(0)+\gamma f\left(\frac{1}{2}\right)$$

that has degree of precision 2.