MSCS 446 Numerical Analysis I
Written Assignment 3
Adhere to the Homework Guidelines
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1. (A) Which term grows faster as $n \in \mathbb{N}$ goes to infinity

   (a) $n!$ or a^n for $a > 1$? Consider $\lim_{n\to\infty} \frac{a^n}{n!}$

   (b) $n!$ or n^n? Consider $\lim_{n\to\infty} \frac{n!}{n^n}$

   Experiments with calculators or code or graphs do not constitute a mathematical proof. The most efficient method for proving this result is to consider whether the infinite series with these terms converges.

2. (A) Derive the approximation formula

$$f'(x) \approx \frac{1}{2h}\left[4f(x+h) - 3f(x) - f(x+2h)\right]$$

   and show that its error term is of the form $\frac{1}{3}h^2 f'''(\xi)$, that is, $\mathcal{O}(h^2)$.

3. (N) Differentiate the given data located in the Homework_3_Data.ipynb notebook using the forward-difference formula,

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

   (a) Use slicing instead of a loop

   (b) With slicing, you should only require roughly two lines of code to compute the derivative

   (c) How do you know if your answer is "essentially" correct? Hint: Consider graphing the data and the derivative (has one less node!) on the same plot.

4. (A) Write out the expansions for $f(x+h)$ and $f(x-h)$ to reconstruct the approximation

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

   and give the order of the approximation.

5. (A) B&F page 183, #29. (see pages 178 to 179 for the relevance of this expression)