Properties of Semiconductors – Characterization & Measurement

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The Hebrew University of Jerusalem, 3rd Year Physics Lab, January 2023

Semiconductors are used in a wide range of applications, from electronics (such as transistors, diodes, integrated circuits, and solar cells) to medical devices. New semiconductor-based technology has enabled the development of smaller and faster electronic devices, such as computers and smartphones over the past decades. Correspondingly, accurate characterization of the properties of semiconductors is required. In this study, we analyze the doping type and measure the charge-carrier density of a semiconductor using the Hall Effect. Consequently, we recreate the well-known Haynes-Shockley experiment in which we attempt to measure the drift velocity and diffusion coefficient of a semiconductor. The results of the experiments we conducted beautifully agree with the accepted theory – the measured charge-carriers' density and drift velocity correlate well with previously measured values, alongside the drift velocity of the charge-carriers and the diffusion coefficient of the semiconductor.

1. Introduction

Semiconductors are materials that have electrical conductivity between that of a conductor and an insulator. They are widely used in electronic devices, including transistors, diodes, and solar cells. The electrical properties of semiconductors can be greatly modified by introducing impurities, a process known as doping. This makes semiconductors a versatile and important material for electronic devices.

The Hall effect, discovered by physicist Edwin Hall in 1879, is a phenomenon that occurs in semiconductors when a magnetic field is applied perpendicular to an electric current flowing through the material. The Hall effect is an important tool for studying the electrical properties of semiconductors, including the concentration and movement of charge carriers, such as electrons and holes.

The Haynes-Shockley experiment, first performed in 1949 by Walter H. Haynes and William Shockley at Bell Labs, investigated the electrical properties of doped germanium and silicon. The results of the experiment were groundbreaking, showing that the electrical conductivity of a semiconductor could be drastically altered by introducing impurities, paving the way for the development of the modern semiconductor industry.

2. Theoretical Foundation

2.1 Types and Properties of Semiconductors

Semiconductors can be divided into two main categories: n-type and p-type. n-type semiconductors are created by introducing impurities such as phosphorus, arsenic, or antimony into the semiconductor material, creating an excess of free electrons in the material. p-type semiconductors are created by introducing impurities such as boron, aluminum, or gallium into the semiconductor material, creating a deficiency of free electrons in the material and filling this deficiency with electron vacancies called holes.

The energy band structure of a semiconductor is a fundamental property that determines its electronic properties and how it can be used in electronic devices. It is the way that the energy levels of the electrons in the material are arranged. In a semiconductor, the electrons are arranged in energy bands, where the lowest energy band is called the valence band and the next highest energy band is called the conduction band. The energy gap between these two bands is known as the bandgap. The bandgap is an important parameter in determining the electrical properties of the semiconductor, as well as its suitability for specific applications.

2.2 Effect of Light

When light is absorbed by a semiconductor, it can give energy to the electrons in the semiconductor, causing them to move from the valence band to the conduction band, creating an electron-hole pair. This process, known as photoexcitation, is the basis for many optoelectronic devices such as solar cells, photodetectors, and LEDs. The movement of these photoexcited electrons creates a current that can be used to generate electricity or to cause the semiconductor to emit light. Additionally, the absorption of light can also cause an increase in the conductivity of a semiconductor, known as photoconductivity.

2.3 Hall Effect

When a magnetic field is applied perpendicular to a current flowing through a semiconductor (B_z in Figure 3.1), the charge carriers experience a force that pushes them to one side of the material (towards $-\hat{y}$ in Figure 3.1).

This results in the buildup of a potential difference, known as the Hall voltage (denoted as V_{Hall}), across the semiconductor. Using Lorentz force in equilibrium, the electric field created by the Hall effect can be immediately derived:

(Eq. 2.1) $$\bar{E}_{Hall} = \frac{\bar{J}}{ne^-} \times \bar{B} = R_{Hall} \times \bar{B}$$
$$\left(with \;\; R_{Hall} \equiv \frac{1}{ne^-} \right)$$

Where $\bar{B}$ is the magnetic field applied, n the density of the charge carriers, e^- the charge of the electron and J the current density. Integrating E_{Hall} over the semiconductor dimensions produces the relation between V_{Hall} and the electric current I, as seen in (Eq. 2.2) - the magnitude of the Hall voltage is proportional to the strength of the magnetic field and the density of the charge carriers.

