COMP9334
Capacity Planning for Computer Systems and Networks

Week 6B_1: Discrete event simulation (3).

Comparing two systems.

Discrete event simulations so far

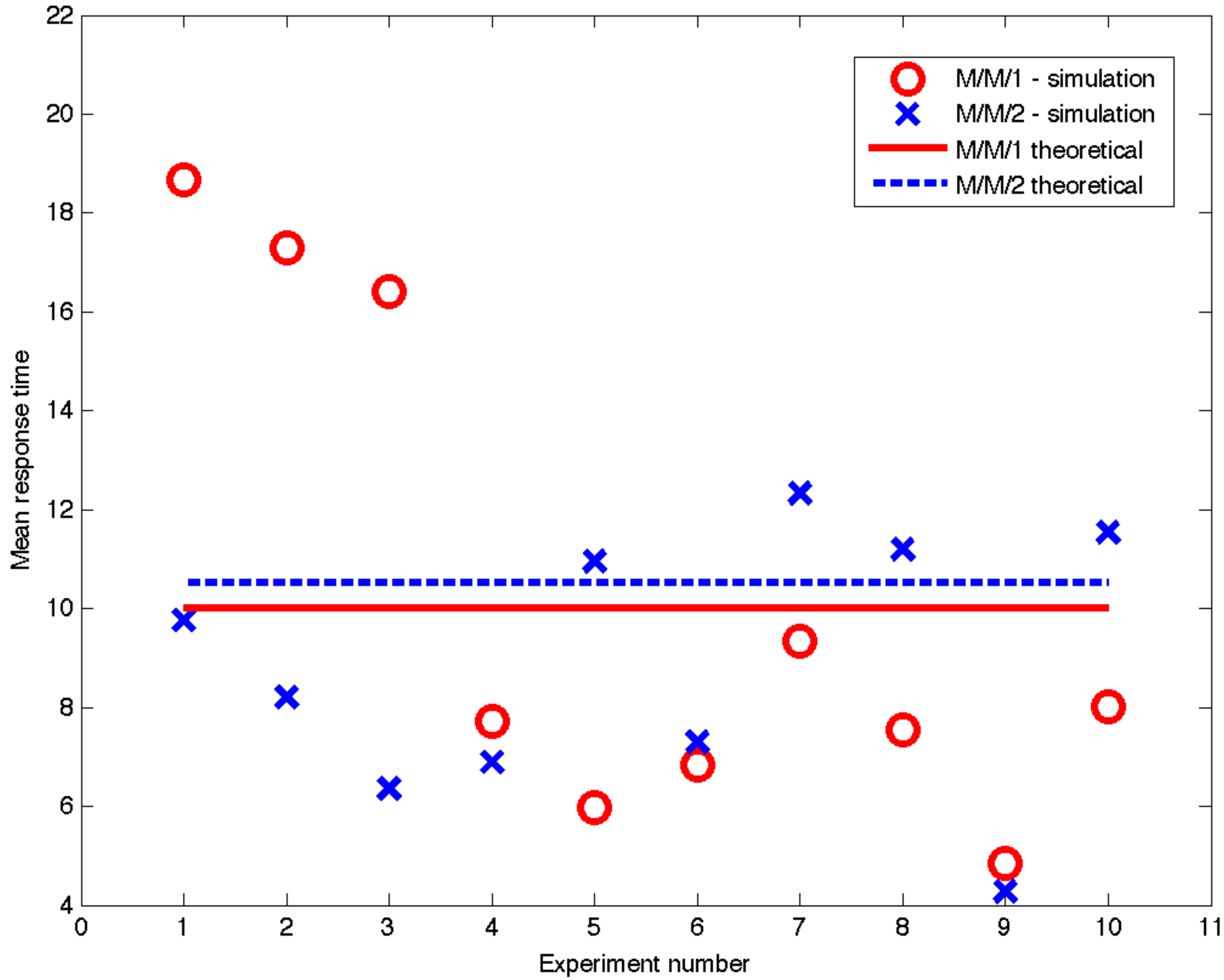
- You have learnt:
  - How to write simulation program
  - You know you cannot get exact mean response time from simulation but you can get a confidence interval
  - You can reduce the width of the confidence interval by
    - Simulate for longer
    - Increase the number of independent replications

- Today, you will learn how you can compare two systems in a statistically sound way

- Before that, we show you that comparing systems can be tricky

Problem: How do we compare 2 alternative choices?

