MSCS 446 Numerical Analysis I
Written Assignment 6
Adhere to the Homework Guidelines
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1. (A) According to Theorem 2.1 on page 51, the bisection method converges with error term

$$e_n = \frac{b-a}{2^n} \qquad \text{n} \geq 1.$$

   (a) How many iterations are required to reach a tolerance of $e_n < \epsilon$?

   (b) On the interval $[2, 5]$, how many iterations are required to reach a tolerance of $e_n < 10^{-8}$?

   (c) Under the best conditions for Newton's method, roughly how many iterations are required to reach a tolerance of $e_n < 10^{-8}$?

2. (A) Competing forms of the interpolating polynomial. DO NOT SIMPLIFY the polynomials, although you may want to perform the computations for terms like

$$((-1) - (-2))((-1) - (1)) = -2$$

   but are not required to do so.

   (a) Given the data, $\{(-2, 13), (-1, 8), (1, 4)\}$, construct the Lagrange form of the interpolating polynomial "by hand."

   (b) Given the data, $\{(-2, 13), (-1, 8), (1, 4)\}$, construct the Newton form of the interpolating polynomial "by hand." (Use the divided-difference table)

   (c) The data point $(0, 5)$ is added. Again, construct both the Lagrange form and the Newton form of the interpolating polynomial. (When you use the divided-difference table for the Newton form be efficient, do not repeat any previously completed computations!) Which form of the interpolating polynomial is "easier" to construct in this scenario?

   (d) The data is changed to $\{(-2, 5), (-1, 7), (1, 6)\}$, notice that x's data values have not changed. Again, construct both the Lagrange form and the Newton form of the interpolating polynomial. Which form of the interpolating polynomial is "easier" to construct in this scenario?

3. (A) It is suspected that the table below comes from a cubic polynomial. How can this be tested? Explain

x	-2	-1	0	1	2	3
y	1	4	11	16	13	-4

4. (A) Complete the following divided difference table and use it to obtain (and write!) a polynomial of degree 3 that interpolates the function values indicated. Did I mention that you should also WRITE the polynomial? You may leave your answer in nested form.

x	$f[\ ]$	$f[\ ,\ ]$	$f[\ ,\ ,\ ]$	$f[\ ,\ ,\ ,\ ]$
-1	2			
1	-4		2	
3	6			
		2		
5	10			

5. (N) Use Newton's method and the bisection method to find the root of

$$f(x) = e^{6x} + 3(\ln 2)^2 e^{2x} - (\ln 8)e^{4x} - (\ln 2)^3$$

on $[-1, 0]$ with an error tolerance of 10^{-8}. Using a max-iteration (MAXITS) of 30, store the error for each method and create a semilogy error plot comparing the rates of convergence. Re-run the experiment but modify Newton's method with a repeated root multiplier(s) of $m = 2, 3, 4$. Again, create a semilogy error plot comparing the rates of convergence. You may display the best-case scenario choice for m.