Information theory

Lecture 02

Sec [0] = Theoretical questions;

1- The sample space is

A. A subset of sample space
B. a function from sample space
C. The set of all possible outcomes

2- If A, B are independent events, then

A. A and B^c are independent but A^c and B are not independent
B. A^c and B^c are independent and A and B^c are independent and A^c and B are independent
C. A^c and B^c are independent but A and B^c are not independent

3- Conditional probability is not symmetric

A. True
B. False
C. It depends on the event.

4- If A, B are independent, then

A. p(AIB) = p(B)
B. p(AIB) = p(A)
C. p(AIB) = p(B n A)

5- A random variable (X) is

A. A subset of sample space
B. a function from sample space
C. The set of all possible outcomes

Sec [1] = problems;

1- Let p(A) = 0.3 and p(B) = 0.7 and p(A n B) = 0.5 , what is the value of p(A u B)

A. 0.5
B. 0.2
C. 1.0

①
$$p(A \cup B) = p(A) + p(B) - p(A \cap B)$$
$$= 0{,}3 + 0{,}7 - 0{,}5 =$$
$$= 0{,}5$$

2- Let p(A) = 0.3 and p(B) = 0.7 and p(A n B) = 0.5 , If A, B are mutually exclusive , what is the value of p(A u B) ?

A. 0.5
B. 0.2
C. 1.0

②
$$P(A \cup B) = p(A) + p(B) - p(A \cap B)$$
$$= 0{,}3 + 0{,}7 - 0$$
$$= 1{,}0$$

3- Let p(A) = 0.3 and p(B) = 0.7 and p(A n B) = 0.5 , If A, B are independent

what is the value of p(A n B) ?

A. 0.30
B. 0.21
C. 1.00

③
$$p(A|B) = \frac{P(A \cap B)}{P(B)} \quad \therefore \quad P(A \cap B) = P(A|B)\, P(B)$$

when A, B are independet $p(A|B) = P(A)$.

$$P(A \cap B) = p(A)\, p(B)$$
$$= 0{,}3 \times 0{,}7$$
$$= 0{,}21$$

4- Let p(A) = 0.3 and p(B) = 0.7 and p(A n B) = 0.5 , If A, B are independent

what is the value of p(A | B) ?

A. 0.71
B. 0.21
C. 0.30

4. $p(A \mid B)$ when A, B are independet is $p(A)$

$\therefore \; p(A|B) = p(A)$

$= 0{,}3$

5 - Let p(A) = 0.3 and p(B) = 0.7 and p(A n B) = 0.5 , what is the value of p(A | B)

A. 1.66
B. 0.71
C. 0.30

5. $p(A|B) = \dfrac{p(A \cap B)}{P(B)}$

6- Let p(A) = 0.3 and p(B) = 0.7 and p(A n B) = 0.5 , what is the value of p(A | A)

A. 1 / 1
B. 1 / 3
C. 1 / 4

P (الحاجه | نفس الحاجه) = 1

Sec [2] = Lecture questions;

1- when you Roll a dice , Let E be the event of getting an odd number. What is the probability of evet E ?

A. 1 / 2
B. 1 / 3
C. 1 / 4

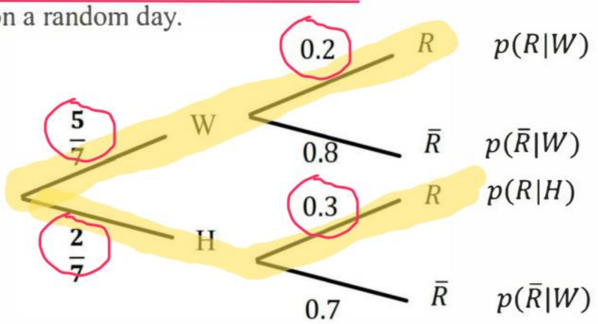
$$p(E) = p(1) + P(3) + P(5) = \frac{3}{6} = \frac{1}{2}$$

2- After a year of observation, we find probability of rain (R) on a workday (W) is 0.2, and probability of rain on a weekend day (H) is 0.3. What is the overall probability of rain on a random day ?

- **Example**: Does it rain more on weekends? After a year of observation, we find probability of rain (R) on a workday (W) is 0.2, and probability of rain on a weekend day (H) is 0.3. What is the overall probability of rain on a random day.

