

۱- هر کدام از دستگاه معادلات زیر را با استفاده از سگوار حل کنید.

$$\text{الف) } \begin{cases} -x + y + 2z = 7 \\ -y + z = 1 \\ x + y = -1 \end{cases}$$

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ -1 \end{bmatrix}$$

$$x_1 = \frac{\det \begin{bmatrix} 7 & 1 & 2 \\ 1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}}{y} \rightarrow x_1 = \frac{0}{y}$$

$$\det A = -1 \times \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + 1 \times \begin{vmatrix} 7 & 1 \\ -1 & -1 \end{vmatrix} + 2 \times \begin{vmatrix} 7 & 1 \\ 1 & 1 \end{vmatrix}$$

$$\det A = 9$$

$$x_2 = \frac{\det \begin{bmatrix} -1 & 7 & 2 \\ 0 & 1 & 1 \\ 1 & -1 & 0 \end{bmatrix}}{y} = x_2 = \frac{-5}{y}$$

$$\det A_1 = -1 \times \begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} + (-1) \times \begin{vmatrix} 7 & 1 \\ -1 & -1 \end{vmatrix}$$

$$\det A_1 = -1 \times (-1) - 1 \times (-6) = 7$$

$$x_3 = \frac{\det \begin{bmatrix} -1 & 1 & 7 \\ 0 & -1 & 1 \\ 1 & 1 & -1 \end{bmatrix}}{y} = x_3 = \frac{1}{y}$$

$$\det A_2 = -1 \times \begin{vmatrix} 7 & 1 \\ -1 & -1 \end{vmatrix} + 1 \times \begin{vmatrix} -1 & 1 \\ 1 & 0 \end{vmatrix}$$

$$\det A_2 = -1 \times (-6) + 1 \times (-1) = 5$$

$$\det A_3 = -1 \times \begin{vmatrix} 7 & 1 \\ 1 & -1 \end{vmatrix} + 1 \times \begin{vmatrix} -1 & 1 \\ 0 & 1 \end{vmatrix}$$

$$\det A_3 = -1 \times (-6) + 1 \times (-1) = 5$$

$$\text{ب) } \begin{cases} x_1 - x_2 + x_3 = 7 \\ x_2 + 2x_3 + x_4 = 1 \\ x_1 + x_2 + x_3 = -2 \\ x_1 + x_2 + x_3 + x_4 = 0 \end{cases} \rightarrow A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 1 \\ -2 \\ 0 \end{bmatrix}$$

$$\det A = \det \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ -1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

برای سطر اول به سطر دوم اضافه می‌کنیم

$$\det \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$\det A = -1 \times \begin{vmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{vmatrix} \rightarrow -1 \times (1 \times \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 1 \times \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix}) = -1 \times (-1 - 1) = 2$$

$$x_1 = \frac{\det A_1}{\det A}$$

$$x_2 = \frac{\det A_2}{\det A}$$

$$x_3 = \frac{\det A_3}{\det A}$$

$$x_4 = \frac{\det A_4}{\det A}$$

$$\det \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = 1 \times \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} - 1 \times \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} + 1 \times \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$1 \times (-1 - 1) - 1 \times (-1 - 1) + 1 \times (-1 - 1) = -2 + 2 - 2 = -2$$

۳- دترمینان هر یک از ماتریس های زیر را بیابید.

الف)  $\det A \cdot \det B = \det \begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \det A \cdot \det B \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix} \rightarrow \det B = 2+1=3$

$\det A \cdot \det B = 6 \quad \det A = 2-2=0$

$\det A = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{vmatrix} \rightarrow \det A = 1 \cdot \begin{vmatrix} 1 & 0 \\ 1 & 2 \end{vmatrix} + 1 \cdot \begin{vmatrix} 0 & 0 \\ 1 & 2 \end{vmatrix} + 2 \cdot \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix}$

ب)  $\begin{vmatrix} 1 & 2 & 0 & 2 & 0 \\ -1 & 2 & 1 & 2 & 0 \\ 0 & 0 & 2 & -1 & -1 \\ 0 & 0 & -1 & 0 & 2 \\ 0 & 0 & 2 & 0 & 1 \end{vmatrix}$

$\det A \cdot \det B = 1 \times 2 = 2$

$\det B = 2 \times \begin{vmatrix} 0 & 2 \\ -1 & 1 \end{vmatrix} - 1 \times \begin{vmatrix} -1 & 2 \\ 2 & 4 \end{vmatrix} + 1 \times \begin{vmatrix} -1 & 0 \\ 2 & 0 \end{vmatrix} \quad \det B = 1$

$\det A = \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} = 2-2=0$

ج)  $M = \begin{bmatrix} A & X & Y \\ 0 & B & 0 \\ 0 & Z & C \end{bmatrix}$

(ماتریس های A, B, C مربع)

$\det = A \begin{vmatrix} B & 0 \\ Z & C \end{vmatrix} - 0 \begin{vmatrix} X & Y \\ Z & 0 \end{vmatrix} - 0 \begin{vmatrix} X & Y \\ B & 0 \end{vmatrix} \rightarrow \det = A \begin{vmatrix} B & 0 \\ Z & C \end{vmatrix}$

$\det(M) = A \cdot \det B \cdot \det C$

$\det \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = (b-a)(c-a)(c-b)$

د- نشان دهید

$(bc-ab-ac+a^2)(c-b) = (a^2-a(b+c)+bc)(c-b) = c a^2 - b a^2 - a c^2 + a b c + b c^2 - b^2 c$

$\det \begin{bmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{bmatrix} = 1 \begin{vmatrix} b & b^2 \\ c & c^2 \end{vmatrix} - 1 \begin{vmatrix} a & a^2 \\ c & c^2 \end{vmatrix} + 1 \begin{vmatrix} a & a^2 \\ b & b^2 \end{vmatrix} = b c^2 - c b^2 - a c^2 + a b c + a b^2 - b^2 c = \det M$