

Instrumentation

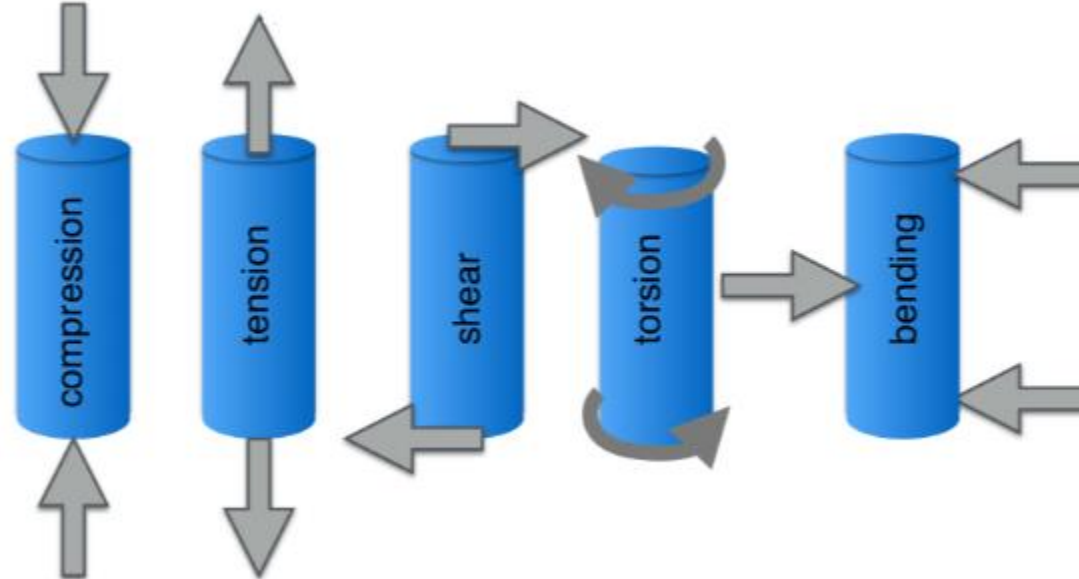


Force and Torque Measurement

Course Instructor: Mohammad Reza Nayeri

Stress and strain

- Every component in a linear motion system experiences some form of loading due to applied forces or motion
- The component's reactions to these loads are described by its mechanical properties.
- There are five fundamental types of loading: *compression*, *tension*, *shear*, *torsion*, and *bending*.



Stress and strain

- **Stress** is the force applied to a material, divided by the material's cross-sectional area.

$$\sigma = \frac{F}{A_0}$$

σ = stress (N/m², Pa)

F = force (N)

A_0 = original cross-sectional area (m²)

- **Strain** is the deformation or displacement of material that results from an applied stress.

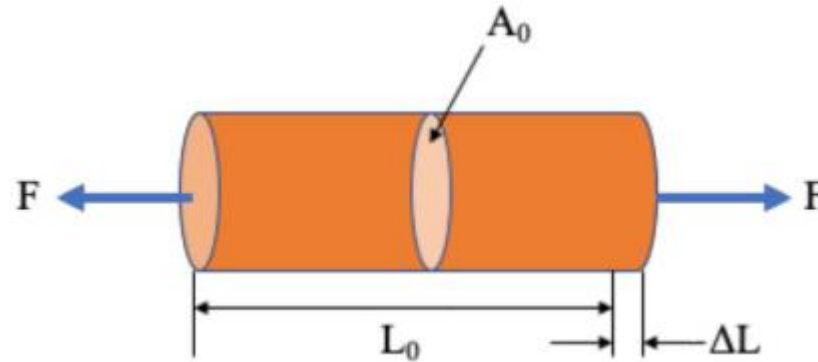
$$\varepsilon = \frac{L - L_0}{L_0}$$

ε = strain

L = length after load is applied (mm)

L_0 = original length (mm)

Note: A material's change in length ($L - L_0$) is sometimes represented as δ .



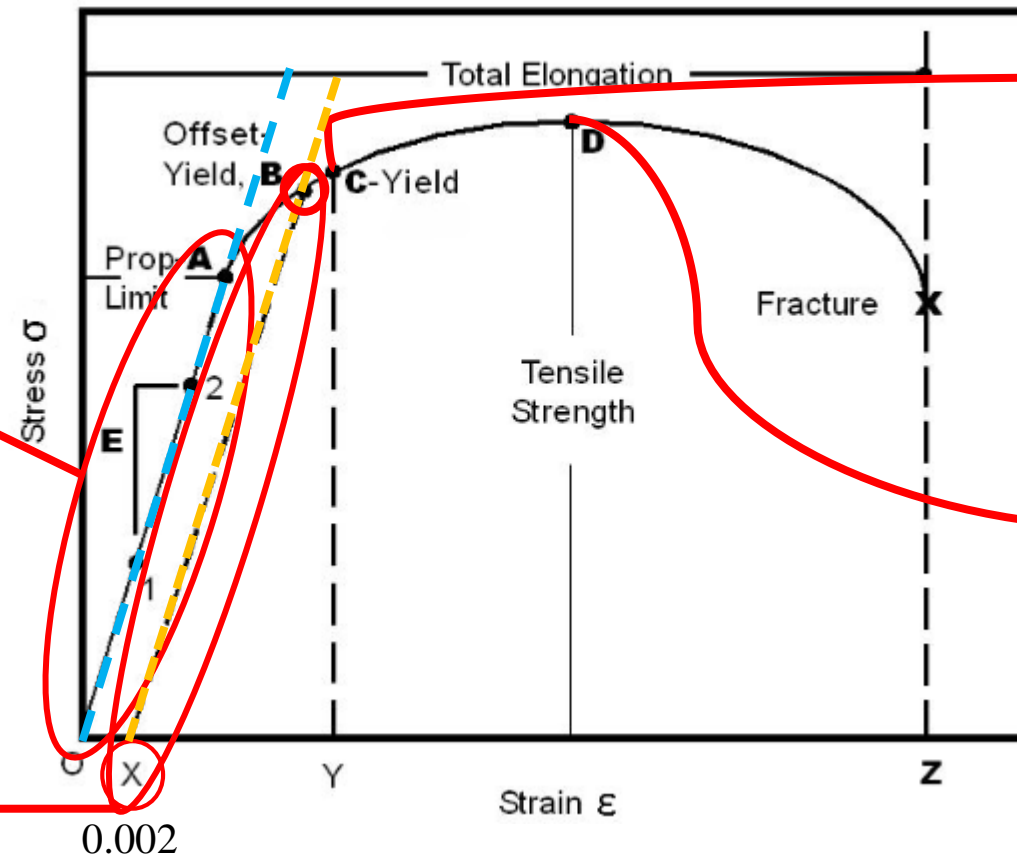
Stress and strain

- The most common way to analyze the relationship between stress and strain for a particular material is with a *stress-strain diagram*.

A. This stress-strain relationship is known as *Hooke's Law*, and in this region, the slope of the stress-strain curve is referred to as the modulus of elasticity (aka Young's modulus), denoted E.

$$E = \frac{\sigma}{\epsilon}$$

For materials that do not have a well-defined yield point, or whose yield point is difficult to determine, an offset yield strength — shown here as point “B” — is used. Offset yield strength is the stress that will cause a specified amount of permanent strain (typically 0.2 percent).



