

# 16.APPLICATIONS OF DERIVATIVES

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Section-A JEE Advanced/IIT-JEE

C.MCQ with One Correct Answer

1) If  $f(x) = x^3 + bx^2 + cx + d$  and  $0 < b^2 < c$ , then in  $(-\infty, \infty)$  (2004S)

- a)  $f(x)$  is a strictly increasing function
- b)  $f(x)$  has a local maxima
- c)  $f(x)$  is a strictly decreasing function
- d)  $f(x)$  is bounded

2) If  $f(x) = x^\alpha \log x$  and  $f(0) = 0$ , then the value of  $\alpha$  for which Rolles's theorem can be applied in  $[0, 1]$  is (2004)

- a)  $-2$
- b)  $-1$
- c)  $0$
- d)  $1/2$

3) If  $P(x)$  is a polynomial of degree less than or equal to 2 and  $S$  is the set of all such polynomials so that  $P(0) = 0, P(1) = 1$  and  $P'(x) > 0 \forall x \in [0, 1]$ , then (2005S)

- a)  $S = \phi$
- b)  $S = ax + (1-a)x^2 \forall a \in (0, 2)$
- c)  $S = ax + (1-a)x^2 \forall a \in (0, \infty)$
- d)  $S = ax + (1-a)x^2 \forall a \in (0, 1)$

4) The tangent to the curve  $y = e^x$  drawn at the point  $(c, e^c)$  intersects the line joining the points  $(c-1, e^{c-1})$  and  $(c+1, e^{c+1})$  (2007-3 Marks)

- a) on the left of  $x = c$
- b) on the right of  $x = c$
- c) at no point
- d) at all points

5) Consider the two curves  $C_1 : y^2 = 4x, C_2 : x^2 + y^2 - 6x + 1 = 0$ . Then, (2008)

- a)  $C_1$  and  $C_2$  touch each other only at one point.
- b)  $C_1$  and  $C_2$  touch each other exactly at two

points

- c)  $C_1$  and  $C_2$  intersect (but do not touch) at exactly two points
- d)  $C_1$  and  $C_2$  neither intersect nor touch each other

6) The total number of local maxima and local minima of the function

$$f(x) = \begin{cases} (2+x)^3 & \text{if } -3 < x \leq -1 \\ x^{2/3} & \text{if } -1 < x < 2 \end{cases}$$

is (2008)

- a) 0
- b) 1
- c) 2
- d) 3

7) Let the function  $g : (-\infty, \infty) \rightarrow (-\frac{\pi}{2}, \frac{\pi}{2})$  be given by  $g(u) = 2\tan^{-1}(e^u) - \frac{\pi}{2}$ . Then,  $g$  is (2008)

- a) even and is strictly increasing in  $(0, \infty)$
- b) odd and is strictly decreasing in  $(-\infty, \infty)$
- c) odd and is strictly increasing in  $(-\infty, \infty)$
- d) neither even nor odd, but is strictly increasing in  $(-\infty, \infty)$

8) The least value of  $a \in \mathbb{R}$  for which  $4ax^2 + \frac{1}{x} \geq 1$ , for all  $x > 0$ , is (JEE Adv. 2016)

- a)  $\frac{1}{64}$
- b)  $\frac{1}{32}$
- c)  $\frac{1}{27}$
- d)  $\frac{1}{25}$

9) If  $f : \mathbb{R} \rightarrow \mathbb{R}$  is a twice differentiable function such that  $f''(x) > 0$  for all  $x \in \mathbb{R}$  and  $f(\frac{1}{2}) = (\frac{1}{2})$ ,  $f(1) = 1$ , then (JEE Adv. 2017)

- a)  $f'(1) \leq 0$
- b)  $0 < f'(1) \leq \frac{1}{2}$
- c)  $\frac{1}{2} < f'(1) \leq 1$

d)  $f'(1) > 1$

d)  $f(x)$  has the maximum value at  $x = 2$

**D. MCQs With One or More than One Correct**

1) Let  $P(x) = a_0 + a_1x^2 + a_2x^4 + \dots a_nx^{2n}$  be a polynomial in a real variable  $x$  with

$0 < a_0 < a_1 < a_2 < \dots a_n$ . The function  $P(x)$  has (1986- 2 Marks)

- a) neither a maximum nor a minimum
- b) only one maximum
- c) only one minimum
- d) only one maximum and only one minimum
- e) none of these

2) If the line  $ax + by + c = 0$  is a normal to the curve  $xy = 1$ , then (1986-2 Marks)

- a)  $a > 0, b > 0$
- b)  $a > 0, b < 0$
- c)  $a < 0, b > 0$
- d)  $a < 0, b < 0$
- e) none of these.

3) The smallest positive root of the equation,  $\tan x - x = 0$  lies in (1987-2 Marks)

- a)  $\left(0, \frac{\pi}{2}\right)$
- b)  $\left(\frac{\pi}{2}, \pi\right)$
- c)  $\left(\pi, \frac{3\pi}{2}\right)$
- d)  $\left(\frac{3\pi}{2}, 2\pi\right)$
- e) None of these

4) Let  $f$  and  $g$  be increasing and decreasing functions, respectively from  $[0, \infty)$  to  $[0, \infty)$ . Let  $h(x) = f(g(x))$ . If  $h(0) = 0$ , then  $h(x) - h(1)$  is (1987-2 Marks)

- a) always zero
- b) always negative
- c) always positive
- d) strictly increasing
- e) None of these.

5) If

$$f(x) = \begin{cases} 3x^2 + 12x - 1 & \text{if } -1 \leq x \leq 2 \\ 37 - x & \text{if } 2 < x \leq 3 \end{cases}$$

then:

(2008)

- a)  $f(x)$  is increasing on  $[-1, 2]$
- b)  $f(x)$  is continuous on  $[-1, 3]$
- c)  $f'(2)$  does not exist

6) Let  $h(x) = f(x) - (f(x))^2 + (f(x))^3$  for every real number  $x$ . Then (1998-2 Marks)

- a)  $h$  is increasing whenever  $f$  is increasing
- b)  $h$  is increasing whenever  $f$  is decreasing
- c)  $h$  is decreasing whenever  $f$  is decreasing
- d) nothing can be said in general.