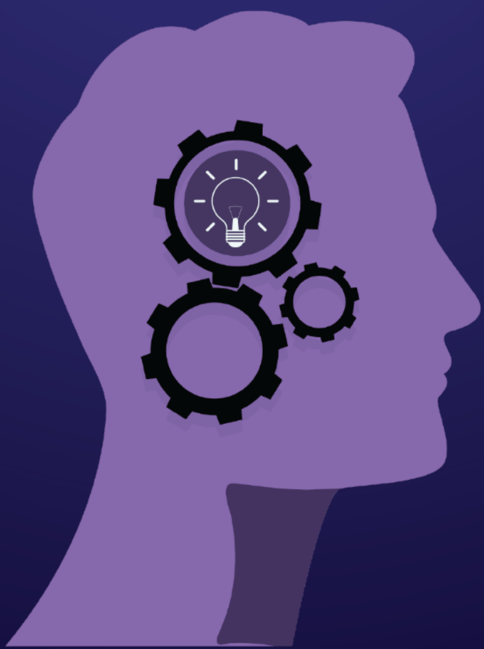
IS 2nd material

Information theory



Lecture 03

Sec [0] = Lecture questions;

- **Example 2.2.1** Let (X, Y) have the following joint distribution:

X

Y	1	2	3	4
1	1/8	1/16	1/32	1/32
2	1/16	1/8	1/32	1/32
3	1/16	1/16	1/16	1/16
4	1/4	0	0	0

Calculate $H(X), H(Y), H(X|Y), H(Y|X), H(X,Y)$

- **Solution**

X

Y

	1	2	3	4	$p(Y)$
1	1/8	1/16	1/32	1/32	1/4
2	1/16	1/8	1/32	1/32	1/4
3	1/16	1/16	1/16	1/16	1/4
4	1/4	0	0	0	1/4
$p(X)$	1/2	1/4	1/8	1/8	1

$$H(X) = \frac{1}{2}\log 2 + \frac{1}{4}\log 4 + \frac{1}{8}\log 8 + \frac{1}{8}\log 8 = \frac{7}{4} \text{ bits}$$

$$H(Y) = 4 \times \left(\frac{1}{4}\log 4\right) = 2 \text{ bits}$$

X

Y

	1	2	3	4
1	1/8	1/16	1/32	1/32
2	1/16	1/8	1/32	1/32
3	1/16	1/16	1/16	1/16
4	1/4	0	0	0

$$H(X,Y) = \left[\frac{1}{8}\log 8 + \frac{1}{16}\log 16 + \frac{1}{32}\log 32 + \frac{1}{32}\log 32\right]$$
$$+ \left[\frac{1}{16}\log 16 + \frac{1}{8}\log 8 + \frac{1}{32}\log 32 + \frac{1}{32}\log 32\right]$$
$$+ 4 \times \left[\frac{1}{16}\log 16\right] + \left[\frac{1}{4}\log 4\right] = \frac{27}{8} \text{ bits}$$
$$H(X|Y) = H(X,Y) - H(Y) = \frac{27}{8} - 2 = \frac{11}{8} \, bits$$
$$H(Y|X) = H(X,Y) - H(X) = \frac{27}{8} - \frac{7}{4} = \frac{13}{8} \, bits$$

Sec [3] = section questions;

$$P(x,y) = \begin{bmatrix} 0.2 & 0.1 & 0.3 \\ 0.01 & 0.03 & 0.05 \end{bmatrix}$$

Calculate H(x),H(Y),H(X,Y),H(X|Y),H(Y|X)

1 For discerete memory channel the Joint probability is tabulated as:

$$P(x,y) = \begin{array}{c|ccc|c} & & y & & P(x) \\ \hline x & 0.2 & 0.1 & 0.3 & 0.6 \\ & 0.01 & 0.03 & 0.05 & 0.09 \\ \hline P(y) & 0.21 & 0.13 & 0.35 & \end{array}$$

$$H(x) = 0.6 * \log_2\left(\frac{1}{0.6}\right) + 0.09 * \log_2\left(\frac{1}{0.09}\right) = 0.755$$

$$H(y) = 0.21 * \log_2\left(\frac{1}{0.21}\right) + 0.13 * \log_2\left(\frac{1}{0.13}\right) + 0.35 * \log_2\left(\frac{1}{0.35}\right) = 1.385$$

$$H(x,y) = 0.2 * \log_2\left(\frac{1}{0.2}\right) + 0.1 * \log_2\left(\frac{1}{0.1}\right) + 0.3 * \log_2\left(\frac{1}{0.3}\right) + 0.01 * \log_2\left(\frac{1}{0.01}\right)$$
$$+ 0.03 * \log_2\left(\frac{1}{0.03}\right) + 0.05 * \log_2\left(\frac{1}{0.05}\right) = 1.752$$