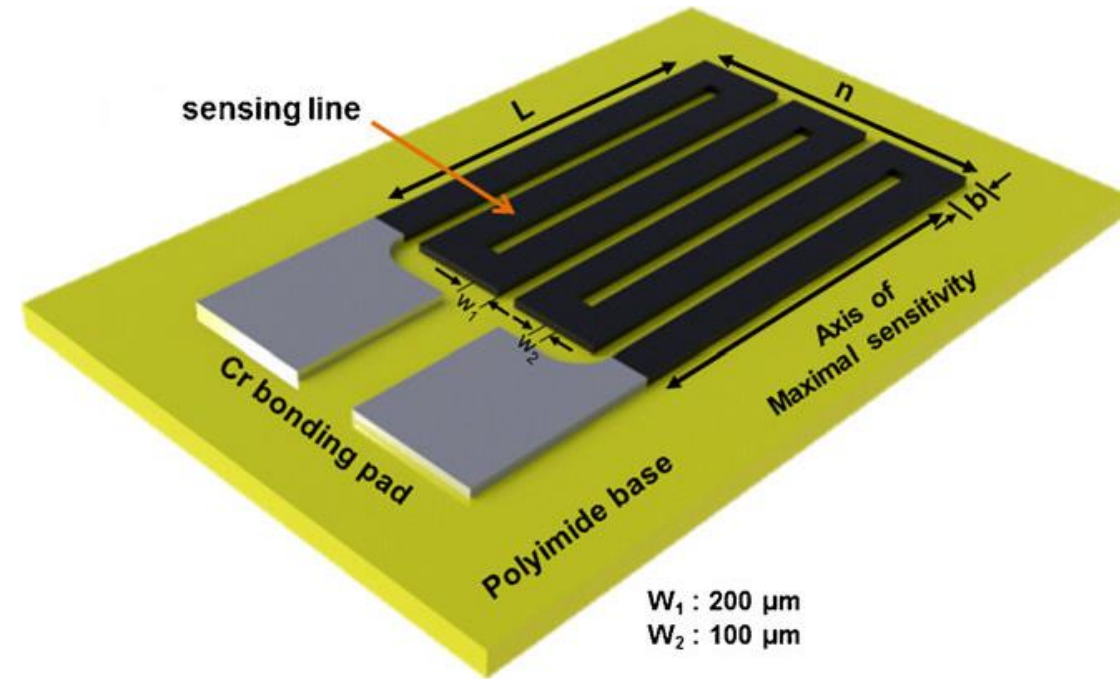
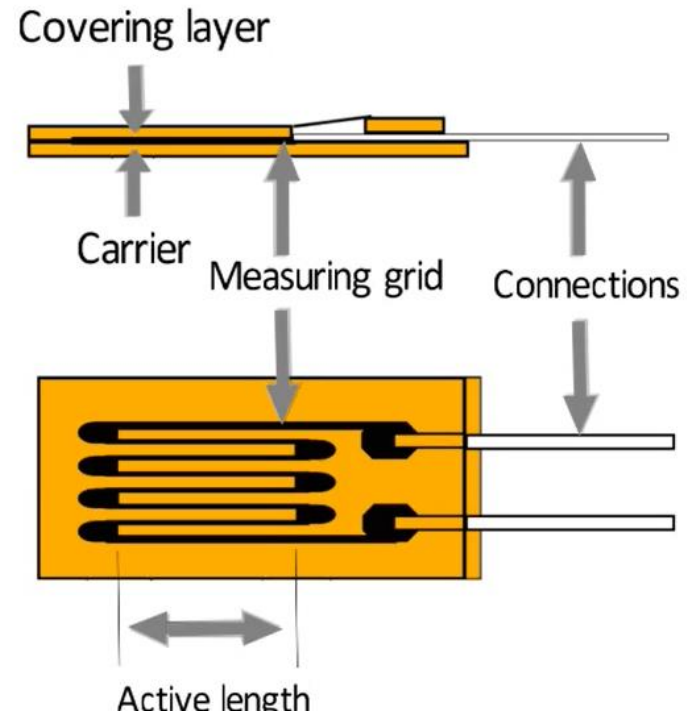
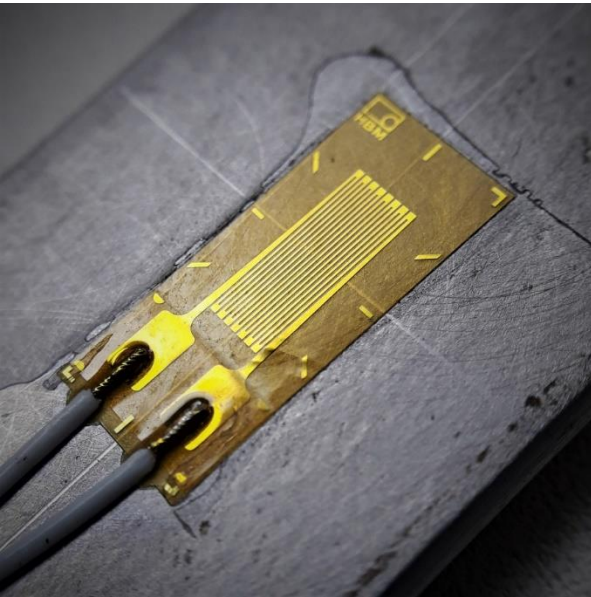
“C,” is the point where strain increases faster than stress and the material experiences some amount of permanent deformation. (Yield point)

“D,” where the curve begins to fall, the material's ultimate tensile strength has been reached. This point denotes the maximum stress that can be applied to a material in tension before failure occurs.

$$\sigma = \frac{F}{A_0} \rightarrow F = \sigma \cdot A_0 = E\epsilon \cdot A_0 \rightarrow F = (E \cdot A_0)\epsilon \rightarrow \mathbf{F = K\epsilon}$$

