An example of what Quiz 1 can look like

Q1: Say you are given the LU factorization of a matrix A with

$$L = \begin{bmatrix} 1 & 0 & 0 \\ -0.25 & 1 & 0 \\ 0.75 & 1 & 1 \end{bmatrix}, \qquad U = \begin{bmatrix} 4 & 0 & 6 \\ 0 & 2 & 4.5 \\ 0 & 0 & -2 \end{bmatrix},$$

and $A = LU$.

a: Find the matrix A.

b: Use the factorization to compute the determinant of A.

c: Use the factorization to solve the for x in $Ax = \mathbf{1}$.

d: Use the factorization to find the inverse of A.

e: Use the factorization to find a different factorization $A = LDU$ where D is a diagonal matrix, L is lower triangular with 1's on the diagonal, and U is upper triangular with 1's on the diagonal.

Q2: Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^3$ where,

$$f(x, y) = \begin{bmatrix} x^2 y \\ e^{x+y} \\ xe^y \end{bmatrix}.$$

a: Compute $||f(1, -1)||$

b: Compute the trace of the matrix $f(x, y)f(x, y)^T$ at $x = 1$, $y = 1$.

c: Compute the Jacobian matrix for $f(\cdot, \cdot)$.

d: Can the rank of your answer to c be 3 for some values of x and y? Explain.

e: Consider now the function $g(x) = 2x^2y + 3e^{x+y} - xe^y$. Represent $g(x)$ as $g(x) = u \cdot f(x, y)$. What is the vector u?