# COMP9334 Project Report

Name: Xue WEN

zID: z5149548

## 1. Inter-arrival, service time and network latency distribution

(1) For inter-arrival probability distribution, I use a library of random.expovariate generate a series of pseudo-random numbers, which are exponentially distributed.

```
def getArrivalTime(lam,time_end):
    #random.seed(0)
    arrival_list = []
    arrival_time = 0
    while arrival_time < time_end:
        inter_arrival_time = random.expovariate(lam)
        inter_arrival_time = round(inter_arrival_time,5)
        arrival_time = round((arrival_time + inter_arrival_time),5)
        arrival_list.append(arrival_time)
    for i in range(len(arrival_list)):
        if arrival_list[i] > time_end:
            arrival_list.pop(i)
    arrival_list.sort()
    return arrival_list
```

Fig 1. Generate the arrival time list

In order to see the correctness of my simulation program in this part, I wrote a program named chart.py to plot the following figure. The data I used to plot this figure is as follow:

| Parameter | Value |
|---|---|
| lam($\lambda$) | 9.72 |
| seed | 1 |
| time_end | 1000 |

The plot is as following:

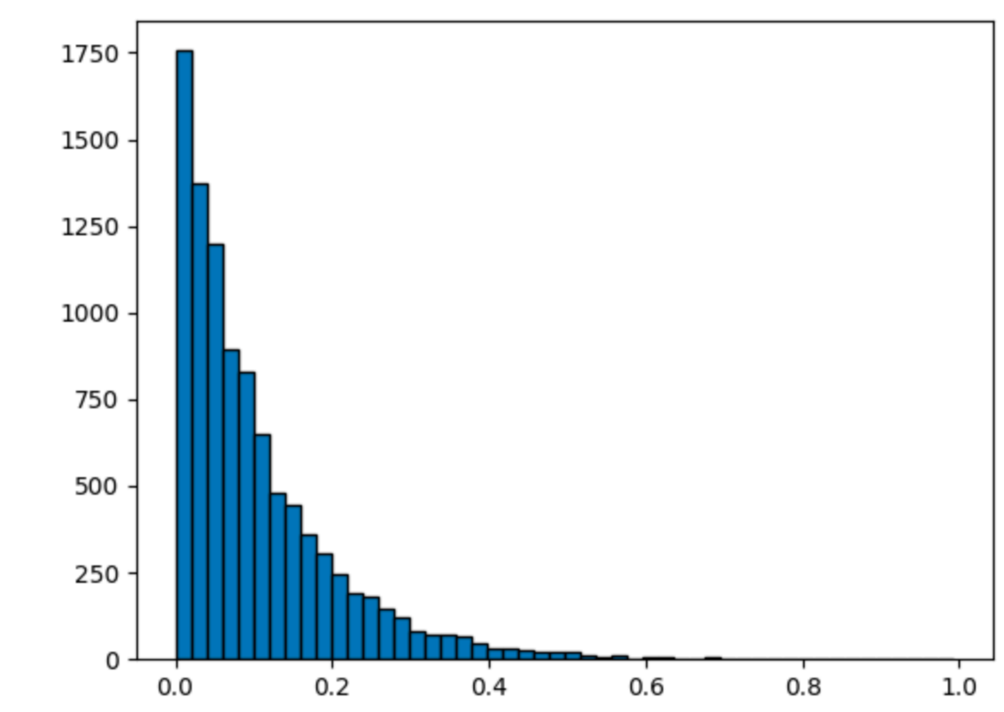



Fig 2. Histogram of exponentially distributed pseudo-random number

(2) For service time distribution, I use the following code to generate a series of pseudo-random numbers, which are following the probability density function given by the project documentation.

```python
def getServiceTime(alpha1,alpha2,beta,requests_number):
    # calculate gama
    #random.seed(0)
    gama = (1.0 - beta) / ((alpha2 ** (1.0 - beta)) - (alpha1 ** (1.0 - beta)))
    service_list_random = []
    for i in range(0, requests_number):
        prob = random.random()
        prob = round(prob, 5)
        time = (prob * (1.0 - beta) / gama + alpha1 ** (1.0 - beta)) ** (1.0 / (1.0 - beta))
        service_list_random.append(round(time,5))
    #print(len(service_list_random))
    return service_list_random
```

Fig 3. Generate the service time list

In order to see the correctness of my simulation program in this part, I wrote a program named chart.py to plot the following figure. The data I used to plot this figure is as follow:

| Parameter | Value |
|---|---|
| alpha1 | 0.01 |
| alpha2 | 0.4 |
| beta | 0.86 |
| seed | 1 |
| requests_number | length(arrival_list)=9715 |

The plot is as following:

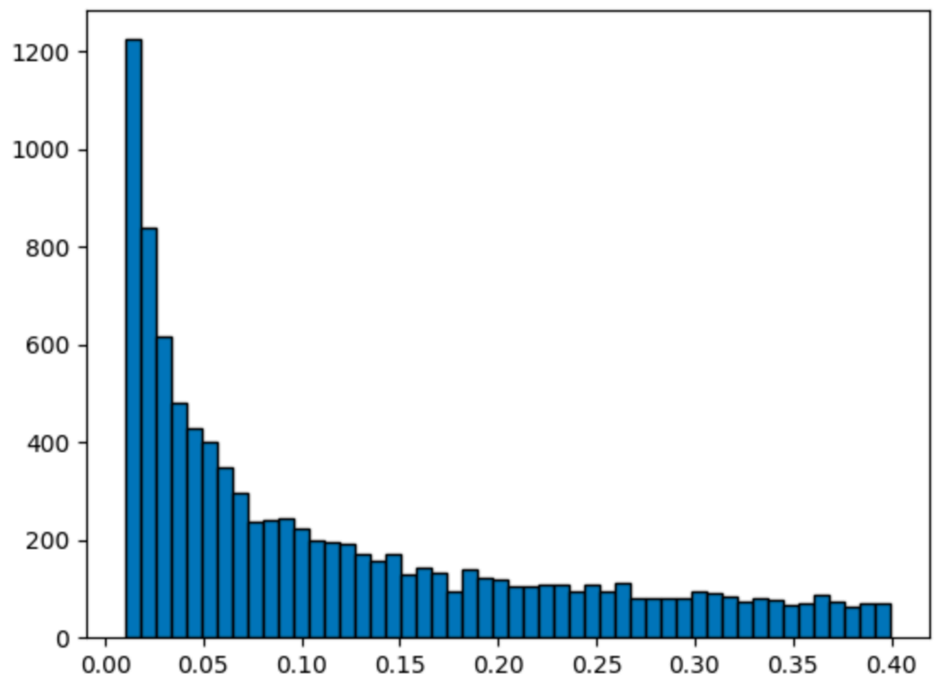



Fig 4. Histogram of service time probability distribution

(3) For network latency distribution, I use the following code to generate a series of pseudo-random numbers, which are following the uniform distribution.

```
def getNetworkLatency(v1,v2,requests_number):
    #random.seed(0)
    network_latency = []
    for i in range(0, requests_number):
        network = random.uniform(v1,v2)
        network = round(network,5)
        network_latency.append(network)
    #print(len(network_latency))
    return network_latency
```

Fig 5. Generate the network latency list

In order to see the correctness of my simulation program in this part, I wrote a program named chart.py to plot the following figure. The data I used to plot this figure is as follow:

| Parameter | Value |
|---|---|
| v1 | 1.2 |
| v2 | 1.47 |
| seed | 1 |
| fogTimeLimit | 0.2 |
| requests_number | length(arrival_list)=9715 |

The plot is as following:

