Solution to COMP9334 Revision Questions for Week 4B_1

Question 1

Let us first compute the second moment of each customer type. Let us use C_a, σ_a, $E[S_a]$ and $E[S_a^2]$ to denote, respectively, the coefficient of variation, standard deviation, mean and second moment of the service time of customer of type a. Recall that the coefficient of variation of a random variable is its standard deviation divided by mean, i.e $C_a = \frac{\sigma_a}{E[S_a]}$. By using the relation:

$$E[S_a^2] = E[S_a]^2 + \sigma_a^2, \tag{1}$$

it can be showed that

$$E[S_a^2] = E[S_a]^2(1 + C_a^2) \tag{2}$$

Since we know $E[S_a] = 0.1$ and $C_a = 1.5$, we can compute $E[S_a^2]$ using the above equation.

Similarly, let C_b, σ_b, $E[S_b]$ and $E[S_b^2]$ to denote, respectively, the coefficient of variation, standard deviation, mean and second moment of the service time of customer of type b. We have

$$E[S_b^2] = E[S_b]^2(1 + C_b^2) \tag{3}$$

With $E[S_b] = 0.08$ and $C_b = 1.2$, we can compute $E[S_b^2]$ using the above equation.

Requests of type a and b have equal priorities

This is an M/G/1 queue without priority. The arrival rate is 10 requests per second ($= \lambda$) . Since 30% of the request are type a and the remaining are type b, we have the mean service time $E[S]$ and second moment $E[S^2]$ of the aggregate are, respectively,

$$\begin{aligned} E[S] &= 0.3E[S_a] + 0.7E[S_b] & (4)\\ E[S^2] &= 0.3E[S_a^2] + 0.7E[S_b^2] & (5) \end{aligned}$$

The mean response time is therefore $E[S] + \frac{\lambda E[S^2]}{2(1-\rho)}$ where $\rho = \lambda E[S]$.

Requests of type b have non-preemptive priority over type a

Let

$$\begin{aligned} R &= \frac{1}{2}(0.3\lambda E[S_a^2] + 0.7\lambda E[S_b^2]) & (6)\\ \rho_a &= 0.3\lambda E[S_a] & (7)\\ \rho_b &= 0.7\lambda E[S_b] & (8) \end{aligned}$$

Response time of type b is

$$E[S_b] + \frac{R}{1 - \rho_b} \tag{9}$$

Response time of type a is

$$E[S_a] + \frac{R}{(1 - \rho_b)(1 - \rho_a - \rho_b)} \tag{10}$$

Requests of type b have preemptive priority over type a

Let

$$R_b = \frac{1}{2}(0.7\lambda E[S_b^2]) \tag{11}$$

$$R_a = \frac{1}{2}(0.3\lambda E[S_a^2] + 0.7\lambda E[S_b^2]) \tag{12}$$

$$\rho_a = 0.3\lambda E[S_a] \tag{13}$$

$$\rho_b = 0.7\lambda E[S_b] \tag{14}$$

Response time of type b is

$$E[S_b] + \frac{R_b}{1-\rho_b} \tag{15}$$

Response time of type a is

$$E[S_a]\frac{1}{1-\rho_b} + \frac{R_a}{(1-\rho_b)(1-\rho_a-\rho_b)} \tag{16}$$

The numerical answers are summarised in the following table.

Mean response time	Part 1	Part 2	Part 3
type a	0.8246	1.7787	1.9059
type b	0.8246	0.3150	0.2042

Observe that the response time for type b customers have become better because it has a higher priority, this is of course at the expense of type a customers which have a lower priority.