9-04-2024 Shift1 Q16-30

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A. Multiple Choice

1) Let $\int \frac{2-\tan x}{3+\tan x} dx = \frac{1}{2}\left(\alpha x + \log_e |\beta \sin x + \gamma \cos x|\right) + C$, where C is the constant of integration. Then $\alpha + \frac{\gamma}{\beta}$ is equal to :
   a) 1
   b) 7
   c) 4
   d) 3

2) A ray of light coming from the point $\mathbf{P}(1,2)$ gets reflected from $\mathbf{Q}$ on the $x - axis$ and then passes through the point $\mathbf{R}(4,3)$. If the point $\mathbf{S}(h,k)$ is such that $PQRS$ is a parallelogram, then hk^2 is equal to :
   a) 80
   b) 70
   c) 60
   d) 90

3) $\overrightarrow{OA} = 2\overrightarrow{a}, \overrightarrow{OB} = 6\overrightarrow{a} + 5\overrightarrow{b}$ and $\overrightarrow{OC} = 3\overrightarrow{b}$, where O is the origin. If the area of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is $15 sq.units$, then the area $(in sq.units)$ of the quadrilateral $OABC$ is equal to :
   a) 38
   b) 32
   c) 40
   d) 35

4) Let $f(x) = x^2 + 9, g(x) = \frac{x}{x-9}$ and $a = (f \circ g)(10), b = (g \circ f)(3)$. If e and l denote the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = 1$, then $8e^2 + l^2$ is equal to :
   a) 16
   b) 12
   c) 8
   d) 6

5) The parabola $y^2 = 4x$ divides the area of the circle $x^2 + y^2 = 5$ in two parts. The area of the smaller part is equal to :
   a) $\frac{1}{3} + \sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
   b) $\frac{2}{3} + \sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
   c) $\frac{1}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$
   d) $\frac{2}{3} + 5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

B. Numericals

1) Let A be a non-singular matrix of order 3. If $det\,(3adj\,(2adj\,((detA)\,A))) = 3^{-13}.2^{-10}$ and $det\,(3adj\,(2A)) = 2^m.3^n$, then $|3m + 2n|$ is equal to __________.

2) The sum of the square of the modulus of the elements in the set $\{z = a+ib : a, b \in \mathbf{Z}, z \in \mathbf{C}, |z-1| \leq 1, |z - 5| \leq |z - 5i|\}$ is __________.

3) Let the centre of a circle, passing through the points $(0, 0), (1, 0)$ and touching the circle $x^2 + y^2 = 9$, be (h, k). Then for all possible values of the coordinates of the centre (h, k), $4\left(h^2 + k^2\right)$ is equal to __________.

4) Let $f : (0, \pi) \rightarrow \mathbf{R}$ be a function given by

$$f(x) = \begin{cases} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}} & 0 < x < \frac{\pi}{2} \\ a - 8 & x = \frac{\pi}{2} \\ (1 + |\cot x|)^{\frac{b}{a}|tan x|} & \frac{\pi}{2} < x < \pi \end{cases}$$

where $a, b \in \mathbf{Z}$. If f is continuous at $x = \frac{\pi}{2}$, then $a^2 + b^2$ is equal to __________.

5) The remainder when 428^{2024} is divided by 21 is __________.

6) Let $\lim_{n\to\infty}\left(\frac{n}{\sqrt{n^4+1}} - \frac{2n}{(n^2+1)\sqrt{n^4+1}} + \frac{n}{\sqrt{n^4+16}} - \frac{8n}{(n^2+4)\sqrt{n^4+16}} + \cdots + \frac{n}{\sqrt{n^4+n^4}} - \frac{2n\cdot n^2}{(n^2+n^2)\sqrt{n^4+n^4}}\right)$ be $\frac{\pi}{k}$, using only the principal values of the inverse trigonometric functions.Then k^2 is equal to __________.

7) If a function f satisfies $f(m + n) = f(m) + f(n)$ for all $m, n \in \mathbf{N}$ and $f(1) = 1$, then the largest natural number λ such that $\sum_{k=1}^{2022} f(\lambda + k) \leq (2022)^2$ is equal to __________.

8) Let the set of all positive values of λ, for which the point of local minimum of the function $\left(1 + x\left(\lambda^2 - x^2\right)\right)$ satisfies $\frac{x^2+x+2}{x^2+5x+6} < 0$, be (α, β). Then $\alpha^2 + \beta^2$ is equal to __________.

9) Let $A = \{2, 3, 6, 7\}$ and $B = \{4, 5, 6, 8\}$. Let R be a relation defined on $A \times B$ by $(a_1, b_1)\,R\,(a_2, b_2)$ if and only if $a_1 + a_2 = b_1 + b_2$. Then the number of elements in R is __________.

10) Let a, b and c denote the outcome of three independent rolls of a fair tetrahedral die, whose four faces are marked $1, 2, 3, 4$. If the probability that $ax^2 + bx + c = 0$ has all real roots is $\frac{m}{n}$, $gcd\,(m, n) = 1$, then $m + n$ is equal to __________.