第九周作业参考答案

P130　习题3.5

11 (2)　$y' = -2x \cdot e^{-x^2}$　　$y'' = (4x^2-2)\,e^{-x^2}$

$$K\big|_{x=0} = \frac{y''\big|_{x=0}}{\left(1+(y'\big|_{x=0})^2\right)^{\frac{3}{2}}} = -2 < 0$$

$$\Rightarrow \rho\big|_{x=0} = \frac{1}{\left|K\big|_{x=0}\right|} = \frac{1}{2}$$

由 $y'\big|_{x=0} = 0$　$\Rightarrow$ $(0,1)$ 处切线为 $y=1$，法线 $x=0$　$\Rightarrow$ 曲率中心为 $(0,\frac{1}{2})$

12 (2)　$K = \dfrac{y''x' - y'x''}{(x'^2+y'^2)^{\frac{3}{2}}} \xRightarrow{\text{代入}} K\big|_{t=\frac{\pi}{2}} = \dfrac{2}{\pi}$

13　$y' = \dfrac{1}{x}$　　$y'' = -\dfrac{1}{x^2}$

$$K = \frac{y''}{(1+y'^2)^{\frac{3}{2}}} = -\frac{x}{(1+x^2)^{\frac{3}{2}}} \qquad (x>0)$$

$$\Rightarrow \frac{d}{dx}(|K|) = \left(\frac{x}{(1+x^2)^{\frac{3}{2}}}\right)' = \frac{(1-2x^2)}{(1+x^2)^{\frac{5}{2}}} \qquad (x>0)$$

$x = \frac{\sqrt{2}}{2}$ 时，$|K|$ 取最大值 $\frac{2\sqrt{3}}{9}$

$$\Rightarrow \rho_{min} = \frac{1}{|K|_{max}} = \frac{3\sqrt{3}}{2}$$

$y = \ln x$ 在 $(\frac{\sqrt{2}}{2}, -\frac{1}{2}\ln 2)$ 曲率半径最小为 $\frac{3\sqrt{3}}{2}$

P140 习题3.6

1(1) $y = x^2+x+3+\frac{4}{x-1}$

$= x^2+x+3-4(1+x+x^2+\cdots+x^n+o(x^n))$

$= -1-3x-3x^2-4x^3-\cdots-4x^n+o(x^n)$ 在 $x=1$ 附近

3. $y = \ln\cos x = \ln(1+(\cos x-1)) = (\cos x-1)-\frac{1}{2}(\cos x-1)^2+\frac{1}{3}(\cos x-1)^3+o(x^6)$ $x=0$ 附近

保留6阶以内

$= \left(-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}\right)-\frac{1}{2}\left(-\frac{x^2}{2!}+\frac{x^4}{4!}\right)^2+\frac{1}{3}\left(-\frac{1}{2!}x^2\right)^3+o(x^6)$

$= -\frac{x^2}{2}-\frac{x^4}{12}-\frac{x^6}{45}+o(x^6)$ $x=0$ 附近

5(2) $y=\frac{1}{x} = -\frac{1}{1-(1+x)}$

$= -1-(1+x)-(1+x)^2-\cdots-(1+x)^n+\boxed{(-1)^{n+1}\cdot\frac{(x+1)^{n+1}}{[-1+\theta(x+1)]^{n+2}}}$ $0<\theta<1$ $x=-1$ 附近

6(1) $\cos x = 1-\frac{x^2}{2!}+\frac{x^4}{4!}+o(x^4)$ $x\to 0$

$e^{-\frac{1}{2}x^2} = 1-\frac{1}{2}x^2+\frac{(-\frac{1}{2}x^2)^2}{2!}+o(x^4) = 1-\frac{1}{2}x^2+\frac{1}{8}x^4+o(x^4)$ $x\to 0$

$\lim\limits_{x\to 0}\frac{\cos x-e^{-\frac{1}{2}x^2}}{\sin^4 x} = \lim\limits_{x\to 0}\frac{\left(1-\frac{x^2}{2!}+\frac{x^4}{4!}\right)-\left(1-\frac{1}{2}x^2+\frac{1}{8}x^4\right)+o(x^4)}{x^4} = -\frac{1}{12}$

6(3) $\ln(1+\frac{1}{x}) = \frac{1}{x}-\frac{1}{2x^2}+o(\frac{1}{x^2})$ $x\to\infty$

$\lim\limits_{x\to\infty}[x-x^2\ln(1+\frac{1}{x})] = \lim\limits_{x\to\infty}[x-x^2(\frac{1}{x}-\frac{1}{2x^2}+o(\frac{1}{x^2}))] = \lim\limits_{x\to\infty}\frac{1}{2}+o(1) = \frac{1}{2}$

$\forall x_0 \in [0,2]$ 在 x_0 附近展开

8. $f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{f''(\xi)}{2!}(x-x_0)^2$ $\quad \xi$ 介于 x, x_0 间

$$\begin{cases} f(0) = f(x_0) - f'(x_0)\,x_0 + \frac{f''(\xi_1)}{2!}x_0^2 & \xi_1 \in (0, x_0) \\ f(2) = f(x_0) + f'(x_0)(2-x_0) + \frac{f''(\xi_2)}{2!}(2-x_0)^2 & \xi_2 \in (x_0, 2) \end{cases}$$

作差：$\quad f(2) - f(0) = 2f'(x_0) + \frac{1}{2}\left[f''(\xi_2)(2-x_0)^2 - f''(\xi_1)x_0^2\right]$

$$\Rightarrow |f'(x_0)| \leq \frac{1}{2}|f(2)-f(0)| + \frac{1}{4}|f''(\xi_1)|x_0^2 + \frac{1}{4}|f''(\xi_2)|(2-x_0)^2$$

$$\leq \frac{1}{2}\times 2 + \frac{1}{4}\times 1\times x_0^2 + \frac{1}{4}\times 1\times(2-x_0)^2$$

$$= 1 + \frac{1}{4}\left(x_0^2 + (2-x_0)^2\right) \qquad 2x_0^2 - 4x_0 + 4 \leq 4 \quad x_0 \in [0,2]$$

$$\leq 2$$

$\Rightarrow |f'(x)| \leq 2 \qquad \forall x \in [0,2]$ #