(Eq. 2.2) $$V_{Hall} = \frac{R_{Hall}}{t} IB$$

Where t, as seen in Figure 3.1, is the height of the semiconductor, and the charge-carrier-density is hidden inside the term R_{Hall}. As a result of this relation, given a known magnetic field B and current I, it is quite straightforward to measure the Hall voltage and deduce the charge carrier density of the semiconductor.

On top of that, the Hall effect can be used to determine the type of semiconductor - whether it is n-type or p-type. In n-type semiconductors, electrons are the majority charge carrier, so the Hall voltage will be positive, whereas in p-type semiconductors, holes are the majority charge carrier, and the Hall voltage will be negative. Additionally, the Hall effect can also be used to measure the mobility of charge carriers in a semiconductor, which is a measure of how easily they can move through the material in response to an applied electric field.

2.4 Haynes-Shockley

The Haynes-Shockley experiment (Figure 3.2) demonstrates that diffusion of charge carriers in a semiconductor can result in a current flowing through the semiconductor. In the experiment, a piece of semiconductor gets a pulse of holes, induced by a short laser pulse or voltage.

Consider a n-type semiconductor with length d. The equations for electron and hole currents (reduced to 1D) are:

(Eq. 2.3) $$j_n = \mu_n nE + D_n \frac{\partial n}{\partial x}$$

(Eq. 2.4) $$j_p = \mu_p pE - D_p \frac{\partial p}{\partial x}$$

Where j stands for the current density of electrons or holes, μ stands for the mobility of the charge carriers, E the applied electric field, n and p are the number densities of the charge carriers, D the diffusion coefficient, and x is the horizontal position. The first term of the equations is the drift current, and the second term is the diffusion current.

Now, considering the continuity equation[1], assuming a constant electric field E inside the semiconductor, and

[1] Continuity equation: $\frac{\partial n}{\partial t} = -\frac{n-n_0}{\tau_n} + \frac{\partial j_n}{\partial x} \quad \wedge \quad \frac{\partial p}{\partial t} = -\frac{p-p_0}{\tau_p} - \frac{\partial j_p}{\partial x}$

using the Einstein relation[2] will output the final equation for the charge carriers:

$$n(x,t) = \frac{A}{\sqrt{4\pi Dt}} e^{-\frac{t}{\tau}} e^{-\frac{(x+\mu Et - x_0)^2}{4Dt}} \quad \text{(Eq. 2.5)}$$

Where D is the diffusion coefficient, τ a time constant of the system, μ the mobility of charge-carriers, t the time, x_0 the initial coordinate of the pulse of electrons. This equation practically describes a Dirac δ function immediately after the pulse, which then takes the shape of a gaussian curve. Parameters μ, D, τ can be obtained from the shape of the signal. The diffusion coefficient can be obtained from:

$$D = (\mu E)^2 \frac{(\delta t)^2}{16 \ln(2) t_0} \quad \text{(Eq. 2.6)}$$

Where δt is the gaussian pulse's width.

Furthermore, plotting τ vs distance between the pulse and the current measurement probe (denoted by d in Figure 3.2) and extracting the slope of that graph, will grant us the drift velocity of the charge carriers, as:

$$v_{drift} = \frac{d}{\tau} \quad \text{(Eq. 2.7)}$$

3. Experiment Setup

3.1 Hall Effect

In the first part of the experiment, we tried to measure the density of the charge carriers in the semiconductor, in the context of the Hall effect (see Figure 3.1).

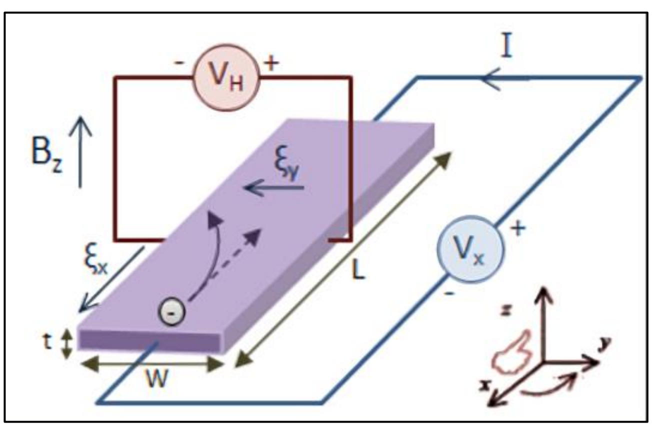

Figure 3.1 – A sketch of the Hall Effect experimental system.

