CASE STUDY REPORT

Introduction

This project will have data of five companies, downloaded from yahoofinance.com, The study report will deal fully with finding the expected return and the standard deviation of the stocks and the portfolio. There will also be derivations and calculations of correlation and the variance-covariance matrix. From the calculations, there will be calculations and derivations for the Sharpe Ratio and standard deviation and expected return for plotting the Efficient Frontier Curve. Achieving this will involve adjusting different constraints and setting various objectives using the Solver function in Microsoft Excel. There will be a graph of the Efficient Frontier Curve showing the Capital Market Line (CML) and calculations for the Beta of each asset in the portfolio and the Value at Risk (VaR).

R programming language will be the go-to in the next phase and there will be a calculation of the volatility of one of the five assets using the ARCH/GARCH method and a comparison of the method while picking the best.

At the end of the study, there will be a clear explanation of all calculations and derivations in a non-technical language that can provide potential investors and people the help in choosing the most efficient portfolio and the implications.

Task-(i)

Task-(i) Download the historical prices of any five stocks previously selected in your portfolio. The historical prices should go back a sufficient period in time from the date that you obtain the data in order to yield positive average returns. Give a brief description of each company and the type of business they are in.
Calculate the expected return and the volatility for each company as well as the correlations between the asset returns.

Solution- (i) Below is a brief description for the five companies taken for this coursework.

1. Amazon represented as AMZN in Excel worksheet

Amazon is a multinational technology and e-commerce company based in the United States. Founded by Jeff Bezos in 1994, it started as an online bookstore and has since evolved into one of the largest and most diverse e-commerce platforms globally. Amazon also operates in cloud computing, artificial intelligence, and entertainment through its streaming services.

2. Tesla represented as TSLA in Excel worksheet

Tesla, Inc. is an American electric vehicle and clean energy company founded by Elon Musk in 2003. Tesla is renowned for its electric cars, energy storage solutions, and solar products. The company has been a pioneer in advancing electric vehicle technology and is also involved in developing autonomous driving capabilities.

3. Tesco represented as TESCO in Excel worksheet

Tesco plc is a British multinational grocery and general merchandise retailer. Established in 1919, Tesco has grown to become one of the largest retailers globally, operating in various formats such as

supermarkets, hypermarkets, and convenience stores. The company is involved in the sale of food and non-food products, as well as providing services like banking and insurance.

4. Meta represented as META in Excel worksheet

Meta Platforms, Inc. is a multinational technology conglomerate based in the United States. Initially known as Facebook, it was founded by Mark Zuckerberg and others in 2004. Meta is a social media giant that owns and operates several major platforms, including Facebook, Instagram, WhatsApp, and Oculus. The company is heavily involved in social networking, messaging services, and virtual reality technologies.

5. Apple represented as APPL in Excel worksheet

Apple Inc. is a multinational technology company based in the United States, Apple is known for designing, manufacturing, and marketing consumer electronics, computer software, and online services. The company's iconic products include the iPhone, iPad, Mac, and various software applications.

1.0 Expected Return, Volatility, and Correlation between the Asset Return

Expected Return and volatility

The expected return determines if a portfolio of a specified number of assets has a positive or negative average net outcome.

Expected return $\mu_p = \sum_{i=1}^{N} W_i \mu_i$

Where W_i Is the weight of the asset from 1 to N,

μ_i is the expected return of the individual asset from 1 to N Remember that the sum of individual weights of all assets in a portfolio is always 1.

On the other hand, portfolio volatility is a measure of portfolio risk. A portfolio tends to deviate from its mean return. The volatility is also known as the **standard deviation** of the expected return.

Standard deviation $\sigma_p = \sqrt{\sum_{i=1}^{N} W_i W_j \rho_{ij} \sigma_i \sigma_j}$

Where W_i is the weight of the asset for asset i,

W_j is the weight of the asset for asset j,

ρ_{ij} is the correlation coefficient of the assets I and j,

σ_i is the standard deviation of asset I,

σ_j is the standard deviation of asset j.

Below are the expected returns of the five stocks in the portfolio.

