P190. 10. 取 $f(x)=x$，$g(x)=f(x)+J(x)$. $J(x)=\begin{cases}0 & ,x\neq c\\ 1 & ,x=c\end{cases}$

~~再~~ $F(x)=\int_a^x f(t)\,dt$ $\quad G(x)=\int_a^x g(t)\,dt$

则 $\forall x\in\mathbb{R}$，$F(x)=G(x)$

$F(x)$ 在 $x=c$ 处可导 $\Rightarrow$ $G(x)$ 在 $x=c$ 处可导 $\not\Rightarrow$

$g(x)$ 在 $x=c$ 处连续

P190. 11. (3) $\frac{d}{dx}f(x)=\frac{d}{dx}\int_0^{x^2}e^{-t^2}dt-\frac{d}{dx}\int_0^{x}e^{-t^2}dt$

$$=2xe^{-x^4}-e^{-x^2}$$

(4) $\frac{d}{dx}f(x)=\cos\left(\int_0^x\sin\left(\int_0^y\sin t^2dt\right)dy\right)\sin\left(\int_0^x\sin t^2\,dt\right)$

P190. 12. (2) $y=f(x)=\int_1^x e^{-t^2}dt\Rightarrow f'=e^{-x^2}$

$x=f^{-1}(y)\Rightarrow\frac{dx}{dy}=(f^{-1})'=\frac{1}{f'(x)}=e^{x^2}$

$\Rightarrow(f^{-1})'(0)=(f^{-1})'\big|_{y=0}=(f^{-1})'\big|_{x=1}=e$

P190. 13. $F'(x)=\frac{d}{dx}x\int_0^x f(t)dt=\int_0^x f(t)dt+xf(x)$

P190. 14. $G'(x)=\dfrac{xf(x)\int_0^x f(t)dt-f(x)\int_0^x tf(t)dt}{\left(\int_0^x f(t)dt\right)^2}$

$$=\frac{f(x)}{(\quad)^2}\times\int_0^x(x-t)f(t)dt$$

~~由~~ $\forall x>0$，$(x-t)f(t)\geq 0$ $\forall t\in[0,x]$ 成立

$\Rightarrow$ $G'(x)\geq 0$

P190. 15. (3) $\int_1^2\ln x\,dx=(x\ln x-x)\big|_1^2=\boxed{2\ln 2-1}$

(4) $\int_2^3\frac{dx}{2x^2+3x-2}=\frac{1}{5}\ln\left|\frac{2x-1}{x+2}\right|\Big|_2^3=\boxed{\frac{1}{5}\ln\frac{4}{3}}$

16. (1) 当 $-1 \le x < 0$ 时, $F(x) = -\int_{-1}^{x} dt = -x-1$

(2) 当 $x=0$ 时, $F(x) = -\int_{-1}^{0} dt = -1$

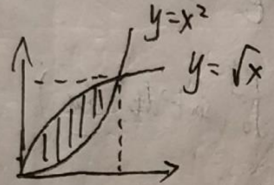
(3) 当 $0 < x \le 1$ 时, $F(x) = -\int_{-1}^{0} dt + \int_{0}^{x} dt = x-1$

$$\Rightarrow F(x) = \begin{cases} -x-1, & -1 \le x < 0 \\ x-1, & 0 \le x \le 1 \end{cases}$$

$F(x)$ 在 $[-1,0)$, $[0,1]$ 上可微, 在 $x=0$ 处,

$F'_+(0) = \lim\limits_{x\to 0^+} \dfrac{F(x)-F(0)}{x-0} = 1$, $F'_-(0) = \lim\limits_{x\to 0^-} \dfrac{F(x)-F(0)}{x} = -1$

17. (1)



$$\begin{cases} y = x^2 \\ y = \sqrt{x} \end{cases} \Rightarrow \begin{cases} x=0 \\ y=0 \end{cases} \text{或} \begin{cases} x=1 \\ y=1 \end{cases}$$

$$A = \int_0^1 (\sqrt{x} - x^2)dx = \left(\frac{2}{3}x^{\frac{3}{2}} - \frac{1}{3}x^3\right)\Big|_0^1 = \frac{1}{3}$$