- Week 5A's Revision Problem, #2
- From Queueing theory, we expect the M/M/1 system to be better but simulation doesn't seem to suggest that?



Comparing two systems: motivation

- An application of simulation is to compare two systems

- For example, in Week 5A's revision question, you used simulation to compare the mean response time of
  - System 1: M/M/1 queue with $\lambda = 0.9$ and $\mu = 1$
  - System 2: M/M/2 queue with $\lambda = 0.9$ and $\mu = 0.5$ for both server

- If you use analytical method, you can find the steady state mean response time of both systems exactly and you compare two numbers

- If you use simulation, you get a confidence interval for each system instead. How do you compare them?

Example: Comparing two systems

- Let us assume our goal is to use simulation to compare:
  - System 1: M/M/1 queue with $\lambda = 0.9$ and $\mu = 1$
  - System 2: M/M/2 queue with $\lambda = 0.9$ and $\mu = 0.5$ for both server

- For each system we carry out 3 independent replications
  - That is, we use 6 sets of independent random numbers together

- After removing the transient, the estimated mean response times are:
  - System 1: 6.8769, 8.5769, 10.6340
  - System 2: 8.8087, 7.4616, 9.1565

- In order to compare them, let us pair up these results
  - 1st experiment for System 1 with 1st experiment for System 2
  - 2nd experiment for System 1 with 2nd experiment for System 2 etc.

A paired-*t* confidence interval

- Let us summarise the data in a table
  - EMRT = estimated mean response time

	EMRT System 1	EMRT System 2	EMRT System 2 - EMRT System 1
Rep. 1	6.8769	8.8087	1.9318
Rep. 2	8.5769	7.4616	-1.1154
Rep. 3	10.6340	9.1565	-1.4775

- We compute the 100 (1-α)% confidence interval of the difference between 2 systems (= last column)

- Let us denote the computed confidence interval by [p,q]
  - Case 1: p,q > 0 ➔ System 1 is better than System 2 with probability (1-α)
  - Case 2: p,q < 0 ➔ System 2 is better than System 1 with probability (1-α)
  - Case 3: q > 0 & p < 0 ➔ Systems 1 and 2 are not different with probability (1-α)

Example: Paired-*t* confidence interval

- We compute the 95% confidence interval of the data showed in the last slide, the confidence interval is:
  - [-4.8721,4.4314]
- Therefore, with 95% probability that the mean response times of the two systems are not different
- Hmmm, we have a problem here, we know from queueing theory that System 1 has a better mean response time than System 2, but our simulation does not seem to be able to distinguish them.
- What can we do?

Let us increase the number of replications

- Since increasing the number of replications can reduce the width of the confidence interval, let us try that.
- Let us try 5, 10, 20, 30 replications

# independent replications	95% Confidence interval of EMRT System 2 - EMRT System 1
5	[-4.9540, 5.0242]
10	[-1.5347, 2.8020]
20	[-1.2724, 1.9870]
30	[-0.6001, 1.8046]

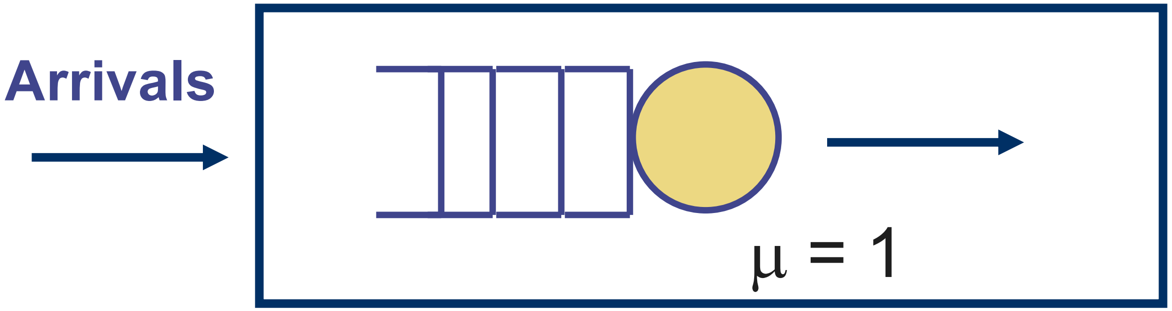
- Increasing the number of replications does reduce the width of the confidence interval
- However, we still cannot conclude which system is better

Let us have a look at how we did our experiments …

- We did our experiment with independent random numbers

Job #	arrival time	service time
1	0.6329	0.3431
2	3.6392	0.4319
3	4.3919	0.1049
…	…	….

