5.10(j)

5 divides 11^n -6

**For**

Base case:

n =0

When n = 0, = 1, 1 - 6 = -5, 5 divides -5.

True for n = 0

Induction steps:

Assume P(n) is divisible by 5: 6 is divisible by 5

6 = 5k for an integer k, = 5k + 6

P(n+1) = = \* 11 = 11 \* (5k+6)

– 6 = 11\*(5k+6) – 6 = 55k + 66 6 = 55k + 60 = 5(11k + 12)

(multiple of 5)

Hence, – 1 is divisible by 5

**By the principle induction, P(n) is true for**

5.12(i)

Base case: n = 1

n! = 1, =

1 , True for n = 1

Induction steps:

Assume

Prove

Since ,

e

Hint ((1 + )

Since =.

From hint, , hence e

In this case e ,

since e and e

It proves that by induction, is true for all n 1

5.18(a)

Base case: n =1

H(1) = 1 = (1+1) \*1 – 1. True for n = 1

Inductive steps:

Assume is true, prove

For

If we prove that when H(n+1), both sides will increase by the same value, we can state that

When H(n+1), the increment is:

LHS:

…

RHS:

LHS equals RHS

We prove that

So the statement is true for all n

5.60

(a) the perimeter highlighted by the thick line in the graph is 42

(b) The base case when P(n), n =1, P(1) =4, which is true for base case

Inductive steps:

We want to prove using direct prove

When P(n) has an even number of total lines, there are four possibilities if we add another square:

1. Added square is completely separated from other squares, in other words, no lines are overlapped. We simply add 4 to the total number, which would still make it an even number.
2. Added square has one line overlapped with other squares. At this time, we would have to first subtract 1 because of overlap, then we have to add 3 into total number because the rest of the lines are individual lines that should be counted. Overall, we add 2 to the total number, which makes it an even number.
3. Added square has two lines overlapped with other squares, we have to first subtract 2 and then add 2 lines. Overall, the total size doesn’t change, which makes it an even number.
4. Added square has three lines overlapped with other squares. We have to first subtract 3 and then add 1 line. Overall, we have to subtract 2 from the total number, which makes it an even number.

For above four possibilities, the total number will always end up with an even number, so by using a direct proof, when P(n) is true, P(n+1) has to be true.

In this case, we prove that the total number will always be even.

6.6

(a)

Base case:

n = 1, H(n) = 1

1 <

Inductive steps:

We want to prove

Assume H(n) is true. If both sides increase by the same amount or the left hand side increase by a smaller amount, then we know that H(n+1) is true.

LHS: H(n+1) – H(n) =

RHS: H(n+1) – H(n) =

=

=

=

=

Right hand side increment < left hand side increment

The increment of left hand side is bigger than the right hand side. In this case,

.

(b)

Base case:

n = 1, 1 < 3/2, so the base case is true.

Assume H(n) is true, we want to prove the implication

Same as above, the increment of LHS is

Increment of RHS:

RHS:

From above, we see that both sides increased by the same value for every term added, and since the base case, 1< 3/2 is true, will always be true because LHS will never be larger than RHS.

It is a stronger claim because we can now prove that RHS always grows at the same rate as LHS rather than growing slower than LHS in (a).

6.45(a)

Base case:

n = 2. P(n) is true. If there are only two cities, a and b, there are direct flights between a and b. If there is a special city, we could go directly to the special city from the current location (either a to b or b to a).

Inductive steps:

Assume P(n) is true, prove

When there are n+1 cities, we know that there is a one-way flight between every pairs of cities. If we want to go to that special city, we could either take the one-way flight, or transfer once at another city. (since there is a one-way flight between every pairs of cities, we are sure that any other cities has their own one-way flights to the special city)

Thus, P(n+1) is true, and is true. The statement is now proved by induction, there must at least one special city that can be reached directly or via one stop.