[2] Einstein relation: $\mu = \frac{eD}{k_B T}$

We conducted 3 separate measurements of the Hall voltage within the mentioned setup:

- Changing the electric current (I in Figure 3.1) in the range 0.02-4.5 [mA], while keeping the magnetic field (B_z in Figure 3.1) at a constant 1002 [G].
- Changing the electric current in the range 0.02-5.64 [mA], while keeping the magnetic field at a constant 102 [G].
- Changing the magnetic field B in the range 499-3000 [G], while keeping the electric current I at a constant 4.91 [mA].

Plotting the results and extracting the slope allows us to evaluate the relation given by Eq. 2.2, and to calculate the density of charge carriers in the semiconductor, using the definition of R_{Hall} in Eq. 2.1.

3.2 Haynes-Shockley

In the second part of the experiment, we conduct the classical Haynes-Shockley experiment, using a light source to induce the electron pulse; We placed a p-doped Germanium semiconductor under the influence of an electric field $\overrightarrow{E_s} = E\hat{x}$, using two points of contact of distance d apart. We injected the semiconductor with excess carriers using a rectangular pulse via a laser. We then measured the signal inside the semiconductor using an oscilloscope. A sketch of the setup can be seen below:

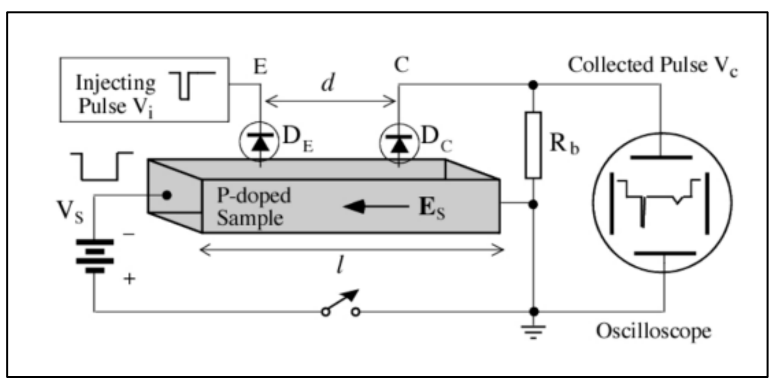

Figure 3.2 – Sketch of the Haynes-Shockley experimental system. The contact points D_E and D_C were used to insert the excess carriers, while the other two contact points were used to generate the field E_S.

In our system, the injecting pulse V_i was replaced by a high frequency light source, which due to the properties of semiconductors discussed in chapter 2, cause the desired pulse of excess electrons.

For distances d ranging from 0.2-7 [mm], we measured the voltage over time, which logically equals the spatial distribution of the pulse of charge carriers, caused by the injected pulse.

From the measured spatial distribution of the pulse, we extracted the time constant τ using numerical methods, and plotted τ vs d – a graph whose slope, according to Eq. 2.7, gives us the drift velocity of the charge carriers.

Additionally, we numerically extracted the diffusion coefficient D from the measured data, using the relation in Eq. 2.6.