ايام الاسبوع **7**
منهم **5** شغل

ايام الاسبوع **7**
منهم **2** اجازة



اول ما بترسم الرسمه
يبقى انت كده حليت
المسالة

$$p(R) = p(R|W)p(W) + p(R|H)p(H)$$

قانون ال **total prob**

$$p(R) = 0.2 \times \frac{5}{7} + 0.3 \times \frac{2}{7} = 0.2286$$

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3- In the previous question : Suppose on a certain day it rains. What is the probability that this day was a weekend (H) day?

قانون ال**bayes** $p(A_j|B) = \dfrac{p(B|A_j)p(A_j)}{\sum_{i=1}^{n} p(B|A_i)p(A_i)}$ $\dfrac{\text{الحدث المطلوب في احتماله}}{\text{قانون التوتال في المقام}}$

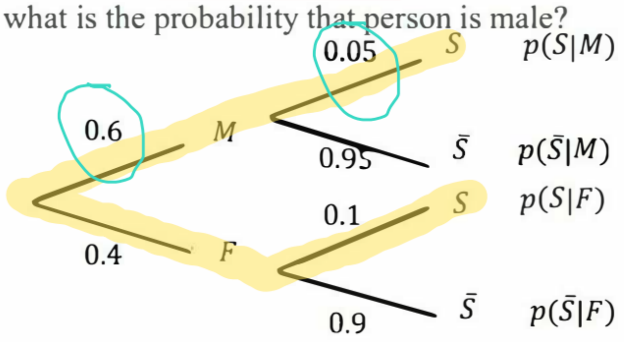
$$p(H|R) = \frac{p(R|H)p(H)}{p(R)}$$

$$p(H|R) = \frac{0.3 \times \frac{2}{7}}{0.2286} = 0.375$$

4- The probability of having a side effect for a drug is 10% for females. The probability of not having the side effect for males is 95%. 40% of the study was females. Given a person having a side effect what is the probability that person is male?

Example: The probability of having a side effect for a drug is 10% for females. The probability of not having the side effect for males is 95%. 40% of the study was females. Given a person having a side effect what is the probability that person is male?

having is 5% for male



$$p(M|S) = \frac{p(S|M)p(M)}{p(S|M)p(M) + p(S|F)p(F)}$$

$$p(D|P) = \frac{0.05 \times 0.6}{0.05 \times 0.6 + 0.1 \times 0.4} = 0.4615$$

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x	3	4	5
p(x)	1/2	1/3	1/6

X is a random variable with it's probability , answer the questions below

5-the expected value is …

$$E(X) = 3 \times \frac{1}{2} + 4 \times \frac{1}{3} + 5 \times \frac{1}{6} = \frac{11}{3}$$

بضرب كل X في p(x) بتاعتها و اجمع

6-the Variance is …

$$\sigma^2 = E((X - \mu)^2) = E(X^2) - \mu^2$$

⑥ $\left[3^2 \times \frac{1}{2} + 4^2 \times \frac{1}{3} + 5^2 \times \frac{1}{6}\right] - \left(\frac{11}{3}\right)^2 = \frac{5}{9}$

7-the standard deviation is …

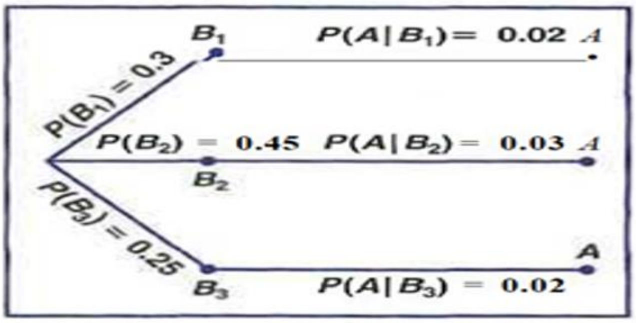
⑦ $\sigma = \sqrt{\sigma^2}$ $\therefore \sqrt{\frac{5}{9}}$

Sec [3] = section questions;

In a certain assembly plant, three machines, Bi, *B2,* and P3, make 30%, 45%, and 25%, respectively, of the products. It is known from past, experience that 2%, 3%, and 2% of the products made by each machine, are defective. Now, suppose that a finished product is randomly selected. What is the probability that if is defective?