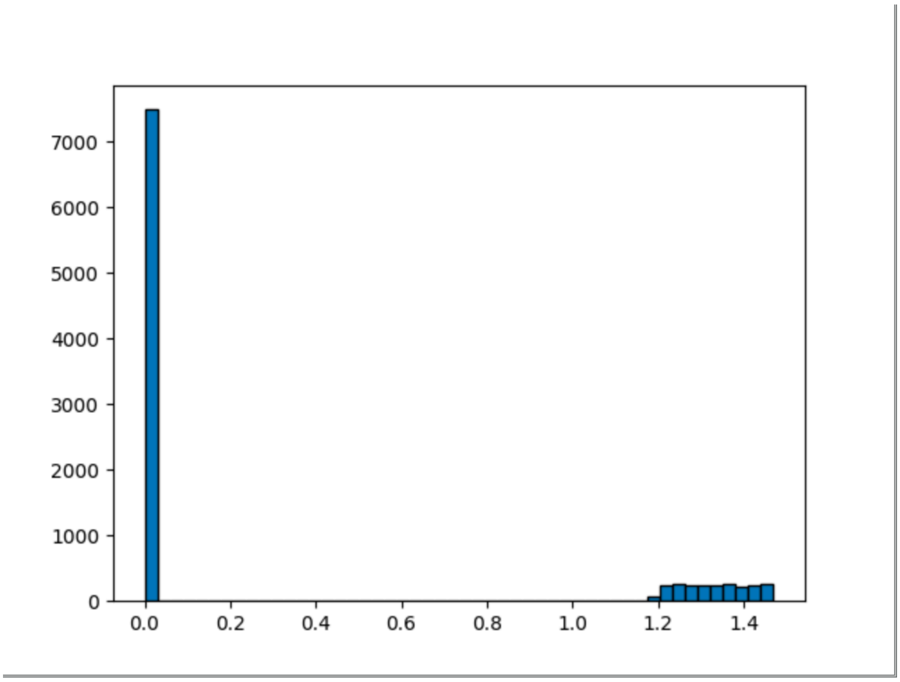



Fig 6. Histogram of uniform distribution

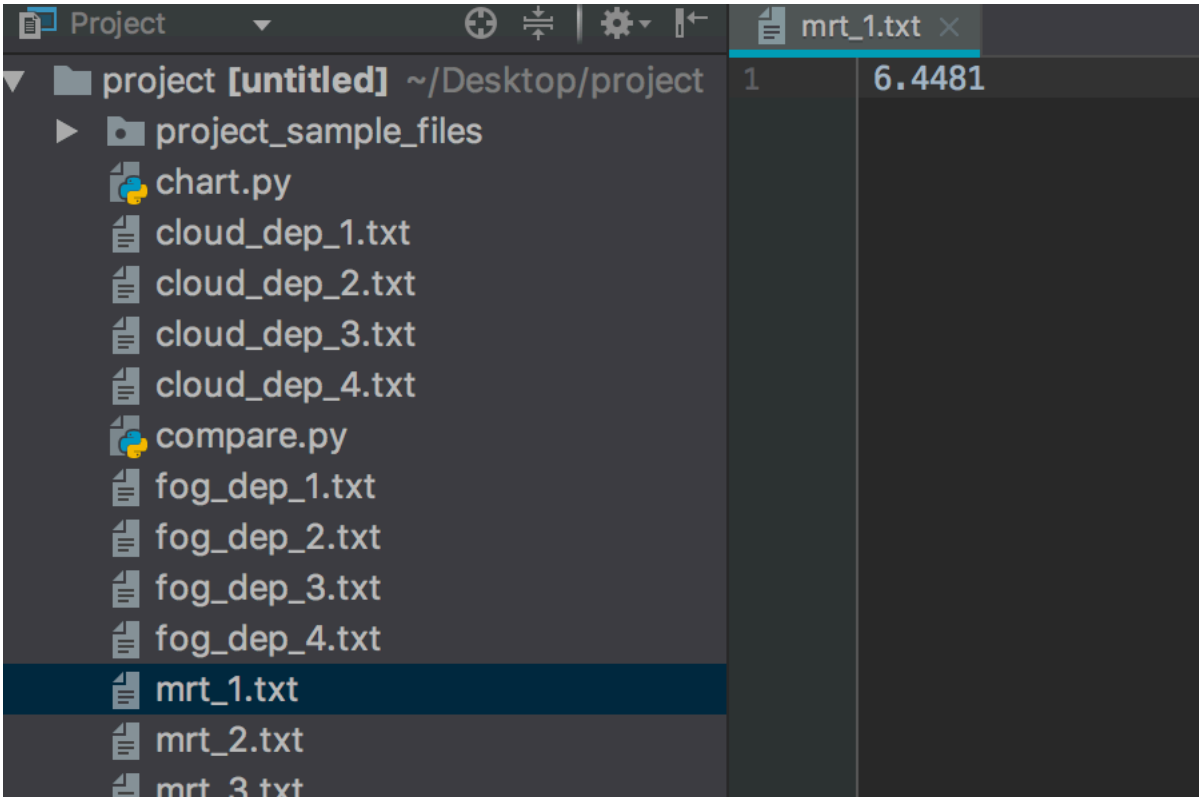
## 2. Simulation Code

In order to see the correctness of my simulation program, there are 3 samples as follow:

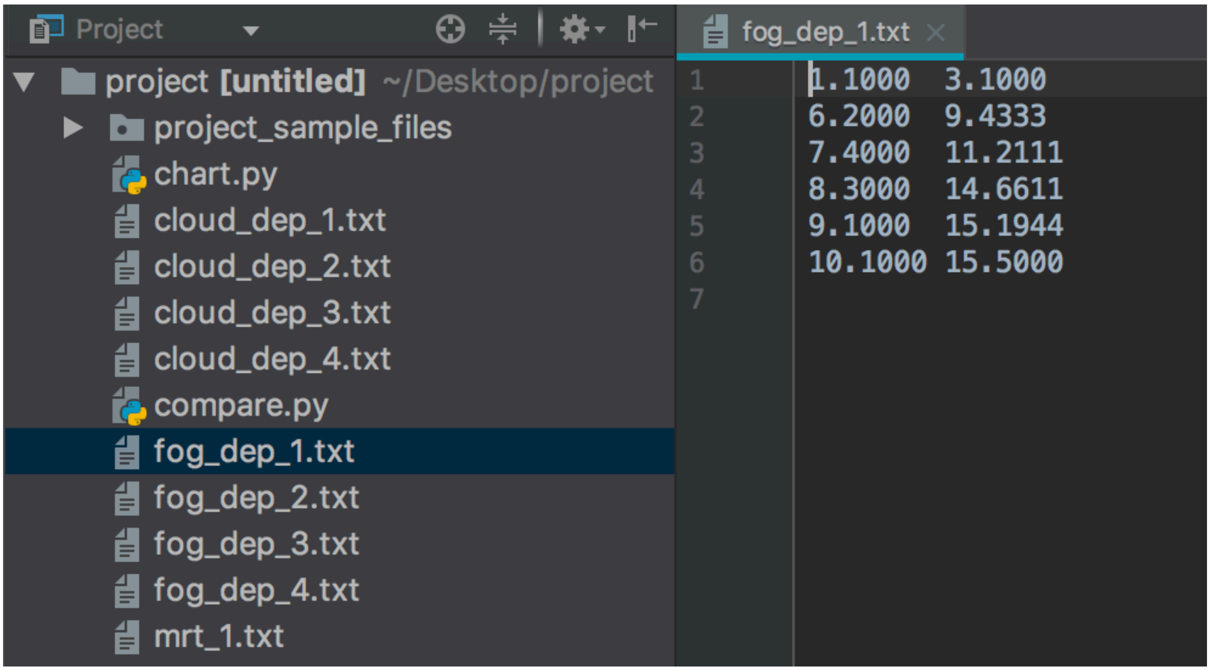
Example 1:

| arrival | service | network | fogTimeLimit | fogTimeToCloudTime |
|---|---|---|---|---|
| 1.100 | 4.100 | 1.500 | 2.000 | 0.600 |
| 6.200 | 5.200 | 1.300 | | |
| 7.400 | 1.300 | 0.000 | | |
| 8.300 | 2.000 | 0.000 | | |
| 9.100 | 3.200 | 1.600 | | |
| 10.100 | 4.100 | 1.800 | | |

