10.10

GCD(356250895, 802137245)

89635455 = 802137245 – 2 * 356250895

GCD(356250895, 89635455)

87344530 = 356250895 - 3 * 89635455

GCD(89635455, 87344530)

2290925 = 89635455 - 87344530

GCD(87344530, 2290925)

289380 = 87344530 – 38 * 2290925

GCD(2290925, 289380)

265265 = 2290925 – 7 * 289380

GCD(289380, 265265)

24115 = 289380 - 265265

GCD(265265, 24115)

265265 = 11 * 24115

24115 = 2801 *356250895 – 1244 * 802137245

10.27(a)

GCD(6, 15) = 3

In this case, we only can measure anything that is a multiple of 3

(i)

First fully fill in 15 gallon jugs, then use 15 gallon jugs to fill 6 gallon jugs twice. Leftover water will be exactly 3 gallons.

(ii)

can't be done, 4 is not a multiple of 3

(iii)

can't be done, 5 is not a multiple of 3

10.40(c)

$Prove\ 2^{70} + 3^{70}\ is\ divisble\ by\ 13$

$$2^{70}\ mod\ 13 = 2^4 * 2^{66}\ mod\ 13$$

$$= 3 * 2^{66}\ mod\ 13$$

$$= 3 * 2^{4*16} * 2^2\ mod\ 13$$

$$= 3^{17} * 2^2\ mod\ 13$$

$$= 3^3 * 3^{14} * 4\ mod\ 13$$

$$= 1 * 3^{14} * 4\ mod\ 13$$

$$= 1^5 * 3^2 * 4\ mod\ 13$$

$$= 36\ mod\ 13 = 10$$

$$3^{70}\ mod\ 13 = 3^3 * 3^{67}\ mod\ 13$$

$$= 1 * 3^{67}\ mod\ 13$$

$$= 1^{13} * 3\ mod\ 13$$

$$= 3$$

In this case, if we add $2^{70} + 3^{70}$, we will get a leftover 13, which is clearly divisible by 13.

We can conclude that $2^{70} + 3^{70}\ is\ divisble\ by\ 13$

11.6

(a)The graph has 5 vertices each of degree 3

5 * 3 = 15, odd number degrees, according to handshaking rules, the graph doesn't exit.

(a) The graph has 4 vertices of degrees 1, 2, 3, 4

Since we only have 4 vertices, the maximum degrees we can get is 3, it is not possible to get a 4 on the graph. Thus, such graph doesn't exist.

(b)The graph has 4 edges and vertices of degrees 1, 2, 3, 4

According to handshaking rules, degree-sum = 2 * number of edges = 2 * 4 = 8

However, the total number of edges we need to satisfy is 1+2+3+4 = 10

$8 \neq 10$

Thus, such graph doesn't exit.

(b)The graph has 6 vertices of degrees 1,2,3,4,5,5

Since we only have 6 vertices, in order to satisfy requirement of having two degrees of 5, there must be 2 nodes that are connected to all other nodes. At this time, all nodes in the graph already have 2 degrees each, we cannot possibly have a node that only have 1 degree.

Thus, such graph doesn't exit.

11.40

(a)Not possible. Because among those 10 dominos, there are five squares that have 1.

Since the requirement is that touching dominos meet at the same number, an odd number of 1 will always have a one leftover. So we cannot make a ring that satisfy the requirement.

(b)$\sum_{i=1}^{n+1} i$,

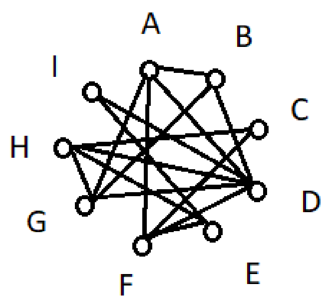
If we have n, total number is 1 + 2 + 3 + … + n+1

(c)

In order to place all the dominos in a ring, we have to make sure any specific patterns has a even total number. In this way, they all can be connected either head to tail or tail to head. (a multiple of 2, which is a even number).

In this case, any n that is an even number can place all dominos in a ring.

12.73(I)



{

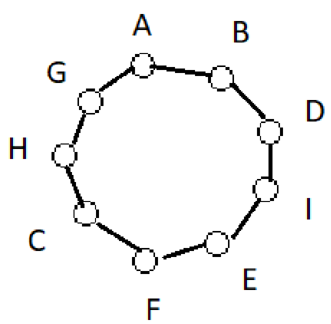
(A,B), (A,D), (A,F), (A,G)   (B,A), (B,D), (B,G)   (C,H), (C,F)

(D,A), (D,B), (D,F), (D,G), (D,H), (D,I)   (E,I) (E,F)   (F,A), (F,C), (F,D), (F,E)

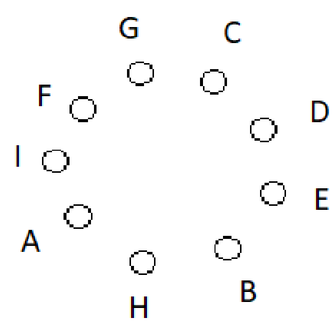
(G,A), (G,B), (G,D), (G,H)   (H,C), (H,D), (H,E), (H,G)   (I,D), (I,E)

}

(i)simply put all friends next to each other, if any two are enemies, put them apart.



Friends on the left and on the right



enemies on the left and on the right

(ii) simply put all enemies next to each other, if any two are friends, put them apart.