18. (3)* $\lim\limits_{n\to\infty} \sum\limits_{i=0}^{n-1} \dfrac{1}{\sqrt{1+(\frac{i}{n})^2}} \dfrac{1}{n} = \lim\limits_{n\to\infty} \sum\limits_{i=0}^{n-1} f(\frac{i}{n})$

由 $f(x) = \dfrac{1}{\sqrt{1+x^2}}$ 在 $[0,1]$ 上的可积性,得:

$$\lim_{n\to\infty} \sum_{i=0}^{n-1} \frac{1}{\sqrt{1+(\frac{i}{n})^2}} \frac{1}{n} = \int_0^1 \frac{dx}{\sqrt{1+x^2}} = \ln(x+\sqrt{1+x^2})\Big|_0^1$$

$$\boxed{= \ln(1+\sqrt{2})}$$

19. (1) $\int_a^b e^{-nb^2}dx \le \int_a^b e^{-nx^2}dx \le \int_a^b e^{-na^2}dx$

$$\left.\begin{aligned} \lim_{n\to\infty} \int_a^b e^{-nb^2}dx &= \lim_{n\to\infty} (b-a)e^{-nb^2} = 0 \\ \lim_{n\to\infty} \int_a^b e^{-na^2}dx &= \lim_{n\to\infty} (b-a)e^{-na^2} = 0 \end{aligned}\right\} \Rightarrow \lim_{n\to\infty} \int_a^b e^{-nx^2}dx = 0.$$

(2) $\int_0^1 \dfrac{x^n}{1+x}dx = \dfrac{1}{1+\xi} \int_0^1 x^n dx = \dfrac{1}{1+\xi} \dfrac{1}{n+1}$, $\xi \in (0,1)$

$$\Rightarrow \left|\int_0^1 \frac{x^n}{1+x}dx\right| \le \frac{1}{n+1} \to 0.$$

21. $\int_T^{a+T} f(x)\,dx = \int_0^a f(t+T)\,dt = \int_0^a f(x)\,dx$

$t = x-T$

$$\int_a^{a+T} f(x)dx = \cancel{\int_0^a f(x)\,dx + \int_a^{a+T} f(x)\,dx + \int_{a+T}} + \int_a^0 f(x)\,dx$$

$$= \int_0^T f(x)\,dx + \int_T^{a+T} f(x)\,dx + \int_a^0 f(x)\,dx$$

$$= \int_0^T f(x)\,dx + \int_0^a + \int_a^0$$

$$= \int_0^T f(x)\,dx$$

22. (1) $\int_0^{2\pi} |\cos x|\,dx = 4\int_0^{\frac{\pi}{2}} \cos x\,dx = \boxed{4}$

(8) $\int_0^a \frac{dx}{x+\sqrt{a^2-x^2}} \overset{t=asint}{=\!=} \int_0^{\frac{\pi}{2}} \frac{\cos t\,dt}{\sin t+\cos t} = \left(\frac{1}{2}t + \frac{1}{2}\ln|\sin t+\cos t|\right)\Big|_0^{\frac{\pi}{2}}$

$\boxed{= \frac{\pi}{4}}$

(11) $\int_{-1}^1 x^4\sqrt{1-x^2}\,dx \overset{x=\sin t}{=\!=} 2\int_0^{\frac{\pi}{2}} \sin^4 t \cos^2 t\,dt$

$$= 2\int_0^{\frac{\pi}{2}} \sin^4 t\,dt - 2\int_0^{\frac{\pi}{2}} \sin^6 t\,dt$$

$$= 2\cdot\frac{3\cdot 1}{4\cdot 2}\cdot\frac{\pi}{2} - 2\cdot\frac{5\,3\,1}{6\,4\,2}\cdot\frac{\pi}{2} = \boxed{\frac{\pi}{16}}$$