System 1

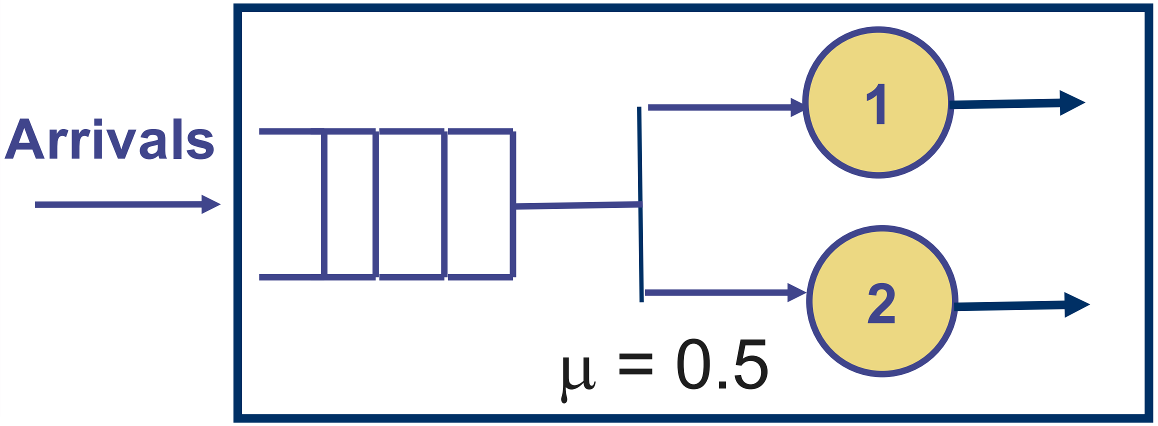


Identical probability distribution but different numbers

Different probability distribution and independently generated

Job #	arrival time	service time
1	0.2548	0.3228
2	4.1194	0.3877
3	6.4435	0.0728
…	…	….

System 2



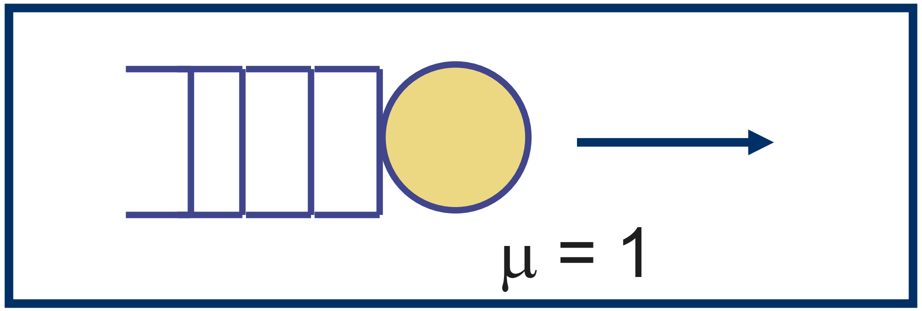
Common random numbers method

- An alternative is to compare two systems under similar condition

Job #	arrival time	service time
1	0.6329	0.3432
2	3.6392	0.4320
3	4.3919	0.1050
…	…	….

