9.7.11

EE24BTECH11058 - P.Shiny Diavajna

Question: Find a particular solution of the differential equation, given that $y = -1$ when $x = 0$.

$$(x - y)(dx + dy) = dx - dy \tag{0.1}$$

Theoretical solution:

$$(x - y - 1)dx = (-x + y - 1)dy \tag{0.2}$$

$$\frac{dy}{dx} = -\left(\frac{x - y - 1}{x - y + 1}\right) \tag{0.3}$$

$$\text{Let } x - y = t \tag{0.4}$$

$$1 - \frac{dy}{dx} = \frac{dt}{dx} \tag{0.5}$$

Substitute equations (0.4) and (0.5) in (0.3)

$$1 - \frac{dt}{dx} = -\left(\frac{t - 1}{t + 1}\right) \tag{0.6}$$

$$\frac{dt}{dx} = \frac{2t}{t + 1} \tag{0.7}$$

$$\frac{t + 1}{2t} dt = dx \tag{0.8}$$

Integrating on both sides

$$\int \left(\frac{1}{2} + \frac{1}{2t}\right) dt = \int dx \tag{0.9}$$

$$\frac{t}{2} + \frac{1}{2} \ln |t| = x + c \tag{0.10}$$

Substitute $t = x - y$

$$\frac{x - y}{2} + \frac{1}{2} \ln |x - y| = x + c \tag{0.11}$$

Given, x=0,y=-1.On substitution in (0.11) ,c=$\frac{1}{2}$

$$\ln |x - y| = x + y + 1 \tag{0.12}$$

Method of finite differences : The finite difference method is rooted in the fundamental concept of approximating derivatives using finite differences.

The derivative of $y(x)$ can be approximated as

$$\frac{dy}{dx} = \frac{y(x+h) - y(x)}{h} \tag{0.13}$$

$$y(x+h) = y(x) + h\left(\frac{dy}{dx}\right) \tag{0.14}$$

Where h is a small value very close to zero.

Substitute (0.3) in (0.14)

$$y(x+h) = y(x) - h\left(\frac{x-y-1}{x-y+1}\right) \tag{0.15}$$

Let (x_0, y_0) be a point on the curve.
Let some $x_1 = x_0 + h$.Then,

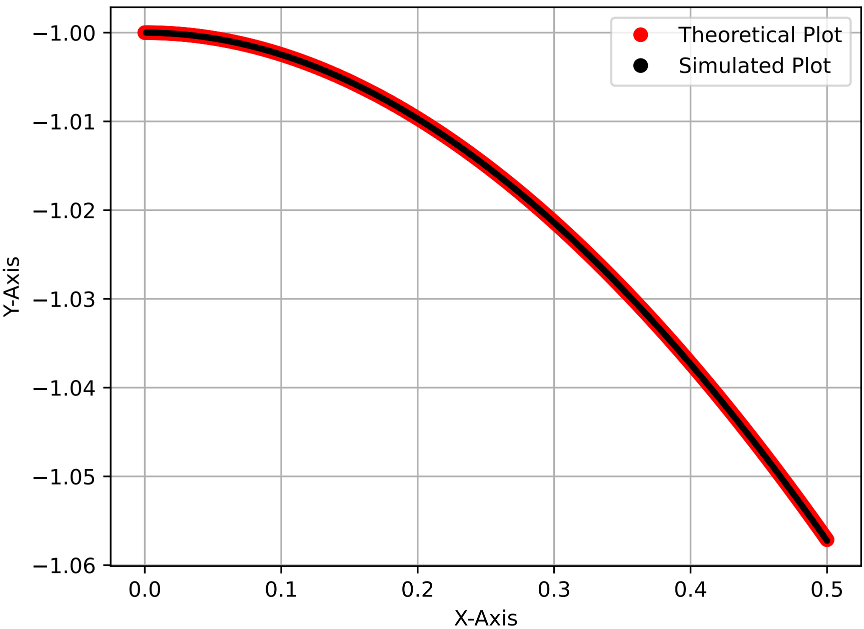
$$y_1 = y_0 - h\left(\frac{x-y-1}{x-y+1}\right) \tag{0.16}$$

On Generalizing the above equation, we have

$$x_{n+1} = x_n + h \tag{0.17}$$

$$y_{n+1} = y_n - h\left(\frac{x_n - y_n - 1}{x_n - y_n + 1}\right) \tag{0.18}$$

This curve is generated by applying the finite difference method to the given problem and taking the values of $x_0 = 0, y_0 = -1$ and $h = 0.001$ and running the iterations for 500 times



Theoretical Plot
Simulated Plot
−1.00
−1.01
−1.02
−1.03
−1.04
−1.05
−1.06
Y-Axis
0.0
0.1
0.2
0.3
0.4
0.5
X-Axis