9.5.17.3

EE24BTECH11058 - P.Shiny Diavajna

Question:

$$\left(x^3 + 2y^2\right) dx + (2xy)\, dy = 0 \tag{0.1}$$

Theoretical solution:

$$\frac{dy}{dx} = -\left(\frac{x^2}{2y} + \frac{y}{x}\right) \tag{0.2}$$

$$2y\frac{dy}{dx} + \frac{2y^2}{x} = -x^2 \tag{0.3}$$

$$\text{Let } y^2 = t \tag{0.4}$$

$$2y\frac{dy}{dx} = \frac{dt}{dx} \tag{0.5}$$

Substitute equations (0.4) and (0.5) in (0.3)

$$\frac{dt}{dx} + \frac{2t}{x} = -x^2 \tag{0.6}$$

$$\tag{0.7}$$

Integrating factor

$$e^{\int \frac{2}{x}\, dx} = x^2 \tag{0.8}$$

solution of the differential equation:

$$t\left(x^2\right) = \int -x^4\, dx \tag{0.9}$$

$$t\left(x^2\right) = -\frac{x^5}{5} + c \tag{0.10}$$

Substitute $t = y^2$

$$x^2y^2 = -\frac{x^5}{5} + c \tag{0.11}$$

$$y = \sqrt{\frac{c}{x^2} - \frac{x^3}{5}} \tag{0.12}$$

Let (1,1) be a point on the curve,Then $c = \frac{6}{5}$

$$y = \sqrt{\frac{6}{5x^2} - \frac{x^3}{5}} \tag{0.13}$$

Method of finite differences : The finite difference method is rooted in the fundamental concept of approximating derivatives using finite differences.

The derivative of $y(x)$ can be approximated as

$$\frac{dy}{dx} = \frac{y(x+h) - y(x)}{h} \tag{0.14}$$

$$y(x+h) = y(x) + h\left(\frac{dy}{dx}\right) \tag{0.15}$$

Where h is a small value very close to zero.

Substitute (0.2) in (0.15)

$$y(x+h) = y(x) - h\left(\frac{x^2}{2y} + \frac{y}{x}\right) \tag{0.16}$$

Let (x_0, y_0) be a point on the curve.
Let some $x_1 = x_0 + h$.Then,

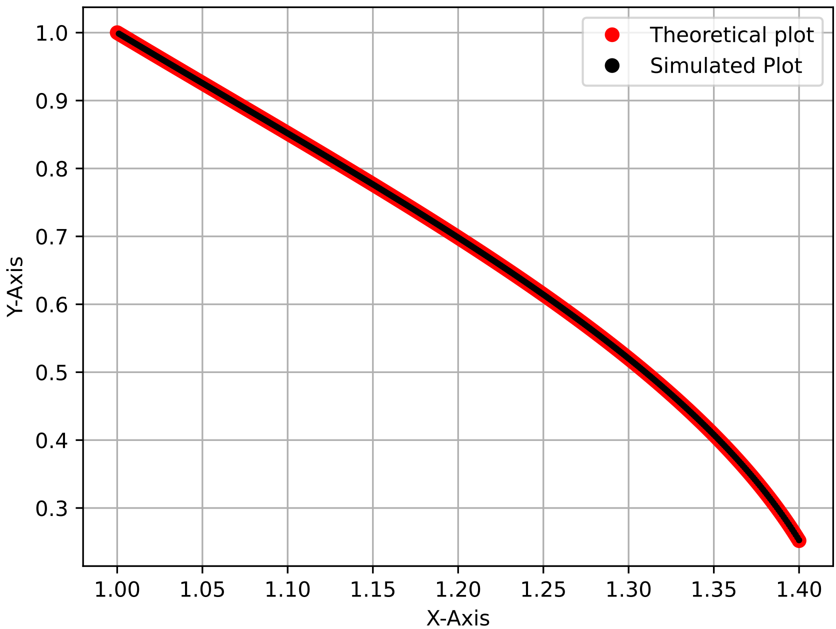
$$y_1 = y_0 - h\left(\frac{x^2}{2y} + \frac{y}{x}\right) \tag{0.17}$$

On Generalizing the above equation, we have

$$x_{n+1} = x_n + h \tag{0.18}$$

$$y_{n+1} = y_n - h\left(\frac{{x_n}^2}{2y_n} + \frac{y_n}{x_n}\right) \tag{0.19}$$

This curve is generated by applying the finite difference method to the given problem and taking the values of $x_0 = 1, y_0 = 1$ and $h = 0.001$ and running the iterations for 400 times


Theoretical plot
Simulated Plot
Y-Axis
X-Axis
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
1.00
1.05
1.10
1.15
1.20
1.25
1.30
1.35
1.40