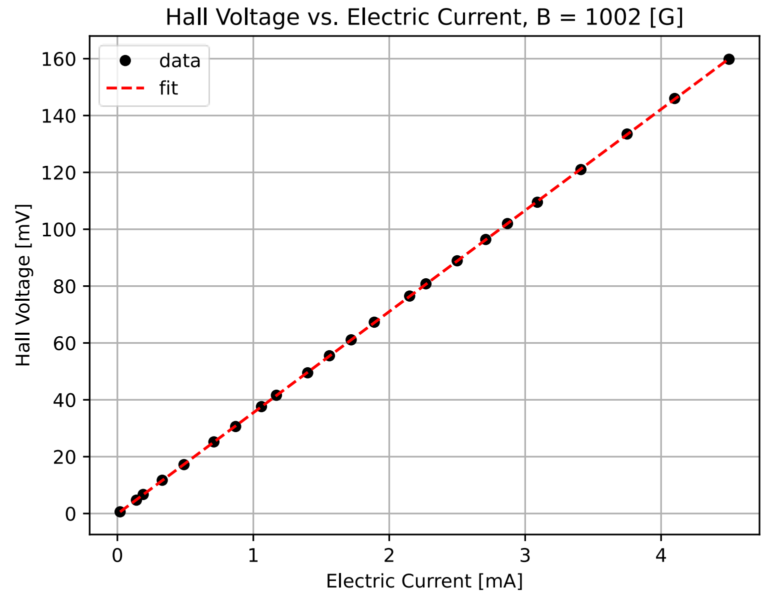

Figure (4.1): Hall voltage V_{Hall} vs the electric current I, at a constant magnetic field $B = 1002[G]$. It is clear, and numerically proven ($R^2 = 1$), that indeed the relation is linear.

4. Results

4.1 Hall Effect Experiment

The desired linear relation predicted by Eq. 2.2 was confirmed in both scenarios – changing current and changing magnetic field, as can be seen in Figures 4.1 and 4.2.

However, there appears to be a slight curve in the residuals of the linear fit to the changing magnetic field experiment. We will address issues regarding this in the discussion.

It is important to note that the errors of measurement were so small, that for the sake of a clean graph they are ignored, but later added on for precise evaluation of the extracted coefficients[3] vs their theoretical values.

A calculation of the slope of these linear graphs, which by Eq. 2.2 should yield $\frac{R_{Hall}}{t}$, gave us the values presented in Figure 4.5. From these values we went on to calculate n, e.g., the charge carrier density of the Germanium semiconductor, based on Eq. 2.1. The results can be seen in Figure 4.6.

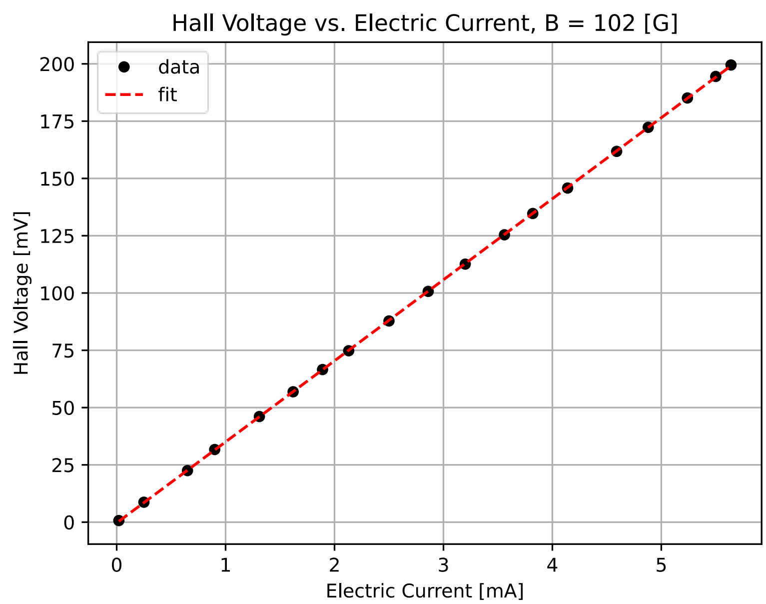

Figure (4.2): Hall voltage V_{Hall} vs the electric current I, at a constant magnetic field $B = 102[G]$. It is clear, and numerically proven ($R^2 = 1$), that indeed the relation is linear.

[3] For our complete error assessment, see Appendices.