Stress and strain

Strain gauge



Strain gauge

Tensile force

extend
reduce the cross-sectional area → increase the resistance

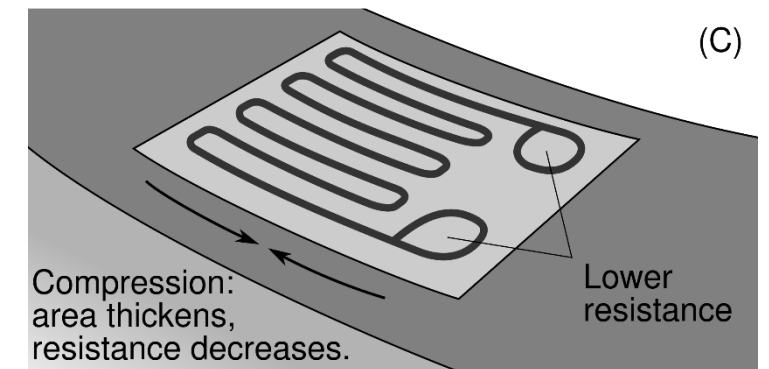
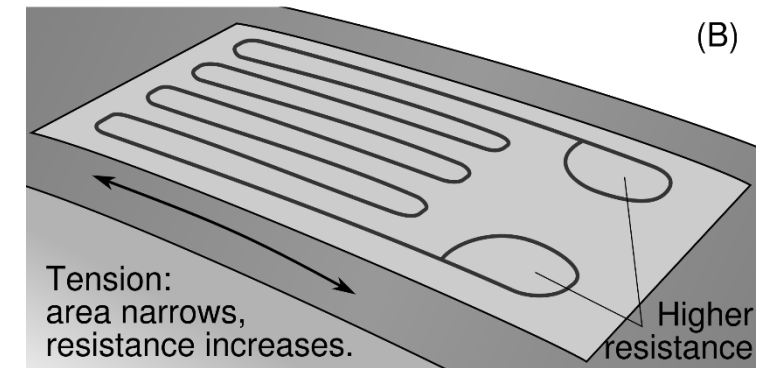
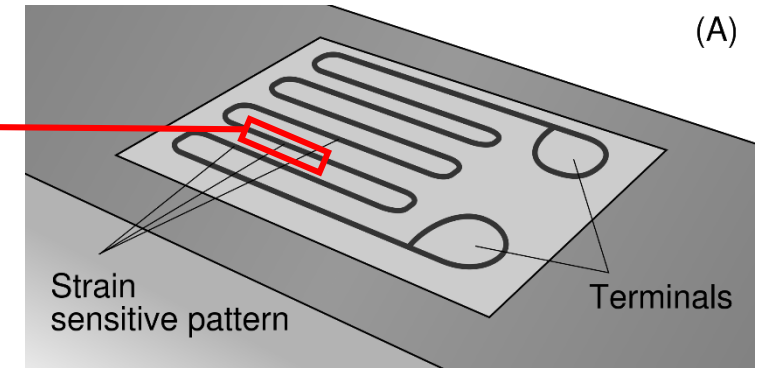
Compression force

shrink
increase the cross-sectional area → reduce the resistance

$$R = \rho \frac{L}{A}$$

$$R = \rho \frac{L}{A}$$

$$R = \rho \frac{L}{A}$$



Strain gauge

Taylor series: (Ignore high order terms)

$$R = R_0 + \Delta R \quad R_0 = \rho \frac{L_0}{A_0}$$

$$\Delta R = \frac{\partial R}{\partial L} \Delta L + \frac{\partial R}{\partial A} \Delta A + \frac{\partial R}{\partial \rho} \Delta \rho$$

$$\rightarrow \Delta R = \frac{\rho}{A} \Delta L - \frac{\rho L}{A^2} \Delta A + \frac{L}{A} \Delta \rho$$

$$\rightarrow \frac{\Delta R}{R} = \frac{\Delta L}{L} - \frac{\Delta A}{A} + \frac{\Delta \rho}{\rho}$$

$$\frac{\Delta L}{L} = \varepsilon \quad (\varepsilon: \text{strain})$$

$$\frac{\Delta A}{A} = -2\mu\varepsilon \quad (\mu: \text{poission ratio})$$

$$\frac{\Delta R}{R} = (1 + 2\mu)\varepsilon + \frac{\Delta \rho}{\rho}$$

???: (Hint: Assume the cross section of a wire is a circle!)



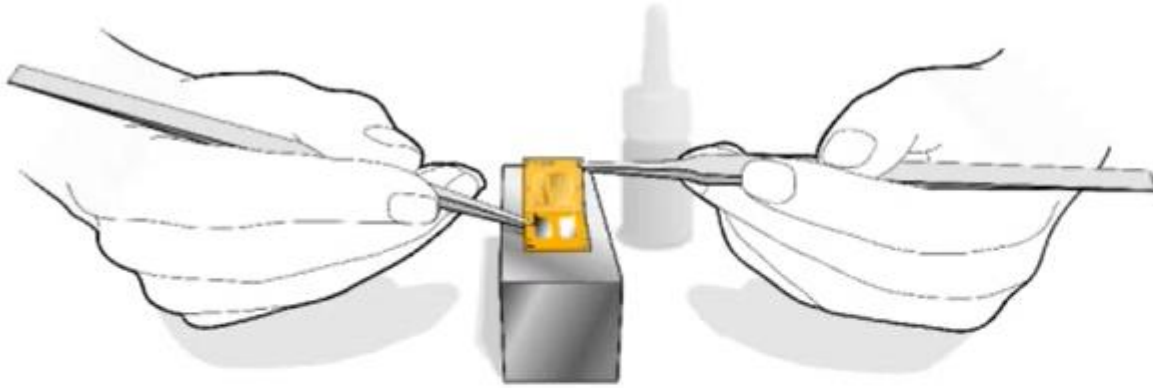
$$R = \rho \frac{L}{A}$$

$$\text{Gauge Factor}(GF) = \frac{\frac{\Delta R}{R}}{\varepsilon}$$

$$\rightarrow GF = (1 + 2\mu) + \frac{\frac{\Delta \rho}{\rho}}{\varepsilon}$$

if $\Delta \rho = 0 \rightarrow \text{GF} = 1 + 2\mu$

Strain gauge



Manufacturing of transducers

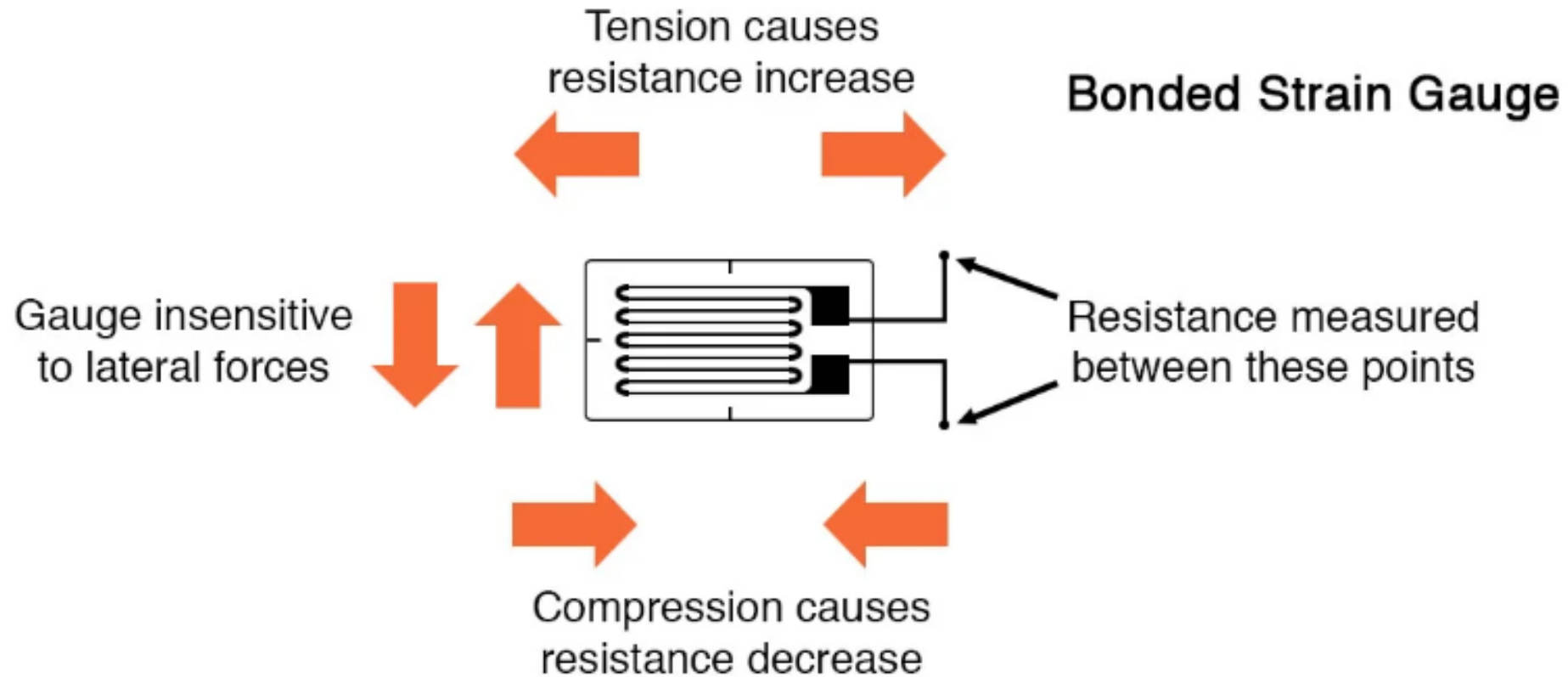
- Force transducers
- Load cells
- Torque and pressure transducers
- ...