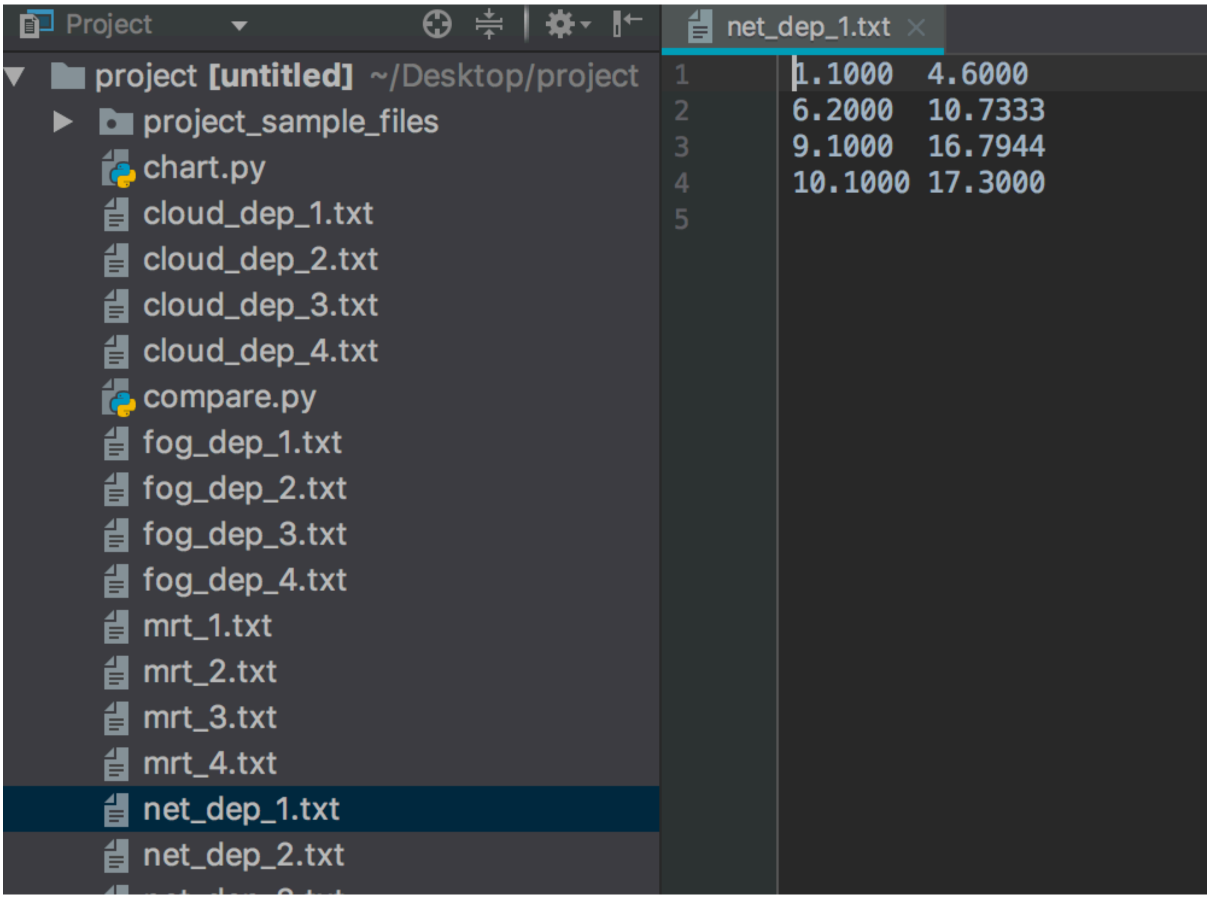
The mean response time is as following:



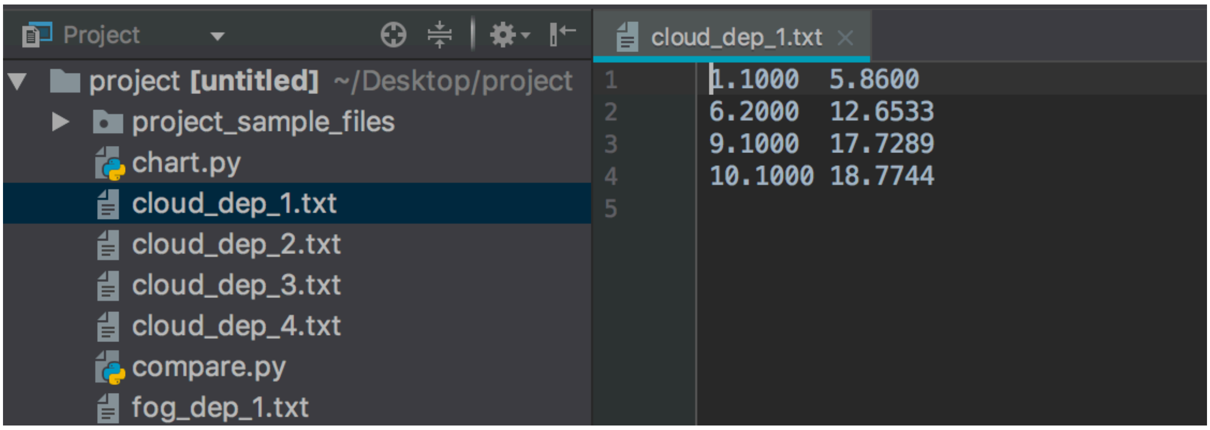

The fog departure time is as following:




The network departure time is as following:




The cloud departure time is as following:

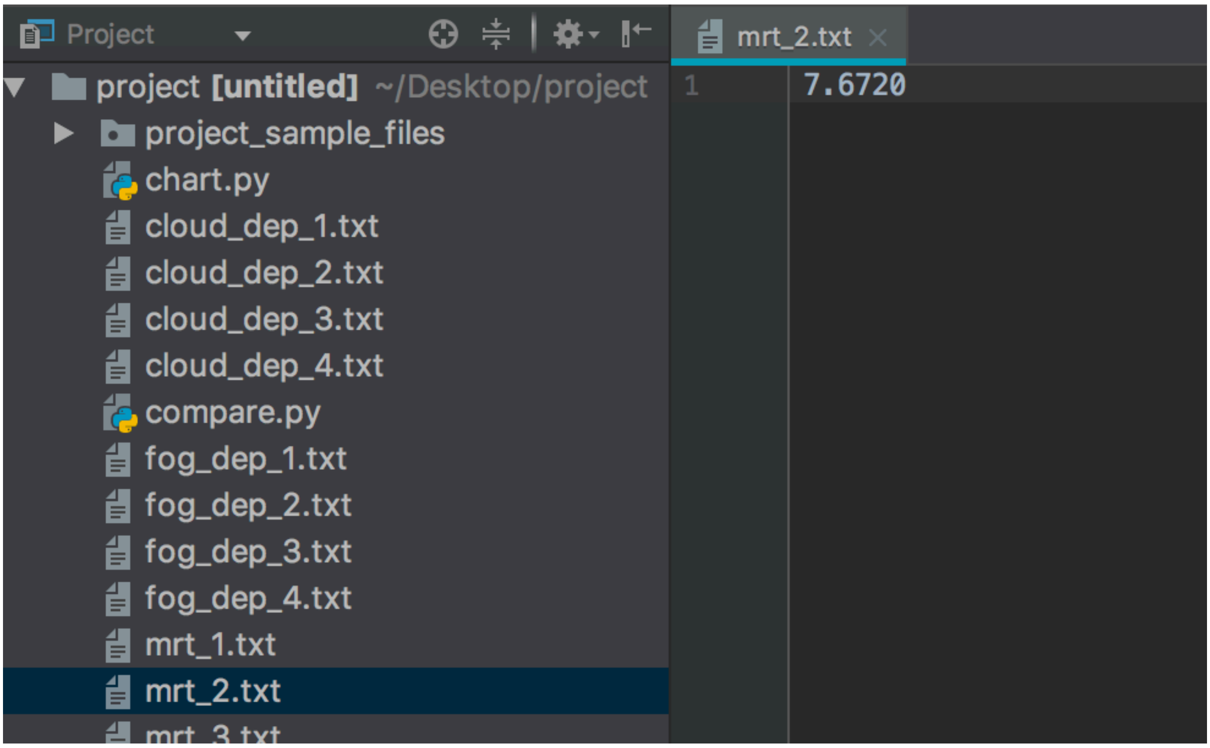



Example 2:

