7-7.3-3

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Question

The equation of a circle with origin as centre and passing through the vertices of an equilateral triangle whose median is of length 3a is

Table

Symbol	Description	Value
O	Centre of the circle	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
$3a$	median of the triangle	-
r	radius of the circle	-
u	$-O$	$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$
f	$\|u\|^2 - r^2$	-

Table: Variables Used

Theory

- General equation of a conic is $g(x) = x^\top V x + 2u^\top x + f = 0$
- For a circle $V = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$
- Therefore, general equation of a circle is

$$\|x\|^2 + 2u^\top x + f = 0 \tag{1}$$

$$u = -O, f = \|u\|^2 - r^2 \tag{2}$$

Theory

- The circumcircle of a triangle is the unique circle that passes through all three vertices of the triangle. The center of this circle is called the circumcenter.
- In an equilateral triangle, the centroid, circumcenter, and incenter are all the same point.
- The centroid divides each median in a 2:1 ratio (measured from the vertex to the midpoint of the opposite side).
- Therefore, the centre of the circle divides the median in the ratio 2:1. Hence,radius of the cirle is 2a.

Solution

The general equation of a circle (3)

$$\|x\|^2 + u^\top x + f = 0 \tag{4}$$

$$u = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{5}$$

$$r = 2a \tag{6}$$

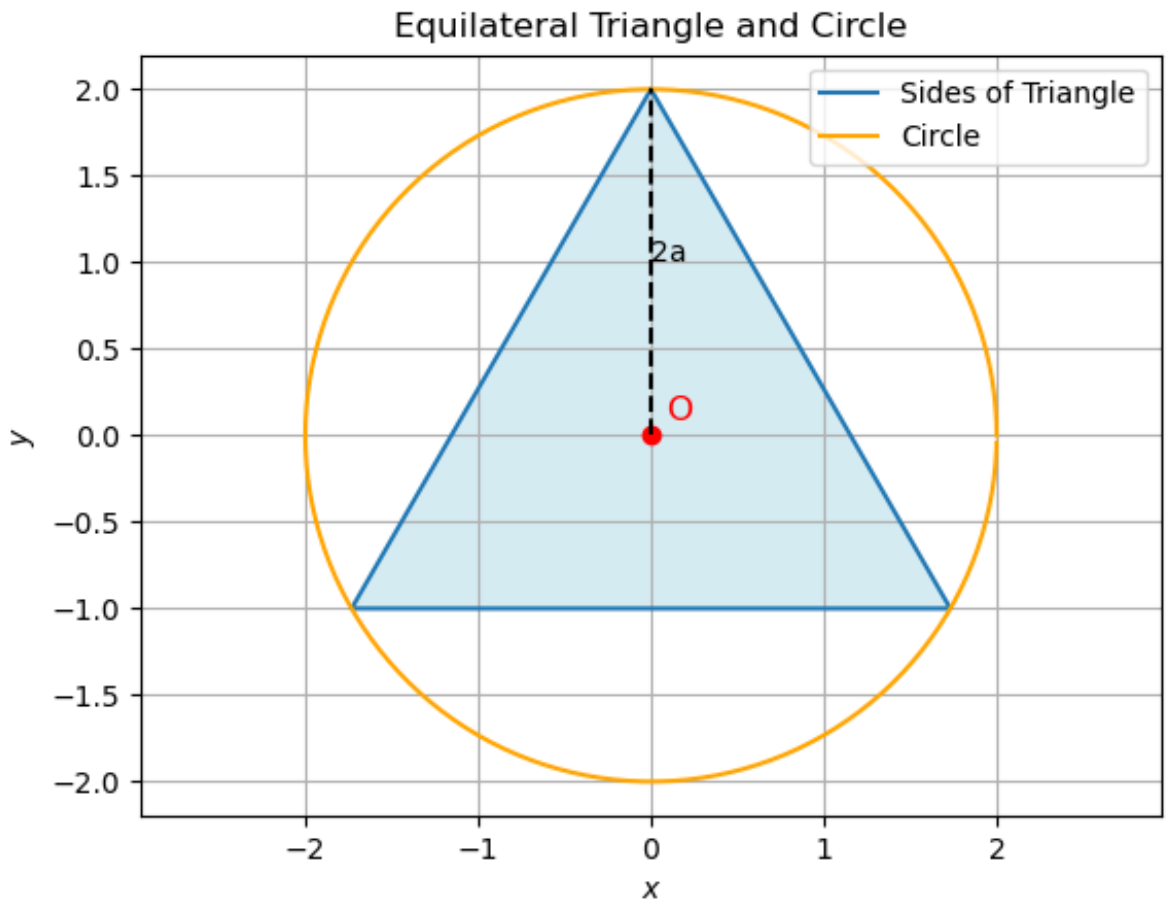
$$f = \|u\|^2 - r^2 \tag{7}$$

$$f = -4a^2 \tag{8}$$

$$\|x\|^2 - 4a^2 = 0 \tag{9}$$

$$x^2 + y^2 = 4a^2 \tag{10}$$

Figure


Equilateral Triangle and Circle
Sides of Triangle
Circle
2a
O
2.0
1.5
1.0
0.5
0.0
−0.5
−1.0
−1.5
−2.0
y
−2
−1
0
1
2
x