Arrivals →

System 1



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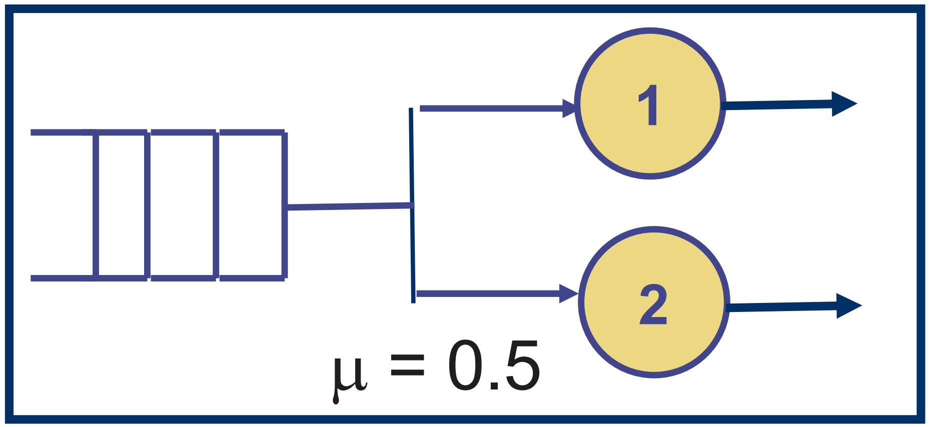
Multiply by 2

Job #	arrival time	service time
1	0.6329	0.6864
2	3.6392	0.8640
3	4.3919	0.2100
…	…	….

Arrivals →

System 2



Common random numbers method

- A method to reduce the variance when comparing two alternative systems is to subject them to similar experimental condition

- In each replication, generate only one arrival time and one service time sequence
  - Apply this to both systems
  - Note: Service time may need to be adjusted according to service rate

- In next replication, generate a new arrival time and a new service time sequence
  - Apply this to both systems

- This method can reduce the variance if the behaviour of the two systems is positively correlated

Applying common random numbers to our problem (1)

- Let us apply the common random numbers method to compare
  - System 1: M/M/1 queue with λ = 0.9 and μ = 1
  - System 2: M/M/2 queue with λ = 0.9 and μ = 0.5 for both server
- Let us carry out 5 replications
- In each replication, we generate one arrival time sequence and one service time sequence (adjusted by service rate) and apply to both systems

Applying common random numbers to our problem (2)

- Let us compare the estimated mean response time (EMRT) from the 5 replications:

	EMRT System 1	EMRT System 2	EMRT System 2 - EMRT System 1
Rep. 1	8.3022	8.8087	0.5065
Rep. 2	6.8809	7.4616	0.5807
Rep. 3	8.5769	9.1565	0.5796
Rep. 4	10.6340	11.3409	0.7069
Rep. 5	16.2648	16.6485	0.3837

- Observation: The EMRT of System 2 is higher than that of System 1 in all 5 replications
- If we compute the 95% confidence interval of the last column, we get [0.4046,0.6983]
- There is a 95% probability that System 1 is better than System 2

Comparing two methods

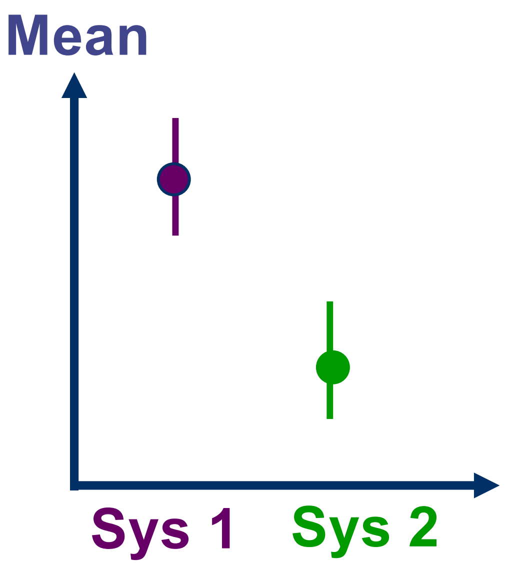
- Let us compare using common random number (CRN) method or not

# independent replications	95% Confidence interval of EMRT System 2 - EMRT System 1	
	Not using CNR	Using CNR
5	[-4.9540, 5.0242]	[0.4046, 0.6983]
10	[-1.5347, 2.8020]	[0.4705, 0.6103]
20	[-1.2724, 1.9870]	[0.5127, 0.5942]
30	[-0.6001, 1.8046]	[0.5026, 0.5786]