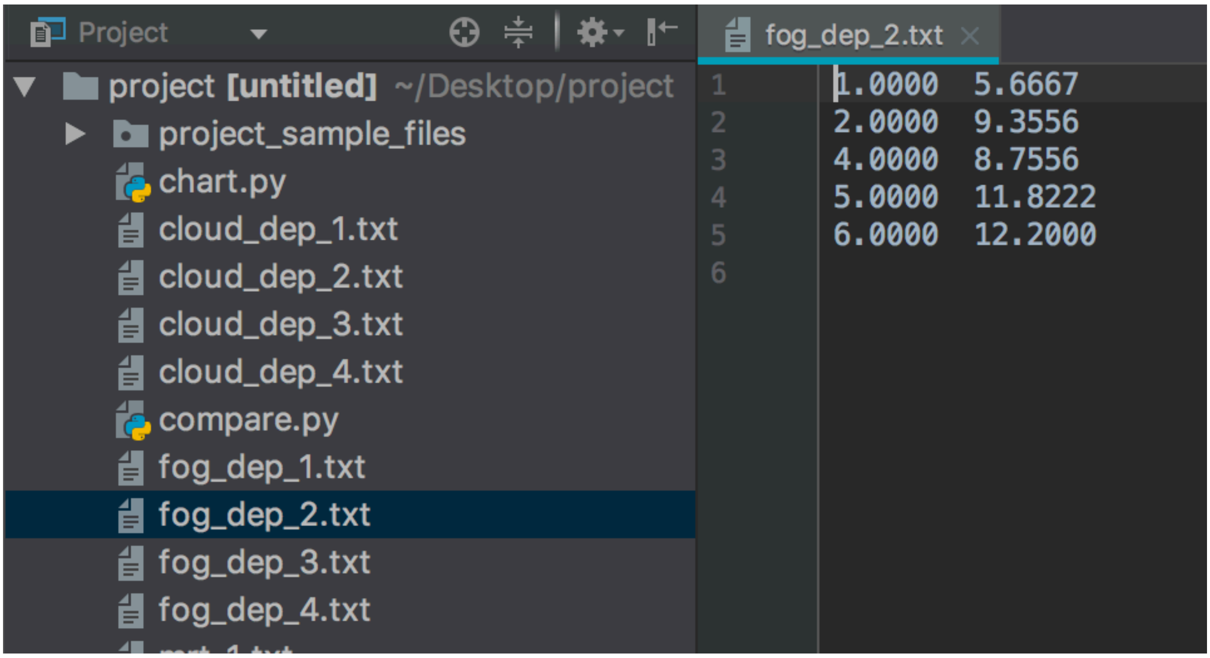
| arrival | service | network | fogTimeLimit | fogTimeToCloudTime |
|---|---|---|---|---|
| 1.000 | 3.700 | 1.500 | 2.500 | 0.700 |
| 2.000 | 5.100 | 1.400 | | |

| | | | | |
|---|---|---|---|---|
| 4.000 | 1.300 | 0.000 | | |
| 5.000 | 2.400 | 0.000 | | |
| 6.000 | 4.500 | 1.600 | | |

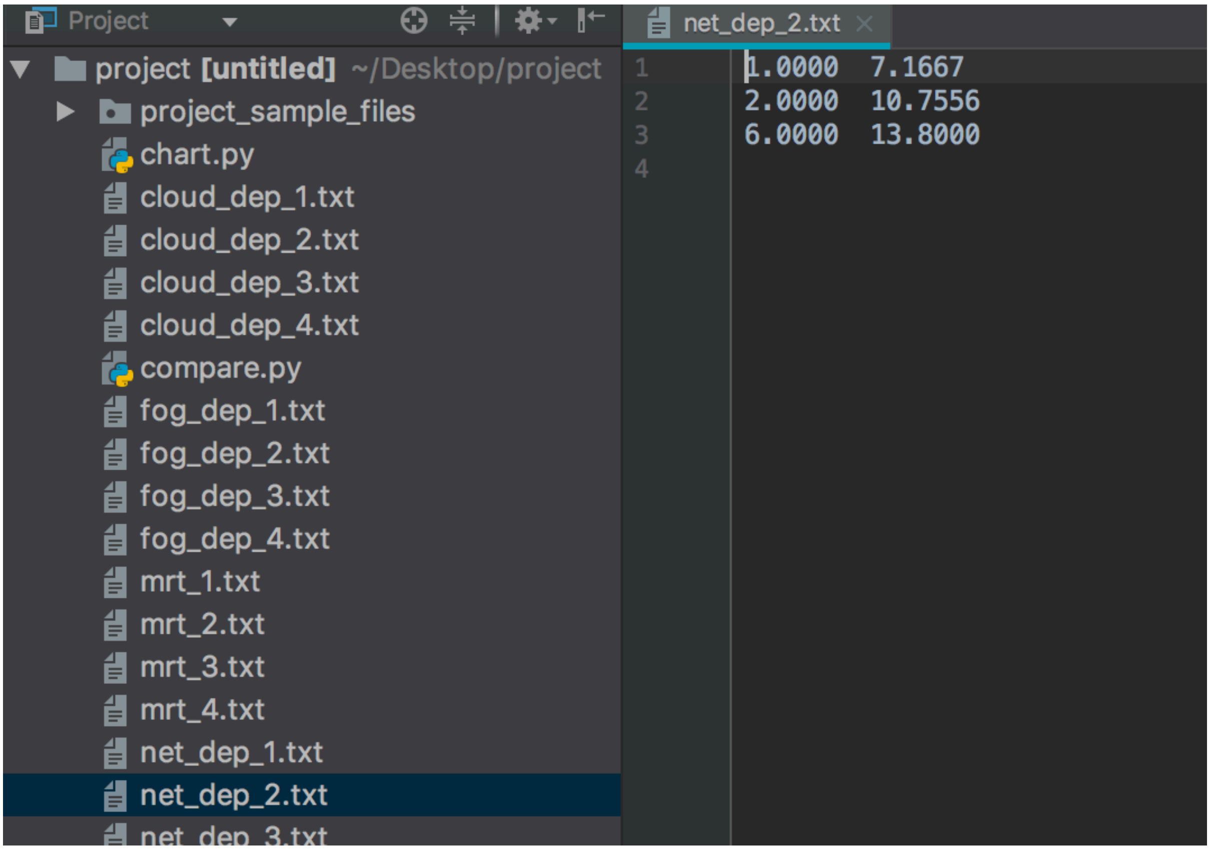
The mean response time is as following:



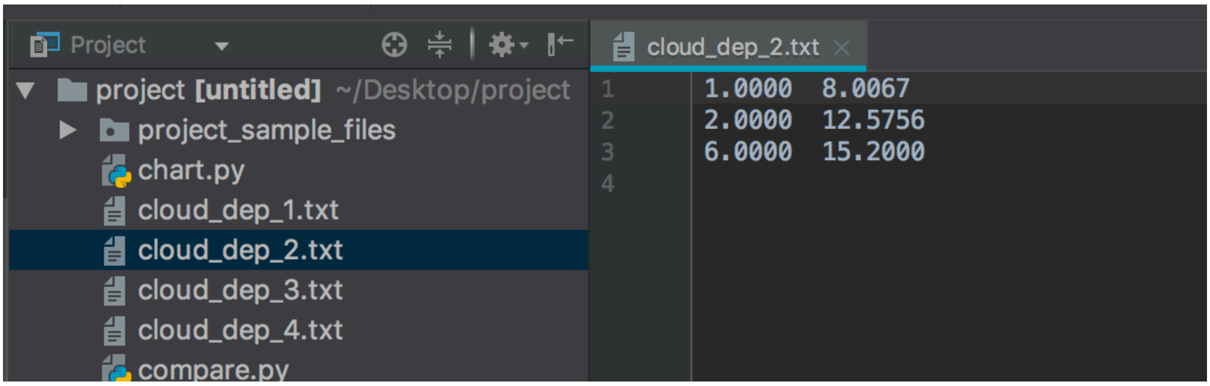

The fog departure time is as following:




The network departure time is as following:




The cloud departure time is as following:

