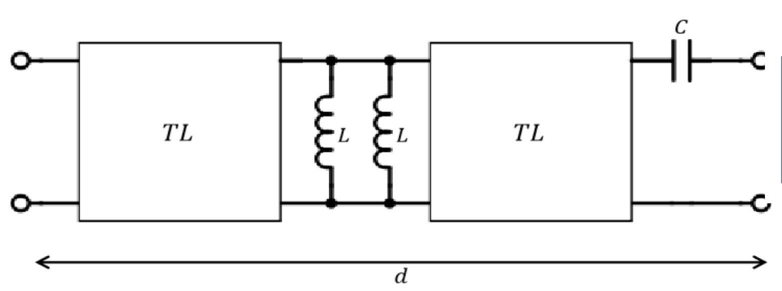
Comparing topological excitations and defect induced local states in CRLH-TL metamaterials

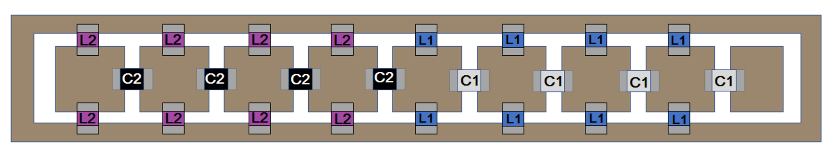
Abstract

Metamaterials are materials constructed of multiple complex elements giving them properties arising from their structural characteristics. One such type is CRLH-TL metamaterial which provides the ability to map Maxwell's equations to Dirac's equation.

We compare between topological excitation in 2 mass metamaterial system and local stated induced by a defect in a 1 mass metamaterial system, showing their resemblance.

Metamaterial Transmission Line





Our metamaterial is built from 8 cells (with length d) , each assembled from a capacitor (C) and a transmission line (TL) divided into two by two inductors (L).

Properties

Our metamaterials have effective permittivity and permeability given by:

$$\epsilon_r = \frac{1}{p\epsilon_0}\left(C - \frac{1}{\omega^2 L_s d}\right) \qquad \omega_1 = \frac{1}{\sqrt{L_s C d}}$$

$$\mu_r = \frac{p}{\mu_0}\left(L - \frac{1}{\omega^2 C_s d}\right) \qquad \omega_2 = \frac{1}{\sqrt{L C_s d}}$$

$$\begin{cases} R\ chirality & \omega < \omega_1, \omega_2 \\ L\ chirality & \omega > \omega_1, \omega_2 \\ Band\ gap & else \end{cases}$$

From Maxwell's Eq. to Dirac Eq.

By defining:

$$\phi = \begin{pmatrix} \sqrt{\epsilon_0} E_z \\ \sqrt{\mu_0} H_y \end{pmatrix} \qquad V(x) = \frac{\omega}{2c}\left[\epsilon_r(x) + \mu_r(x) - \langle \epsilon_r(x) + \mu_r(x) \rangle \right]$$

$$E = -\frac{\omega}{2c}\langle \epsilon_r(x) + \mu_r(x) \rangle \qquad m(x) = \frac{\omega}{2c}\left[\epsilon_r(x) - \mu_r(x)\right]$$

One can map the Maxwell's eq. to Dira eq.

$$\begin{aligned} -\partial_x E_z &= i\omega\mu_0\mu_r(x)H_y \\ \partial_x H_y &= -i\omega\epsilon_0\epsilon_r(x)E_z \end{aligned} \iff \left[-i\sigma_x\partial_x + m(x)\sigma_z + V(x)\right]\phi = E\phi$$

It can be seen that $\omega_1 > \omega_2$ correlates to $m < 0$ and $\omega_1 < \omega_2$ correlates to $m > 0$.