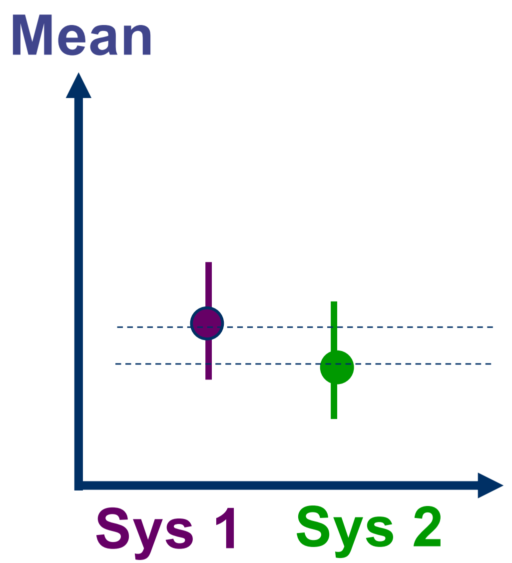
- Observations
  - By using CRN, all 95% confidence interval does not include 0
  - The width of the confidence interval for CRN method is a lot lower!

Approximate visual test

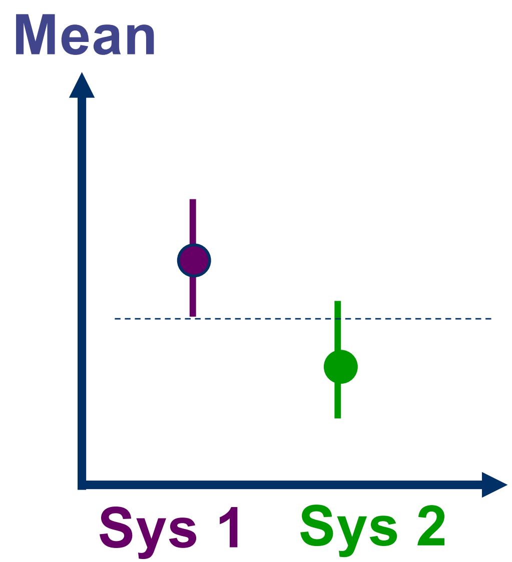
- Let us assume that you know the mean response time and its confidence interval (CI) for 2 systems: **System 1** and **System 2**
- Consider the following 3 possibilities:



CIs do not overlap Mean of System 1 > Mean of Sys. 2



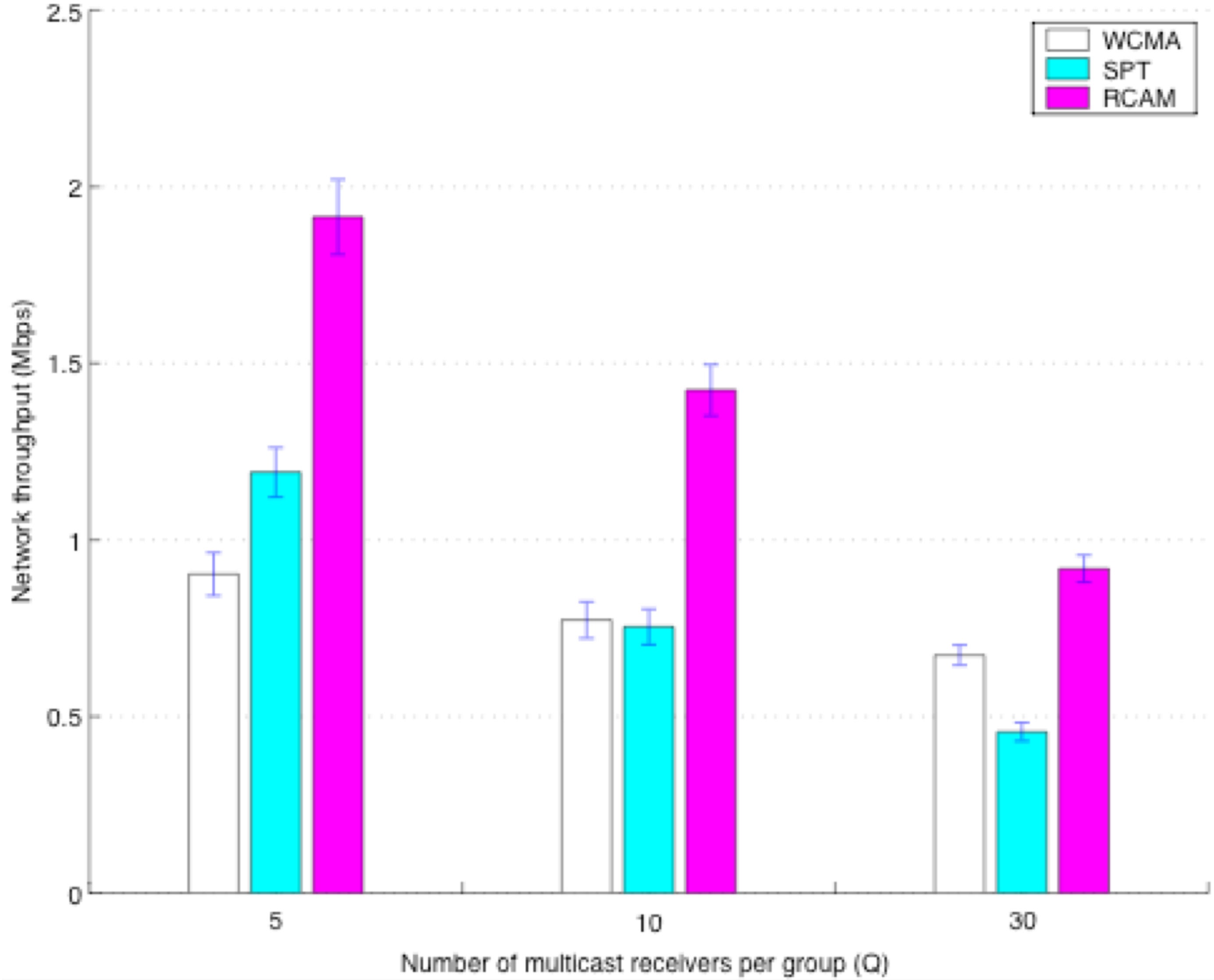
CIs overlap and mean of a system is in the CI of the other: System are *not* different



