16.APPLICATIONS OF DERIVATIVES

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I. Section-A JEE Advanced/ IIT-JEE

C. MCQs with One Correct Answer

24. If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$ **(2004S)**

(a) $f(x)$ is a strictly increasing function
(b) $f(x)$ has a local maxima
(c) $f(x)$ is a strictly decreasing function
(d) $f(x)$ is bounded

25. If $f(x) = x^{\alpha} logx$ and $f(0) = 0$, then the value of α for which Rolles's theorem can be applied in [0,1] is **(2004S)**

(a) -2
(b) -1
(c) 0
(d) 1/2

26. If $P(x)$ is a polynomial of degree less than or equal to 2 and S is the set of all such polynomials so that $P(0) = 0, P(1) = 1$ and $P'(x) > 0\ \forall x \in [0, 1]$, then **(2005S)**

(a) $S = \phi$
(b) $S = ax + (1 - a)x^2\ \forall$ a $\in (0, 2)$
(c) $S = ax + (1 - a)x^2\ \forall$ a $\in (0, \infty)$
(d) $S = ax + (1 - a)x^2\ \forall$ a $\in (0, 1)$

27. The tangent to the curve $y = e^x$ drawn at the point $(\mathbf{c}, \mathbf{e^c})$intersects the line joining the points $(\mathbf{c-1}, \mathbf{e^{c-1}})$ and $(\mathbf{c+1}, \mathbf{e^{c+1}})$ **(2007-3 marks)**

(a) on the left of x=c
(b)on the right of$x$=c
(c) at no point
(d)at all points

28. Consider the two curves $C_1 : y^2 = 4x, C_2 : x^2 + y^2 - 6x + 1 = 0$.) Then, **(2008)**

(a) C_1 and C_2 touch each other only at one point.
(b) C_1 and C_2 touch each other exactly at two points
(c) C_1 and C_2 intersect (but do not touch) at exactly two points
(d) C_1 and C_2 neither intersect nor touch each other

29. The total number of local maxima and local minima of the function

$$f(x) = \begin{cases} (2+x)^3, -3 < x \leq -1 \\ x^{2/3}, -1 < x < 2 \end{cases} \quad 3 \text{ is}$$ **(2008)**

(a) 0
(b) 1
(c) 2
(d) 3

30. Let the function $g : (-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $g(u) = 2tan^{-1}(e^u) - \frac{\pi}{2}$. Then,g is **(2008)**

(a) even and is strictly increasing in (0,∞)
(b) odd and is strictly decreasing in $(-\infty, \infty)$
(c) odd and is strictly increasing in $(-\infty, \infty)$
(d) neither even nor odd,but is strictly increasing in $(-\infty, \infty)$

31. The least value of a $\in \mathbb{R}$ for which $4\alpha x^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is **(JEE Adv. 2016)**

(a) $\frac{1}{64}$
(b) $\frac{1}{32}$
(c) $\frac{1}{27}$
(d) $\frac{1}{25}$

32. If $f : R \to R$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in R$ and $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)$, $f(1) = 1$, then **(JEE Adv. 2017)**

(a) $f'(1) \leq 0$
(b) $0 < f'(1) \leq \frac{1}{2}$
(c) $\frac{1}{2} < f'(1) \leq 1$
(d) $f'(1) > 1$

D. MCQs With One or More than One Correct

1. Let $P(x) = a_0 + a_1x^2 + a_2x^4 + ...a_nx^{2n}$ be a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < .....a_n$. The function $P(x)$ has **(1986- 2 Marks)**

(a) neither a maximum nor a minimum
(b) only one maximum
(c) only one minimum
(d) only one maximum and only one minimum
(e) none of these.

2. If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then **(1986-2 Marks)**

(a)$a > 0, b > 0$
(b)$a > 0, b < 0$
(c)$a < 0, b > 0$
(d)$a < 0, b < 0$
(e) none of these.

3. The smallest positive root of the equation, $tanx - x = 0$ lies in **(1987-2 Marks)**

(a) $\left(0, \frac{\pi}{2}\right)$
(b) $\left(\frac{\pi}{2}, \pi\right)$
(c) $\left(\pi, \frac{3\pi}{2}\right)$
(d) $\left(\frac{3\pi}{2}, 2\pi\right)$
(e) None of these

4. Let f and g be increasing and decreasing functions, respectively from $[0, \infty)$ to $[0, \infty)$. Let $h(x) = f(g(x)$. If $h(0) = 0$, then $h(x) - h(1)$ is **(1987-2 Marks)**

(a) always zero
(b) always negative
(c) always positive
(d) strictly increasing
(e) None of these.

5. If $f(x) = \begin{cases} 3x^2 + 12x - 1, -1 \leq x \leq 2 \\ 37 - x, 2 < x \leq 3 \end{cases}$ then: **(2008)**

(a) $f(x)$ is increasing on [-1,2]
(b) $f(x)$ is continuous on [-1,3]
(c) $f'(2)$ does not exist
(d) $f(x)$ has the maximum value at $x = 2$

6. Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number x. Then **(1998-2 Marks)**

(a) h is increasing whenever f is increasing
(b) h is increasing whenever f is decreasing
(c) h is decreasing whenever f is decreasing
(d) nothing can be said in general.