Experimental stress analysis

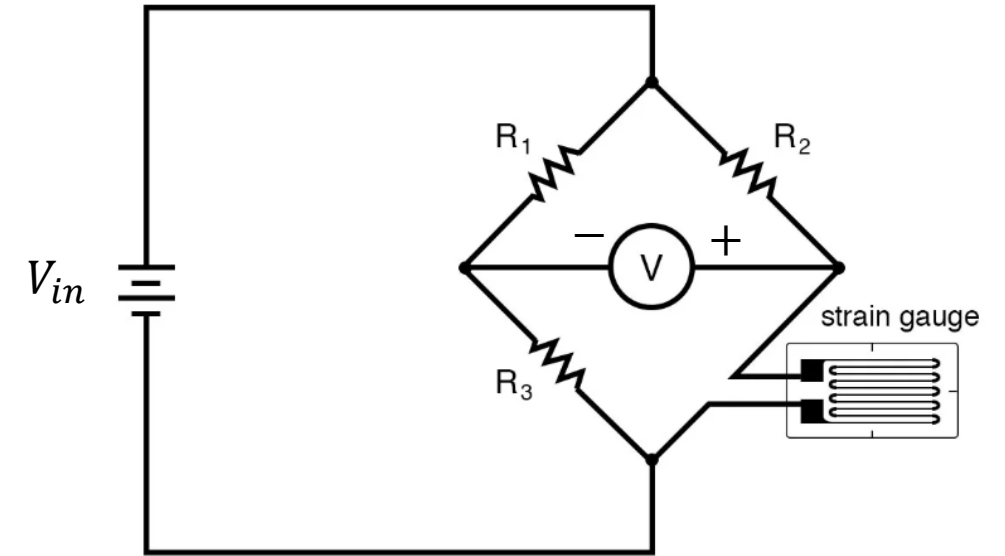
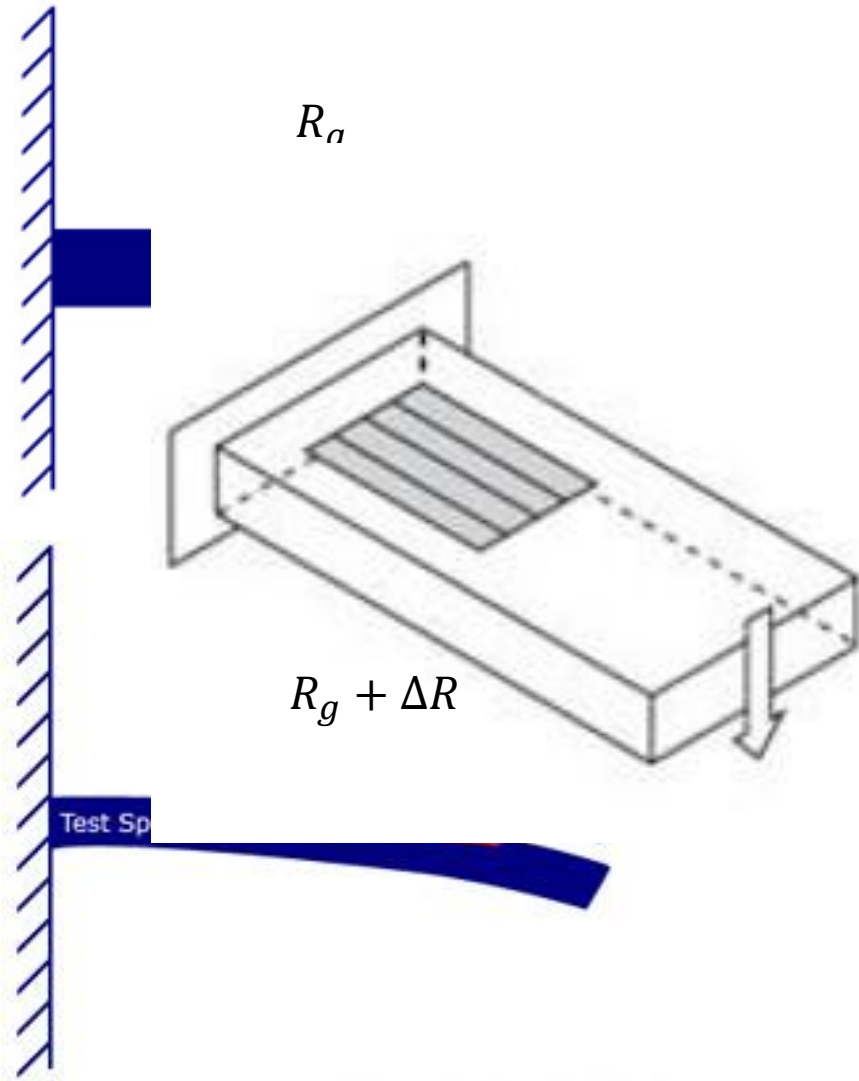
- Determination of the absolute value and direction of mechanical stresses
- Fatigue life analysis
- Residual stress analysis

