27-07-2022 Q1-15

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1) The domain of the function
$$f(x) = \sin^{-1}\left[2x^2 - 3\right] + \log_2\left(\log_{1/2}\left(x^2 - 5x + 5\right)\right)$$
where $[t]$ is the greatest integer function,is :
   a) $\left(-\sqrt{\frac{5}{2}}, \frac{5-\sqrt{5}}{2}\right)$
   b) $\left(\frac{5-\sqrt{5}}{2}, \frac{5+\sqrt{5}}{2}\right)$
   c) $\left(1, \frac{5-\sqrt{5}}{2}\right)$
   d) $\left(1, \frac{5+\sqrt{5}}{2}\right)$
2) Let S be the set of all (α,β) ,$\pi < \alpha,\beta < 2\pi$, for which the complex number $\frac{1-i\sin\alpha}{1-2i\cos\beta}$ is purely real. Let $Z_{\alpha\beta} = \sin 2\alpha + i\cos 2\beta, (\alpha,\beta) \in S$. Then $\sum_{(\alpha,\beta)\in S}\left(iZ_{\alpha\beta} + \frac{1}{i\bar{Z}_{\alpha\beta}}\right)$ is equal to:
   a) 3
   b) $3i$
   c) 1
   d) $2-i$
3) If α,β are the roots of the equation
$$x^2 - \left(5 + 3^{\sqrt{\log_3 5}} - 5^{\sqrt{\log_5 3}}\right) + 3\left(3^{(\log_3 5)^{\frac{1}{3}}} - 5^{(\log_5 3)^{\frac{2}{3}}} - 1\right) = 0$$
then the equation,whose roots are $\alpha + \frac{1}{\beta}$ and $\beta + \frac{1}{\alpha}$,
   a) $3x^2 - 20x - 12$
   b) $3x^2 - 10x - 4$
   c) $3x^2 - 10x + 2$
   d) $3x^2 - 20x + 16$
4) Let A $= \begin{pmatrix} 4 & -2 \\ \alpha & \beta \end{pmatrix}$ If $A^2 + \gamma A + 18I = 0$, then det(A) is equal to
   a) -18
   b) 18
   c) -50
   d) 50
5) If for $p \neq q \neq 0$, the function
$$f(x) = \frac{\sqrt[7]{p(729 + x)} - 3}{\sqrt[3]{729 + qx} - 9}$$

is continuous at x=0, then :

a) $7pqf(0) - 1 = 0$
b) $63qf(0) - p^2 = 0$
c) $21qf(0) - p^2 = 0$
d) $7pqf(0) - 9 = 0$

6) Let

$$f(x) = 2 + |x| - |x - 1| + |x + 1|$$

x $\in$ **R** Consider
$(S1) : f'\left(-\frac{3}{2}\right) + f'\left(-\frac{1}{2}\right) + f'\left(\frac{1}{2}\right) f'\left(\frac{3}{2}\right) = 2$
$(S2) : \int_{-2}^{2} f(x)\, dx = 12$ Then,

a) both (S1) and (S2) are correct
b) both (S1) and (S2) are wrong
c) only (S1) is correct
d) only (S2) is correct

7) Let the sum of an infinite G.P. , whose first term is a and the commom ratio is r, be 5. Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an AP, whose first term is $10ar$, n^{th} term is a_n and the common difference is $10ar^2$ is equal to :

a) $21a_{11}$
b) $22a_{11}$
c) $15a_{16}$
d) $14a_{16}$

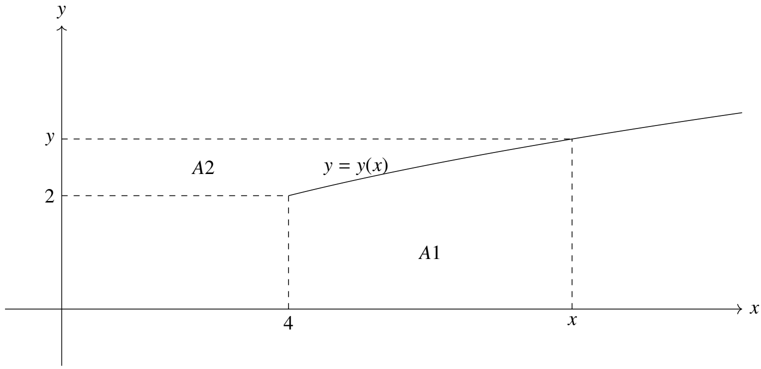
8) The area of the region enclosed by $y \leq 4x^2, x^2 \leq 9y$ and $y \leq 4$, is equal to :

a) $\frac{40}{3}$
b) $\frac{56}{3}$
c) $\frac{112}{3}$
d) $\frac{80}{3}$

9) $\int_0^2 \left(|2x^2 - 3x| + \left[x - \frac{1}{2}\right]\right) dx$ where $[t]$ is the greatest integer funnction, is equal to :

a) $\frac{7}{6}$
b) $\frac{19}{12}$
c) $\frac{31}{12}$
d) $\frac{3}{2}$

10) Consider a curve $y = y(x)$ in the first quadrant as shown in the figure.Let the area A_1 is twice the area A_2. Then the normal to the curve perpendicular to the line $2x - 12y = 15$ does **NOT** pass through the point.



a) $(6, 21)$
b) $(8, 9)$
c) $(10, -4)$
d) $(12, -15)$

11) The equation of the sides AB, BC and CA of a triangle ABC are $2x+y = 0, x+py = 39$ and $x-y = 3$ respectively and $\mathbf{P}(2, 3)$ is its circumcentre. Then which of the following is NOT true:
a) $(AC)^2 = 9p$
b) $(AC)^2 + p^2 = 136$
c) $32 < area(\Delta ABC) < 36$
d) $34 < area(\Delta ABC) < 38$

12) A Circle C_1 passes through the origin $\mathbf{O}$ and has diameter 4 on the positive $x-axis$. The line $y = 2x$ gives a chord OA of a circle C_1. Let C_2 be the circle with OA as a diameter. If the tangent to C_2 at the point $\mathbf{A}$ meets the $x-axis$ at $\mathbf{P}$ and $y-axis$ at $\mathbf{Q}$, then $QA : AP$ is equal to :
a) $1 : 4$
b) $1 : 5$
c) $2 : 5$
d) $1 : 3$

13) If the length of the latus rectum of a parabola,whose focus is (a, a) and the tangent at its vertex is $x + y = a$, is 16, then $|a|$ is equal to
a) $2\sqrt{2}$
b) $2\sqrt{3}$
c) $4\sqrt{2}$
d) 4

14) If the Length of the perpendicular drawn from the point $\mathbf{P}\left(a, 4, 2\right), a > 0$ on the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1}$ is $2\sqrt{6}$ units and $\mathbf{Q}\left(\alpha_1, \alpha_2, \alpha_3\right)$ is the image of the point $\mathbf{P}$ in this line, then $a + \sum_{i=1}^{3} \alpha_i$ is equal to :
a) 7
b) 8
c) 12

d) 14

15) If the line of intersection of the planes $ax + by = 3$ and $ax + by + cz = 0, a > 0$ makes an angle 30° with the plane $y - z + 2 = 0$, then the direction cosines of the line are :

a) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$

b) $\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0$

c) $\frac{1}{\sqrt{5}}, -\frac{2}{\sqrt{5}}, 0$

d) $\frac{1}{2}, -\frac{\sqrt{3}}{2}, 0$