Solution to COMP9334 Revision Questions for Week 4A

Question 1

First note that one counter is not sufficient to serve all the customers. If we consider all the customers together, each customer carries on average 20.5 items, which takes $\frac{20.5}{15}$ to complete. Since the customer arrival rate is 1, the utilisation will be above 1 if only one counter is used.

Let us refer to the two counters as Counter 1 and Counter 2. Let us assume that Counter 1 serves customers with x items or less and Counter 2 serves customers with more than x items, where $1 \leq x \leq 39$.

Let $\lambda(=1)$ denote the overall arrival rate and $\mu = 15$ be the service rate of each counter.

Let us consider Counter 1 first. Since only customers with x items or less go to Counter 1, the arrival rate at Counter 1 is $\lambda\frac{x}{40}$. The customers arriving at Counter 1 bring with them 1, 2, ..., x items uniformly distributed. Let S_1 denote the service time at Counter 1. We have

$$E[S_1] = \sum_{i=1}^{x} \frac{i}{\mu}\frac{1}{x} \tag{1}$$

$$E[S_1^2] = \sum_{i=1}^{x} \left(\frac{i}{\mu}\right)^2 \frac{1}{x} \tag{2}$$

Let $\rho_1 = \lambda\frac{x}{40}E[S_1]$, by the P-K formula, the mean waiting time at Counter 1 is

$$W_1 = \begin{cases} \lambda\frac{x}{40}\frac{E[S_1^2]}{2(1-\rho_1)} & \text{if } \rho_1 < 1 \\ \infty & \text{if } \rho_1 \geq 1 \end{cases} \tag{3}$$

Similarly, the arrival rate to Counter 2 is $\lambda_2 = \lambda\frac{40-x}{40}$. Let S_2 denote the service time at Counter 2, then

$$E[S_2] = \sum_{i=x+1}^{40} \frac{i}{\mu}\frac{1}{(40-x)} \tag{4}$$

$$E[S_2^2] = \sum_{i=x+1}^{40} \left(\frac{i}{\mu}\right)^2 \frac{1}{(40-x)} \tag{5}$$

Let $\rho_2 = \lambda\frac{40-x}{40}E[S_2]$, by the P-K formula, the mean waiting time at Counter 2 is

$$W_2 = \begin{cases} \lambda\frac{40-x}{40}\frac{E[S_2^2]}{2(1-\rho_2)} & \text{if } \rho_2 < 1 \\ \infty & \text{if } \rho_2 \geq 1 \end{cases} \tag{6}$$

The mean waiting time of the customers is

$$W = \frac{x}{40}W_1 + \frac{40-x}{40}W_2 \tag{7}$$

Note that W is a function of x. We write a computer program (Matlab file *week04A_q1.m*) to calculate how W varies with x. Figure 1 shows how W varies with x. It can be seem that the minimum value of W is achieved at $x = 28$.

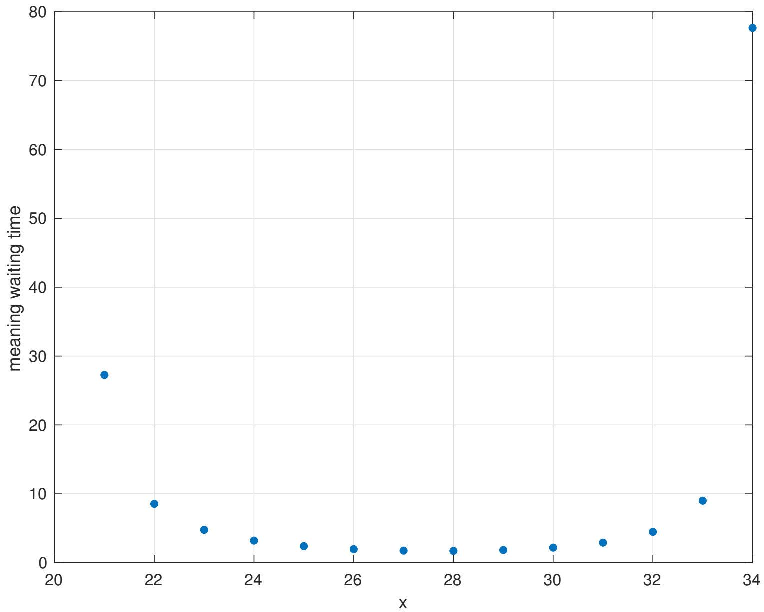

Figure 1: For Question 1

Question 2

The system behaves as an M/G/1 queueing system.

Since there are 10 sessions each generating Poisson traffic at a rate of 150 packets/minute, the packet arrival rate to the communication line is 1500 packets/minute or 25 packets/s ($= \lambda$).

With a transmission rate of 50 kbits/s, a 100-bit packet requires a transmission time (= service time in queueing theory terminology) 0.002s and a 1000-bit packet requires a transmission time of 0.02s. (Recall that transmission time is packet size divided by transmission rate.)

Given that 10% of the packets are 100 bits long and the rest are 1000 bits long, the mean service time $E[S]$ (where S denotes the service time random variable)

$$E[S] = 0.1 * 0.002 + 0.9 * 0.02 = 0.0182s \tag{8}$$

and the second moment of the service time is

$$E[S^2] = 0.1 * 0.002^2 + 0.9 * 0.02^2 = 3.6040 \times 10^{-4} s^2 \tag{9}$$

The mean waiting time W, according to the P-K formula, which applies to M/G/1 queueing system, is

$$W = \frac{\lambda E[S^2]}{2(1 - \lambda E[S])} = 8.3ms \tag{10}$$

By Little's Law, the mean queue length is given by the product of the throughout of the queue and the mean waiting time,

$$\lambda * W = 0.21 \ \ packets \tag{11}$$