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Question 1

(a) According to Service demand law:

$$D(j) = \frac{V(j)}{X(0)} \qquad V(j) = \frac{B(j)}{T} \qquad X(0) = \frac{C(0)}{T}$$

$B(j) = B(Disk) = 2565$ seconds

$B(cpu) = 4729$ seconds

$T = 90$ minutes $= (90 \times 60)$ seconds $= 5400$ seconds.

$C(0) = 676$

$$\because \frac{V(j)}{X(0)} = \frac{B(j)}{T} \times \frac{T}{C(0)} = \frac{B(j)}{C(0)}$$

$$\therefore D(disk) = \frac{2565}{676} \approx 3794 \text{ ms}$$

$$D(cpu) = \frac{4729}{676} \approx 6996 \text{ ms}.$$

(b) Yes, it is possible to determine the bottleneck of the system ~~with~~ without calculating the service demands..

First, Assume the cpu. is the bottleneck. we can get

$U(cpu) = U(0) = 1$. $X(0) = X(cpu)$

$\because U(cpu) = S(cpu)\,X(0)$

$$\therefore X(0) = \frac{U(cpu)}{S(cpu)} = \frac{1}{S(cpu)} = U(cpu) = \frac{C}{B(cpu)} = \frac{676}{4729} = 0.143$$

Second, Assume ~~the~~ the disk is the bottleneck, we can get

$U(disk) = U(0) = 1$. $X(0) = X(disk)$

$\because U(disk) = S(disk)\,X(0)$

$$\therefore X(0) = \frac{U(disk)}{S(disk)} = \frac{1}{S(disk)} = U(disk) = \frac{C}{B(disk)} = \frac{676}{2565} = 0.264$$

$X(0) \le min(0.143, 0.264)$.

Therefore, the bottleneck of the system is 0.143.

(c) Since the bottleneck analysis is

$$X(0) \le min\left[\frac{1}{maxD_i}, \frac{N}{\sum_{i=1}^{k} D_i}\right]$$

if there is thinking time, then bottleneck analysis is

$$X(0) \le min\left[\frac{1}{maxD_i}, \frac{N}{\sum_{i=1}^{k} D_i + thinking\ time}\right]$$

$\because D(disk) = 3.794$ ms $\quad D(cpu) = 6.996$ ms

$\therefore \frac{1}{maxD_i} = \frac{1}{6.996} = 0.1429$ (jobs/s)

$\because$ there are 30 interactive users and thinking time per job is 31 second.

$$\therefore \frac{N}{\sum_{i=1}^{k} D_i + thinking\ time} = \frac{30}{3.794 + 6.996 + 31} = \frac{30}{41.79} = 0.7179 \text{ (jobs/s)}$$

Therefore, the asymptotic bound should be 0.1429 (jobs/s).

(d) the number of interactive users = max system Throughput * (thinking time + min Response time).

$\therefore$ ~~min Response time~~ $= \frac{N}{\cancel{Asymptotic\ b}}$

$$\therefore min\ Response\ time = \frac{N}{Asymptotic\ bound} - Thinking\ Time.$$

$$= \frac{30}{0.1429} - 31$$

$$= 178.94 \text{ (s)}.$$

Question 2

(a). For System 1 : $\lambda_1 = P \cdot \lambda = 20P$. $\quad \rho_1 = \frac{\lambda_1}{\mu_1} = \frac{20P}{10}$.

For System 2 : $\lambda_2 = (1-P)\lambda = 20(1-P)$. $\quad \rho_2 = \frac{\lambda_2}{\mu_2} = \frac{20(1-P)}{15}$.

$\because$ System 1 and 2 have the same utilisation.

$\therefore \frac{20P}{10} = \frac{20(1-P)}{15}$

$\Rightarrow P = 0.4$.

(b). For system 1 : $\lambda_1 = 20 \times 0.4 = 8$. $\quad \mu_1 = 10$.

For system 2 : $\lambda_2 = 20 \times 0.6 = 12 \quad \mu_2 = 15$

therefore we can get.