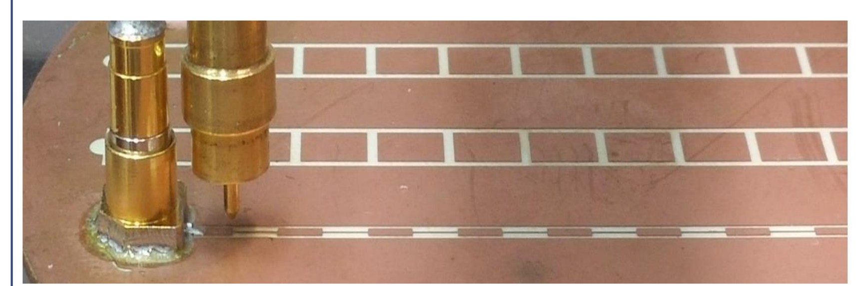
When connecting two metamaterials with opposite masses the local solution is the Jackiw-Rebbi soliton.

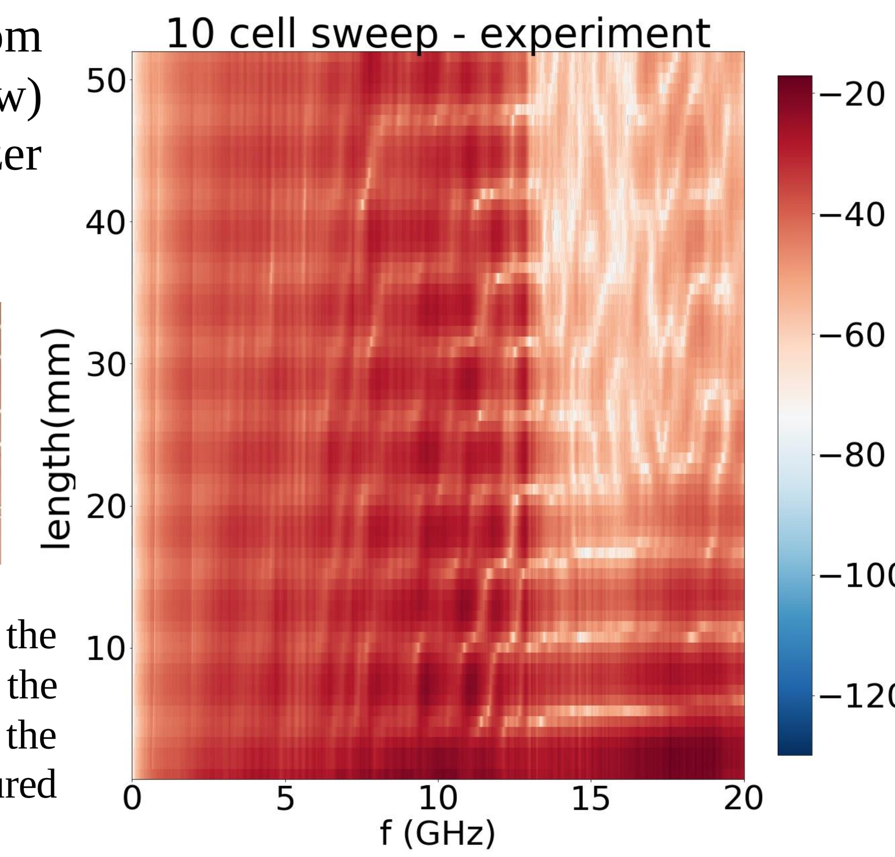
$x = 0$ is the transition point between the two metamaterials.

$$\phi(x) = \sqrt{\frac{m_{left}\, m_{right}}{m_{left} + m_{right}}} \begin{pmatrix} 1 \\ -i \end{pmatrix} e^{-|m(x) \cdot x|}$$

