9.5.6

EE24BTECH11058 - P.Shiny Diavajna

Question:

$$xdy - ydx = \sqrt{x^2 + y^2}dx \tag{0.1}$$

Theoretical solution:

$$\frac{dy}{dx} = \frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} \tag{0.2}$$

$$\text{Let } \frac{y}{x} = t \tag{0.3}$$

$$\frac{dy}{dx} = x\frac{dt}{dx} + t \tag{0.4}$$

Substitute equations (0.2) and (0.3) in (0.1)

$$x\frac{dt}{dx} = \sqrt{1 + t^2} \tag{0.5}$$

$$\frac{1}{\sqrt{1 + t^2}}dt = \frac{1}{x}dx \tag{0.6}$$

Integrate on both sides to obtain the solution

$$\int \frac{1}{\sqrt{1 + t^2}} dt = \int \frac{1}{x} dx \tag{0.7}$$

$$\ln\left(t + \sqrt{1 + t^2}\right) = \ln x \tag{0.8}$$

$$t + \sqrt{1 + t^2} = x \tag{0.9}$$

Substitute $t = \frac{y}{x}$

$$y + \sqrt{x^2 + y^2} = x^2 \tag{0.10}$$

$$y = \frac{x^2 - 1}{2} \tag{0.11}$$

Method of finite differences : The finite difference method is rooted in the fundamental concept of approximating derivatives using finite differences.

The derivative of $y(x)$ can be approximated as

$$\frac{dy}{dx} = \frac{y(x + h) - y(x)}{h} \tag{0.12}$$

$$y(x + h) = y(x) + h\left(\frac{dy}{dx}\right) \tag{0.13}$$

Where h is a small value very close to zero.

Substitute (0.1) in (0.12)

$$y(x+h) = y(x) + h\left(\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right) \tag{0.14}$$

Let (x_0, y_0) be a point on the curve.
Let some $x_1 = x_0 + h$.Then,

$$y_1 = y_0 + h\left(\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}}\right) \tag{0.15}$$

On Generalizing the above equation, we have

$$x_{n+1} = x_n + h \tag{0.16}$$

$$y_{n+1} = y_n + h\left(\frac{y_n}{x_n} + \sqrt{1 + \frac{{y_n}^2}{{x_n}^2}}\right) \tag{0.17}$$

This curve is generated by applying the finite difference method to the given problem and taking the values of $x_0 = 1, y_0 = 0$ and $h = 0.001$ and running the iterations for 500 times