Strain gauge



Strain gauge

Quarter – Bridge Circuit



$$R_1 = R_3, \quad R_2 = R_g(\text{No load})$$

$$\text{No load: } V = 0$$

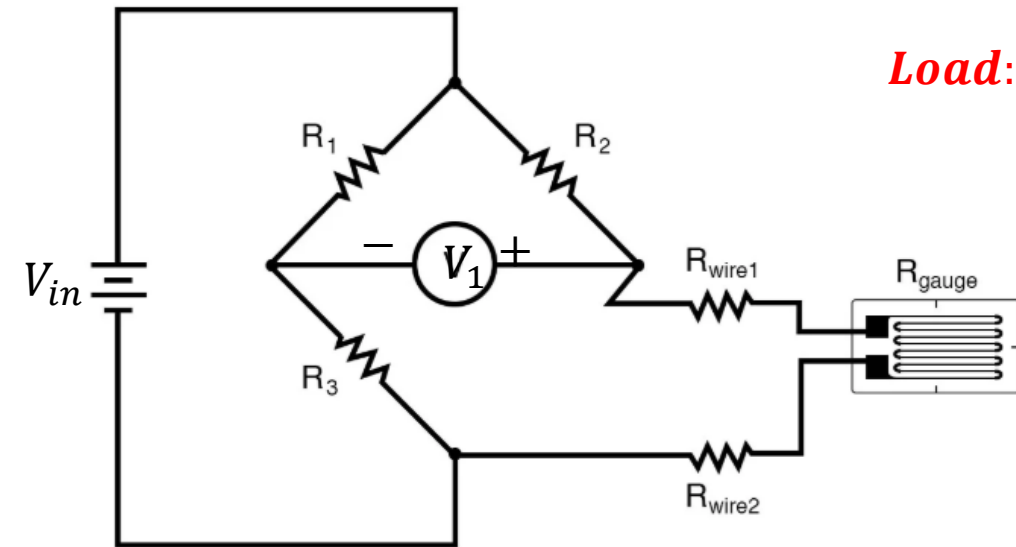
$$\text{Load: } V = \left[\frac{R_g + \Delta R}{R_2 + R_g + \Delta R} - \frac{R_3}{R_1 + R_3} \right] V_{in} = \left[\frac{R_2 + \Delta R}{2R_2 + \Delta R} - \frac{1}{2} \right] V_{in}$$

$$R_2 \gg \Delta R \rightarrow V \cong \left[\frac{\Delta R}{4R_2} \right] V_{in}$$

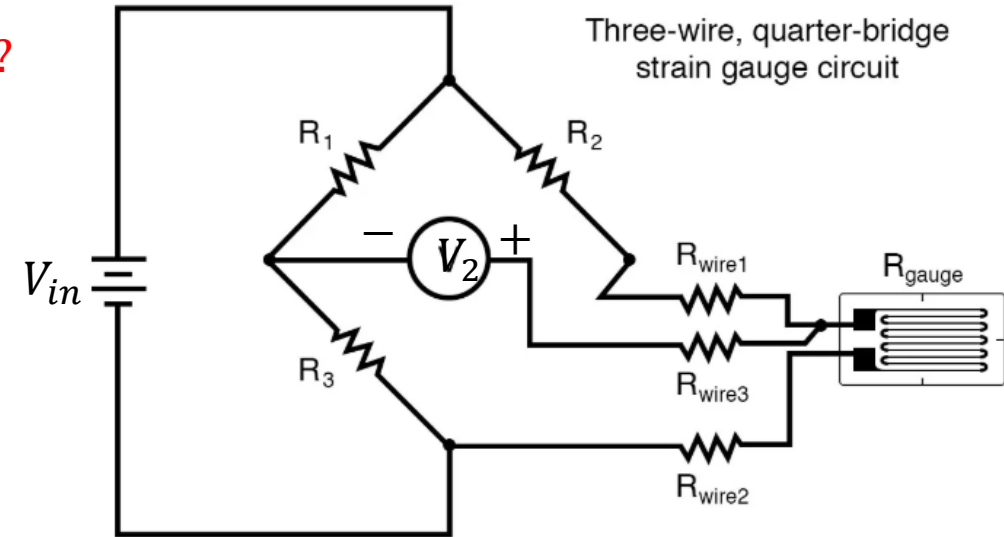
Strain gauge

Quarter – Bridge Circuit

Wire Resistances



Load: $R_{gauge} = R_g + \Delta R$??



$$R_1 = R_3, R_2 = R_g(\text{No load}), \\ R_{wire1} = R_{wire2} = R_w$$

$$\text{No load: } V_1 = \left[\frac{R_g + 2R_w}{R_2 + R_g + 2R_w} - \frac{R_3}{R_1 + R_3} \right] V_{in}$$

$$\rightarrow V_1 = \frac{R_w}{2(R_g + R_w)} V_{in}$$

$$R_1 = R_3, R_2 = R_g(\text{No load}), \\ R_{wire1} = R_{wire2} = R_{wire3} = R_w$$

$$\text{No load: } V_2 = \left[\frac{R_g + R_w}{R_2 + R_w + R_g + R_w} - \frac{R_3}{R_1 + R_3} \right] V_{in}$$

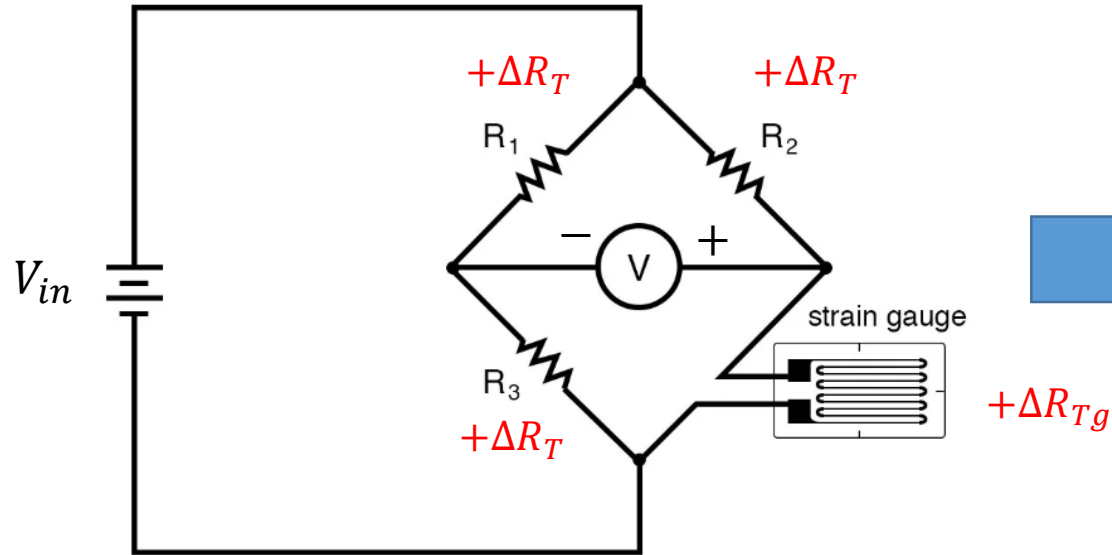
$$\rightarrow V_2 = 0$$

Strain gauge

Quarter – Bridge Circuit

Resistance Change in Temperature

Load: $R_{gauge} = R_g + \Delta R$??