Data Acquisition

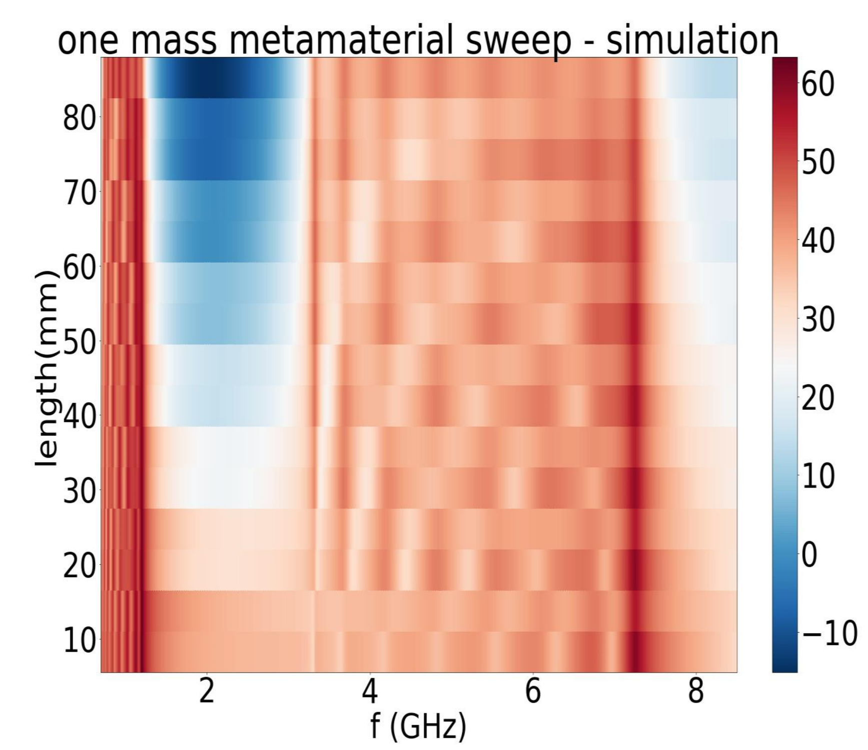
- Method – we scanned the system from above using a probe (shown below) connected to a vector network analyzer (VNA).