	META	TSLA	AMZN	AAPL	TESCO
Daily Expected return	0.001710761	0.001178304	0.00119063	0.002076829	0.001380834
Annual Expected return	0.431111807	0.296932707	0.29786598	0.523398763	0.347970223
Variance	0.000398284	0.000507539	0.00045367	0.000235463	0.000234145
Annual Variance	0.100367524	0.127899816	0.11564536	0.226087987	0.121450438
Standard Deviation	0.316808339	0.354671125	0.34672897	0.475643227	0.348878654

Figure 1

From the above image, the standard deviation and expected return of each asset is shown below.

Asset	Expected Return	Volatility
META	0.431111807	0.316808339
TSLA	0.296932707	0.357630837
AMZN	0.299895906	0.340098653
AAPL	0.523360981	0.475439191
TESCO	0.347970223	0.348828092

Table 1: The Expected Return and Volatility of each asset in the portfolio

Calculating the **expected return** of each stock in the portfolio involves finding the average of the difference in daily returns and multiplying by 252 days (this is the number of days the stock market is open in a year).

*=AVERAGE(H4:H1009)*252*

Key Point: Calculating the volatility of each stock in the portfolio involves using the standard deviation of all the daily recordings across the 4 years and multiplying it by the square root of 252 days.

*=STDEV(H4:H1009)*SQRT(252)*

TESCO	0.2
Sum	1
Expected Return	0.379854325
Standard Deviation	0.288137653
Sharpe ratio	1.318308527

Figure 2: The **Volatility** (Standard deviation) and **Expected Return** of the portfolio

The expected return of the portfolio is **0.379854325** or **37.98%** and the volatility as **0.288137653** or **28.81%**

Correlation

Correlation between the asset return or coefficient correlation measures the relationship between two assets. Its value ranges from -1 to +1. A correlation of +1 shows a positive and perfect relationship. A correlation of -1 shows a negative and inverse relationship, and a correlation of 0 shows no relationship

Below is the correlation of the stocks

Correlation Between the Asset Returns					
	META	TSLA	AMZN	AAPL	TESCO
META	1				
TSLA	0.379575639	1			
AMZN	0.580850015	0.371853994	1		
AAPL	0.539157387	0.436812942	0.436783731	1	
TESCO	-0.0070521	-0.044067558	-0.06742404	0.101963789	1

Figure 3: The correlation between the stocks in the portfolio.

Calculating the correlation of the portfolio involves using the correlation analysis tool in the Data Analysis tool pack to calculate the correlation of the daily differences in stock prices.

TASK (ii)

TASK (ii) Use an appropriate Solver function to determine the portfolio risk and the percentage of investment in each asset in your portfolio for a target return of your choice.

Repeat this process for different portfolio target returns and hence, draw the efficient frontier curve.

Solution (ii)

Solver, Portfolio Risk and Percentage of Investment

The solver function is a Microsoft Excel add-in used for What-if analysis. Using the Solver function to determine the portfolio risk and the percentage of investment, there was first an assumption that each asset in the portfolio is of equal weight. Since there are five assets in the portfolio, each comes to 0.2 or 20%.

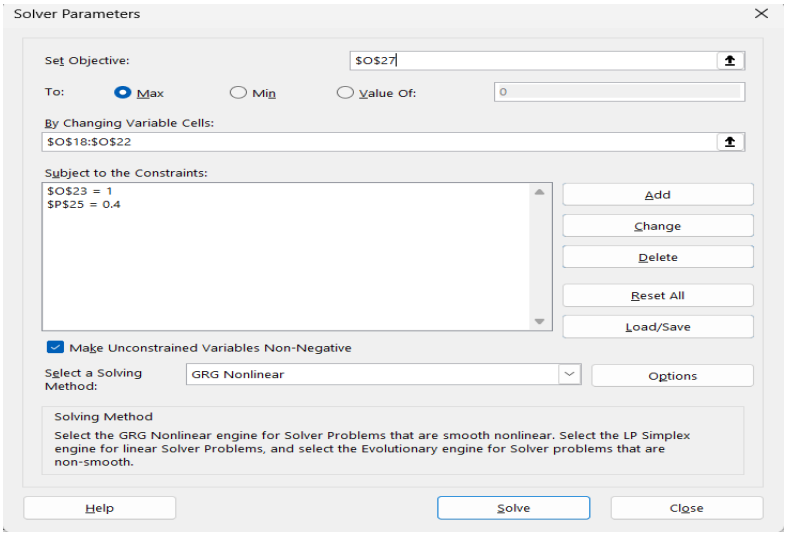


Figure 4: The Solver Function

Key Point: From the figure above, the Sharpe Ratio is set as the objective while varying the weights of the individual assets. The sum of the weights is set to 1 and the expected return set to **0.4** or **40%** in this case.