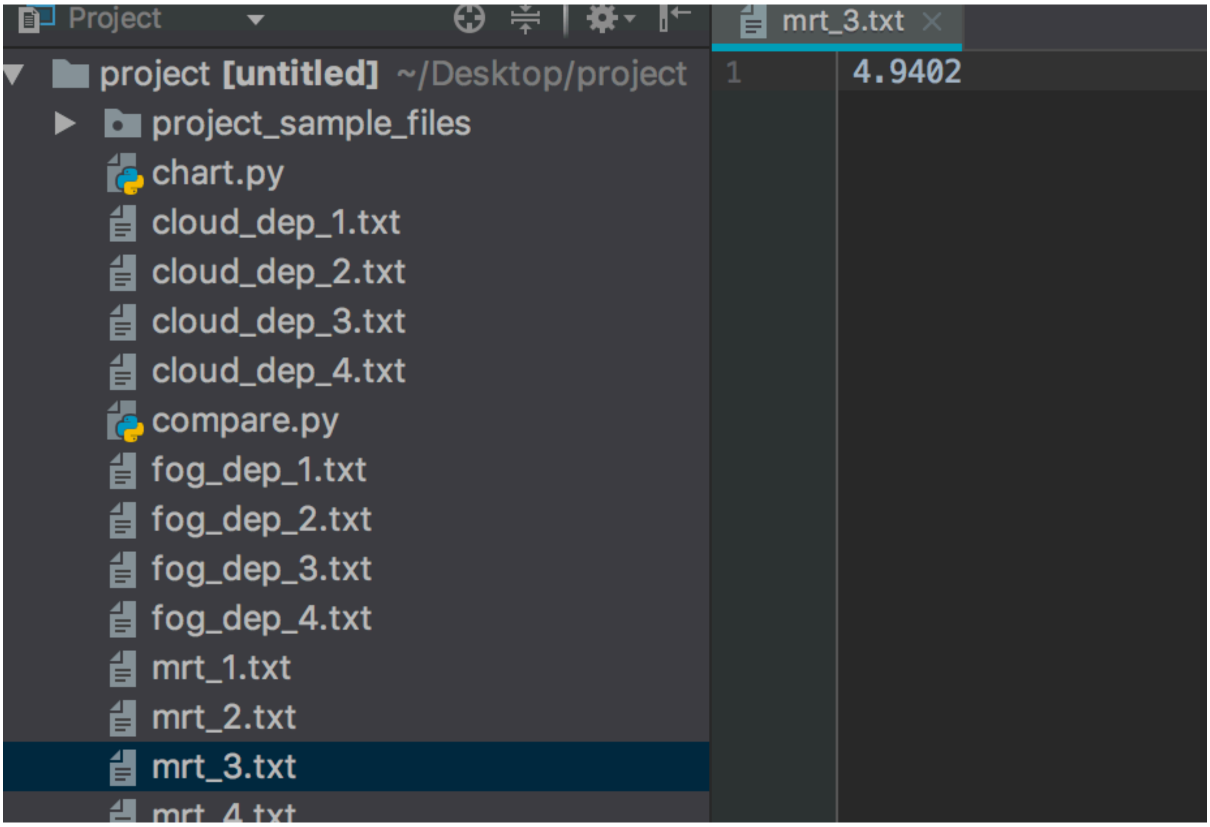



Example 3:

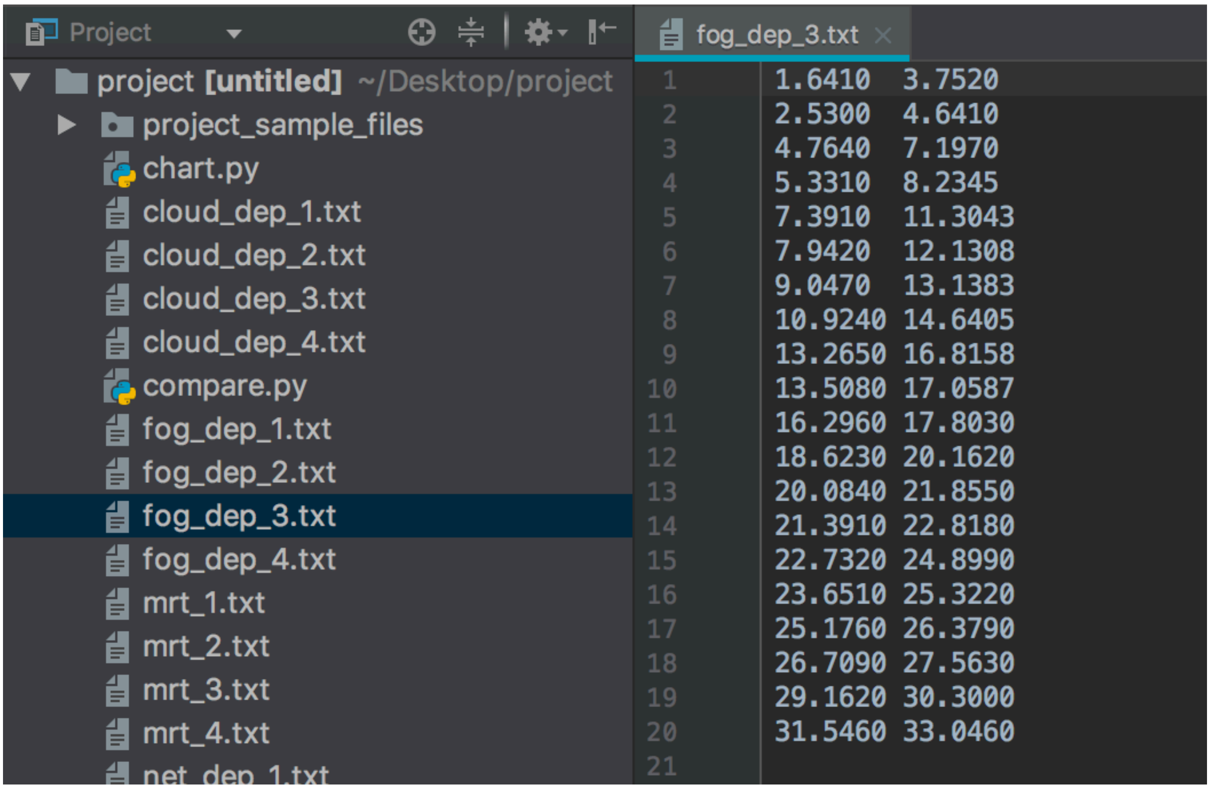
| arrival | service | network | fogTimeLimit | fogTimeToCloudTime |
|---|---|---|---|---|
| 1.641 | 3.222 | 0.622 | 1.500 | 0.800 |
| 2.530 | 1.893 | 1.847 | | |
| 4.764 | 4.058 | 0.860 | | |
| 5.331 | 2.664 | 0.370 | | |
| 7.391 | 1.754 | 1.810 | | |
| 7.942 | 4.695 | 1.959 | | |
| 9.047 | 4.380 | 0.878 | | |
| 10.924 | 2.751 | 0.222 | | |
| 13.265 | 3.112 | 0.516 | | |
| 13.508 | 2.935 | 0.817 | | |
| 16.296 | 1.039 | 0.000 | | |
| 18.623 | 1.506 | 0.524 | | |
| 20.084 | 2.355 | 1.206 | | |
| 21.391 | 1.152 | 0.000 | | |
| 22.732 | 4.222 | 0.443 | | |

| | | | | |
|---|---|---|---|---|
| 23.651 | 0.974 | 0.000 | | |
| 25.176 | 1.130 | 0.000 | | |
| 26.709 | 0.854 | 0.000 | | |
| 29.162 | 1.138 | 0.000 | | |
| 31.546 | 2.178 | 1.016 | | |

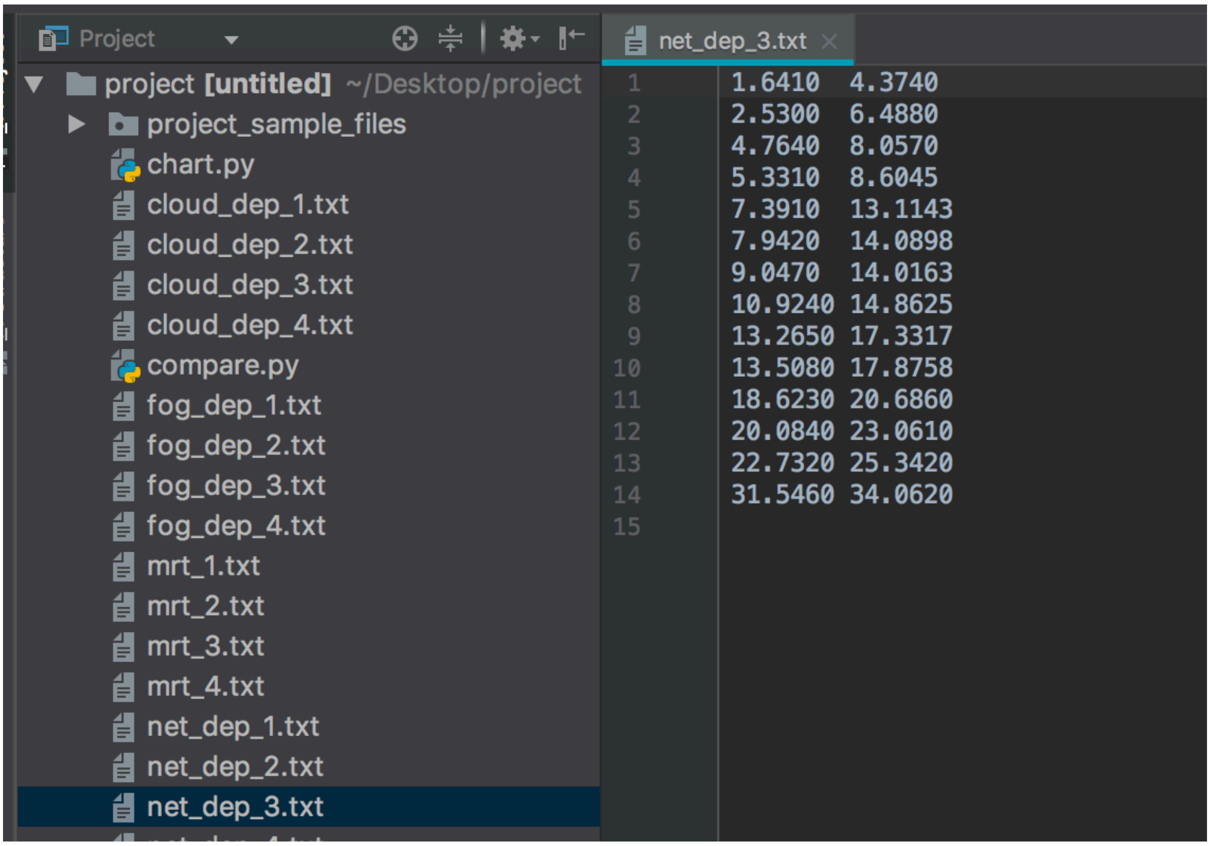
The mean response time is as following:



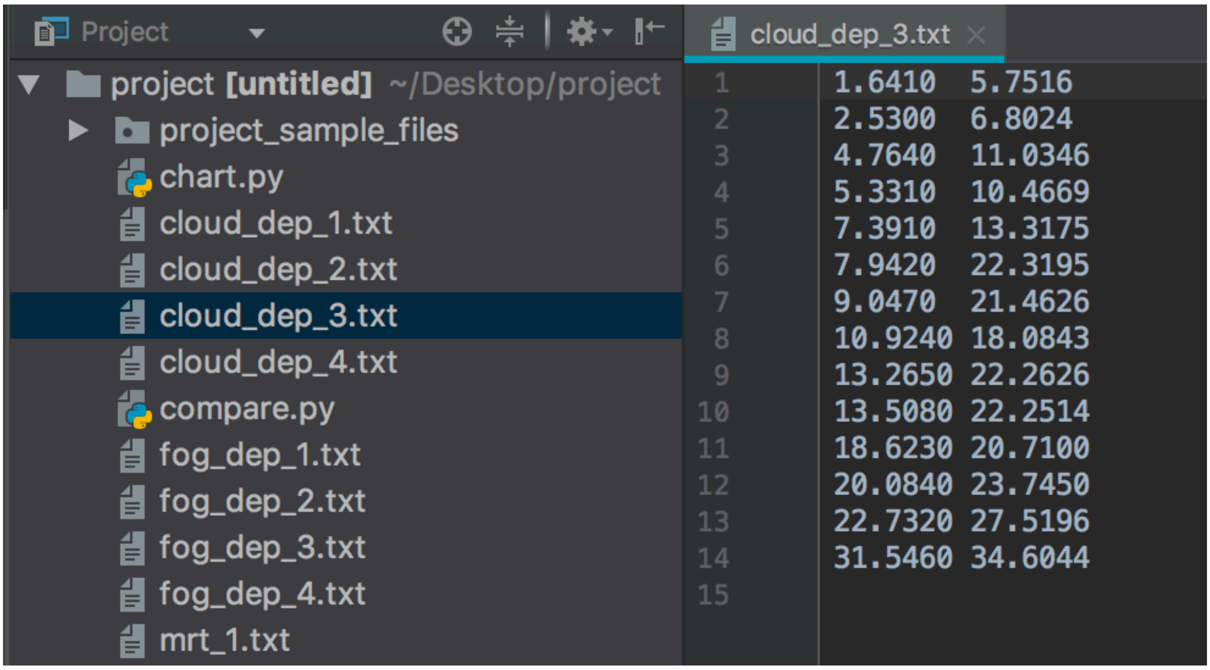

The fog departure time is as following:




The network departure time is as following:

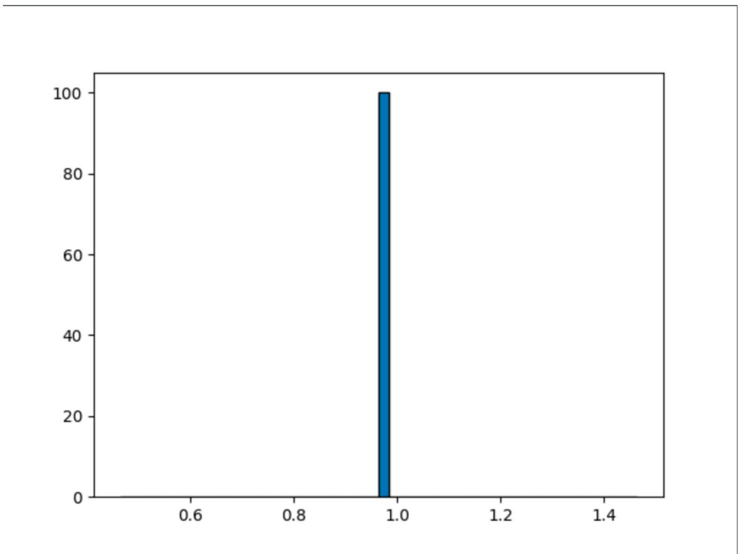



The cloud departure time is as following:




## 3. Reproducible

In my simulation program, given a fixed seed, my program will give identical outputs. In order to prove that, I would like to run a simulation 100 times with the same seed(1), and plot the mean response time in the figure below.



From the figure, we can see that the mean response times are the same over 100 simulations. Thus, given the same seed, my simulation program will always provide the same output.

# 4. Find suitable value for fogTimeLimit

## (1) Find mean response time and suitable value of fogTimeLimit

The parameters are below:

| $\alpha1$ | $\alpha2$ | $\beta$ | $\lambda$ | $v1$ | $v2$ | fogTimeToCloudTIme | seed | time_end |
|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.4 | 0.86 | 9.72 | 1.2 | 1.47 | 0.6 | 1 | 10000 |

| fogTimeLimit | Mean Response Time |
|---|---|
| 0 | 1.5827 |
| 0.10 | 0.8283 |
| 0.11 | 0.8284 |
| 0.12 | 0.8398 |
| 0.13 | 0.8672 |
| 0.14 | 0.9121 |
| 0.15 | 0.9864 |
| 0.16 | 1.0975 |
| 0.17 | 1.264 |
| 0.18 | 1.5483 |
| 0.19 | 2.1048 |

| | |
|---|---|
| 0.20 | 3.2986 |

From the above table, we can see that when fogTimeLimit is 0.1 or 0.11, the mean response time is better. In order to determine which one is better, I have done the following.

| $\alpha 1$ | $\alpha 2$ | $\beta$ | $\lambda$ | $v1$ | $v2$ | fogTimeToCloudTIme | seed | time_end |
|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.4 | 0.86 | 9.72 | 1.2 | 1.47 | 0.6 | 1 | 20000 |

| fogTimeLimit | Mean Response Time |
|---|---|
| 0.10 | 0.8227 |
| 0.11 | 0.82 |

| $\alpha 1$ | $\alpha 2$ | $\beta$ | $\lambda$ | $v1$ | $v2$ | fogTimeToCloudTIme | seed | time_end |
|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.4 | 0.86 | 9.72 | 1.2 | 1.47 | 0.6 | 1 | 30000 |

| fogTimeLimit | Mean Response Time |
|---|---|
| 0.10 | 0.8238 |
| 0.11 | 0.822 |

| $\alpha 1$ | $\alpha 2$ | $\beta$ | $\lambda$ | $v1$ | $v2$ | fogTimeToCloudTIme | seed | time_end |
|---|---|---|---|---|---|---|---|---|
| 0.01 | 0.4 | 0.86 | 9.72 | 1.2 | 1.47 | 0.6 | 1 | 40000 |

| fogTimeLimit | Mean Response Time |
|---|---|
| 0.10 | 0.8238 |
| 0.11 | 0.8219 |

From above we can see that the value of fogTimeLimit that gives the best system response time is 0.11.

