

# 16.APPLICATIONS OF DERIVATIVES

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Section-A JEE Advanced/IIT-JEE

C.MCQ with One Correct Answer

- 1) If  $f(x) = x^3 + bx^2 + cx + d$  and  $0 < b^2 < c$ , then in  $(-\infty, \infty)$  (2004S)
  - a)  $f(x)$  is a strictly increasing function
  - b)  $f(x)$  has a local maxima
  - c)  $f(x)$  is a strictly decreasing function
  - d)  $f(x)$  is bounded
- 2) If  $f(x) = x^\alpha \log x$  and  $f(0) = 0$ , then the value of  $\alpha$  for which Rolles's theorem can be applied in  $[0, 1]$  is (2004)
  - a)  $-2$
  - b)  $-1$
  - c)  $0$
  - d)  $1/2$
- 3) If  $P(x)$  is a polynomial of degree less than or equal to 2 and  $S$  is the set of all such polynomials so that  $P(0) = 0, P(1) = 1$  and  $P'(x) > 0 \forall x \in [0, 1]$ , then (2005S)
  - a)  $S = \phi$
  - b)  $S = ax + (1-a)x^2 \forall a \in (0, 2)$
  - c)  $S = ax + (1-a)x^2 \forall a \in (0, \infty)$
  - d)  $S = ax + (1-a)x^2 \forall a \in (0, 1)$
- 4) The tangent to the curve  $y = e^x$  drawn at the point  $(c, e^c)$  intersects the line joining the points  $(c-1, e^{c-1})$  and  $(c+1, e^{c+1})$  (2007-3 Marks)
  - a) on the left of  $x=c$
  - b) on the right of  $x=c$
  - c) at no point
  - d) at all points
- 5) Consider the two curves  $C_1 : y^2 = 4x, C_2 : x^2 + y^2 - 6x + 1 = 0$ . Then, (2008)
  - a)  $C_1$  and  $C_2$  touch each other only at one point.
  - b)  $C_1$  and  $C_2$  touch each other exactly at two points
  - c)  $C_1$  and  $C_2$  intersect (but do not touch) at exactly two points
  - d)  $C_1$  and  $C_2$  neither intersect nor touch each other
- 6) The total number of local maxima and local minima of the function
 
$$f(x) = \begin{cases} (2+x)^3, & -3 < x \leq -1 \\ x^{2/3}, & -1 < x < 2 \end{cases} \quad \text{is} \quad (2008)$$
  - a) 0
  - b) 1
  - c) 2
  - d) 3
- 7) Let the function  $g : (-\infty, \infty) \rightarrow \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$  be given by  $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$ . Then,  $g$  is (2008)
  - a) even and is strictly increasing in  $(0, \infty)$
  - b) odd and is strictly decreasing in  $(-\infty, \infty)$
  - c) odd and is strictly increasing in  $(-\infty, \infty)$
  - d) neither even nor odd, but is strictly increasing in  $(-\infty, \infty)$
- 8) The least value of  $a \in \mathbb{R}$  for which  $4ax^2 + \frac{1}{x} \geq 1$ , for all  $x > 0$ , is (JEE Adv. 2016)
  - a)  $\frac{1}{64}$
  - b)  $\frac{1}{32}$
  - c)  $\frac{1}{27}$
  - d)  $\frac{1}{25}$
- 9) If  $f : R \rightarrow R$  is a twice differentiable function such that  $f''(x) > 0$  for all  $x \in R$  and  $f\left(\frac{1}{2}\right) = \left(\frac{1}{2}\right), f(1) = 1$ , then (JEE Adv. 2017)
  - a)  $f'(1) \leq 0$
  - b)  $0 < f'(1) \leq \frac{1}{2}$
  - c)  $\frac{1}{2} < f'(1) \leq 1$
  - d)  $f'(1) > 1$

### D. MCQs With One or More than One Correct

- 1) Let  $P(x) = a_0 + a_1x^2 + a_2x^4 + \dots a_nx^{2n}$  be a polynomial in a real variable  $x$  with  $0 < a_0 < a_1 < a_2 < \dots a_n$ . The function  $P(x)$  has (1986- 2 Marks)
  - a) neither a maximum nor a minimum
  - b) only one maximum
  - c) only one minimum
  - d) only one maximum and only one minimum
  - e) none of these
  
- 2) If the line  $ax + by + c = 0$  is a normal to the curve  $xy = 1$ , then (1986-2 Marks)
  - a)  $a > 0, b > 0$
  - b)  $a > 0, b < 0$
  - c)  $a < 0, b > 0$
  - d)  $a < 0, b < 0$
  - e) none of these.
  
- 3) The smallest positive root of the equation,  $\tan x - x = 0$  lies in (1987-2 Marks)
  - a)  $\left(0, \frac{\pi}{2}\right)$
  - b)  $\left(\frac{\pi}{2}, \pi\right)$
  - c)  $\left(\pi, \frac{3\pi}{2}\right)$
  - d)  $\left(\frac{3\pi}{2}, 2\pi\right)$
  - e) None of these
  
- 4) Let  $f$  and  $g$  be increasing and decreasing functions, respectively from  $[0, \infty)$  to  $[0, \infty)$ . Let  $h(x) = f(g(x))$ . If  $h(0) = 0$ , then  $h(x) - h(1)$  is (1987-2 Marks)
  - a) always zero
  - b) always negative
  - c) always positive
  - d) strictly increasing
  - e) None of these.
  
- 5) If  $f(x) = \begin{cases} 3x^2 + 12x - 1, & -1 \leq x \leq 2 \\ 37 - x, & 2 < x \leq 3 \end{cases}$  then: (2008)
  - a)  $f(x)$  is increasing on  $[-1, 2]$
  - b)  $f(x)$  is continuous on  $[-1, 3]$
  - c)  $f'(2)$  does not exist
  - d)  $f(x)$  has the maximum value at  $x = 2$
  
- 6) Let  $h(x) = f(x) - (f(x))^2 + (f(x))^3$  for every real number  $x$ . Then (1998-2 Marks)
  - a)  $h$  is increasing whenever  $f$  is increasing
  - b)  $h$  is increasing whenever  $f$  is decreasing
  - c)  $h$  is decreasing whenever  $f$  is decreasing
  - d) nothing can be said in general.