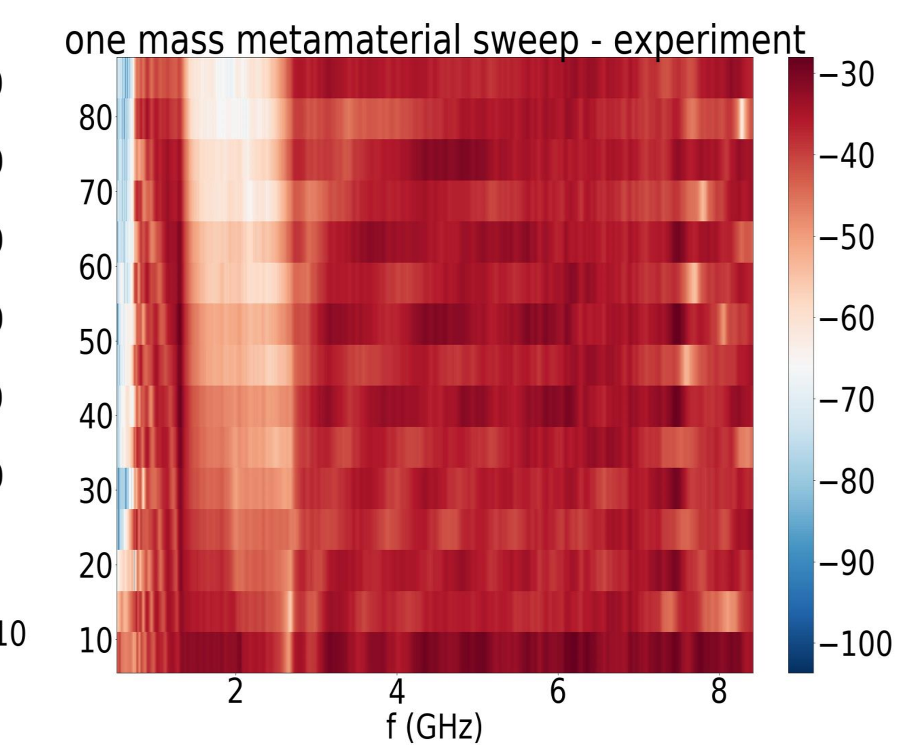




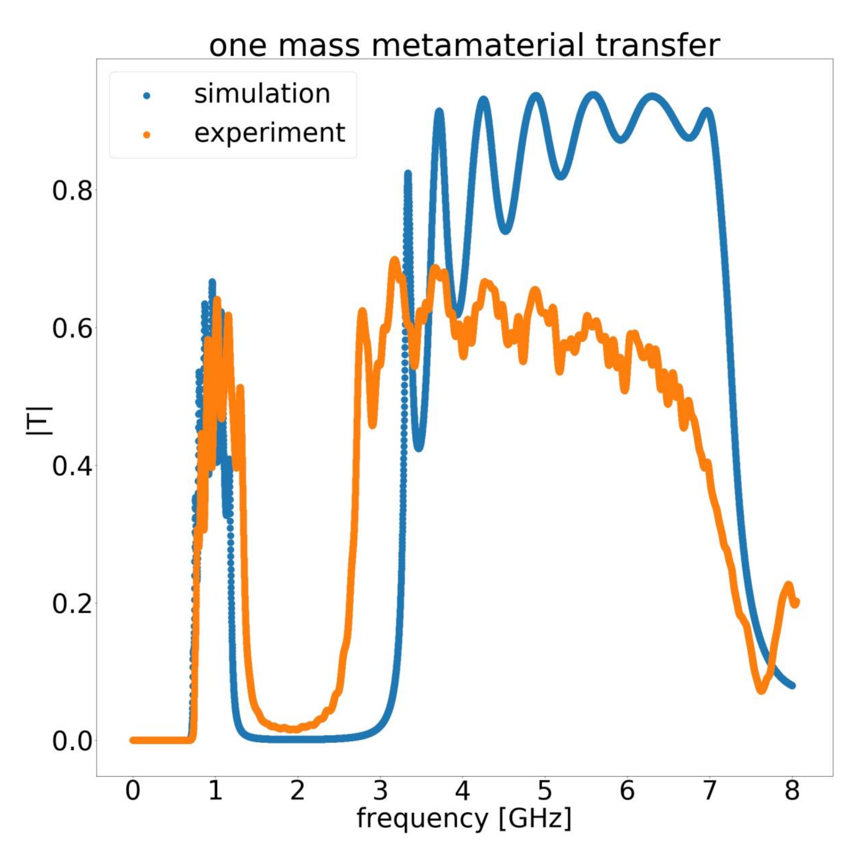
The figure to the right, is raw measurement of the system in the picture above. The graph describes the measured voltage (in log scale) as a function of the probe's location over the system for all measured frequencies.

