

COMP9334 Solution to the revision problems for Week 8A

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March 28, 2019

1. (a) Let us define a few notation:

- T_{\max} = computation time limit = 4000s
- P = total number of cycles required = 10^7 Mcycles.
- p_i = Speed (in Mcycles/s) of company i , e.g. $p_1 = 1000$ Mcycles/s
- c_i = Per-second charge for company i , e.g. $c_1 = \$0.1/\text{s}$.
- s_i = Set up cost for company i , e.g. $s_1 = 500$.

The decision variables are y_i and x_i (for $i = 1, 2, 3$):

$$y_i = \begin{cases} 1 & \text{if Company } i \text{ is chosen} \\ 0 & \text{otherwise} \end{cases}$$

$$x_i = \text{fraction of cycles to be bought from Company } i \text{ and is } \geq 0$$

Based on these decision variables, we know that

- Completion time T is:

$$T = \max_i \frac{Px_i}{p_i}$$

- Total cost C is:

$$C = \sum_{i=1}^n \left(\frac{c_i Px_i}{p_i} + s_i y_i \right)$$

The problem formulation is

$$\begin{aligned} & \min C \\ & \text{subject to} \\ & T_{\max} \geq \frac{Px_i}{p_i} & 1 \leq i \leq n \\ & C = \sum_{i=1}^n \left(\frac{c_i Px_i}{p_i} + s_i y_i \right) \\ & x_i \leq y_i & 1 \leq i \leq n \\ & \sum_{i=1}^3 x_i = 1 \\ & x_i \geq 0 & 1 \leq i \leq n \\ & y_i \in \{0, 1\} & 1 \leq i \leq n \end{aligned}$$

- (b) By using an integer programming solver, we find that we should buy 8×10^6 Mcycles from Company 2 and 2×10^6 Mcycles from Company 3. This costs a total of \$1833.
The files used to solve this problem are `hw_grid.mod`, `hw_grid.dat` and `hw_grid_batch`.

2. (a) Since Database 1 can only be placed at one of the three locations, there are three possible combinations for x_{11} , x_{21} and x_{31} :

- i. $x_{11} = 1$, $x_{21} = 0$ and $x_{31} = 0$
- ii. $x_{11} = 0$, $x_{21} = 1$ and $x_{31} = 0$
- iii. $x_{11} = 0$, $x_{21} = 0$ and $x_{31} = 1$

This means $x_{11} + x_{21} + x_{31} = 1$.

- (b) The decision variables are

$$x_{ij} = \begin{cases} 1 & \text{if Database } j \text{ is stored in Location } i \\ 0 & \text{otherwise} \end{cases}$$

We use the notation:

- D_i = capacity of Location i
- t_i = access time of Location i
- s_j = size of Database j
- f_j = frequency of accessing Database j
- F = total number of accesses per second

The (binary) integer programming problem is:

$$\min \frac{1}{F} \sum_{i=1}^3 \sum_{j=1}^8 f_j t_i x_{ij}$$

subject to

$$\begin{aligned} \sum_{j=1}^8 s_j x_{ij} &\leq D_i \quad \text{for } 1 \leq i \leq 3 \\ \sum_{i=1}^3 x_{ij} &= 1 \quad \text{for } 1 \leq j \leq 8 \\ x_{ij} &\in \{0, 1\} \end{aligned}$$

Note the first set of constraints enforces capacity limit of each location. The second set of constraints ensures that each database is stored at exactly one location.

- (c) The location is as showed below.

- Databases 2,3,4 at Location 1.
- Databases 1,7,8 at Location 2.
- The rest at Location 3.

The files used to solve this problem are `hw_db.mod`, `hw_db.dat` and `hw_db_batch`.