3.53

(a) True when $x \in \mathbb{N}$, $x \in \mathbb{Z}$, $x \in \mathbb{R}$, $x \in \mathbb{Q}$

(b) True when $x \in \mathbb{R}$

(c) True when

$(x \in \mathbb{N}, \mathrm{y} \in \mathbb{R})$, $(x \in \mathbb{N}, \mathrm{y} \in \mathbb{N})$, $(x \in \mathbb{N}, \mathrm{y} \in \mathbb{Q})$, $(x \in \mathbb{N}, \mathrm{y} \in \mathbb{Z})$,

$(x \in \mathbb{Z}, \mathrm{y} \in \mathbb{Q})$, $(x \in \mathbb{Z}, \mathrm{y} \in \mathbb{N})$, $(x \in \mathbb{Z}, \mathrm{y} \in \mathbb{R})$, $(x \in \mathbb{Z}, \mathrm{y} \in \mathbb{Z})$,

$(x \in \mathbb{Q}, \mathrm{y} \in \mathbb{Q})$, $(x \in \mathbb{Q}, \mathrm{y} \in \mathbb{R})$,

$(x \in \mathbb{R}, \mathrm{y} \in \mathbb{R})$.

(d)

True when $(y \in \mathbb{N}, x \in \mathbb{R})$

4.9

(a) P = n^3 + 5 is odd, Q = n is even.

1. Assume Q is false. Q is odd, Q = n = 2k + 1.

2. P = (2k+1)^3 + 5 = **2 * (4k^3 + 6k^2 + 3k + 3)**

3. since n^3 + 5 equals two times some integer, so it must be even in this case.

4. P is false.

5. Q is false and P is false, (p being True and Q being false cannot happen), we can conclude that if P then Q, $p \to Q$

(b) (3 does not divide n) → (3 divides n^2 + 2)

1. Assume Q is false. 3 does not divide n^2 + 2.

2. if n^2 can be divided by 3, then n^2 + 2 cannot.

(for example, 3 does not divide (3 + 2))

3. if n^ 2 can be divided by 3, n also can be divided by 3.

(n = 3k, n^2 = 9k^2 = **3 (3k^2)**, it is three times some integer)

4. in this case, n = 3k

5. 3k can be divided by 3.

6. P is false, n can be divided by 3

7. Q is false and P is false, (p being True and Q being false cannot happen), we can conclude that if P then Q.

4.15

(e) For $n \in \mathbb{Z}$, n^2 + 3n + 4 is even

Using direct proof, assume $n \in \mathbb{Z}$ is true

1. When n is odd, n = 2k + 1.

2. n^2 + 3n + 4 = 4k^2 + 4k + 1 + 6k + 3 + 4 = 4k^2 + 10k + 8 = **2 * (2k^2 + 5k + 4)**

3. it equals two times some integer, n^2 + 3n + 4 has to be even

4. When n is even, n = 2k

5. n^2 + 3n + 4 = 4k^2 + 6k + 4 = **2 * (2k^2 + 3k + 2)**

6. it equals two times some integer, n^2 + 3n + 4 has to be even

7. When n = 0, n^2 + 3n + 4 = 4.

8. In conclusion, as long as $n \in \mathbb{Z}$, n^2 + 3n + 4 will be an even number

(w) If a and b are positive real numbers with ab < 10,000, then min(a,b) < 100.

Using contraposition

P is ab < 10,000, Q = min(a,b) < 100. $P \rightarrow Q$

1. Assume min(a,b) < 100 is false. min(a,b) >= 100.
2. In this case, assume a and b are smallest number 100, a = 100, b = 100.
3. a * b = 10,000 $\rightarrow \neg$(ab < 10,000)
4. When min(a,b) < 100 is false, ab < 10,000 has to be false.
5. When Q is false, P cannot be true
6. In conclusion, $P \rightarrow Q$

4.26

(b)$\neg(P(n) \rightarrow Q(n))$

Prove:

Direct prove to show that not P(n) implies not Q(n).

Disprove:

Find an example where not P(n) is true and not Q(n) is false.

(d) $\forall x: ((\forall n: P(n) \rightarrow Q(x))$

Prove:

Contraposition/ contradiction.

Disprove:

For all x, find an example (n) where $(\forall n: P(n) \rightarrow Q(x))$ is false.

(f) $\exists x: ((\exists n: P(n) \rightarrow Q(x))$

Prove:

Find an x for which $((\exists n: P(n) \rightarrow Q(x))$ is true.

Disprove:

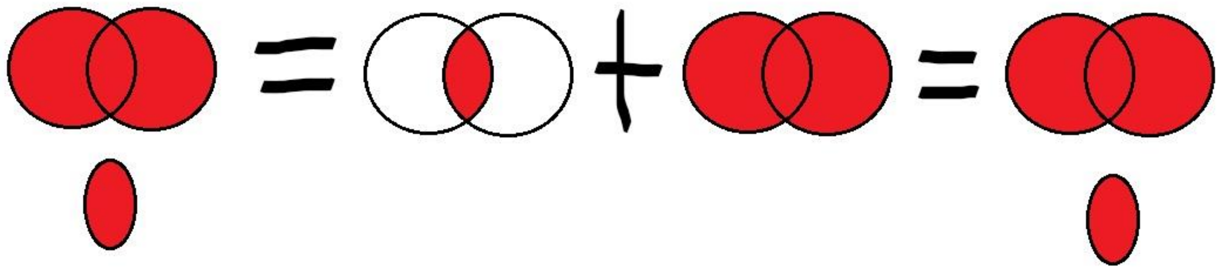
$\neg\exists x: ((\exists n: P(n) \rightarrow Q(x))$ = $\forall x: \neg((\exists n: P(n) \rightarrow Q(x))$.

Prove $\forall x: \neg((\exists n: P(n) \rightarrow Q(x))$ is true, which in other words, prove that for all x, there is no n such that $P(n) \rightarrow Q(x)$ is true.

4.36

(j)

$|A| + |B|$ = $|A \cap B|$ + $|A \cup B|$ = $|A| + |B|$



4.45

(b)

(ii)

1. assume f(n) does not go to 1.

2. Then there is a C for every n_c, $n \geq n\_c$, where f(n) < C

3. If $n \geq n\_c$, then $f(n) \geq f(n_c)$

4. But $f(n\_c) \geq C, f(n) \geq C$ this is a contradiction.

5. For example, as n goes up to infinity, f(n) will become n/n rather than (n+3) / (n+1), which is 1 in this case.

6. In conclusion, $f(n) \rightarrow 1$ is true.

7. above conclusion disproves part (i) and (iii).

5.7

(f) (1 - 1/2) (1 - 1/3) (1 - 1/4) … (1 - 1/n) = 1/n

1. Assume (induction hypothesis) is true.

2. P(n) starts with 2. (1 – 1/2) = ½, P(2) is true (first term is true).

3. As we know, P(n) = 1/n,

so P(n+1) = (1/n) (1 - 1/n+1)

= (1/n) (n/n+1)

= 1/ (n+1)

4. from above expression, we see that P(n+1) is true.

5. For such statement, $P(n) \rightarrow P(n+1)$

6. By induction, P(n) is true $for\ \forall x \geq 2$