(2) Transient Removals

In this part, I use seed(1), seed(10), seed(20) to generate the figure of mean response time.

seed(1):

| $\alpha 1$ | $\alpha 2$ | $\beta$ | $\lambda$ | $v1$ | $v2$ |
|---|---|---|---|---|---|
| 0.01 | 0.4 | 0.86 | 9.72 | 1.2 | 1.47 |
| fogTimeToCloudTIme | seed | time_end | fogTimeLimit | | |
| 0.6 | 1 | 1000 | 0.11 | | |

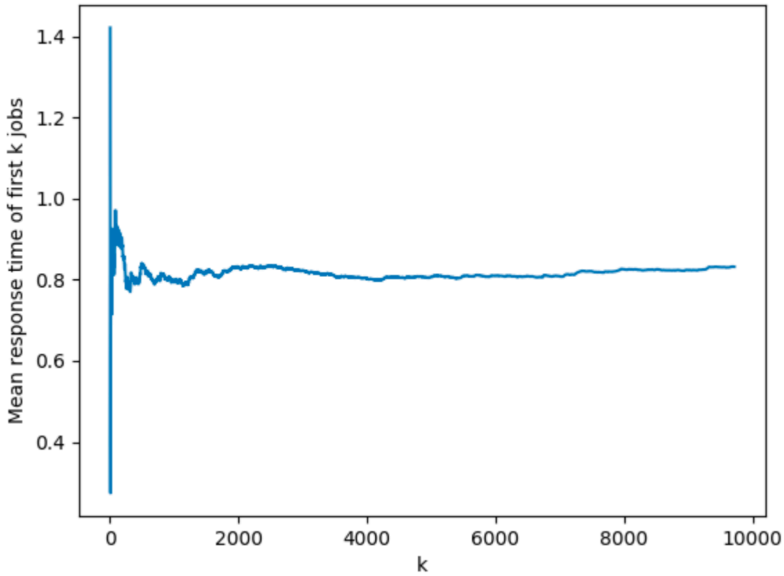



Fig 7. Mrt of k jobs when fogTimeLimit=0.11 (seed=1)

seed(10):

| $\alpha 1$ | $\alpha 2$ | $\beta$ | $\lambda$ | $v1$ | $v2$ |
|---|---|---|---|---|---|
| 0.01 | 0.4 | 0.86 | 9.72 | 1.2 | 1.47 |
| fogTimeToCloudTIme | seed | time_end | fogTimeLimit | | |
| 0.6 | 10 | 1000 | 0.11 | | |

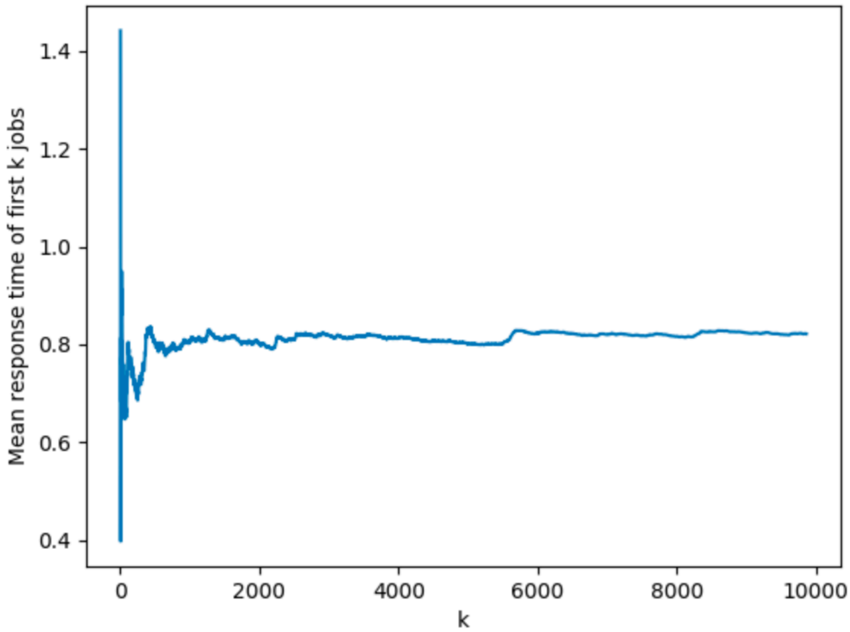



Fig 8. Mrt of k jobs when fogTimeLimit=0.11 (seed=10)

seed(20)

| $\alpha 1$ | $\alpha 2$ | $\beta$ | $\lambda$ | $v1$ | $v2$ |
|---|---|---|---|---|---|
| 0.01 | 0.4 | 0.86 | 9.72 | 1.2 | 1.47 |
| fogTimeToCloudTIme | seed | time_end | fogTimeLimit | | |
| 0.6 | 20 | 1000 | 0.11 | | |

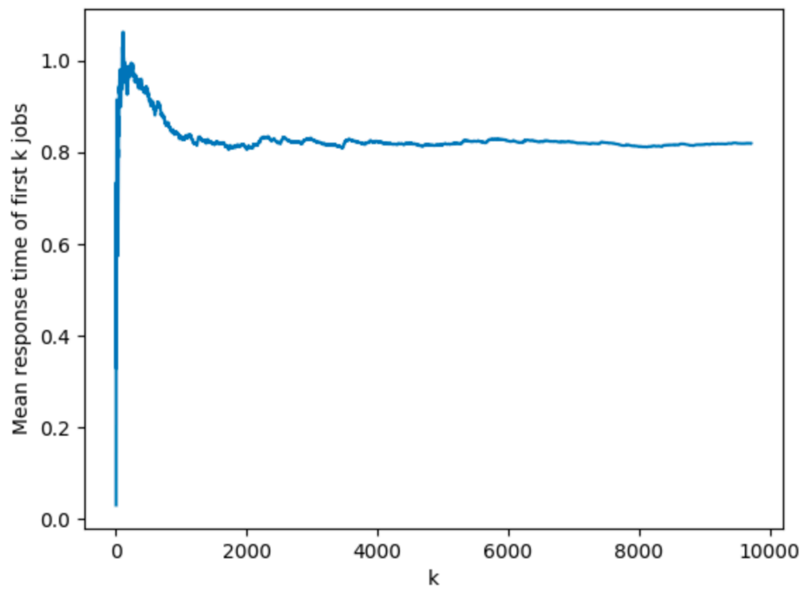



Fig 9. Mrt of k jobs when fogTimeLimit=0.11 (seed=20)

The early part of the simulation displays transient. The later part of the simulation converges or fluctuates around the steady state value. Since we are interested in the steady state value, we should not use the transient part of the data to compute the steady state value. We should remove the transient part and only use the steady state part to compute the mean. One method to identify the transient part is to use visual inspection

It can be seen from the line chart that first 2000 jobs can be removed.

And the figure after transient removals are below:

seed(1):

fogTimeLimit = 0.11

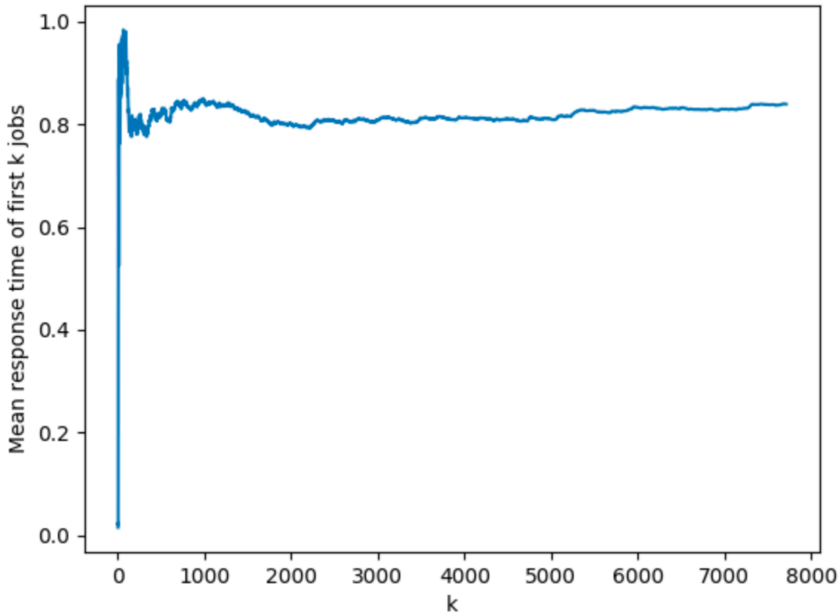



Fig 10. Transient removal when fogTimeLimit=0.11 (seed=1)