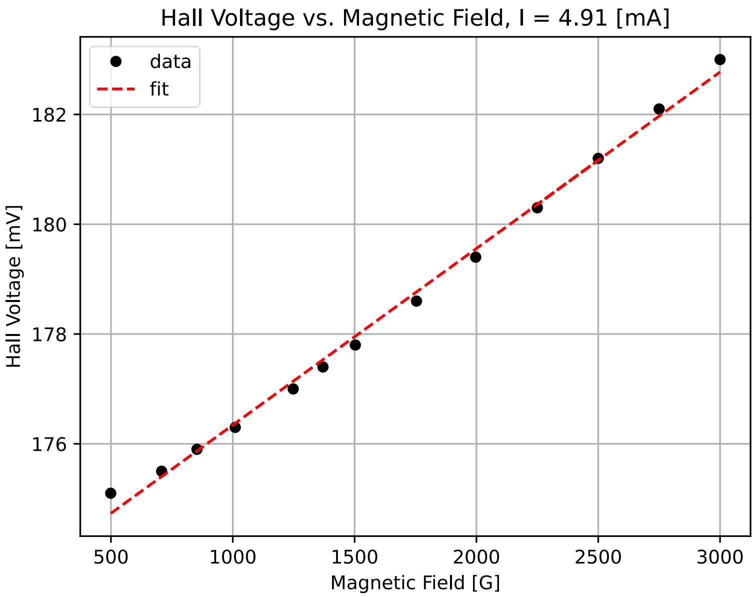

Figure (4.3): Hall voltage V_{Hall} vs the magnetic field B, at a constant electric field I. The graph seems linear, with high statistical confidence ($R^2 = 0.91$).

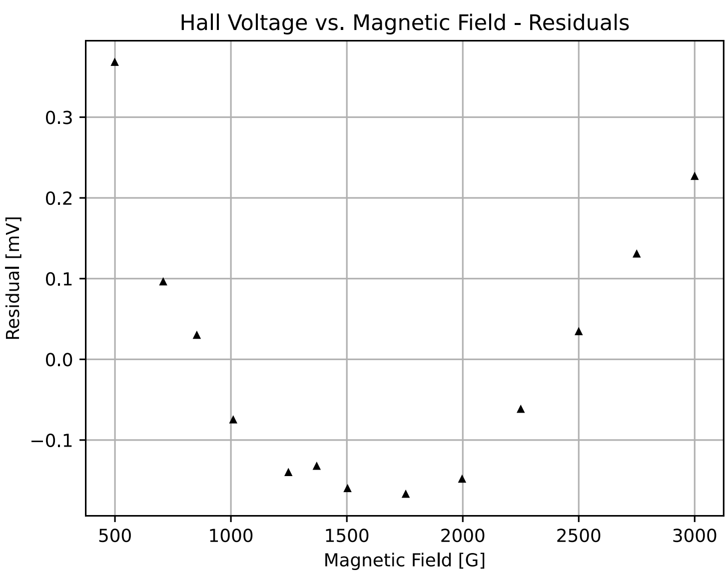

Figure (4.4): Residuals of the graph presented in Figure 4.3. Although their magnitude is almost negligible, the residuals are not random. This issue will be addressed in the discussion.

$\boldsymbol{Experiment}$	$\boldsymbol{Measured\ R_{Hall}/t}$ $\left[{}^{m^3}/_{C}\right]$
$B = 1002\ [G]$	24.5 ± 0.6
$B = 102\ [G]$	23.7 ± 0.7
$I = 4.91\ [mA]$	23.4 ± 0.6

Figure (4.5): A table presenting the measured $\frac{R_{Hall}}{t}$ coefficients in all 3 experiments, which were obtained from the slopes of Figures 4.1 and 4.2 and 4.3.

The statistical fit between the results of the 3 measurements is good, as the average N_σ[4] between any 2 measurements of the 3 stands on:

$$\overline{N_\sigma} = 0.91$$

$\boldsymbol{Experiment}$	$\boldsymbol{n_{experiment}}\ [cm^{-3}]$	$\boldsymbol{n_{theory}}[cm^{-3}]$
$B = 1002\ [G]$	$7.5 \pm 0.7 \cdot 10^{13}$	$2.33 \cdot 10^{13}$ (without doping)
$B = 102\ [G]$	$7.9 \pm 0.6 \cdot 10^{13}$	
$I = 4.91\ [mA]$	$7.1 \pm 0.8 \cdot 10^{13}$	

Figure (4.6): A table presenting the measured charge-carrier density in all 3 experiments, alongside the theoretical value for the charge-carrier-density of Germanium that was not subjected to doping.