$$H(x|y) = \sum P(x,y) * \log_2 \frac{P(y)}{P(x,y)}$$

$$= 0.2 * \log_2 \frac{0.21}{0.2} + 0.01 * \log_2 \frac{0.21}{0.01} + 0.1 * \log_2 \frac{0.13}{0.1} + 0.03 * \log_2 \frac{0.13}{0.03} + 0.3 * \log_2 \frac{0.35}{0.3}$$
$$+ 0.05 * \log_2 \frac{0.35}{0.05} = 0.3664$$

$$H(y|x) = \sum P(x,y) * \log_2 \frac{P(x)}{P(x,y)}$$

$$= 0.2 * \log_2 \frac{0.6}{0.2} + 0.1 * \log_2 \frac{0.6}{0.1} + 0.3 * \log_2 \frac{0.6}{0.3} + 0.01 * \log_2 \frac{0.09}{0.01} + 0.03 * \log_2 \frac{0.09}{0.03}$$
$$+ 0.05 * \log_2 \frac{0.09}{0.05} = 0.997$$

p(x, y)		y: 0	y: 1	y: 2
x	0	3/24	2/24	1/24
	1	2/24	5/24	2/24
	2	6/24	1/24	2/24

Calculate H(x),H(Y),H(X,Y),H(X|Y),H(Y|X)

P(x, y)	0	1	2	P (x)
0	3/24	2/24	1/24	6/24
1	2/24	5/24	2/24	9/24
2	6/24	1/24	2/24	9/24
P(y)	11/24	8/24	5/24	

$$* \; H(x) = \frac{6}{24} * \log_2 \frac{24}{6} + \frac{9}{24} * \log_2 \frac{24}{9} + \frac{9}{24} * \log_2 \frac{24}{9} = 1.56.$$

$$* \; H(y) = \frac{11}{24} * \log_2 \frac{24}{11} + \frac{8}{24} * \log_2 \frac{24}{8} + \frac{5}{24} * \log_2 \frac{24}{5} = 1.51$$

$$* \; H(x,y) = 4 * \frac{2}{24} \log_2 \frac{24}{2} + 2 * \frac{1}{24} * \log_2 \frac{24}{1} + \frac{3}{24} * \log_2 \frac{24}{3} + \frac{5}{24} * \log_2 \frac{24}{5} + \frac{6}{24} * \log \frac{24}{6}$$

$$= 2.9$$

$$* \; H(x|y) = \frac{3}{24} \log_2 \left(\frac{11/24}{3/24}\right) + \frac{2}{24} \log_2 \left(\frac{12/24}{2/24}\right) + \frac{6}{24} \log_2 \left(\frac{11/24}{6/24}\right) + \frac{2}{24} \log_2 \left(\frac{8/24}{2/24}\right)$$

$$+ \frac{5}{24} * \log_2 \left(\frac{8/24}{5/24}\right) + \frac{1}{24} * \log_2 \left(\frac{8/24}{1/24}\right) + 2 * \frac{2}{24} \log_2 \left(\frac{5/24}{2/24}\right) + \frac{1}{24} \log_2 \left(\frac{5/24}{1/24}\right)$$

$$= 1.4$$

$$* \; H(y|x) = \frac{3}{24} \log_2 \left(\frac{6/24}{3/24}\right) + \frac{2}{24} \log_2 \left(\frac{6/24}{2/24}\right) + \frac{1}{24} \log_2 \left(\frac{6/24}{1/24}\right) + 3 * \left[\frac{2}{24} \log_2 \left(\frac{9/24}{2/24}\right)\right]$$

1.362

$$+ \frac{5}{24} * \log_2 \left(\frac{9/24}{5/24}\right) + \frac{6}{24} * \log_2 \left(\frac{9/24}{6/24}\right) + \frac{1}{24} \log_2 \left(\frac{9/24}{1/24}\right) = \cancel{1.362}$$

Sec [5] = 2023_Final_info theory;

2. Channel capacity equals the maximum over all input distributions of the ___________.
   A) mutual information   B) relative entropy   C) joint entropy   D) conditional entropy

Answer is: A

4. Let $p(X) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right)$, $p(Y) = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$, and $H(X,Y) = \frac{27}{8}$, What is the value of $H(X|Y)$?
   A) 9/8   B) 11/8   C) 13/8   D) 14/8