(13) $\int_{-1}^1 e^{|x|}\arctan e^x\,dx = \int_0^1 e^x \arctan e^x\,dx + \int_{-1}^0 e^{-x}\arctan e^x\,dx$

$$= \int_0^1 e^x \arctan e^x\,dx + \int_0^1 e^t \arctan\frac{1}{e^t}\,dt$$

$$= \frac{\pi}{2}\int_0^1 e^x\,dx = \boxed{\frac{\pi}{2}(e-1)} \approx 2.699$$

23. $\int_0^{\pi} x f(\sin x)\,dx = \int_0^{\frac{\pi}{2}} x f(\sin x)\,dx + \int_{\frac{\pi}{2}}^{\pi} x f(\sin x)\,dx$

对后一项，令 $t = \pi - x$，则 $= \int_{\frac{\pi}{2}}^{0} (\pi - t) f(\sin(\pi - t))\,d(\pi - t)$

$= \int_0^{\frac{\pi}{2}} (\pi - t) f(\sin t)\,dt$

$\Rightarrow \int_0^{\pi} x f(\sin x)\,dx = \pi \int_0^{\frac{\pi}{2}} x f(\sin x)\,dx$ □

$\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x}\,dx = \pi \int_0^{\frac{\pi}{2}} \frac{\sin x\,dx}{1+\cos^2 x} = -\pi \arctan(\cos x)\Big|_0^{\frac{\pi}{2}}$

$= \boxed{\frac{\pi^2}{4}}$

24. 由 $x^2 - \frac{x^6}{3!} < \sin x^2 < x^2$

$\Rightarrow \int_0^1 \sin x^2\,dx > \int_0^1 x^2 - \frac{x^6}{3!}\,dx = \frac{13}{42} > \frac{1}{6}$

$\int_0^1 \sin x^2\,dx < \int_0^1 x^2\,dx = \frac{1}{3}$

由 $\sin x^2 > \frac{2}{\pi} x^2 \Rightarrow \int_0^1 \sin x^2\,dx > \frac{2}{\pi}\int_0^1 x^2\,dx = \frac{2}{3\pi} > \frac{1}{6}$

26. 在 $[0, 100]$ 上，$\frac{1}{200} \le \frac{1}{x+100} \le \frac{1}{100}$，$e^{-x}$ 可积，非负，故

$\frac{1}{200}\int_0^{100} e^{-x}\,dx \le \int_0^{100} \frac{e^{-x}}{x+100}\,dx \le \frac{1}{100}\int_0^{100} e^{-x}\,dx$

$\Rightarrow \le \frac{1-e^{-100}}{200} \le \int_0^{100} \frac{e^{-x}}{x+100}\,dx \le \frac{1-e^{-100}}{100} \le \frac{1}{100}$

$\Rightarrow \frac{1}{202} \le I \le \frac{1}{100}$

26.12) $I=\int_0^{100}\frac{e^{-x}}{x+100}dx$

$$=-\frac{e^{-x}}{x+100}\Big|_0^{100}+\int_0^{100}\frac{e^{-x}}{-(x+100)^2}dx$$

$$=-\frac{e^{-x}}{x+100}\Big|_0^{100}+\frac{e^{-x}}{(x+100)^2}\Big|_0^{100}+\int_0^{100}\frac{2e^{-x}dx}{(x+100)^3}$$

取 $\left(\frac{e^{-x}}{(x+100)^2}-\frac{e^{-x}}{x+100}\right)\Big|_0^{100}$ 作为近似值，则误差

$$|R|=\left|\int_0^{100}\frac{2e^{-x}dx}{(x+100)^3}\right|\le\int_0^{100}\frac{2dx}{(x+100)^3}=\frac{1}{100^2}-\frac{1}{200^2}<\frac{1}{100^2}$$

即误差不超过 $\frac{1}{10000}$，精确到 0.0001

$\Rightarrow I\approx 0.0099$