It is important to note that the calculation is done assuming $t \cong 1 \pm 0.5$ [mm], which is an educated guess, as it was impractical to measure it in our experimental setup.

It can be clearly seen that the measured values are not far from the theoretical value (a factor of $\simeq 3$). However, let us not forget that the semiconductor in our experiment was doped, and it makes sense that its charge carrier density after doping would increase at a factor of 3.

From the positive sign of the R_{Hall} constant, we conclude that the semiconductor used in this part of the experiment is n-doped.

[4] $N_\sigma \equiv \frac{v_d^{measured} - v_d^{theoretical}}{\sqrt{\Delta v_d^{measured^2} + \Delta v_d^{theoretical^2}}}$

4.2 Haynes-Shockley Experiment

The following graphs were produced using the oscilloscope for the voltage between the semiconductor`s edges in the time domain:

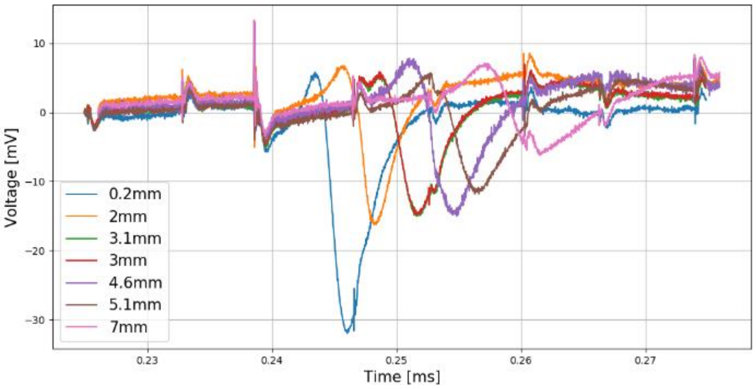

Figure (4.7): Time domain plot of the signal across the semiconductor. The different colored plots represent different measurements with changed distance d between the contact points D_E and D_C.

The measurements were more variant than expected, but we managed to get a clear view of the moment the pulse was generated (see peak close to $t = 0.24\ [ms]$ in Figure 4.7) and the resulting gaussian response within the semiconductor (see lowest peaks in Figure 4.7). Using this we constructed a graph for the distance between the electron pulse and the resulting gaussian, as a function of the distance d:

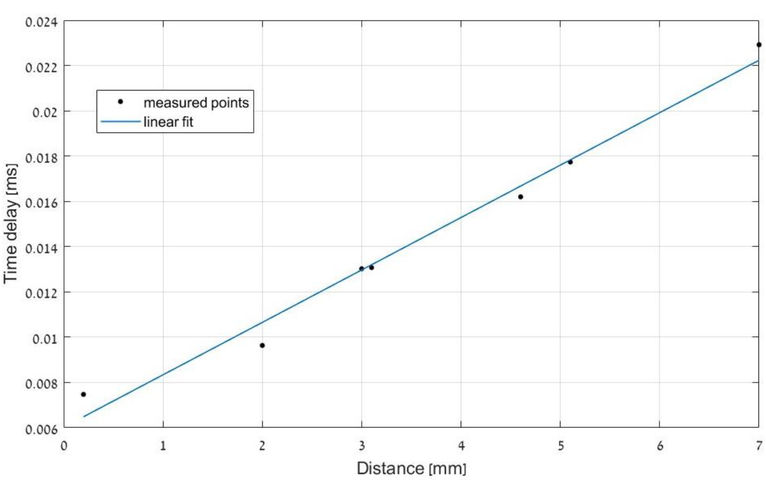

Figure (4.8): plot of the Time delay Δt between the pulse and the corresponding signal change inside the semiconductor as a function of the distance d. The blue plot is our theoretical fit according to Eq. 2.7. It is noticeable that the fit is statistically strong ($R^2 = 0.983$). Errors in both axes are visually negligible.

