Let $G = (V, E)$ be a (uni) directed sequence graph. Each vertex $v \in V$ is a sequence of length $|v|$. $v[k]$ is the k-th residue on the vertex with $0 \le k < |v|$. To find the optimal alignment between query P and the graph, classical algorithms compute score $\hat{H}_{ivk}$,where i is the position on P and k is the position on vertex v. In this note, we will use the diagonal formulation: we let $H_{v,s,i-k} = k$ if $\hat{H}_{ivk} = s$. Given H_{vsd}, the position on the query sequence i equals $H_{vsd} - d$.

Algorithm 1 computes the optimal alignment score between a query sequence P and the graph G. The pseudocode probably has the initial condition wrong and it does not consider edge cases, but the basic idea is there. The most difficult part to implement the algorithm in its current form is the data structure for H_{vsd}. Dynamically allocating from the heap may be slow. Perhaps we can manually allocate H_{vsd} from a large memory block.

Algorithm 1: Global graph wavefront alignment

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Input: Target sequence graph $G = (V, E)$, query sequence $P$ and scoring: $b$ for mismatch, $q$ for linear
        gap; $n_0 \in V$ and $n_1$ are the start and end segments, respectively.
Output: Best alignment score
Function GwfAlignFor($G, P, b, q, r$):
    push($D, (v_0, 0)$)
    $s \leftarrow 0$
    while $true$ do
        $D \leftarrow$ GwfExtend($H, s, D$)
        if $H_{v_1,s,|P|-|v_1|} = |v_1|$ then
            return s
        $s \leftarrow s + 1$
        GwfNext($H, s, D, b, q$)
Function GwfExtend($H, s, D$):
    $A \leftarrow D$
    $B \leftarrow []$
    while $|A| \neq 0$ do
        $(v, d) \leftarrow$ pop(A)
        $k \leftarrow H_{vsd}$
        $i \leftarrow k + d$
        if $k < |v|$ and $i < |P|$ then
            while $i < |P|$ and $k < |v|$ and $P[i] = v[k]$ do
                $i \leftarrow i + 1$
                $k \leftarrow k + 1$
            $H_{vsd} \leftarrow k$
            if $k = |v|$ then
                push($A, (v, d)$)
            else
                push($B, (v, d - 1)$);push($B, (v, d)$);push($B, (v, d + 1)$)
        else if $k = |v|$ and $i < |P|$ then
            $count \leftarrow 0$
            for $(v, w)$ in $E$ do
                if $P[i + 1] = w[0]$ then
                    push($A, (w, i + 1)$)
                    $count \leftarrow count + 1$
                else
                    push($B, (w, i)$);push($B, (w, i + 1)$)
            if $deg(v) = 0$ or $count < deg(v)$ then
                push($B, (v, d + 1)$)
    Remove duplicated entries in $B$
    return $B$
Function GwfNext($H, s, D, b, q$):
    for $(v, d)$ in $D$ do
        if $H_{vsd} > 0$ then
            $H_{vsd} \leftarrow \max\{H_{v,s-b,d} + 1, H_{v,s-q,d-1}, H_{v,s-q,d+1} + 1\}$
        else
            $i \leftarrow d$
            $H_{vsd} \leftarrow H_{v,s-q,d-1}$
            for $(u, v)$ in $E$ do
                $H_{vsd} \leftarrow \max\{H_{vsd}, H_{u,s-b,i-|u|+1} - |u|, H_{u,s-q,i-|u|} - |u|\}$
```