Multivariate Continuity Equation

Overview: The continuity equation explains how something, like mass, charge, or probability, is conserved as it moves over time and space. It is used in physics, engineering, and statistics to track how these quantities change or flow. In statistics, it can show how a probability distribution shifts over time while keeping the total probability constant.

The continuity equation can be applied to probability density functions (PDFs) to model how the distribution of a variable changes over time. It ensures that the total probability remains constant as the PDF evolves, accounting for factors like drift or flow.

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho v) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial (\rho v)}{\partial x} = 0$$

Where:

- *p (x,t)* is the density of population
- *v (x,t)* is the migration velocity of the population

Real World Application: The continuity equation can model the movement and density of people across Metro Manila, helping identify congestion hotspots like terminals or busy intersections. By simulating population flow, planners can test interventions such as new routes or staggered work hours to reduce overcrowding. This approach enables better resource allocation and more effective solutions to improve urban mobility in Metro Manila.

Logistic Growth with Diffusion Equation

Overview: Logistic growth with diffusion is a mathematical model that combines two key processes: logistic growth, which describes how a population or quantity increases over time under limiting factors, and diffusion, which represents how this quantity spreads across space. The model is represented by a partial differential equation that accounts for both the rate of growth and the spread of a phenomenon over time and space.

The logistic growth with diffusion equation is widely used to model scenarios such as the spread of diseases, population dynamics, or political opinions, where both growth and spatial movement are involved.

$$\frac{\partial u}{\partial t} + D\nabla^2 u + ru\left(1 - \frac{u}{K}\right) = 0$$

Where:

- *u (x,t)* is the proportion of voters supporting a candidate at a given time t and location x
- *r* is the growth rate of supporters in a given region
- *K* is the maximum support a candidate can reach in a region (due to limited voter base)
- *D* is the diffusion coefficient
- $\nabla^2 u$ is the Laplacian operator representing the spatial spread of support

Real World Application: The logistic growth with diffusion model can be applied to political polling in the Philippines to predict how voter support for a candidate spreads across different provinces and cities over time. In the early stages of a campaign, the model captures rapid growth in support, but as a candidate's popularity reaches saturation in certain areas, the growth slows down. The diffusion term accounts for how support spreads from urban centers like Metro Manila to surrounding regions, influenced by media, rallies, and social interactions. By using this model, political campaigns in the Philippines can forecast where to focus resources, improve outreach strategies, and better understand regional voting patterns in preparation for national elections.