CIs overlap and mean of any one is not in the CI of the other: do *t*-test

Ex: Multicast protocol design for wireless mesh networks

- Comparing 3 multicast protocols (WCMA, **SPT** and **RCAM**) for wireless mesh networks
- The thin vertical line shows the confidence interval
- What conclusion can you draw?



- Source: Chou et al, "Maximizing Broadcast and Multicast Traffic Load through Link-Rate Diversity in Wireless Mesh Networks", you can download it from my web site: http://www.cse.unsw.edu.au/~ctchou/

Simulation tools and some applications (1)

- You do not always have to write your own simulation programs from scratch
- There are plenty of simulation tools available
  - Many with GUI
- Simulation tools are used in a lot in computer networking research
  - Protocol #1 is the existing protocol, you have designed Protocol #2. You want to see whether Protocol #2 is better or not.
  - You have two options (Option #1 and Option #2) to design a network. Which option is better?

Simulation tools and some applications (2)

- Some examples of publicly available simulation tools
  - General purpose: OMNet++
    - http://www.omnetpp.org/
  - For networking research: ns3
    - http://www.isi.edu/nsnam/ns/
- Some commercial tools
  - For network design: OPNET, Qualnet
    - http://www.opnet.com/
    - http://web.scalable-networks.com/content/qualnet
- **Important note: These tools save you time in writing simulation program but don't forget that you still need to analyse your simulation results using statistically sound methods!**

Summary

- Simulation is not just a computer programming exercise
- You need to make sure that your program is correct
- It is also important to analyse your results statistically
- Unfortunately, a lot of published research papers in computer networking did not do sound statistical analysis
  - Optional reading: Pawlikowski et al, “On credibility of simulation studies of telecommunication networks”, IEEE Communications Magazine, Pages 132-139, January 2002.

References

- The primary reference is Law and Kelton, “Simulation Modelling and Analysis”
  - Comparing two alternatives, Section 10.1, 10.2 (10.2.1 only)
  - Common random numbers, Section 11.2
- Raj Jain, “The Art of Computer Systems Performance Analysis” has materials on
  - Comparing two alternatives, Sections 13.3, 13.4 (13.4.1 and 13.4.3 only)
- Note that we have only touched on the basic of statistical analysis of simulation data. The above two books (outside the specified sections) will provide you with more in depth discussion on the topic.