One Mass Metamaterial



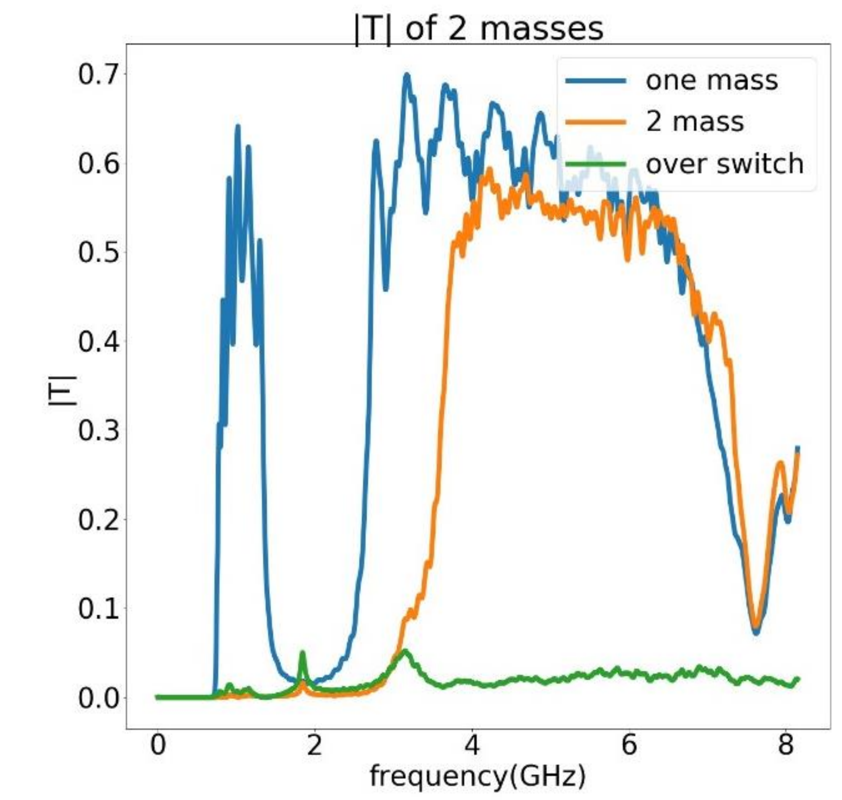


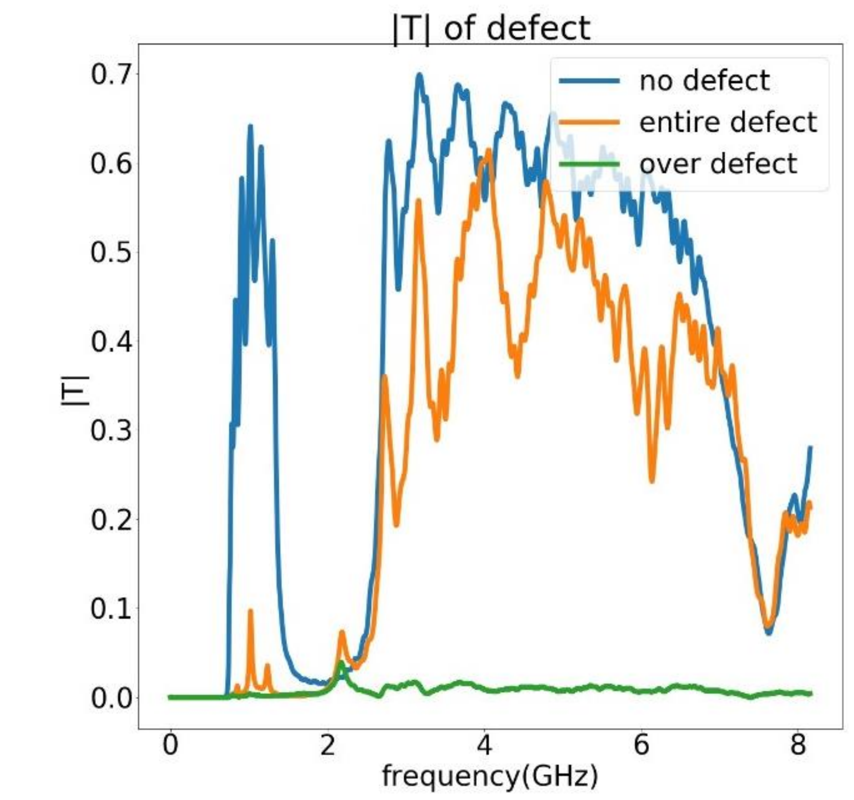
Voltage as a function of length and frequency of one mass metamaterial system, $C = 1pF$, $L = 15nH$. A band gap can be seen centered around 2GHz frequency in both simulation (right) and experiment (left).

