MSCS 446 Numerical Analysis I
Written Assignment 9
Adhere to the Homework Guidelines

Dr. Keith Wojciechowski

1. (A) In this problem the quadratic Legendre polynomial is $q_2(x) = 3x^2 - 1$ (the factor of $\frac{1}{2}$ will be ignored for computational simplicity). Let $p(x) = 6x^3 + 5x^2 + x$ and answer the following.

   (a) Show that $\int_{-1}^{1} q_2(x)g(x)\,dx = 0$ for all linear polynomials, $g(x) = ax + b$, over the interval $[-1, 1]$.

   (b) Find a linear polynomial $q(x)$ and $r(x)$ such that $p(x) = q_2(x)q(x) + r(x)$.
   (If you forgot polynomial division: `www.purplemath.com/modules/polydiv2.htm`)

   (c) Confirm that $\int_{-1}^{1} p(x)\,dx = \int_{-1}^{1} r(x)\,dx$ and briefly explain why this equality must be true.

2. (N) Compare the trapezoidal rule versus Simpson's rule versus Gaussian quadrature for approximating the error function, erf(1). Use absolute error plotted on semilogy plot and compare for grid-sizes ranging from 3, 5, 9, 17, 33, ..., 1025 nodes. (`2**np.arange(1,11)+1`)

   (a) Use
   $$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\,dt.$$

   (b) To use Gaussian quadrature, you will need to transform the interval to $[0, 1]$. Do not forget to adjust the weights as well.

   (c) Overlay the results on the same plot.

   (d) Briefly explain what you are witnessing and whether or not this result is to be expected.

3. (N) Use Gaussian quadrature formulas on the test cases below. Display the results on a semilogy plot for an increasing number of grid-sizes (you decide how many for the numerical experiment).

   (a) $$\int_0^1 \frac{\ln(1-x)}{x}\,dx = -\frac{\pi^2}{6}$$

   (b) $$\int_0^1 \frac{\ln(1+x)}{x}\,dx = \frac{\pi^2}{12}$$

   (c) $$\int_0^1 \frac{\ln(1+x^2)}{x}\,dx = \frac{\pi^2}{24}$$

4. (A) Find the degree of precision for the quadrature rule

$$\int_0^1 g(t)\,dt \approx \frac{1}{24}\left[55g(0) - 59g(-1) + 37g(-2) - 9g(-3)\right]$$

5. (A) Determine, "by hand", the nodes and weights for the Gaussian quadrature rule with $N = 3$.