MSCS 446 Numerical Analysis I
Written Assignment 4
Adhere to the Homework Guidelines
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1. (N) Estimate $\mathrm{C}(1) = \int_0^1 \cos\left(\frac{\pi}{2}x^2\right)\,dx$

   (a) using the right-hand rule with $n = 8, 16, 32, 64, 128$ nodes.

   (b) using the left-hand rule with $n = 8, 16, 32, 64, 128$ nodes.

   (c) using the midpoint rule with $n = 8, 16, 32, 64, 128$ nodes.

   (d) using the composite trapezoidal rule with $n = 8, 16, 32, 64, 128$ nodes.

   (e) using composite Simposon's rule with $n = 8, 16, 32, 64, 128$ subintervals.

   Which method is the most accurate in terms of absolute error? Justify your answer by displaying the output of $I_n(f)$ as well as the semilogy error plot with the absolute error. Use `from scipy.special import fresnel` with `fresnel(1)[1]` for the true value.

2. (A) Describe how to apply Newton's method to approximate the following numbers:

   (a) $\sqrt[3]{5}$

   (b) π

   (c) e

   (d) $\ln(2)$

   "Describe" means determine a function, write the Newton iteration formula, and provide a reasonable x_0.

3. (N) Find the first positive value ($\alpha = 2.066393863$) for which

$$e^{-0.2x^2} = \frac{\sin(x)}{x}$$

   using Steffensen's method with a tolerance of 10^{-8}.

   Steffensen's method is similar to Newton's method except that $f'(x)$ gets replaced by

$$f'(x) \approx \frac{f(x + f(x)) - f(x)}{f(x)}.$$

4. (N) The amount of money required to pay off a 30-year mortgage is

$$A(r) = \frac{P}{(r/12)} \left(1 - \left(1 + \frac{r}{12}\right)^{-360}\right).$$

In this equation A is the mortgage amount, P is the payment (installments) amount, and r is the monthly interest rate. Suppose you plan to take out a \$199,000 mortgage but can only afford a payment of \$1200 per month. What interest rate do you need the bank to offer? (Note that you could use Steffensen's method from the previous problem if you want to avoid taking a derivative.)

5. (A) Consider using Newton's method to estimate $\sqrt{R}$ where $R \in (0, \infty)$. The Babylonian method for completing this task is given by

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{R}{x_n}\right).$$

   (a) Similar to problem 2 of this assignment, determine the function, $f(x)$, for applying Newton's method to estimate $\sqrt{R}$.

   (b) Simplify Newton's method from part (a) to obtain the Babylonian method above.

   (c) If R can be written as $R = AB$, e.g. $6 = 2 \cdot 3$ so $\sqrt{6} = \sqrt{2 \cdot 3}$, then let $x_0 = A$ in the Babylonian method and show (or explain) that you can reduce the number of iterations by one if you choose $x_0 = \frac{A+B}{2}$ instead. Notice that this reasoning applies if you choose $x_0 = B$ as well.