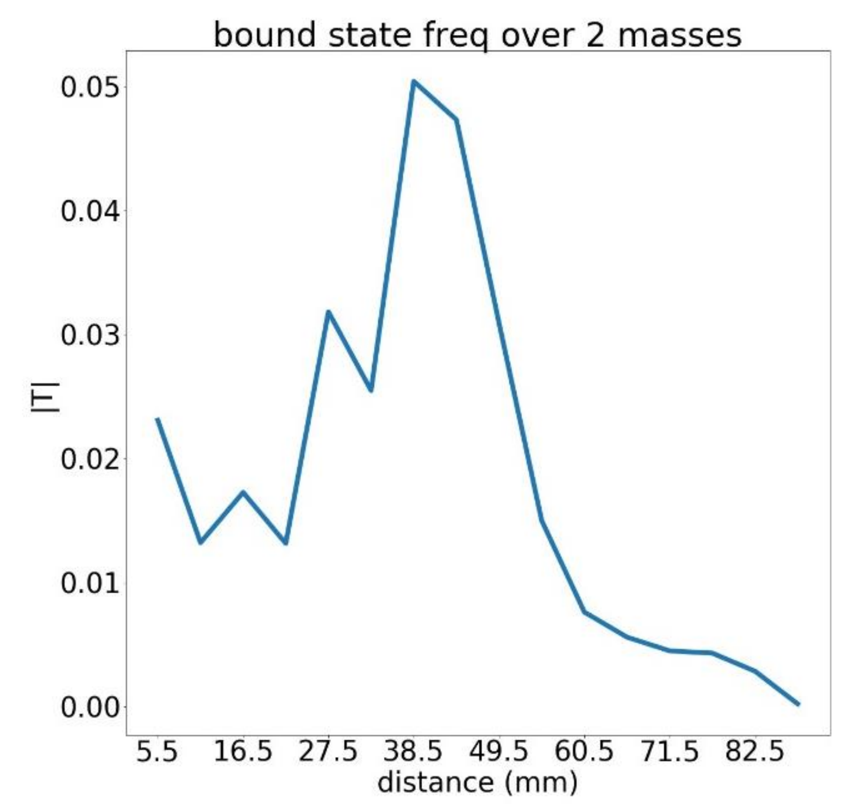


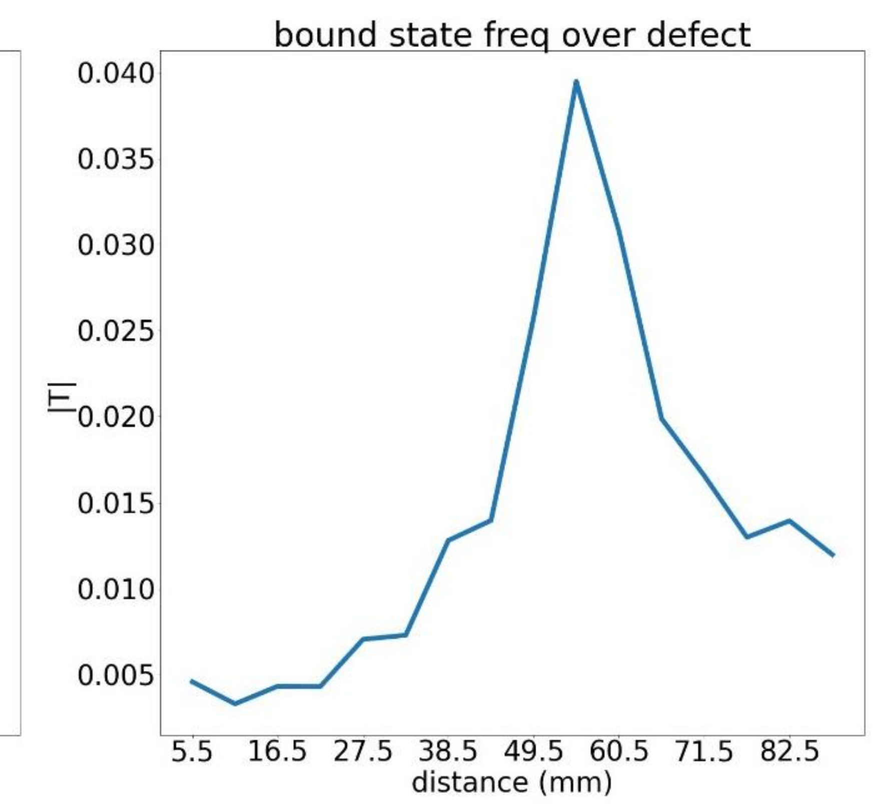
Transmission at the end of line as a function of frequency. Simulation and experiment.

Two Masses Vs. Defect









Two masses metamaterial system –

first mass: $C1 = 1pF,\ L1 = 15nH$

second mass: $C2 = 43pF,\ \ L2 = 2nH$

One mass metamaterial with defect at cell no. 5 system –

mass: $C = 1pF,\ L = 15nH$

Defect: $C = 1pF,\ \ L = 2nH$.

Conclusion

- We showed that the theory of CRLH metamaterials agrees with measurements.
- We have shown that a bound state can be created by introducing a defect (impurity).
- We found that differentiating between topological excitation and impurity induced local state is hard.

Acknowledgments

We thank Tom Dvir for the guidance and Prof. Nadav Katz for assisting with the experiment and supplying with facilities and infrastructure.