Solution:

A: the product, is defective,
B*1*: the product is made by machine B1,
B2: the product is made by machine *B2,*
B3: the product is made by machine: B3.
Applying the rule of elimination, we can write



$$P(A) = P(B_1)P(A|B_1) + P(B_2)P(A|B_2) + P(B_3)P(A|B_3).$$

and hence

$$P(A) = 0.006 + 0.0135 + 0.005 = 0.0245.$$

A manufacturing firm employs three analytical plans for the design and development of a particular product. For cost reasons, all three are used at varying times. In fact, plans 1, 2, and 3 are used for 30%,, 20%, and 50% of the products respectively. The "defect rate:" is different for the three procedures as follows:

$$P(D|P_1)=0.01,\quad P(D|P_2)=0.03, \text{ and}$$

$$P(D|P_3)=0.020$$

where $P(D|P_1)$ is the probability of a, defective product, given plan j. If a random product was observed and found to be defective, which plan was most likely used and thus responsible?

Solution :

From the statement, of the problem
$P(P_1)=0.30,\quad P(P_2)=0.20$ and $P(P_3)=0.50$
we must find $P(P_j \mid D)$ for $j = 1,2,3$. Bayes' rule shows

$$P(P_1 \mid D)=\frac{P(P_1)P(D|P_1)}{P(P_1)P(D|P_1)+P(P_2)P(D|P_2)+P(P_3)P(D|P_3)}$$

$$P(P_1 \mid D)=\frac{0.3*0.01}{0.3*0.01+0.2*0.03+0.5*0.02}=0.158$$

Similarly
$P(P_2 \mid D)=0.316$ and $P(P_3 \mid D)=0.526$

The conditional probability of a defect given plan 3 is the largest of the three; thus a defective for a random product is most, likely the result of the use of plan 3.

Problem 13. **Testing for a rare disease.** A test for a certain rare disease has 90% accuracy: if a person has the disease, the test results are positive with probability 0.9, and if the person does not have the disease, the test results are negative with probability 0.9. A random person drawn from a certain population has probability 0.001 of having the disease. Given that the person just tested positive, what is the probability of having the disease?

بتتحل بال bayes rule و فيه مسالة شبهها جت في امتحان 2023 محلولة في صفحه قدام ودي هتتحل بنفس الطريقة

Problem 3. Out of the students in a class, 60% are geniuses, 70% love chocolate, and 40% fall into both categories. Determine the probability that a randomly selected student is neither a genius nor a chocolate lover.

Problem 8. We roll two fair 6-sided dice. Each one of the 36 possible outcomes is assumed to be equally likely.

(a) Find the probability that doubles were rolled.

(b) Given that the roll resulted in a sum of 4 or less, find the conditional probability that doubles were rolled.

(c) Find the probability that at least one die is a 6.

(d) Given that the two dice land on different numbers, find the conditional probability that at least one die is a 6.

Sec [4] = Last year's questions for probability2 ;

10. Time to commute from home to work is considered a --------------- variable.
A) discrete B) continuous C) nominal

Answer is : B

21. For the given probability distribution, what is the standard deviation of x?

x	0	1	2	3	4	5
$p(x)$	0.02	0.2	0.3	0.3	0.1	0.08

A) 0.94 B) 1.2 C) 2.7

(21)

Expected value or $\mu = \sum x\, p(x)$

$= 0 \times 0{,}02 + 1 \times 0{,}2 + 2 \times 0{,}3 + 3 \times 0{,}3 + 4 \times 0{,}1 + 5 \cdots$

$= 2{,}5$

Variance $= \sum x^2 p(x) - \mu^2$

$= 0^2 \times 0{,}02 + 1^2 \times 0{,}2 + 2^2 \times 0{,}3 + 3^2 \times 0{,}3 + 4^2 \times 0{,}1 + 5^2 \times 0{,}08 - (2{,}5)^2$

$= 1{,}45$

Standard diviation $= \sqrt{\text{variance}} = \sqrt{1{,}45} = 1{,}2$ (B)