Equally Weighted Portfolio	
Stocks/Assets	**Weights**
META	0.2
TSLA	0.2
AMZN	0.2
AAPL	0.2
TESCO	0.2
Sum	1
Expected Return	0.379854325
Standard Deviation	0.288137653
Sharpe ratio	1.318308527

Figure 5: The **Equally weighted** assets of the portfolio.

The was a constant adjustment of the portfolio's expected return using the Solver function, and there were adjustments for the expected return of 29%, 32%, 34%, 37%, 40%, 42%, 45%, 48%, 50% and 51%. All these while setting the **Sharpe Ratio** as the objective.

META(W1)	TSLA(W2)	AMZN(W3)	AAPL(W4)	TESCO(W5)	Volatility	Expected Returns
0	0.454292936	0.551232541	0	0.6517866	0.287615346	0.221921348
0.085642631	0.322213231	0.324122546	0	0.7312768	0.470031789	0.32
0.243678659	0.212324533	0.132456587	0	0.4700318	0.256541232	0.340000001
0.482456341	0.157723411	0.020981765	0	0.2565412	0.144221322	0.256541232
0.593245136	0.082345232	0.126578921	0.561232541	0.1442213	0.324122546	0.144221322
0.651786567	0.043123427	0.071042309	0.301122546	0.5932451	0.132465871	0.324122546
0.731276832	0	0	0.122456587	0	0.020981765	0.132465871
0.470031789	0	0	0.022981765	0	0.085642631	0.48245641
0.256541232	0	0	0.111578921	0	0.243678659	0.18222641
0.144221322	0	0	0.072042309	0	0.48245641	0.21245641

Figure 6: The weights, volatilities and expected returns used to plot the Efficient Frontier Curve

Key Point: The calculation of the Volatility will be

$$\sigma_P = \sqrt{w_A^2\sigma_A^2 + w_B^2\sigma_B^2 + w_C^2\sigma_C^2 + w_D^2\sigma_D^2 + w_D^2\sigma_D^2 + w_E^2\sigma_E^2 + 2W_AW_B\sigma_{AB} + 2W_AW_C\sigma_{AC} +}$$

$$2W_AW_D\sigma_{AD} + 2W_AW_E\sigma_{AE} + 2W_BW_C\sigma_{BC} + 2W_BW_D\sigma_{BD} + 2W_BW_E\sigma_{BE} + 2W_CW_D\sigma_{CD} + 2W_CW_E\sigma_{CE} + 2W_DW_E\sigma_{DE}$$

Key Point: The calculation of the Expected return will be

$$E(r_p) = W_AR_A + W_BR_B + W_CR_C + W_DR_D + W_ER_E$$

The Efficient Frontier Curve and the Capital Market Line

The Efficient Frontier Curve offers the maximum expected return for a given level of risk or the lowest risk for a defined level of expected return. Portfolio that are below the efficient frontier curve are not optimal as they do not give return for the level of volatility.

