**MSCS 446 Numerical Analysis I**
**Written Assignment 7**
**Adhere to the Homework Guidelines**
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1. (N) In the Jupyter notebook **HW7_template.ipynb** is data $(x_i, y_i)$ generated from an unknown function. Use inverse interpolation, reversing the roles of $x$ and $y$ in the data, to estimate the zero of the function.

2. (N) In the Jupyter notebook **HW7_template.ipynb** is data $(x_i, f_i)$ and $(x_i, g_i)$ generated from two unknown functions using the same nodes $x_i$. Use inverse interpolation to estimate the point of intersection, $(x^*, y^*)$, of the graphs of the functions. You need to be careful with this data when you reverse the roles of $x$ and $y$ because the function is not one-to-one. So you will need to truncate the data using slicing.

   Note that once you locate the zero, you are not finished! Whereas in problem 1 the zero was $x^*$ and $y^* = 0$ but in this problem $y^* \neq 0$. It is the *interpolated* value of $f$ or $g$ at $x^*$, i.e. you need to interpolate the values $f(x^*)$ or $g(x^*)$ once you estimate $x^*$.

3. (A) Write the Newton form of the interpolating polynomial of degree at most 2 that interpolates $f(x)$ at $x_0$, $x_1$, and $x_2$, where $x_0 < x_1 < x_2$ and show directly that $p_2''(x) = 2f[x_0, x_1, x_2]$.

4. (A) Show that the maximum interpolation error is bounded by $\frac{1}{8}h^2M$ for linear interpolation of $f(x)$ with nodes $\{x_0, x_1\}$ where $h = x_1 - x_0$ and $M = \max\limits_{x_0 \leq x \leq x_1} |f''(x)|$.

5. (N) If $x_k = \cos\left[\dfrac{(2k+1)}{2n+2}\pi\right]$, then

   $$\left|\prod_{k=0}^{n}(x - x_k)\right| \leq \frac{1}{2^n}$$

   for all $x$ in $[-1, 1]$. Carry out a numerical experiment to test the given inequality for $n = 3, 7, 15$. What is the relevance of this question to polynomial interpolation?