Matrix, Linear Algebra, Differential Equation MAT2207

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Matrix

0.1 Definition of Matrix:

A system of any $m \times n$ numbers arranged in a rectangular arrangement of m rows and n columns is called a matrix of order $m \times n$ or an $m \times n$ matrix.

Ex:

$$\begin{bmatrix} 1 & -2 & 4 \\ 3 & 1 & 7 \end{bmatrix} \qquad \text{is a } 2 \times 3 \text{ matrix.}$$

in general form :

$$A = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \dots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \dots & \sigma_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \sigma_{m1} & \sigma_{m2} & \dots & \sigma_{mm} \end{bmatrix} = (\sigma_{ij})_{mxm}$$

0.2 Singular and Non-singular Matrix:

Let A be any square matrix. If $\det(A) = 0$, then A is called a singular matrix, and if $\det(A) \neq 0$, then A is called a non-singular matrix.

Ex: $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ $\qquad B = \begin{bmatrix} 1 & 5 \\ 2 & 12 \end{bmatrix}$

Then $|A| = \det(A) = \det \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix} = 4 - 4 = 0$
So, A is a singular matrix

Again, $|B| = \det(B) = \det \begin{bmatrix} 1 & 5 \\ 2 & 12 \end{bmatrix} = 12 - 10 = 2 \neq 0$
Hence, B is a non-singular matrix.

0.3 Inverse Matrix:

Let A and B be two $n \times n$ square matrices such that $AB = BA = I_n = I$, then B is said to be the inverse of A, and we write $B = A^{-1}$. Also, $A = B^{-1}$.

Ex: Let $A = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix}$.

$$\therefore\ AB = \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & -12+12 \\ 1-1 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$$\text{and } BA = \begin{bmatrix} 1 & -3 \\ -1 & 4 \end{bmatrix} \times \begin{bmatrix} 4 & 3 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4-3 & 3-3 \\ -4+4 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_2$$

$\therefore AB = BA = I_2 = I$
Therefore, we can write $A = B^{-1}$ and $B = A^{-1}$.

[N.B.: The inverse of a matrix exists only when the matrix is non-singular, i.e., $|A| \neq \emptyset$.]

*** Multiplication of two matrices is possible only when the number of columns in the first matrix is equal to the number of rows in the second matrix.

0.4 Echelon Matrix:

Let $A = (a_{ij})_{m\times n}$ be any matrix. Then A is said to be an echelon matrix or is said to be in echelon form if:

1. all the non-zero rows (if any) precede the zero rows,
2. the number of zero entries preceding the first non-zero entry in each row increases by row.

Ex:

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 5 \end{bmatrix} \text{ is an echelon matrix, but } \begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 3 \\ 2 & 5 & 4 \end{bmatrix} \text{ is not an echelon matrix.}$$

0.5 Rank of a Matrix:

Rank of a matrix is the largest non-zero row in the matrix of row echelon form.

Ex: $A = \begin{bmatrix} 1 & 4 & 5 \\ 0 & 3 & 7 \\ 0 & 0 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 6 & 7 \\ 0 & 0 & 0 \end{bmatrix}$

Rank of matrix A = 3, Rank of matrix B = 2

Find the rank of the following matrices:

$$A = \begin{bmatrix} 1 & -1 & -1 & 2 \\ 2 & 1 & 3 & -1 \\ 3 & 2 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 3 & 1 & -2 & -3 \\ 1 & 4 & 3 & -1 & -4 \\ 2 & 3 & -4 & -7 & -3 \\ 3 & 8 & 1 & -7 & -8 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 3 & 0 & 2 & 2 \\ 2 & 1 & -1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & -1 & 1 & 3 \\ 2 & -1 & 0 & 2 \\ 1 & 0 & 1 & 1 \\ 4 & 2 & 0 & 1 \end{bmatrix}$$