C-Code

```c
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
#include "libs/matfun.h"
#include "libs/geofun.h"

void point_gen(FILE *fptr, double **A, double **B, int no_rows,
↪  int no_cols, int num_points) {
    for (int i = 0; i < num_points; i++) {
        double t = (double)i / (num_points - 1);
        double **output = Matadd(A, Matscale(Matsub(B, A,
        ↪  no_rows, no_cols), no_rows, no_cols, t), no_rows,
        ↪  no_cols);
        fprintf(fptr, "%lf,%lf\n", output[0][0], output[1][0]);
        freeMat(output, no_rows);
    }
}
```

C-Code

```
void equi_triangle_gen(double median, FILE *fptr) {
double side = (2 * median) / sqrt(3);
double xA = 0, yA = 2*median/3;
double xB = -side / 2, yB = -median/3;
double xC = side / 2, yC =  -median/3;

int m = 2, n = 1;

double **A = createMat(m, n);
double **B = createMat(m, n);
double **C = createMat(m, n);

A[0][0] = xA;  A[1][0] = yA;
B[0][0] = xB;  B[1][0] = yB;
C[0][0] = xC;  C[1][0] = yC;
```

C-Code

```
    point_gen(fptr, A, B, m, n, 10);
    point_gen(fptr, B, C, m, n, 10);
    point_gen(fptr, C, A, m, n, 10);

    freeMat(A, m);
    freeMat(B, m);
    freeMat(C, m);
}

void circle_point_gen(FILE *fptr, double radius, double *center,
↪   int num_points) {
    double **output;
    for (int i = 0; i < num_points; i++) {
        double angle = (2 * M_PI * i) / num_points;
        output = createMat(2, 1);
        output[0][0] = center[0] + radius * cos(angle);
        output[1][0] = center[1] + radius * sin(angle);
```

C-Code

```
    fprintf(fptr, "%lf,%lf\n", output[0][0], output[1][0]);
        freeMat(output, 2);
    }
}

int main() {
    double a = 1.0; //for graphing
    double median = 3*a;
    double radius = 2*a;
    double center[2] = {0.0, 0.0};

    FILE *fptr = fopen("points.dat", "w");
    if (fptr == NULL) {
        printf("Error opening file!\n");
        return 1;
    }
```

C-Code

```
    equi_triangle_gen(median, fptr);
    circle_point_gen(fptr, radius, center, 300);

    fclose(fptr);
    return 0;
}
```

C code output

0.000000,2.000000
-0.192450,1.666667
-0.384900,1.333333
-0.577350,1.000000
-0.769800,0.666667
-0.962250,0.333333
-1.154701,0.000000
-1.347151,-0.333333
-1.539601,-0.666667
-1.732051,-1.000000
-1.732051,-1.000000
-1.347151,-1.000000
-0.962250,-1.000000
-0.577350,-1.000000
-0.192450,-1.000000

C code output

0.192450,-1.000000
0.577350,-1.000000
0.962250,-1.000000
1.347151,-1.000000
1.732051,-1.000000
1.732051,-1.000000
1.539601,-0.666667
1.347151,-0.333333
1.154701,0.000000
0.962250,0.333333
0.769800,0.666667
0.577350,1.000000
0.384900,1.333333
0.192450,1.666667
0.000000,2.000000

C code output

2.000000,0.000000
1.999561,0.041885
1.998246,0.083751
1.996053,0.125581
1.992986,0.167356
1.989044,0.209057
1.984229,0.250666
1.978545,0.292166
1.971992,0.333537
1.964575,0.374763
1.956295,0.415823
1.947158,0.456702
1.937166,0.497380
1.926325,0.537840
1.914639,0.578064
1.902113,0.618034

C code output

1.888753,0.657733
1.874564,0.697144
1.859553,0.736249
1.843726,0.775031
1.827091,0.813473
1.809654,0.851559
1.791424,0.889270
1.772407,0.926592
1.752613,0.963507
1.732051,1.000000
1.710729,1.036054
1.688656,1.071654
1.665842,1.106783
1.642298,1.141427
1.618034,1.175571
1.593060,1.209198

Python Code

```
import numpy as np
import matplotlib.pyplot as plt

# Load data
points = np.loadtxt("points.dat", delimiter=',')
x_triangle = points[:30, 0]
y_triangle = points[:30, 1]
x_circle = points[30:, 0]
y_circle = points[30:, 1]

# Create the plot
plt.figure()
plt.plot(x_triangle, y_triangle, label='Sides of Triangle')
plt.fill(x_triangle, y_triangle, 'lightblue', alpha=0.5)
plt.plot(x_circle, y_circle, label='Circle', color='orange')
```

Python Code

```
# Define circle center
circle_center = (0, 0)

# Indicate the center with a point and annotate it
plt.plot(circle_center[0], circle_center[1], 'ro')  # Red point
↪  for the center
plt.annotate('O', xy=circle_center, xytext=(5, 5),
↪  textcoords='offset points', fontsize=12, color='red')

# Identify the topmost vertex of the triangle (assuming it's the
↪  first vertex in your data)
top_vertex = (x_triangle[0], y_triangle[0])
```

Python Code

```
# Draw a line from the center to the top vertex
plt.plot([circle_center[0], top_vertex[0]], [circle_center[1],
↪  top_vertex[1]], 'k--')  # Dashed line
plt.annotate('2a', xy=((circle_center[0] + top_vertex[0]) / 2,
↪  (circle_center[1] + top_vertex[1]) / 2),
                        fontsize=10, color='black')

# Plot settings
plt.xlabel("$x$")
plt.ylabel("$y$")
plt.title("Equilateral Triangle and Circle")
plt.grid(True)
plt.legend(loc="upper right")
plt.axis('equal')
plt.show()
```