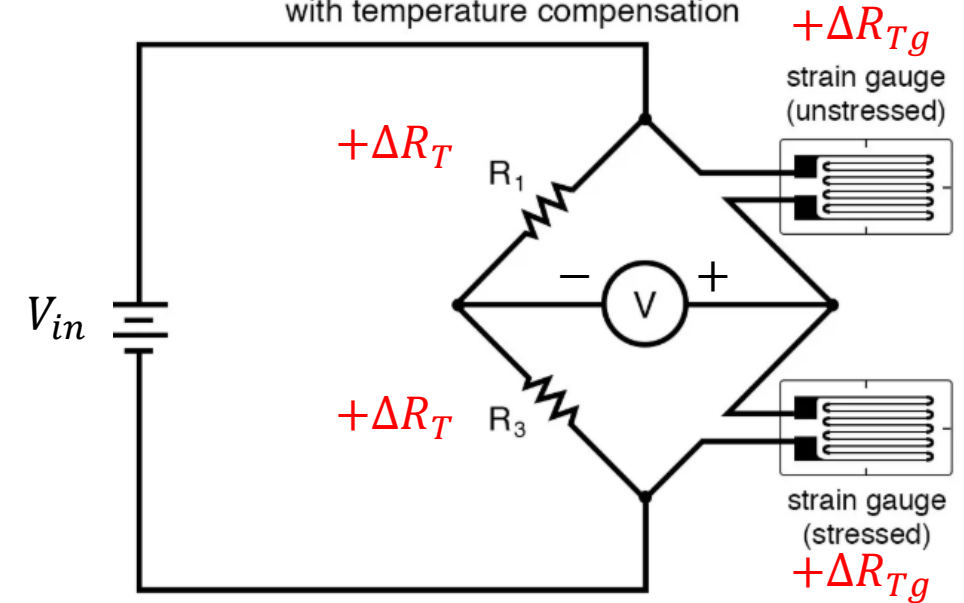
$$R_1 = R_3, \quad R_2 = R_g(\text{No load})$$

No load:

$$V = \left[\frac{R_g + \Delta R_{Tg}}{R_2 + \Delta R_T + R_g + \Delta R_{Tg}} - \frac{R_3 + \Delta R_T}{R_1 + \Delta R_T + R_3 + \Delta R_T} \right] V_{in}$$

$$\rightarrow V \neq 0$$

Quarter-bridge strain gauge circuit with temperature compensation



$$R_1 = R_3$$

No load:

$$V = \left[\frac{R_g + \Delta R_{Tg}}{R_g + \Delta R_{Tg} + R_g + \Delta R_{Tg}} - \frac{R_3 + \Delta R_T}{R_1 + \Delta R_T + R_3 + \Delta R_T} \right] V_{in}$$

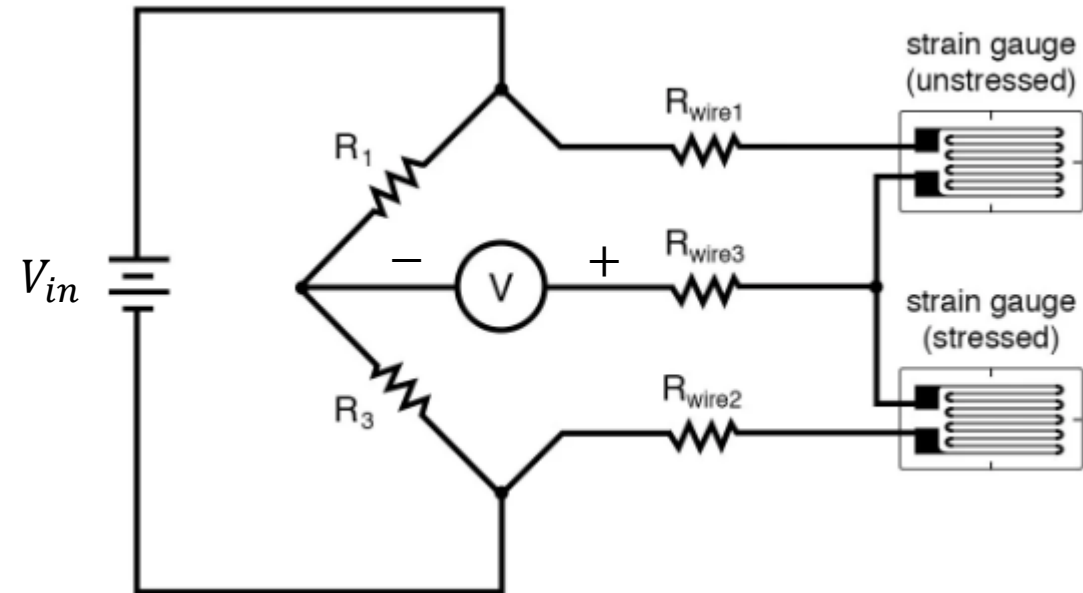
$$\rightarrow V = 0$$

Strain gauge

Quarter – Bridge Circuit

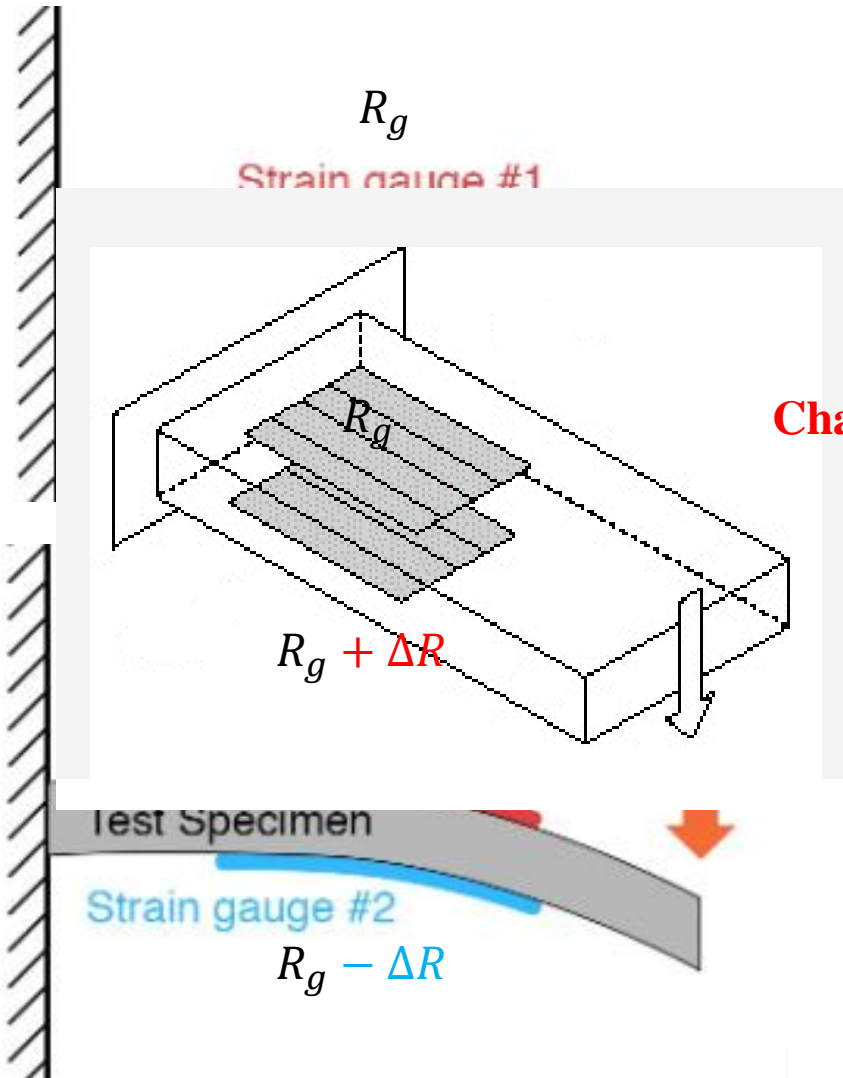
Resistance Change in Temperature

Wire Resistances ??



