习题 3.4.

5. (10) $\tan x \sim x$ $\lim\limits_{x\to 0}\frac{x^2\cos\frac{1}{x}}{\tan x} = \lim\limits_{x\to 0} x\cos\frac{1}{x} = 0$ $(|\cos\frac{1}{x}| \leq 1)$

(12) $\lim\limits_{x\to 1^-}\frac{\ln(1-x)}{\frac{1}{\ln x}} \xrightarrow{L} \lim\limits_{x\to 1^-}(-\frac{1}{1-x})\Big/\left[(-\frac{1}{\ln^2 x})\cdot\frac{1}{x}\right] = \lim\limits_{x\to 1^-}\frac{x\ln^2 x}{1-x}$

$\xrightarrow{L} \lim\limits_{x\to 1^-} -\frac{\ln^2 x+2\ln x}{-1} = 0$

(14) 先求ln原式 $\lim\limits_{x\to 0}\frac{1}{x}\ln\frac{(1+x)^{\frac{1}{x}}}{e} = \lim\limits_{x\to 0}\frac{1}{x}\ln(1+x)^{\frac{1}{x}} - \frac{1}{x}$

$= \lim\limits_{x\to 0}\frac{1}{x}(\ln(1+x)^{\frac{1}{x}} - 1) = \lim\limits_{x\to 0}\frac{1}{x}\cdot\frac{\ln(1+x)-x}{x} = \lim\limits_{x\to 0}\frac{\ln(1+x)-x}{x^2}$

$\overset{L}{=\!=} \lim\limits_{x\to 0}\frac{\frac{1}{1+x}-1}{2x} \overset{L}{=\!=} \lim\limits_{x\to 0}\frac{-(1+x)^{-2}}{2} = -\frac{1}{2}$

故原式 $= e^{-\frac{1}{2}}$

(16) 2

(18) $\lim\limits_{x\to 0}\frac{\arctan x^2}{\sqrt{1+x\sin x}-\sqrt{\cos x}}\cdot(2-\frac{x}{e^x-1})$

$= \lim\limits_{x\to 0}\frac{x^2}{1+x\sin x-\cos x}\cdot\lim\limits_{x\to 0}(\sqrt{1+x\sin x}+\sqrt{\cos x})\cdot\lim\limits_{x\to 0}(2-\frac{x}{e^x-1})$

↓ 2 ↓ 1

由于 $\lim\limits_{x\to 0}(\frac{1-\cos x}{x^2}+\frac{x\sin x}{x^2}) = \frac{3}{2}$

故原式 $= 2\times\frac{2}{3} = \frac{4}{3}$

(20) 0

3.5.

2. 由Hint 即可

7. pf: 不妨设 $f'''(x_0)>0$ $\lim\limits_{x\to x_0^-}\frac{f''(x)-f''(x_0)}{x-x_0}>0$ 得 $f''(x)<0$ $(x\to x_0^-)$

同理 $\lim\limits_{x\to x_0^+}\frac{f''(x)-f''(x_0)}{x-x_0}>0 \Rightarrow f''(x)>0$ $(x\to x_0^+)$

即 x_0 为 $f'(x)$ 极值点、

由def 亦为拐点、

8.(2) $(0,+\infty)$ 凸 $(-\infty,0)$ 凹 无拐点.

(4) $(-\infty,-3)$ $(-1,+\infty)$ 凸

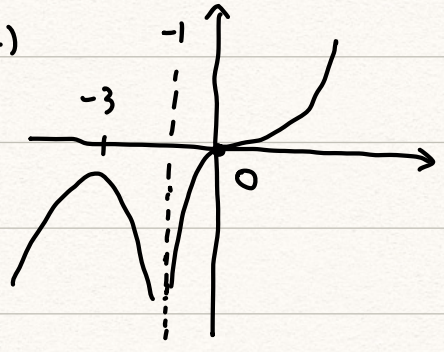
$(-3,-1)$ 凹 拐点为 $-3,-1$

(6) $((2k-1)\pi, 2k\pi)$ 凸

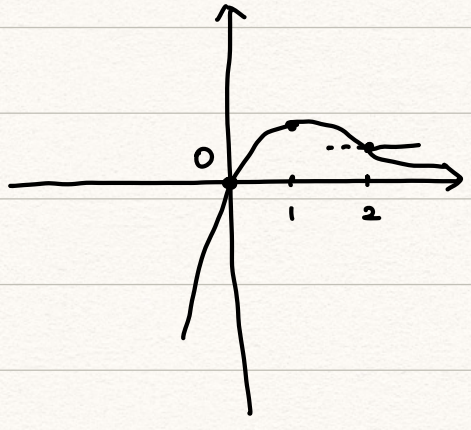
$(2k\pi, (2k+1)\pi)$ 凹 拐点为 $k\pi$ $k\in\mathbb{Z}$

9. $\begin{cases} a=-\frac{3}{2} \\ b=\frac{9}{2} \end{cases}$

10. (2)



(4)



$x=2$ 拐点.

$x=1$ 极值点.