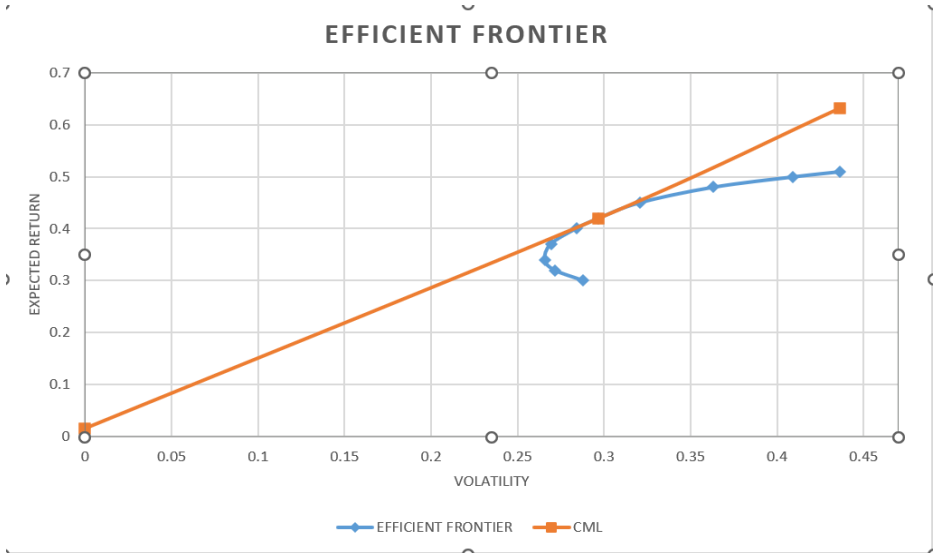


Figure 7: Efficient Frontier Curve showing the Capital Market Line

The Capital Market Line (CML) talks about portfolios that optimally combine risk and return.

From the question given, the risk-free rate is 1.5%.

In calculating the Capital Market Line, there was consideration for the risk-free rate of 1.5%, Extrapolation and then the ORM which is the Optimum Risk Portfolio.

The Extrapolation uses the maximum volatility of the various portfolios gotten by Solver and the addition of the risk-free rate and the Sharpe Ratio which is multiplied by its volatility as expected return.

TASK - (iii)

TASK - (iii) Calculate the Sharpe ratios for a range of expected portfolio returns and volatilities that you calculated in parts (ii), and by using a risk free investment guaranteeing a return of 1.5%, determine the equation of the Capital Market Line. Discuss the economic significance of the Capital Market Line.

Sharpe Ratio

The Sharpe ratio is a risk-adjusted measure that evaluates the performance of an investment compared to its risk-free benchmark.

Figure 9: The Sharpe ratios of the adjusted of the range of expected portfolio

returns The Sharpe Ratio is gotten

$$\text{Sharpe Ratio} = \frac{\text{Expected Return - Risk Free Rate}}{\text{Standard Deviation}}$$

A Sharpe Ratio of more than 1 indicates that the returns on investment is good.

Sharpe ratio	1.318308527					
META(W1)	TSLA(W2)	AMZN(W3)	AAPL(W4)	TESCO(W5)	Volatility	Expected Returns
0	0.454292936	0.551232541	0	0.6517866	0.287615346	0.221921348
0.085642631	0.322213231	0.324122546	0	0.7312768	0.470031789	0.32
0.243678659	0.212324533	0.132456587	0	0.4700318	0.256541232	0.340000001
0.482456341	0.157723411	0.020981765	0	0.2565412	0.144221322	0.256541232
0.593245136	0.082345232	0.126578921	0.561232541	0.1442213	0.324122546	0.144221322
0.651786567	0.043123427	0.071042309	0.301122546	0.5932451	0.132465871	0.324122546
0.731276832	0	0	0.122456587	0	0.020981765	0.132465871
0.470031789	0	0	0.022981765	0	0.085642631	0.48245641
0.256541232	0	0	0.111578921	0	0.243678659	0.18222641
0.144221322	0	0	0.072042309	0	0.48245641	0.21245641

As explained in previous solution (ii)

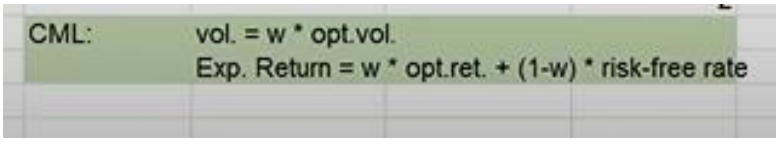
The Capital Market Line (CML) talks about portfolios that optimally combine risk and return.

From the question given, the risk-free rate is 1.5%.

Key Point: In calculating the Capital Market Line, there was consideration for the risk-free rate of 1.5%, Extrapolation and then the ORM which is the Optimum Risk Portfolio.

Key Point: The Extrapolation uses the maximum volatility of the various portfolios gotten by Solver and the addition of the risk-free rate and the Sharpe Ratio which is multiplied by its volatility as expected return.

To Calculate CML I have used the below Formula.



Risk free rate		0.015
	Capital Market Line	
	Volatility	Return
Risk Free	0	0.015
ORP	0.296803571	0.41989
Extrapolation	0.43630789	0.632248
OPR stands for optimum risk portfolio		
OPTIMALLY WEIGHTED ASSETS		
META	0.651180654	
TSLA	0.045624334	
AMZN	0.0713331	
AAPL	0.134201095	
TESCO	0.097660645	
Expected Return	0.41989022	
Standard Deviation	0.29680357	
Sharpe Ratio	1.414707431	

Economic Significance of the Capital Market Line

The capital market line allows investors to balance risky and risk-free assets to maximize return. There cannot be an understatement with the significance of the CML. Investors use the CML to get the best portfolios that maximize performance. Portfolios on the CML are the best as they give the best volatility/return relationship, thereby maximizing profit.