[4] $H(x|y) = H(x,y) - H(y)$

$* \ H(y) = \log_2 4 = 2$ $* \ H(x,y) = \frac{27}{8}$

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$\therefore H(y) = \log_2$ عددهم

$\therefore H(x|y) = \frac{27}{8} - 2 = \frac{11}{8}$

5. The entropy is measured in _______, when the base of the logarithm is **2**.
   A) nats   B) dits   C) bits   D) none of these

Answer is: C

11. The expected number of extra bits required to represent the random variable using a distribution other than the true distribution is _________.
   A) entropy   B) conditional entropy   C) joint entropy   D) relative entropy

Answer is: D

13. The conditional entropy of X given a particular value of Y $\left(H(X|Y=y)\right)$ ___________ $H(X)$.
   A) =   B) <   C) >   D) either (A), (B), or (C)

Answer is: B

16. If $I(X;Y) = 0$, then X and Y are ______________.
   A) independent   B) dependent   C) disjoint   D) reflexive

Answer is: A

24. By the chain rule, $I(X;Y,Z)$ = ___________.

A) $I(X;Z) + I(X;Y|Z)$
B) $I(X;Y;Z) + I(Y;Z|X)$
C) $I(X;Y;Z) + I(X;Z|Y)$
D) $I(X;Y) + I(X;Y|Z)$

Answer is: A

True or false

1. The entropy of a collection of random variables is the sum of the relative entropies.

(F) conditional

3. Conditional Probability is symmetric.

(F) not symmetric

5. Relative entropy satisfies triangle inequality.

(F) does not satisfy

6. $H(X) \leq H(X|Y)$.

(F) $HH(XX|YY) \leq HH(XX)$

8. Entropy is called self-information.

11. $H(X|Y)$ is the amount of uncertainty about the channel input (X) that is resolved after observing the output (Y).

(F) The difference $HH(XX) - HH(XX|YY)$

12. $I(X;Y)$ is the relative entropy between the joint distribution and the product distribution p(x)p(y).

(T)

13. The entropy of a random variable (RV) is an upper bound on the average number of bits required to represent that RV.

(F) lower

15. If $X \rightarrow Y \rightarrow Z$, then $I(X;Y) \geq I(X;Z)$.

(T)

Sec [6] = 2022_Final_info theory;

1. The entropy is measured in ________, when the base of the logarithm is ***e***.

   A) nats B) bits C) nits D) dits

Answer is: A

2. One reason that relative entropy is not a true distance is that it is __________.

   A) symmetric B) non-symmetric C) transitive D) reflexive

Answer is: B

3. The maximum value of the binary entropy function: $\boldsymbol{H(X) = -p \log p - (1-p)\log(1-p)}$ equals __________.

   A) 0.5 B) 1 C) 1.5 D) 2

Answer is: B

6. Let $p(X) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right)$, $p(Y) = \left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$, and $H(X,Y) = \frac{27}{8}$, What is the value of $H(Y|X)$?

   A) 9/8 B) 11/8 C) 13/8 D) 15/8

(4) $H(x|y) = H(x,y) - H(y)$

$* \; H(y) = \log_2 4 = 2$ $\qquad * \; H(x,y) = \frac{27}{8}$

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$\therefore H(y) = \log_2$ عددهم

$\therefore H(x|y) = \frac{27}{8} - 2 = \frac{11}{8}$

12. ____________ distribution has maximum entropy among all distributions.

   A) Normal B) Binomial C) Geometric D) Uniform

Answer is: D

15. The mutual information of a random variable with itself equals the ___________ of a random variable.

   A) entropy B) conditional entropy C) joint entropy D) relative entropy

Answer is: A

17. By the chain rule, $I(X;Y,Z)$ = ___________.

A) $I(X;Y|Z) + I(X;Y|Z)$ B) $I(X;Y;Z) + I(Y;Z|X)$

C) $I(X;Y;Z) + I(X;Z|Y)$ D) $IIII(XXXX;YYYY) + IIII(XXXX;ZZZZ|YYYY)$

Answer is: D

30. Channel capacity equals the maximum over all input distributions of the __________.

A) mutual information B) relative entropy C) joint entropy D) conditional entropy

Answer is: A

True or false

3. The entropy of a collection of random variables is the sum of the relative entropies.

(F) conditional

2. Both data compression and data transmission remove redundancy from the input data to the channel.

(F) Only data compression

4. $H(X) \leq H(X|Y)$.

(F) $HH(XX|YY) \leq HH(XX)$

11. Mutual information is symmetric.

(T)

14. The entropy of a balanced dice is less than the entropy of an unbalanced dice.

(F) more than