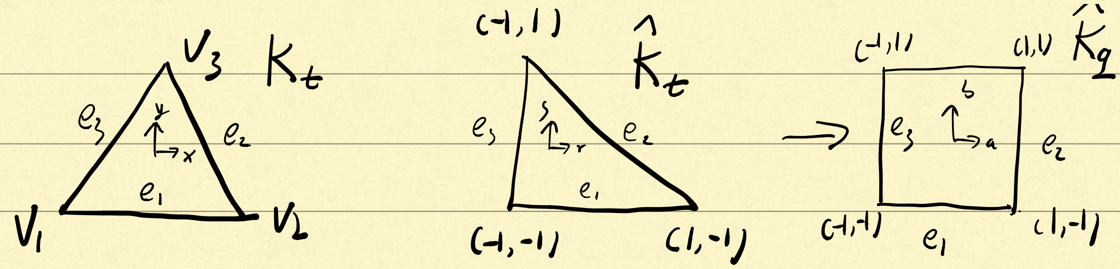

volume: $\begin{pmatrix} x \\ y \end{pmatrix} = A \begin{pmatrix} r+1 \\ s+1 \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

$$\begin{cases} r = \dfrac{(a+1)(1-b)}{2} - 1 \\ s = b \end{cases}$$

$$\frac{\partial(r,s)}{\partial(a,b)} = \begin{vmatrix} \frac{1-b}{2} & -\frac{a+1}{2} \\ 0 & 1 \end{vmatrix}$$

edge: e_1: $\begin{pmatrix} x \\ y \end{pmatrix} = A \begin{pmatrix} r+1 \\ 0 \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

e_2: $\begin{pmatrix} x \\ y \end{pmatrix} = A \begin{pmatrix} r+1 \\ -r+1 \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$ $\qquad r+s=0$

e_3 $\begin{pmatrix} x \\ y \end{pmatrix} = A \begin{pmatrix} 0 \\ s+1 \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$

Volume Integral

$$\int_{K_t} f(x,y)\,dx\,dy = \int_{\hat{K}_t} f\left(A\begin{pmatrix} r+1 \\ s+1 \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}\right) J_k \, ds\, dr$$

$$= J_k \int_{\hat{K}} \tilde{f}(r,s)\, dr\, ds$$

$$= J_k \int_{K_q} \tilde{f}\left(\frac{(a+1)(1-b)}{2} - 1, b\right) \frac{1-b}{2}\, da\, db$$

$$= J_k \int_{k} \tilde{\tilde{f}}(a,b)\, da\, db$$

$$= J_k \sum \sum w_i w_j \tilde{\tilde{f}}(a_i, b_j)$$

$$f\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = f\left(A\begin{pmatrix} r+1 \\ s+1 \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}\right)$$

$$= f\left(A\begin{pmatrix} \frac{(a+1)(1-b)}{2} \\ b \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}\right) \quad \checkmark$$

$$\tilde{\tilde{f}}(a,b) = f\left(A\begin{pmatrix} \frac{(a+1)(1-b)}{2} \\ b \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}\right)\left(\frac{1-b}{2}\right)$$

Face Integral

$$\int_{\partial K_t} f(x,y)\, ds$$

$$= \sum_{i=1}^{3} \int_{e_i} f(x,y)\, ds$$

$$= \sum_{i=1}^{3} \int_{\hat{e}_i} f\left(A\begin{pmatrix} r+1 \\ s+1 \end{pmatrix}\Big|_{e_i} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}\right) \frac{|e_i|}{2}\, ds$$

$$= \frac{|e_1|}{2}\int_{-1}^{1} f\left(A\begin{pmatrix} r+1 \\ 0 \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}\right) dr$$

Gauss Quadrature.

$$+ \frac{|e_2|}{2}\int_{-1}^{1} f\left(A\begin{pmatrix} r+1 \\ -r+1 \end{pmatrix} + \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}\right) dr$$

$$+\frac{|e_3|}{2}\int_{-1}^{1} f\left(A\begin{pmatrix}0\\ s+1\end{pmatrix}+\begin{pmatrix}x_1\\ y_1\end{pmatrix}\right)ds$$