TASK- (iv)

TASK- (iv) Using linear regression analysis, calculate the *beta* for each asset in the portfolio and discuss the significance of this quantity.

Estimate Value at Risk (5%) for your portfolio and discuss how much each asset contributes to your estimated VaR.

Solution- (iv) Beta for each asset in the portfolio using Linear Regression

Beta is the measure of the volatility of an asset in comparison to the market.

The derivation of beta for each asset using linear regression involves the S&P 500 stocks and the S&P market itself.

	BETA
META	1.19484557
TSLA	0.834020628
AMZN	0.858652939
AAPL	1.541669131
TESCO	1.150369994

Figure 10: The Beta of each asset in the portfolio using Linear regression analysis

Calculating the beta of each asset involves finding the slope of the S&P500 stock and each asset in Microsoft Excel. The inserted values will be the daily change in the asset and the S&P 500.
The Beta of an asset is a particularly important measure as it tells its relationship with the market. A beta of more than 1 means that the asset is more volatile than the market while a beta of less than 1 is less volatile than the market. From above, one can deduce that **META, APPL** and **TESCO** are more volatile than the market while **TSLA** and **AMZN** are less volatile than the market.

Value at Risk

The value at Risk measures the extent in which a firm can make financial losses over a specific period. Below is the Value at Risk (5%) of the portfolio.

OPTIMAL WEIGHTED ASSETS			Calculating VAR		
META	0.651180826		Expected Return	0.419890217	
TSLA	0.045624334		Expected Volatility	0.296803571	
AMZN	0.0713331		Time(Year)	1	
AAPL	0.134201095		Confidence Level	0.95	
TESCO	0.097660645		Stress Event	-1.64485363	
Total	1		Asset Value	£1,000,000.00	
Expected Return	0.41989022		VAR	0.068308213	£68,308.21
Standard Deviation	0.29680357				
Sharpe Ratio	1.414707431				

VAR of Individual Asset	
	VAR of Asset
META	0.339332507
TSLA	0.026838532
AMZN	0.039904628
AAPL	0.104948997
TESCO	0.567034861
Total	0.567059525

	META	TSLA	AMZN	AAPL	TESCO
S.D	0.316808339	0.357630837	0.34009865	0.47543919	0.348828092

Figure 11: Calculation of the Value at Risk (VaR) and each asset contribution to the VAR. The VAR of the portfolio is **0.069308213** or **6.8308%.**

Calculation of the VaR is below

The formula used to calculate the VaR of the asset is

$$VAR_{asset} = \text{Z-score} * \text{StdDev of the asset} * \text{weight of the asset}$$

The formula for calculating the VaR of the portfolio is

$$VAR_{portfolio} = \text{Expected Return} + \text{StdDev of the asset} * \text{z-score}$$

TASK- (V)

TASK- (V) Estimate the volatility of a single asset in your portfolio using ARCH/GARCH and its extensions using R. Identify the best model and provide an explanation of your chosen model.

