16.APPLICATIONS OF DERIVATIVES

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Section-A JEE Advanced/IIT-JEE

C.MCQ with One Correct Answer

1) If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$ (2004S)
   a) $f(x)$ is a strictly increasing function
   b) $f(x)$ has a local maxima
   c) $f(x)$ is a strictly decreasing function
   d) $f(x)$ is bounded

2) If $f(x) = x^\alpha \log x$ and $f(0) = 0$, then the value of α for which Rolles's theorem can be applied in $[0, 1]$ is (2004)
   a) -2
   b) -1
   c) 0
   d) $1/2$

3) If $P(x)$ is a polynomial of degree less than or equal to 2 and S is the set of all such polynomials so that $P(0) = 0, P(1) = 1$ and $P'(x) > 0 \ \forall x \in [0, 1]$, then (2005S)
   a) $S = \phi$
   b) $S = ax + (1-a)x^2 \ \forall a \in (0, 2)$
   c) $S = ax + (1-a)x^2 \ \forall a \in (0, \infty)$
   d) $S = ax + (1-a)x^2 \ \forall a \in (0, 1)$

4) The tangent to the curve $y = e^x$ drawn at the point (c, e^c)intersects the line joining the points $\left(c-1, e^{c-1}\right)$ and $\left(c+1, e^{c+1}\right)$ (2007-3 Marks)
   a) on the left of $x = c$
   b) on the right of$x = c$
   c) at no point
   d) at all points

5) Consider the two curves $C_1 : y^2 = 4x$, $C_2 : x^2 + y^2 - 6x + 1 = 0$. Then, (2008)
   a) C_1 and C_2 touch each other only at one point.
   b) C_1 and C_2 touch each other exactly at two points
   c) C_1 and C_2 intersect (but do not touch) at exactly two points
   d) C_1 and C_2 neither intersect nor touch each other

6) The total number of local maxima and local minima of the function

$$f(x) = \begin{cases} (2+x)^3 & \text{if } -3 < x \le -1 \\ x^{2/3} & \text{if } -1 < x < 2 \end{cases}$$

   is (2008)
   a) 0
   b) 1
   c) 2
   d) 3

7) Let the function $g : (-\infty, \infty) \to \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ be given by $g(u) = 2tan^{-1}(e^u) - \frac{\pi}{2}$. Then,$g$ is (2008)
   a) even and is strictly increasing in $(0, \infty)$
   b) odd and is strictly decreasing in $(-\infty, \infty)$
   c) odd and is strictly increasing in $(-\infty, \infty)$
   d) neither even nor odd,but is strictly increasing in $(-\infty, \infty)$

8) The least value of a $\in \mathbb{R}$ for which $4\alpha x^2 + \frac{1}{x} \ge 1$, for all $x > 0$, is (JEE Adv. 2016)
   a) $\frac{1}{64}$
   b) $\frac{1}{32}$
   c) $\frac{1}{27}$
   d) $\frac{1}{25}$

9) If $f : R \to R$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in R$ and $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right)$, $f(1) = 1$, then (JEE Adv. 2017)
   a) $f'(1) \le 0$
   b) $0 < f'(1) \le \frac{1}{2}$
   c) $\frac{1}{2} < f'(1) \le 1$

d) $f'(1) > 1$

D. MCQs With One or More than One Correct

1) Let $P(x) = a_0 + a_1x^2 + a_2x^4 + \ldots a_nx^{2n}$ be a polynomial in a real variable x with
$0 < a_0 < a_1 < a_2 < \ldots a_n$. The function $P(x)$ has (1986- 2 Marks)
   a) neither a maximum nor a minimum
   b) only one maximum
   c) only one minimum
   d) only one maximum and only one minimum
   e) none of these

2) If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then (1986-2 Marks)
   a) $a > 0, b > 0$
   b) $a > 0, b < 0$
   c) $a < 0, b > 0$
   d) $a < 0, b < 0$
   e) none of these.

3) The smallest positive root of the equation, $\tan x - x = 0$ lies in (1987-2 Marks)
   a) $\left(0, \frac{\pi}{2}\right)$
   b) $\left(\frac{\pi}{2}, \pi\right)$
   c) $\left(\pi, \frac{3\pi}{2}\right)$
   d) $\left(\frac{3\pi}{2}, 2\pi\right)$
   e) None of these

4) Let f and g be increasing and decreasing functions, respectively from $[0, \infty)$ to $[0, \infty)$. Let $h(x) = f(g(x))$. If $h(0) = 0$, then $h(x) - h(1)$ is (1987-2 Marks)
   a) always zero
   b) always negative
   c) always positive
   d) strictly increasing
   e) None of these.

5) If

$$f(x) = \begin{cases} 3x^2 + 12x - 1 & \text{if } -1 \leq x \leq 2 \\ 37 - x & \text{if } 2 < x \leq 3 \end{cases}$$

then:

(2008)

   a) $f(x)$ is increasing on $[-1, 2]$
   b) $f(x)$ is continuous on $[-1, 3]$
   c) $f'(2)$ does not exist
   d) $f(x)$ has the maximum value at $x = 2$

6) Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number x. Then (1998-2 Marks)
   a) h is increasing whenever f is increasing
   b) h is increasing whenever f is decreasing
   c) h is decreasing whenever f is decreasing
   d) nothing can be said in general.