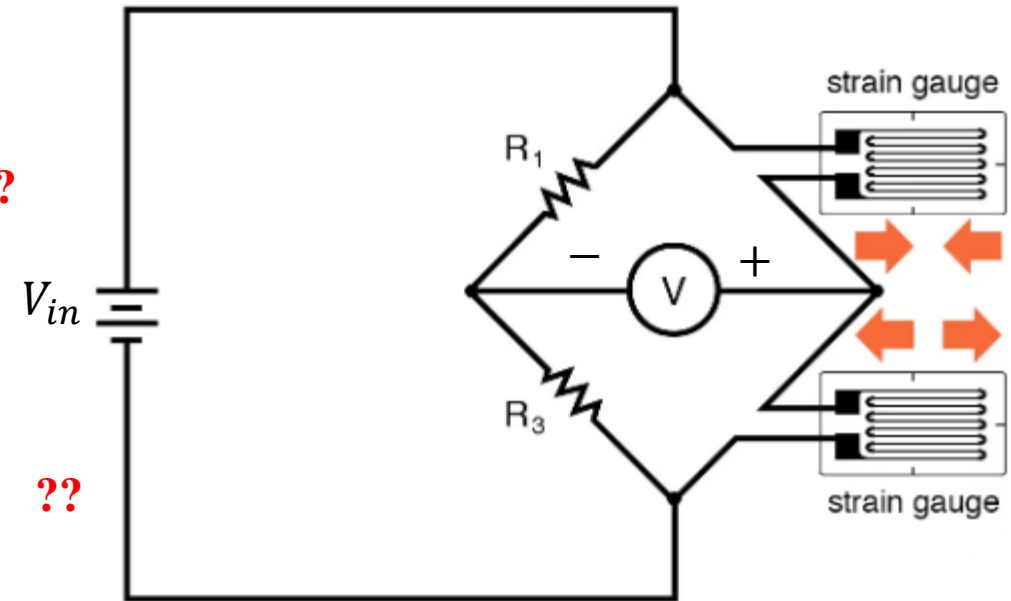
Strain gauge

Half – Bridge Circuit



Wire Resistances ??

Change in Temperature ??



$$R_1 = R_3$$

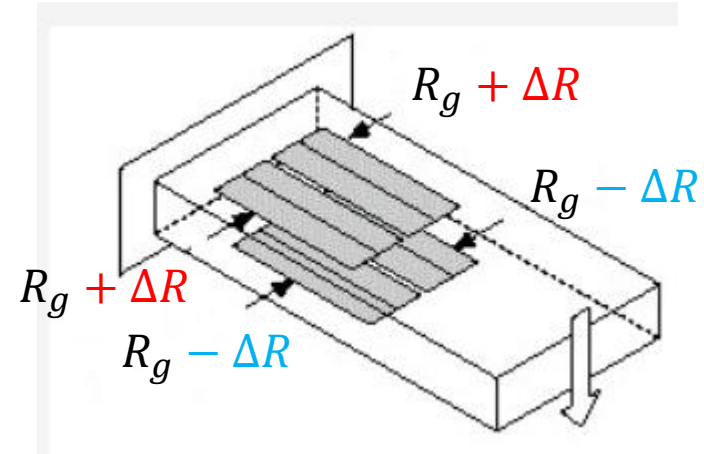
$$\text{No load: } V = 0$$

$$\text{Load: } V = \left[\frac{R_g + \Delta R}{R_g + \Delta R + R_g - \Delta R} - \frac{R_3}{R_1 + R_3} \right] V_{in}$$

$$\rightarrow V = \left[\frac{\Delta R}{2R_g} \right] V_{in}$$

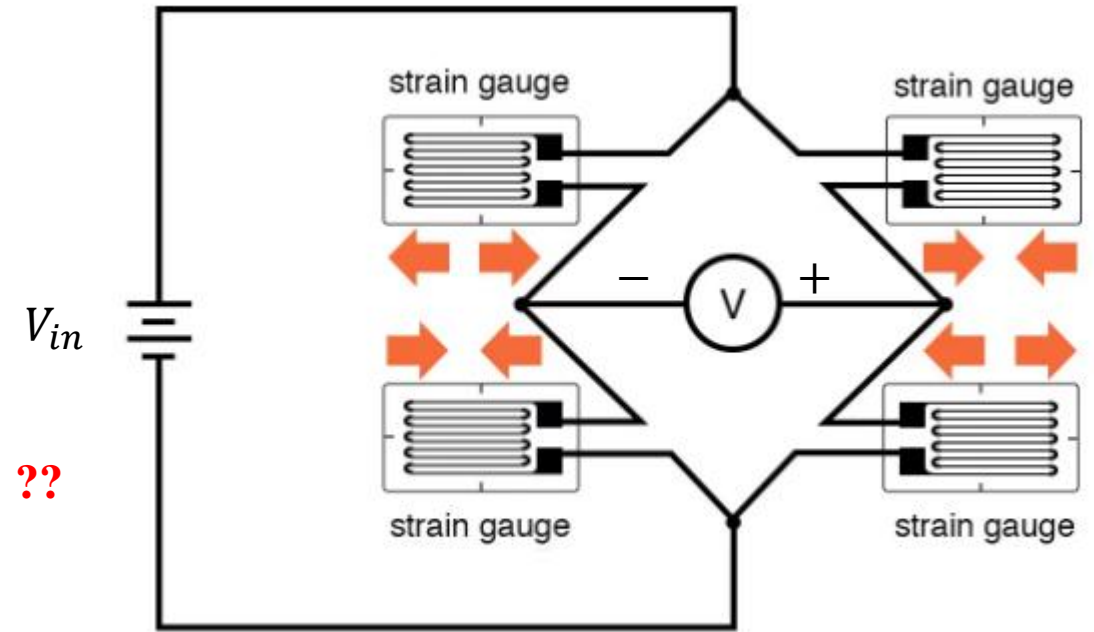
Strain gauge

Full – Bridge Circuit



Wire Resistances ??

Change in Temperature ??