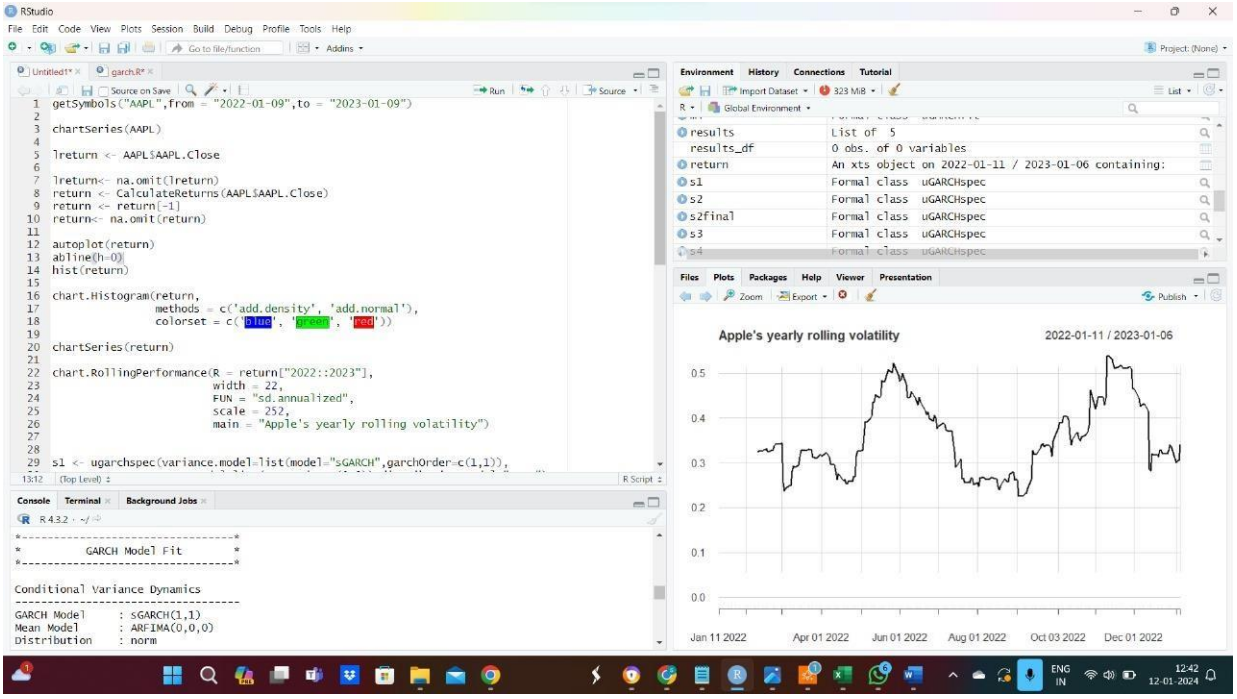
Solution – (V)

Estimating the Volatility of a single asset (AAPL) using ARCH/GARCH

The Autoregressive Conditional Heteroskedasticity (ARCH) model is a statistical model that analyses volatility in a time series data in order to forecast volatility. In the financial world, helps calculate risk by providing a model of volatility that more closely resembles real markets.

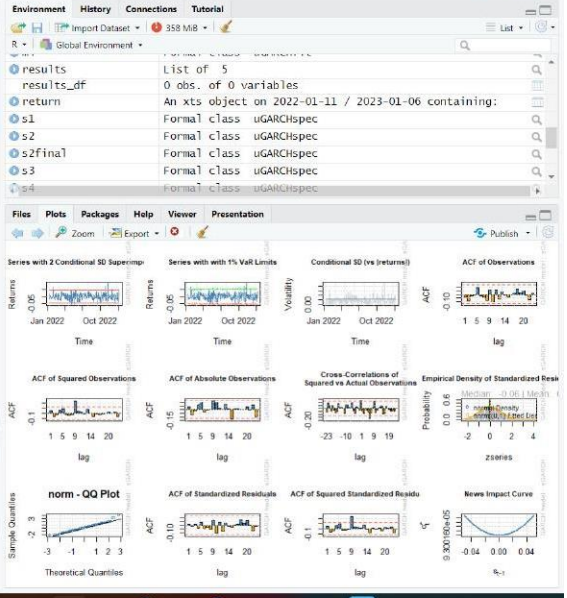
Generalized Autoregressive Conditional Heteroskedasticity (GARCH) models are most suitable for finding the volatility of returns of stocks, currencies, and cryptocurrencies. Some types of GARCH include the standard GARCH (sGARCH), Nonlinear GARCH (nGARCH) and Exponential GARCH (eGARCH). The main difference between the ARCH model and GARCH model is that while the GARCH method considers the volatility of the previous period while the ARCH model does not [16].