From the slope of the graph, we can calculate the drift velocity of the charge-carriers:

$$slope = 0.0023 \pm 0.0002 \left[\frac{s}{m}\right]$$

$$v_d{}^{measured} = (slope)^{-1} = 431 \pm 9 \left[\frac{m}{s}\right]$$

This result is close, with a very strong statistical correlation (of $N_\sigma = 0.62$), to a result from a similar experiment conducted in the University of Padova [2] which yielded a value of[5]:

$$v_d^{theoretical} \approx 423 \pm 9 \left[\frac{m}{s}\right]$$

To extract the diffusion coefficient D, we fit each of the plots in Figure 4.7 to their theoretical fit according to Eq. 2.5 and used the fitted graphs to extract the full width at half maximum (FWHM or δt as depicted in our equations) and their errors. According to Eq. 2.6, we expect a linear relation between the δt^2 and the time delay τ.

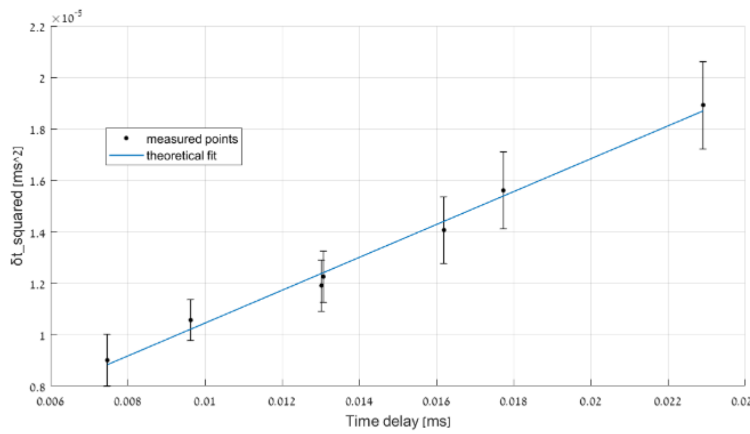

Figure (4.9): FWHM squared of the gaussian peaks as a function of their distance from the initial pulse τ. Each point in the graph was calculated using a numerical fit as to Eq. 2.5. The errors were calculated according to the accuracy of the numerical fit. The fit is statistically significant ($R^2 = 0.98$).

Using the slope of this graph, and using the value that we have extracted for v_d, we get a value for the diffusion coefficient:

$$slope = 0.63 \pm 0.04 [\mu s]$$

[5] The value was extracted from their results, using the relation $v_d^{theoretical} = \frac{\mu V_s}{L}$

$$D_{measured} = 107 \pm 4 \left[\frac{m}{s}\right]$$

The diffusion coefficient measured in the Padova experiment [2] yielded a value of:

$$D_{theoretical} = 101 \pm 7 \left[\frac{m}{s}\right]$$

And upon comparing these values we get:

$$N_\sigma = 0.74$$

Which means both values are within less than only a single standard deviation away, which is an excellent correlation.

5. Discussion

Overall, our experiment yielded good results; In the 1st part, we utilized the Hall Effect and measured the density of charge-carriers in a semiconductor. The result we obtained is in very good correlation with the predicted theoretical value. In the 2nd part of the experiment, we measured the drift velocity of the charge-carriers and diffusion coefficient in the Haynes-Shockley setup, and our results stand within a single standard-deviation from results of other studies of the same setup [2].

However, upon plotting the Hall voltage vs a changing magnetic field, we observed an unusual residual plot, that clearly does not appear to be random. We assume that this non-linear relation is due to impurities in the semiconductor, such as a non-homogenous distribution of charge-carriers, or multiple types of charge carriers. Digging deeper into this relation can be useful for future experiments. On top of the that, for further experimenting with the Hall effect, we advise to carefully measure the dimensions of the semiconductor and use different types of semiconductors (p-doped) to observe the differences in behavior. And finally, we suggest completing the picture with analyzing the effect of doping on the charge-carrier density (as in our result we discovered a charge-carrier density 3 times higher than the density of undoped material).

