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Math 4800 hw1 solutions

Solution:

1. I have read chapter 0 and Appendix B in the book, and practiced Matlab.

2. The matlab code is given below.

Here is my Matlab code for nest.m,

Listing 1: nest.m, Matlab code for computing polynomials

```
%Program Nested multiplication
function y = nest(d,c,x,b)
%Input: degree d of polynomial,
% array of d+1 coefficients c (constant term first),
% x-coordinate x at which to evaluate, and
% array of d base points b, if needed
%if there are only three inputs, initialize b to zero
%Output: value y of polynomial at x
if nargin < 4, b = zeros(d,1); end
%start to calculate values from lowest degree term to highest
y = c(d + 1);
for i = d: -1 : 1
    %evaluate component using Horner's rule
    y = y.*(x - b(i)) + c(i);
end
```

Here is the results obtained by evaluating directly and by using nest.m, we can see the error (difference) and can compare it with tanh(x) values:

Listing 2: Results from nest.m and evaluate directly, compare with tanh(x)

```
>> nest.m
>> y = nest(7, [0,1,0,-1/3,0,2/15,0,-17/315],0.25)
>> p = 0.25 - 1/3.*0.25.^3 + 2/15.*0.25.^5-17/315.*0.25.^7
>> fprintf("evaluating using nest(): %.25f\nevaluating directing: %.25f\n", y, p)
evaluating using nest(): 0.2449185810391865114610255
evaluating directing: 0.2449185810391864837054499
>> error = y - p
error = 2.775557561562891e-17
>> fprintf("%.25f\n", tanh(0.25))
0.2449186624037091308814240
```

From above, we can see our error has been outputed, which is 2.775557561562891e-17.

Listing 3: graph.m, Matlab code for graphing two functions

```
%set an array to store coefficient of polynomial
a = [-17/315 0 2/15 0 -1/3 0 1 0]
%find 21 equally distributed interval from -1.5 to 1.5
x = linspace(-1.5,1.5,21)
%set two functions
y1 = tanh(x)
y2 = a(1) * x.^7 + a(2) * x.^6 + a(3) * x.^5 + a(4) * x.^4 + a(5) * x.^3 + a(6) * x.^2 + a(7) * x.^
    1
```

```

figure;
%plot tanh(x) using blue color and marker 'o', polynomial using red color
%and marker 'x'
plot(x, y1,'-*',x, y2, 'r-x','LineWidth', 1.5, 'MarkerSize', 10)
hold on
%set x and y axis and graph title
xlabel('x-axis␣from␣-1.5␣to␣1.5');
ylabel('y-axis␣from␣-1␣to␣1');
title('{\itResults␣from␣function␣nest}');
hold off
legend('p(x)', 'tanh(x)');
grid on;
```