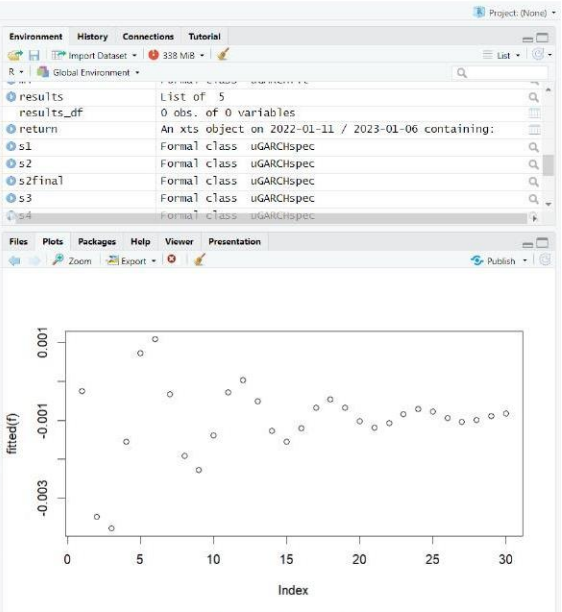
In estimating the volatility of the single asset (AAPL) using ARCH/GARCH, there will be an evaluation of the Akaike Information Criterion (AIC) or Bayesian Information Criterion.



```
getSymbols("AAPL",from = "2022-01-09",to = "2023-01-09")

chartSeries(AAPL)

lreturn <- AAPL$AAPL.Close

lreturn<- na.omit(lreturn)
return <- CalculateReturns(AAPL$AAPL.Close)
return <- return[-1]
return<- na.omit(return)

autoplot(return)
abline(h=0)
hist(return)

chart.Histogram(return,
                methods = c('add.density', 'add.normal'),
                colorset = c('blue', 'green', 'red'))

chartSeries(return)

chart.RollingPerformance(R = return["2022::2023"],
                         width = 22,
                         FUN = "sd.annualized",
                         scale = 252,
                         main = "Apple's yearly rolling volatility")


s1 <- ugarchspec(variance.model=list(model="sGARCH",garchOrder=c(1,1)),
```

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model      : sGARCH(1,1)
Mean Model       : ARFIMA(0,0,0)
Distribution     : norm
```





```
getSymbols("AAPL",from = "2022-01-09",to = "2023-01-09")

chartSeries(AAPL)

lreturn <- AAPL$AAPL.Close

lreturn<- na.omit(lreturn)
return <- CalculateReturns(AAPL$AAPL.Close)
return <- return[-1]
return<- na.omit(return)

autoplot(return)
abline(h=0)
hist(return)

chart.Histogram(return,
                methods = c('add.density', 'add.normal'),
                colorset = c('blue', 'green', 'red'))

chartSeries(return)

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                         main = "Apple's yearly rolling volatility")


s1 <- ugarchspec(variance.model=list(model="sGARCH",garchOrder=c(1,1)),
```
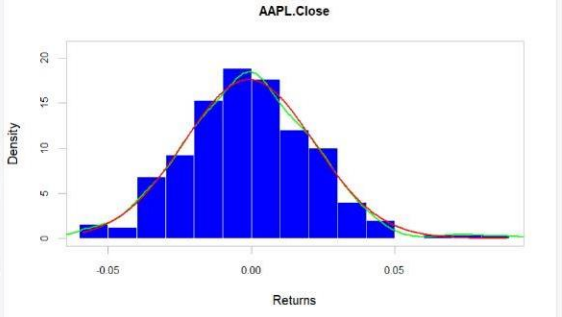

```
*---------------------------------*
*          GARCH Model Fit        *
*---------------------------------*

Conditional Variance Dynamics
-----------------------------------
GARCH Model      : sGARCH(1,1)
Mean Model       : ARFIMA(0,0,0)
Distribution     : norm
```



Summary of findings

From the various findings above, one can deduce that the portfolio is a little risky and there is a high annual expected return. The level of risk is worth the expected return. There is a certain level of correlation between the assets that shows that the combination of the assets are somehow a good fit. Also, the individual assets are riskier than the market. Considering the return of the portfolio with its risk, the portfolio is a particularly good and has remarkably high returns. This type of portfolio will be highly recommended to investors.

The optimum weighted assets of the portfolio favours more investment into Apple (AAPL) than any other asset. It is not surprising that the Capital Market Line (CML) intercepts the Efficient Frontier Curve at the same point that was gotten through solver. The model has favours less of Activison stock. More than 78% investments should be on Apple and stocks.

Conclusion

The portfolio offers an incredibly worthwhile investment return for the risk involved. Investments should be more on Apple and Nvidia stocks.

TASK- (vi) Give a summary of your findings in a non-technical language that could inform a potential investor choosing the best efficient portfolio and discuss the implication of other relevant performance measurements.

Solution- (vi) Submitting this return report for this section