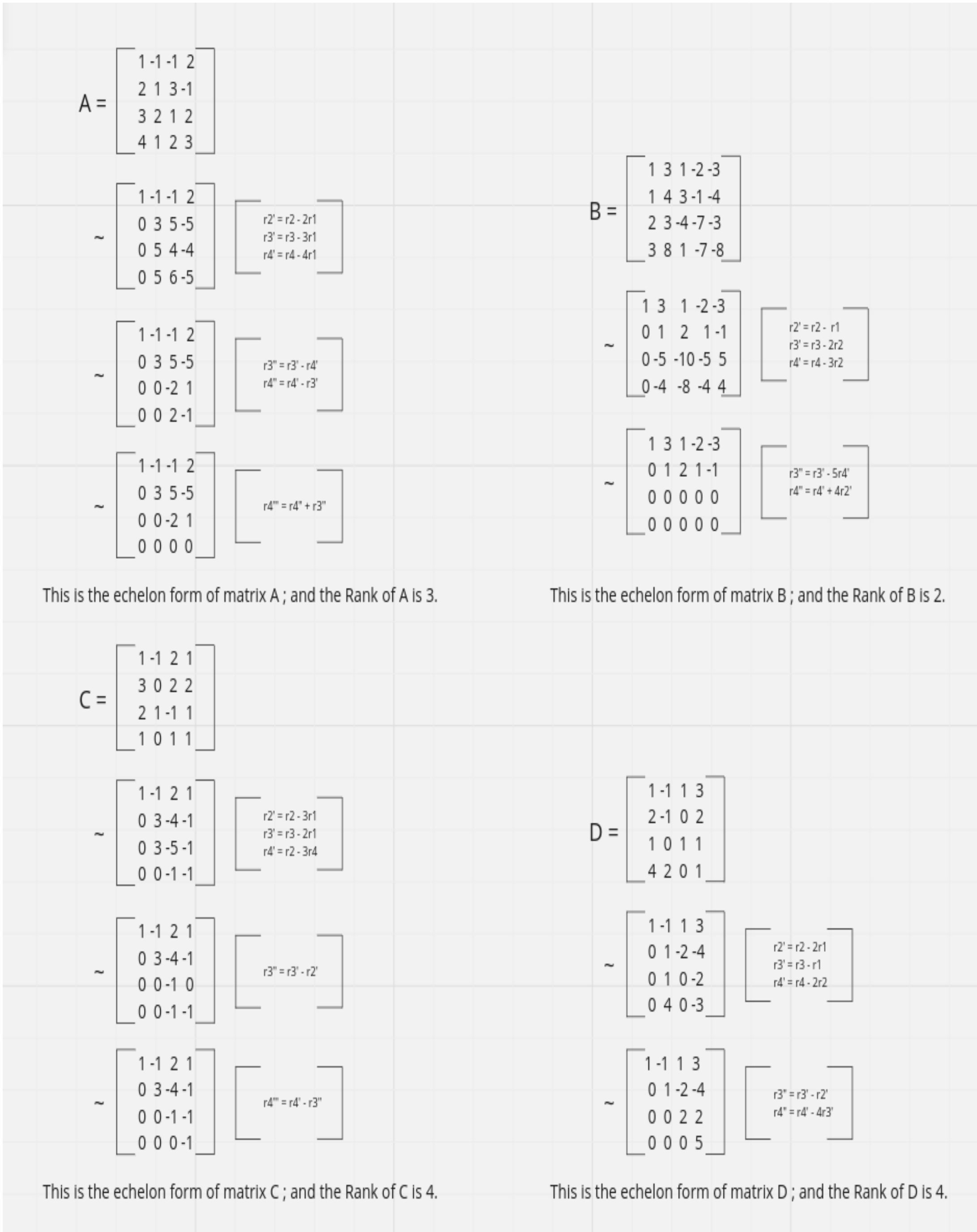

Figure 1: Caption of the image