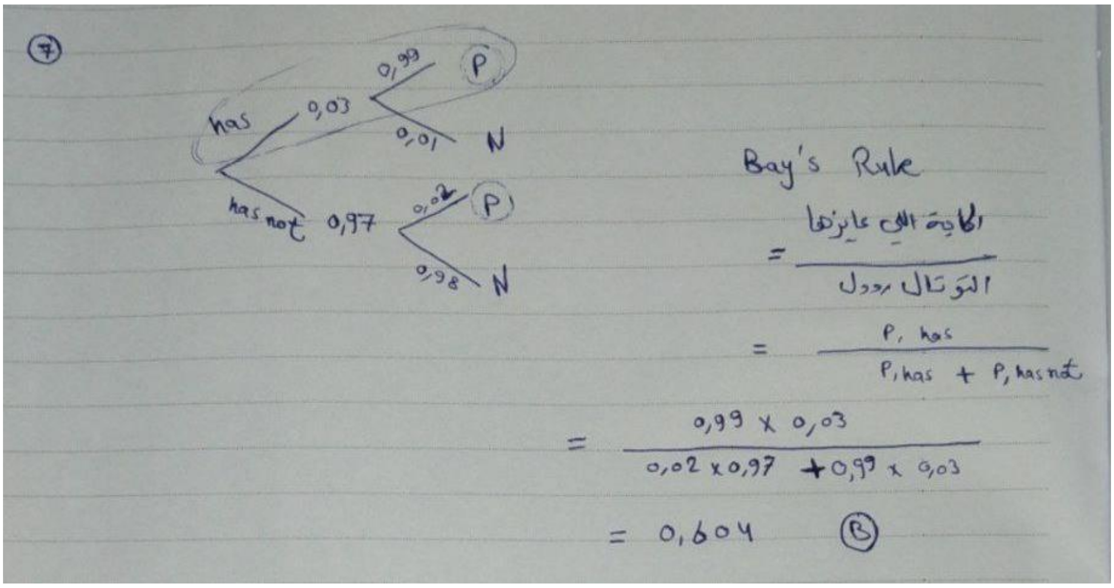
11. A certain medical test is designed to detect a rare disease with only 1% of people suffers from that disease. If the probability of testing positive given that a person has the disease is 95% while the probability of testing negative given that a person does not have the disease is 90%. A randomly selected person takes the test and tests positive. What is the probability that this person actually has the disease?
A) 0.068 B) 0.078 C) 0.088

Answer is : C

بتتحل بال bayes rule و فيه مسالة شبهها جت في امتحان 2023 محلولة في الصفحه الجاية ودي هتتحل بنفس الطريقة

Sec [5] = 2023_Final_info theory;

7. Suppose that 3% of a population has a certain disease (D). A certain test is 99% sure of correctly returning positive if the person has the disease and 98% sure of correctly returning negative if the person does not have it. If a person got a positive test result, what is the probability that the person has the disease?
   A) 0.5 B) 0.6 C) 0.7 D) 0.8



12. A measure of the spread out of the values from the mean is known as ___________.
   A) entropy B) relative entropy C) expected value D) variance

Answer is : D

15. If two events cannot both occur at the same time, these events are ___________.
   A) independent B) dependent C) disjoint D) complementary

Answer is : C

18. The probability that a rolled dice shows an even number and a tossed coin shows a head is ________.
A) 1/2 B) 1/3 **C) 1/4** D) 1/6

Answer is : C

(18) $P(\text{Even}) = P(2) + P(4) + P(6)$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

$P(H) = \frac{1}{2}$

$P(\text{Even} \cap H) = ??$

These Events are Independent Events

$\therefore P(\text{Even} \cap H) = P(\text{Even}) \times P(H)$

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$ (C)

