

16.APPLICATIONS OF DERIVATIVES

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Section-A JEE Advanced/IIT-JEE

C.MCQ with One Correct Answer

1) If $f(x) = x^3 + bx^2 + cx + d$ and $0 < b^2 < c$, then in $(-\infty, \infty)$ (2004S)

- a) $f(x)$ is a strictly increasing function
- b) $f(x)$ has a local maxima
- c) $f(x)$ is a strictly decreasing function
- d) $f(x)$ is bounded

2) If $f(x) = x^\alpha \log x$ and $f(0) = 0$, then the value of α for which Rolles's theorem can be applied in $[0, 1]$ is (2004)

- a) -2
- b) -1
- c) 0
- d) $1/2$

3) If $P(x)$ is a polynomial of degree less than or equal to 2 and S is the set of all such polynomials so that $P(0) = 0, P(1) = 1$ and $P'(x) > 0 \forall x \in [0, 1]$, then (2005S)

- a) $S = \phi$
- b) $S = ax + (1-a)x^2 \forall a \in (0, 2)$
- c) $S = ax + (1-a)x^2 \forall a \in (0, \infty)$
- d) $S = ax + (1-a)x^2 \forall a \in (0, 1)$

4) The tangent to the curve $y = e^x$ drawn at the point (c, e^c) intersects the line joining the points $(c-1, e^{c-1})$ and $(c+1, e^{c+1})$ (2007-3 Marks)

- a) on the left of $x = c$
- b) on the right of $x = c$
- c) at no point
- d) at all points

5) Consider the two curves $C_1 : y^2 = 4x, C_2 : x^2 + y^2 - 6x + 1 = 0$. Then, (2008)

- a) C_1 and C_2 touch each other only at one point.
- b) C_1 and C_2 touch each other exactly at two

points

- c) C_1 and C_2 intersect (but do not touch) at exactly two points
- d) C_1 and C_2 neither intersect nor touch each other

6) The total number of local maxima and local minima of the function

$$f(x) = \begin{cases} (2+x)^3 & \text{if } -3 < x \leq -1 \\ x^{2/3} & \text{if } -1 < x < 2 \end{cases}$$

is (2008)

- a) 0
- b) 1
- c) 2
- d) 3

7) Let the function $g : (-\infty, \infty) \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$ be given by $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$. Then, g is (2008)

- a) even and is strictly increasing in $(0, \infty)$
- b) odd and is strictly decreasing in $(-\infty, \infty)$
- c) odd and is strictly increasing in $(-\infty, \infty)$
- d) neither even nor odd, but is strictly increasing in $(-\infty, \infty)$

8) The least value of $a \in \mathbb{R}$ for which $4ax^2 + \frac{1}{x} \geq 1$, for all $x > 0$, is (JEE Adv. 2016)

- a) $\frac{1}{64}$
- b) $\frac{1}{32}$
- c) $\frac{1}{27}$
- d) $\frac{1}{25}$

9) If $f : R \rightarrow R$ is a twice differentiable function such that $f''(x) > 0$ for all $x \in R$ and $f(\frac{1}{2}) = (\frac{1}{2})$, $f(1) = 1$, then (JEE Adv. 2017)

- a) $f'(1) \leq 0$
- b) $0 < f'(1) \leq \frac{1}{2}$
- c) $\frac{1}{2} < f'(1) \leq 1$

d) $f'(1) > 1$

d) $f(x)$ has the maximum value at $x = 2$

D. MCQs With One or More than One Correct

- 1) Let $P(x) = a_0 + a_1x^2 + a_2x^4 + \dots a_nx^{2n}$ be a polynomial in a real variable x with $0 < a_0 < a_1 < a_2 < \dots a_n$. The function $P(x)$ has (1986- 2 Marks)
- neither a maximum nor a minimum
 - only one maximum
 - only one minimum
 - only one maximum and only one minimum
 - none of these

- 2) If the line $ax + by + c = 0$ is a normal to the curve $xy = 1$, then (1986-2 Marks)
- $a > 0, b > 0$
 - $a > 0, b < 0$
 - $a < 0, b > 0$
 - $a < 0, b < 0$
 - none of these.

- 3) The smallest positive root of the equation, $\tan x - x = 0$ lies in (1987-2 Marks)
- $\left(0, \frac{\pi}{2}\right)$
 - $\left(\frac{\pi}{2}, \pi\right)$
 - $\left(\pi, \frac{3\pi}{2}\right)$
 - $\left(\frac{3\pi}{2}, 2\pi\right)$
 - None of these

- 4) Let f and g be increasing and decreasing functions, respectively from $[0, \infty)$ to $[0, \infty)$. Let $h(x) = f(g(x))$. If $h(0) = 0$, then $h(x) - h(1)$ is (1987-2 Marks)
- always zero
 - always negative
 - always positive
 - strictly increasing
 - None of these.

- 5) If

$$f(x) = \begin{cases} 3x^2 + 12x - 1 & \text{if } -1 \leq x \leq 2 \\ 37 - x & \text{if } 2 < x \leq 3 \end{cases}$$

then:

(2008)

- $f(x)$ is increasing on $[-1, 2]$
- $f(x)$ is continuous on $[-1, 3]$
- $f'(2)$ does not exist

- 6) Let $h(x) = f(x) - (f(x))^2 + (f(x))^3$ for every real number x . Then (1998-2 Marks)
- h is increasing whenever f is increasing
 - h is increasing whenever f is decreasing
 - h is decreasing whenever f is decreasing
 - nothing can be said in general.