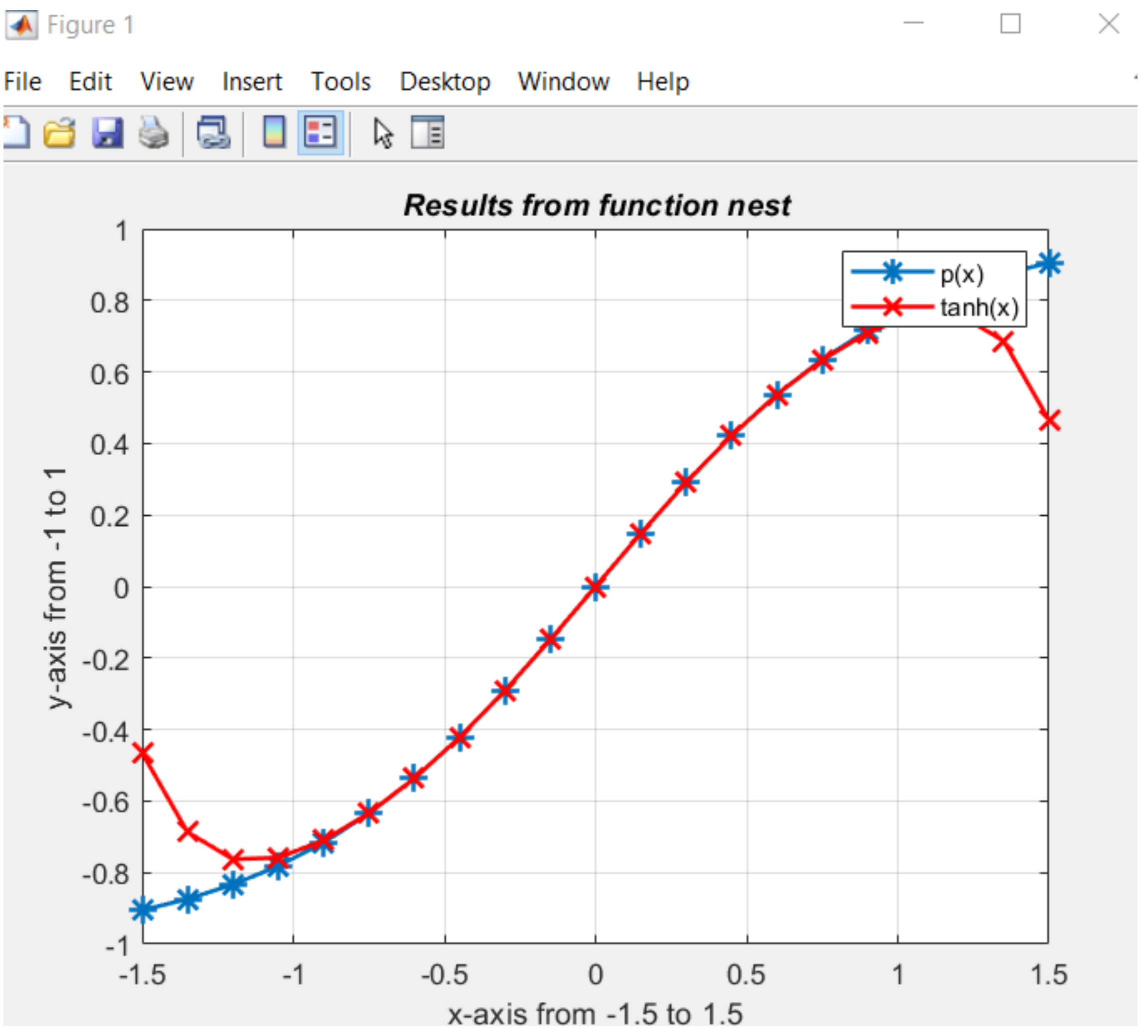

Figure 1: used to compare values of p(x) with tanh(x)

(3) (a) $f(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$

$f(0) = 1$

$f'(0) = \frac{e^x - e^{-x}}{2} = 0$

$f''(0) = \frac{e^x + e^{-x}}{2} = 1$

$f'''(0) = 0$

$f''''(0) = 1$

$P(x) = f(0) + (x - x_0) f'(0) + \frac{(x - x_0)^2}{2!} f''(0) + \frac{(x - x_0)^3}{3!} f'''(0)$

$= 1 + \frac{1}{2} \cdot \frac{1}{100} = 1.005$

$R(x) = (x - x_0)^4 \frac{f''''(0.1)}{4!} = (0.1)^4 \cdot \frac{\frac{e^{0.1} + e^{-0.1}}{2}}{24} = 4.1875 \times 10^{-6}$

Using MatLab to compute actual error, I got 4.1681×10^{-6}

thus, our upper bound is 4.1875×10^{-6}

(b) $f(x) = (1 + x)^{\frac{3}{2}}$

$f(0) = 1$ $f'(0) = \frac{3}{2}(1+x)^{\frac{1}{2}} = \frac{3}{2}$ $f''(0) = \frac{3}{4}(1+x)^{-\frac{1}{2}} = \frac{3}{4}$

$f'''(0) = -\frac{3}{8}(x+1)^{-\frac{3}{2}} = -\frac{3}{8}$ $f''''(0.1) = \frac{9}{16}(1\ )^{-\frac{5}{2}}$

$P(x) = 1 + \frac{3}{2} \cdot (0.1) + \frac{3}{4} \cdot \frac{1}{2} \cdot (0.1)^2 + (-\frac{3}{8}) \cdot \frac{1}{6} \cdot (0.1)^3 = 1.1537$

$R(x) = \frac{f''''(1)}{24} \cdot (0.1)^4 = 2.3438 \times 10^{-6}$

Using MatLab to compute actual error, I got 2.232987×10^{-6}

thus, upper bound is confirmed