System 1 : $T_1 = \frac{1}{\mu_1 - \lambda_1} = \frac{1}{10-8} = 0.5$

System 2 : $T_2 = \frac{1}{\mu_2 - \lambda_2} = \frac{1}{15-12} = 0.33$

$\Rightarrow T = 0.4 T_1 + 0.6 T_2 = 0.4 \times 0.5 + 0.6 \times 0.33 = 0.398$.

(c) For System 1 : $\lambda_1 = 20P \quad \mu_1 = 10$. $\quad T_1 = \frac{1}{10-20P}$

For System 2 : $\lambda_2 = 20(1-P)$. $\quad \mu_2 = 15$. $\quad T_2 = \frac{1}{15-20(1-P)} = \frac{1}{20P-5}$.

we can get $T = PT_1 + T_2(1-P) = \frac{P}{10-20P} + \frac{1-P}{20P-5}$

~~T = 0.3876~~ $T = 0.3876$

About the calculation, please refer to q2c.py in supp.zip

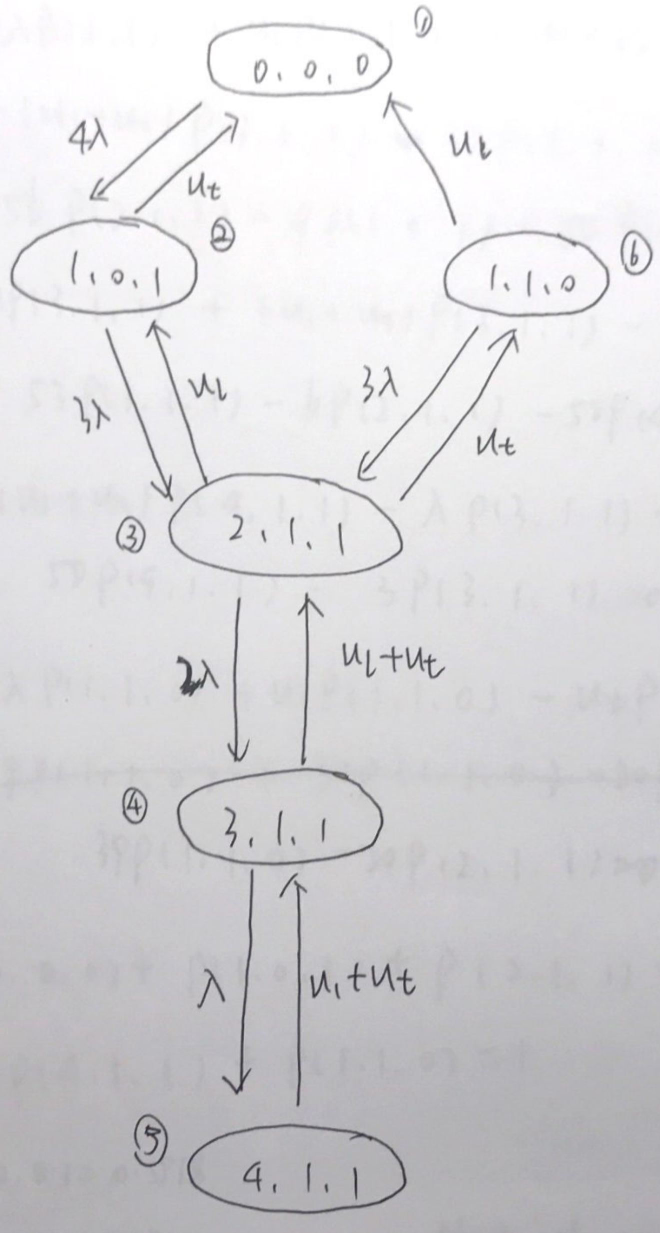
Question 3

(a) from the context, we ~~kno~~ can get there are 6 states

$P(0,0,0)$. $P(1,0,1)$. $P(2,1,1)$. $P(1,1,0)$. $P(3,1,1)$

$P(4,1,1)$

Therefore, the transition diagram is below.