6. Appendices

Vertical system

	All Models
Input coupling	DC, AC (10 Hz cutoff frequency)
Input impedance/capacitance	1 MΩ ± 2%, 16 pF ±3 pF
Input sensitivity range[4]	500 µV/div to 10 V/div
Standard probes	N2142A 1/10 switchable 75 MHz (2 included in EDUX1052A/EDUX1052G) N2140A 1/10 switchable 200 MHz (2 included in DSOX1202A/DSOX1202G) N2140A 1/10 switchable 200 MHz (4 included in DSOX1204A/DSOX1204G)
Probe attenuation factor	0.1X to 10,000X in 1-2-5 sequence; (–20 dB to +80 dB in 0.1 dB steps)
Hardware bandwidth limits	Approximately 20 MHz (selectable)
Vertical resolution	8 bits
Invert signal	Selectable
Maximum input voltage	150 Vrms, 200 Vpk
DC vertical accuracy	± [DC vertical gain accuracy + DC vertical offset accuracy + 0.25% full scale]
DC vertical gain accuracy [1]	+3% full scale (≥ 10 mV/div)
	+4% full scale (< 10 mV/div)
DC vertical offset accuracy	± 0.1 div ± 2 mV ± 1% of offset setting
Skew	Channel to channel: 1 ns (without deskew)
	Channel to external: 2 ns (without deskew)
Offset range	500 uV/div to 200 mV/div: +2 V
	> 200 mV/div to 10 V/div: +100 V

Figure (5.1): Data regarding the readings of our oscilloscope, and their accuracy, was used to calculate errors in some parts of the experiment.

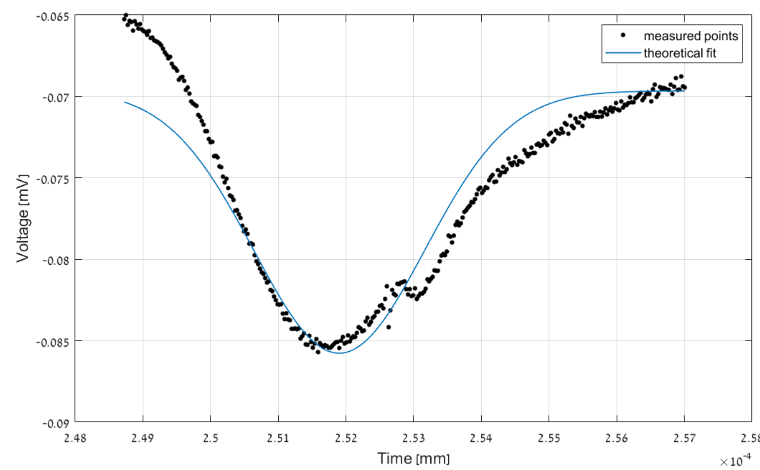

Figure (5.2): Example of data fitting to the gaussian curves according to Eq. 2.5, for $d = 3[\mathrm{mm}]$*, which was used to calculate the FWHM. A graph like this was extracted for each of the curves seen in Figure 4.7.*

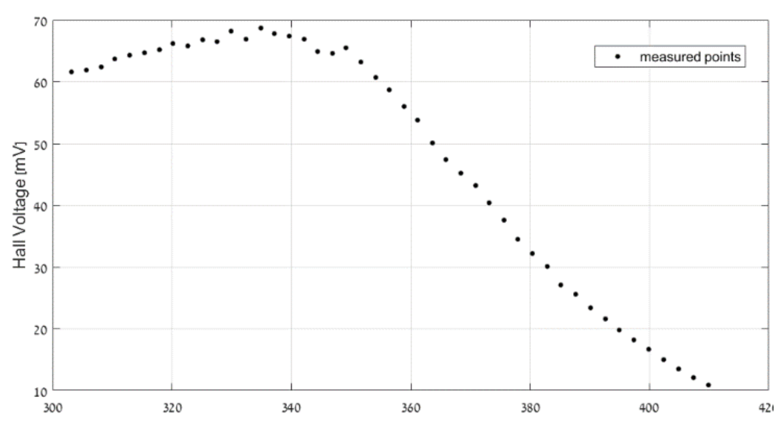

Figure (5.3): Temperature dependance of the Hall voltage. The temperature was measured in volts (via a voltmeter) and converted to kelvin using a conversion chart. We can see that after a certain temperature, the hall voltage starts to plummet. This might be a cause of thermal agitation in the semiconductor, which *leads to increased scattering of the carriers and reduces the difference in charge between the two sides of the conductor.*

7. Bibliography

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