F

3. Conditional Probability is symmetric.

4. An event and its complement are disjoint events.

T

7. If A, B are independent events, then $\boldsymbol{A}$ and $\overline{\boldsymbol{B}}$ are independent.

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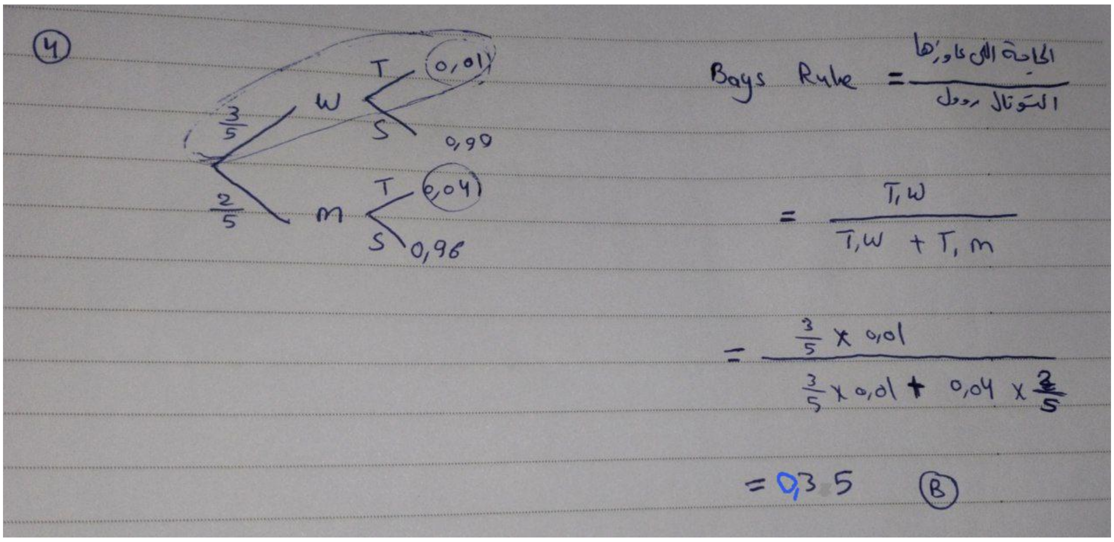
Sec [6] = 2022_Final_info theory;

4. At a certain university, 4% of men are over 6 feet tall and 1% of women are over 6 feet tall. The total student population is divided in the ratio 3:2 in favor of women. If a student is selected at random from among all those over 6 feet tall, what is the probability that the student is a woman?
   A) 0.27 B) 0.35 C) 0.44 D) 0.5

Answer is : B



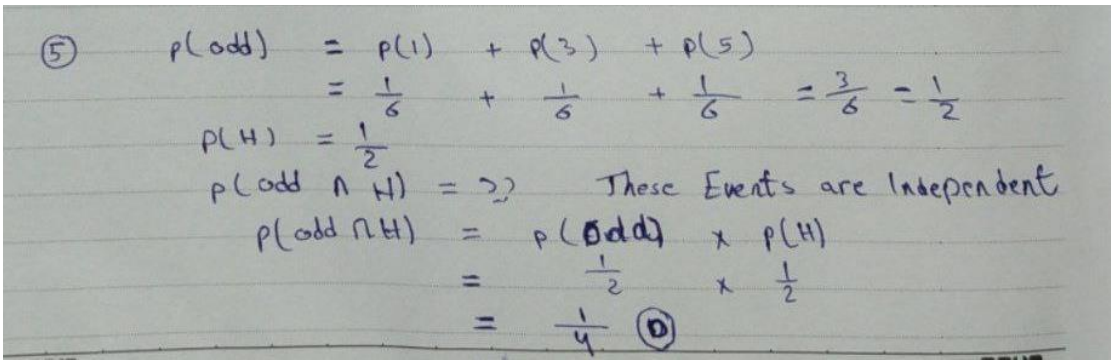
Bays Rule $= \dfrac{\text{الحاجة اللى عاوزها}}{\text{التوتال روول}}$

$= \dfrac{T, W}{T, W + T, m}$

$= \dfrac{\frac{3}{5} \times 0{,}01}{\frac{3}{5} \times 0{,}01 + 0{,}04 \times \frac{2}{5}}$

$= 0{,}35$ (B)

5. A dice is rolled and a coin is tossed, the probability that the dice shows an **odd number** and the coin shows a **head** is __________.
   A) 1/2 B) 1/6 C) 1/3 D) 1/4



5

$p(\text{odd}) = p(1) + p(3) + p(5)$

$= \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{3}{6} = \frac{1}{2}$

$p(H) = \frac{1}{2}$

$p(\text{odd} \cap H) = ??$ These Events are Independent

$p(\text{odd} \cap H) = p(\text{Odd}) \times p(H)$

$= \frac{1}{2} \times \frac{1}{2}$

$= \frac{1}{4}$ (D)

7. If the outcome of one event affects the outcome of another, these events are called _____ events.
   A) complementary B) disjoint C) independent D) dependent

Answer is : D

9. When an experiment is repeated many times, the average number obtained is known as ___________.
   A) variance B) standard deviation C) entropy D) expected value

Answer is : D

1. An event and its complement are disjoint events.

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