## seed(10):

fogTimeLimit = 0.11

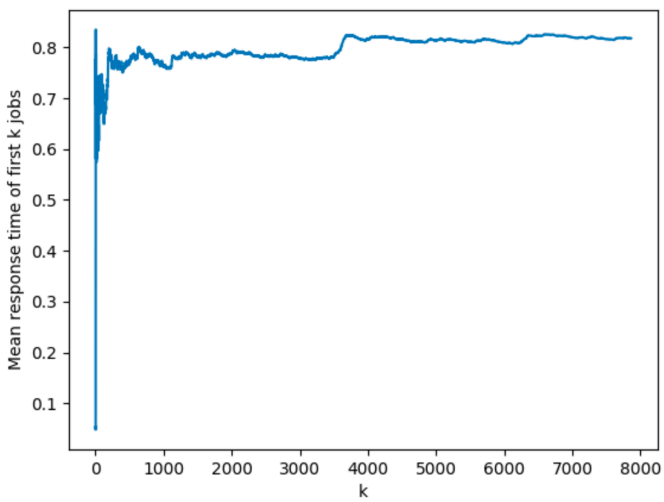



Fig 11. Transient removal when fogTimeLimit=0.11 (seed=10

seed(20):

fogTimeLimit = 0.11

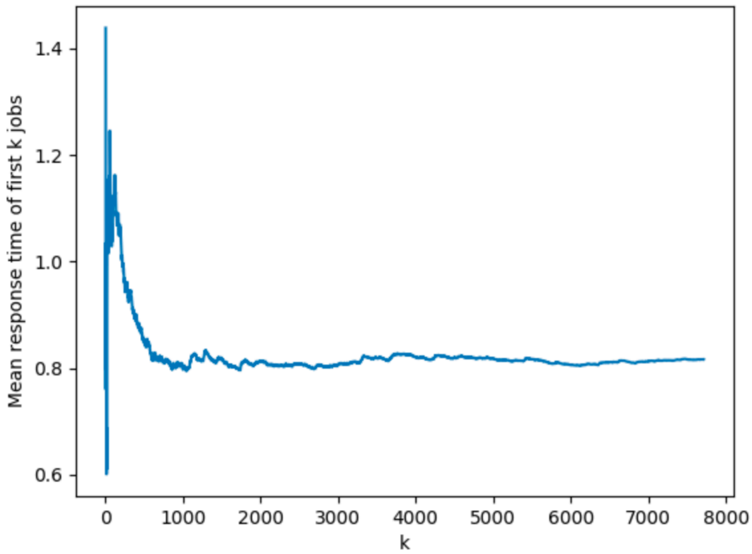



Fig 12. Transient removal when fogTimeLimit=0.11 (seed=20)

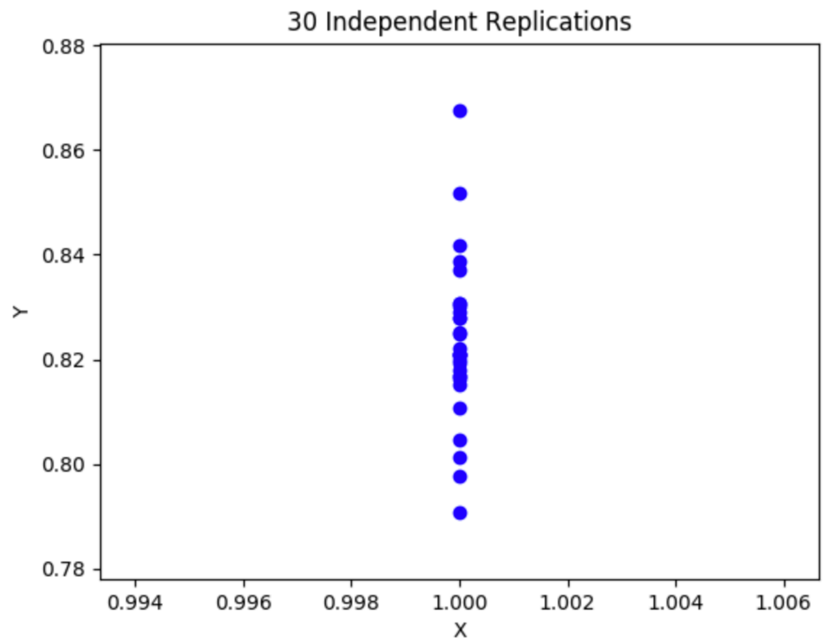
(3) 30 independent replications

| $\alpha 1$ | $\alpha 2$ | $\beta$ | $\lambda$ | $v1$ | $v2$ |
|---|---|---|---|---|---|
| 0.01 | 0.4 | 0.86 | 9.72 | 1.2 | 1.47 |
| fogTimeToCloudTIme | time_end | fogTimeLimit | jobs remove | | |
| 0.6 | 1000 | 0.11 | 2000 | | |

| seed | Mean response time |
|---|---|
| 1 | 0.8387 |
| 2 | 0.82 |
| 3 | 0.8194 |
| 4 | 0.8168 |
| 5 | 0.822 |
| 6 | 0.828 |
| 7 | 0.8045 |
| 8 | 0.8418 |
| 9 | 0.8252 |
| 10 | 0.8179 |
| 11 | 0.8208 |
| 12 | 0.828 |
| 13 | 0.8371 |
| 14 | 0.8013 |
| 15 | 0.8152 |
| 16 | 0.8301 |
| 17 | 0.7976 |
| 18 | 0.8307 |
| 19 | 0.8169 |
| 20 | 0.8164 |
| 21 | 0.8517 |
| 22 | 0.7907 |

| | |
|---|---|
| 23 | 0.8211 |
| 24 | 0.8306 |
| 25 | 0.8209 |
| 26 | 0.8209 |
| 27 | 0.825 |
| 28 | 0.8677 |
| 29 | 0.8107 |
| 30 | 0.8291 |

fogTimeLimit = 0.11




The blue circles show the estimated mean response time from the 30 independent experiments.

(4) Computing the confidence interval

In each replication, after removing the transient part and compute an estimate of the mean steady state response time For fogTimeLimit = 0.11, the mean steady state response time is equal to 0.8232266666666669

Let us call the estimate from the kth replication, T(k)

$$\hat{T} = \frac{\sum_{i=1}^{n} T(i)}{n}$$

the sample standard deviation

$$\hat{S} = \sqrt{\frac{\sum_{i=1}^{n} (\hat{T} - T(i))^2}{n-1}}$$

```
import statistics


y = [0.8387, 0.82, 0.8194, 0.8168, 0.822, 0.828, 0.8045, 0.8418, 0.8252, 0.8179, 0.8208, 0.828,
     0.8371, 0.8013, 0.8152, 0.8301, 0.7976, 0.8307, 0.8169, 0.8164, 0.8517, 0.7907, 0.8211,
     0.8306, 0.8209, 0.8209, 0.825, 0.8677, 0.8107, 0.8291]


mean_mrt = statistics.mean(y)
s = statistics.stdev(y)
print(mean_mrt)
print(s)
```

From the code above, we can get that

For fogTimeLimit = 0.11

T = 0.8232266666666667 (the sample arithmetic mean)

S = 0.01529216462279212 (the sample standard deviation)

There is a probability $(1-\alpha)$ that the mean response time that you want to estimate lies in the interval

$$[\hat{T} - t_{n-1,1-\frac{\alpha}{2}} \frac{\hat{S}}{\sqrt{n}}, \hat{T} + t_{n-1,1-\frac{\alpha}{2}} \frac{\hat{S}}{\sqrt{n}}]$$

- The sample mean of (n = 30) replications for fogTimeLimit = 0.11 is 0.8232
- The sample standard deviation of 30 replications for fogTimeLimit = 0.11 is 0.01529216462279212
- If we want to compute the 95% confidence interval, $\alpha = 0.05$
- Since we did 30 independent replications and want 95% confidence interval, we use t(29, 0975)
  From the t-distribution table, the value of 2.0452, the 95% confidence interval for fogTimeLimit = 0.11 is [0.8232-2.0452*(0.0153/$\sqrt{30}$), 0.8232 + 2.0452*(0.0153/$\sqrt{30}$)]

  Using python to compute this equation, we can get:
  fogTimeLimit = 0.11:

```
# fogTimeLimit = 0.11
a1 = 0.8232 - 2.0452*(0.0153/(30**(1/2)))
print(a1)

b1 = 0.8232 + 2.0452*(0.0153/(30**(1/2)))
print(b1)
```

```
0.8174869689094912
0.8289130310905088
```

| $\alpha 1$ | $\alpha 2$ | $\beta$ | $\lambda$ | $v1$ | $v2$ |
|---|---|---|---|---|---|
| 0.01 | 0.4 | 0.86 | 9.72 | 1.2 | 1.47 |
| fogTimeToCloudTIme | time_end | fogTimeLimit | jobs remove | | |
| 0.6 | 1000 | 0.11 | 2000 | | |

There is a 95% probability that the true mean response time that we want to estimate is in the interval [0.8175, 0.8289] under a conditions from the above table.