Assignment 2: Implicit Surface Reconstruction

Theory Question

Let S be the implicit surface defined by $f(x) = 0$. Formally prove that the normal of S at point $p \in S$ is proportional to $\nabla f(p)$.

Solution

- S is a 0-level surface defined by $f(x) = 0$ and $p \in S$, therefore $f(p) = 0$.
- We can write every curve crossing p as $r(t) = \big(x(t), y(t), z(t)\big)$, for some parameter t.
- $r(t)$ is completely included in S, therefore $F(t) = f\big(x(t), y(t), z(t)\big) = 0$.
- By the rule of chain:

$$0 = F'(t) = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt} + \frac{\partial f}{\partial z} \cdot \frac{dz}{dt} = \nabla f \cdot r'(t)$$

  And as the curve crosses p, this concludes the proof. ■