$\lambda = \frac{1}{600}$

$u_t = \frac{1}{90}$

$u_1 = \frac{1}{60}$

(b)

① $4\lambda P(0,0,0) - \mu_t P(1,0,1) - \mu_L P(1,1,0) = 0$

$12P(0,0,0) - 20P(1,0,1) - 30P(1,1,0) = 0$

② $3\lambda P(1,0,1) + \mu_t P(1,0,1) - 4\lambda P(0,0,0) - \mu_L P(2,1,1) = 0.$

$29P(1,0,1) - 12P(0,0,0) - 30P(2,1,1) = 0$

③ $2\lambda P(2,1,1) + \mu_L P(2,1,1) + \mu_t P(2,1,1) - 3\lambda P(1,0,1)$

$- (\mu_1+\mu_t) P(3,1,1) - 3\lambda P(1,1,0) = 0$

$56P(2,1,1) - 9P(1,0,1) - 50P(3,1,1) - 9P(1,1,0) = 0$

④ $\lambda P(3,1,1) + (\mu_1+\mu_t) P(3,1,1) - 2\lambda P(2,1,1) - (\mu_1+\mu_t) P(4,1,1) = 0$

$53P(3,1,1) - 6P(2,1,1) - 50P(4,1,1) = 0$

⑤ $(\mu_1+\mu_t) P(4,1,1) - \lambda P(3,1,1) = 0$

$50P(4,1,1) - 3P(3,1,1) = 0$

⑥ $3\lambda P(1,1,0) + \mu_L P(1,1,0) - \mu_t P(2,1,1) = 0$

$\cancel{9P(1,1,0) + 30P(1,1,0) - 20P(2,1,1) = 0}$

$39P(1,1,0) - 20P(2,1,1) = 0$

⑦ $P(0,0,0) + P(1,0,1) + P(2,1,1) + P(3,1,1)$

$+ P(4,1,1) + P(1,1,0) = 1.$

(c). $P(0,0,0) = 0.5918$

$P(1,0,1) = 0.3081$

$P(1,1,0) = 0.0313$

$P(2,1,1) = 0.0611$

$P(3,1,1) = 0.0073$

$P(4,1,1) = 0.0004$

About the calculation. ~~please~~ please refer to q3c.m in supp.zip.

(d) the probability that at least 3 machines are available is

$$P = P(0.0.0) + P(1.0.1) + P(1.1.0)$$

$$= 0.5918 + 0.3081 + 0.0313$$

$$= 0.9312$$

(e) We can get the table below

k	P(k)
0	0.5918
1	0.3081 + 0.0313 = 0.3394
2	0.0611
3	0.0073
4	0.0004

therefore, the mean number failed machines is

$$\cancel{nb = 1 \times 0.3385}$$

$$nb = 1 \times 0.3394 + 2 \times 0.0611 + 3 \times 0.0073 + 4 \times 0.0004$$

$$= 0.4851$$

(f). $N_{avg} = 1 \times P(1.0.1) + 1 \times P(1.1.0) + 2 \times P(2.1.1) + 3 \times P(3.1.1) + 4 \times P(4.1.1)$

$$= 1 \times 0.3081 + 1 \times 0.0313 + 2 \times 0.0611 + 3 \times 0.0073 + 4 \times 0.0004$$

$$= 0.4851$$

$$X = 4\lambda P(0.0.0) + 3\lambda P(1.1.0) + 3\lambda P(1.0.1) + 2\lambda P(2.1.1) + \lambda P(3.1.1)$$

$$= 4 \times \frac{1}{600} \times \cancel{0.3081}\ 0.5918 + 3 \times \frac{1}{600} \times (0.3394) + 2 \times \frac{1}{600} \times 0.0611 + \frac{1}{600} \times 0.0073$$

$$= \cancel{0.0058}\ 0.0059$$

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According to the little's Law $R = \frac{N}{X}$.

we ~~that~~ can get that

$$R = \frac{0.4851}{0.0059} = 82.22 \text{ min}$$

Therefore, the mean-time-to-repair (MTTR) for this data centre is ~~82.3~~ 82.22 minutes.