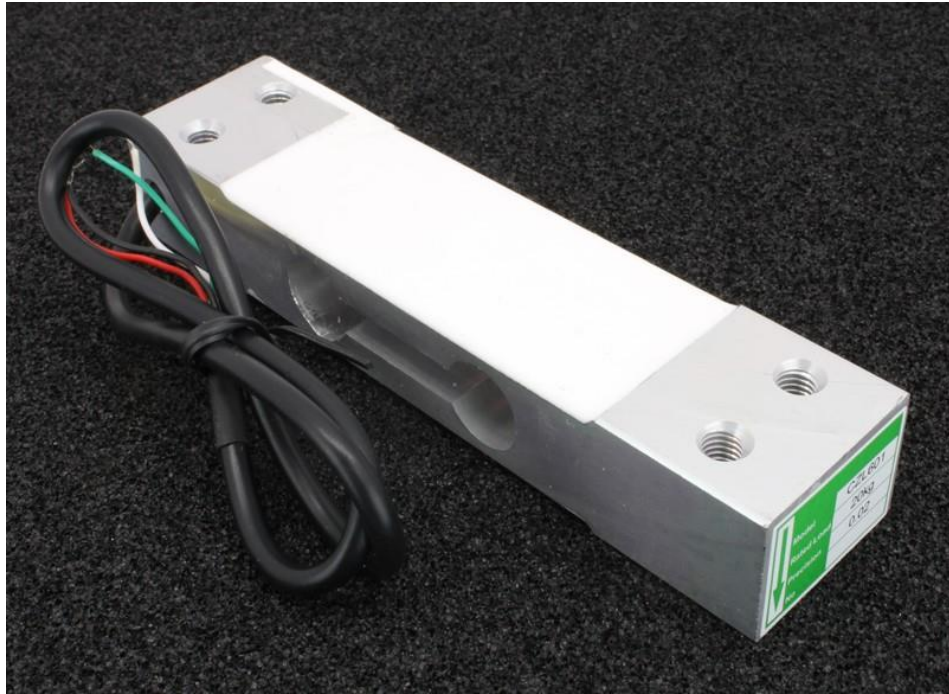


No load: $V = 0$

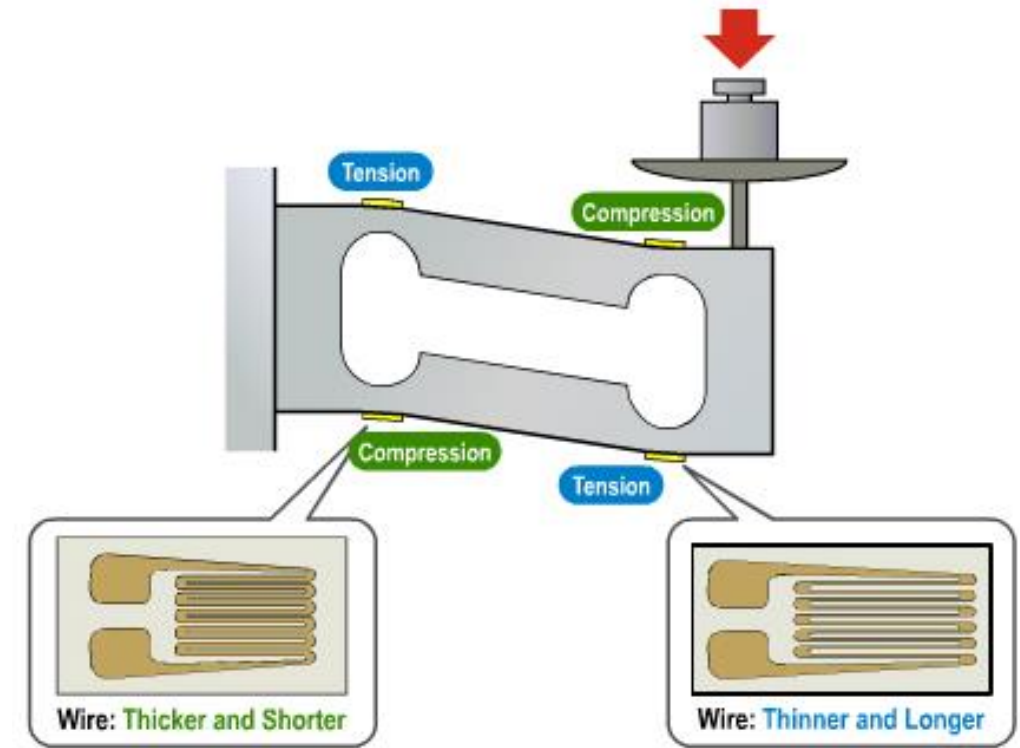
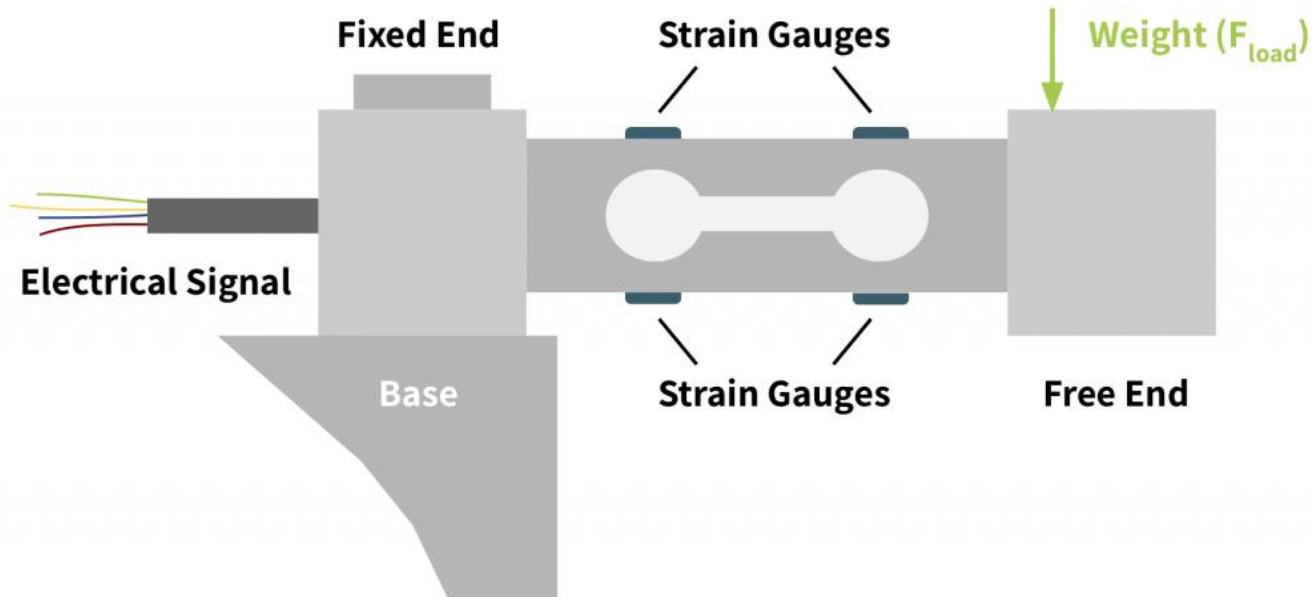
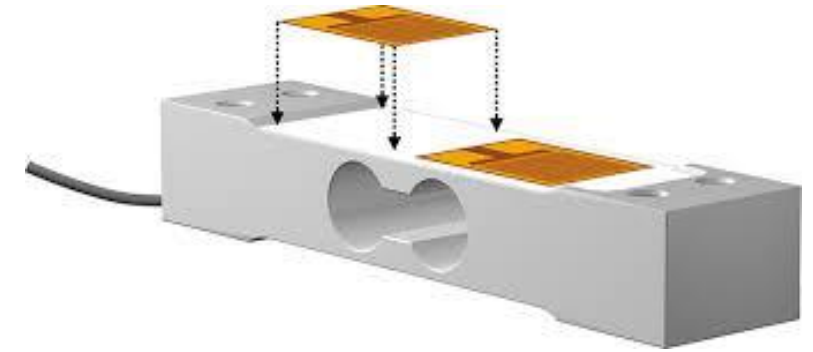
$$\text{Load: } V = \left[\frac{R_g + \Delta R}{R_g + \Delta R + R_g - \Delta R} - \frac{R_g - \Delta R}{R_g + \Delta R + R_g - \Delta R} \right] V_{in}$$

$$\rightarrow V = \left[\frac{\Delta R}{R_g} \right] V_{in} \quad V_{\text{Half-Bridge}} = \left[\frac{\Delta R}{2R_g} \right] V_{in}$$

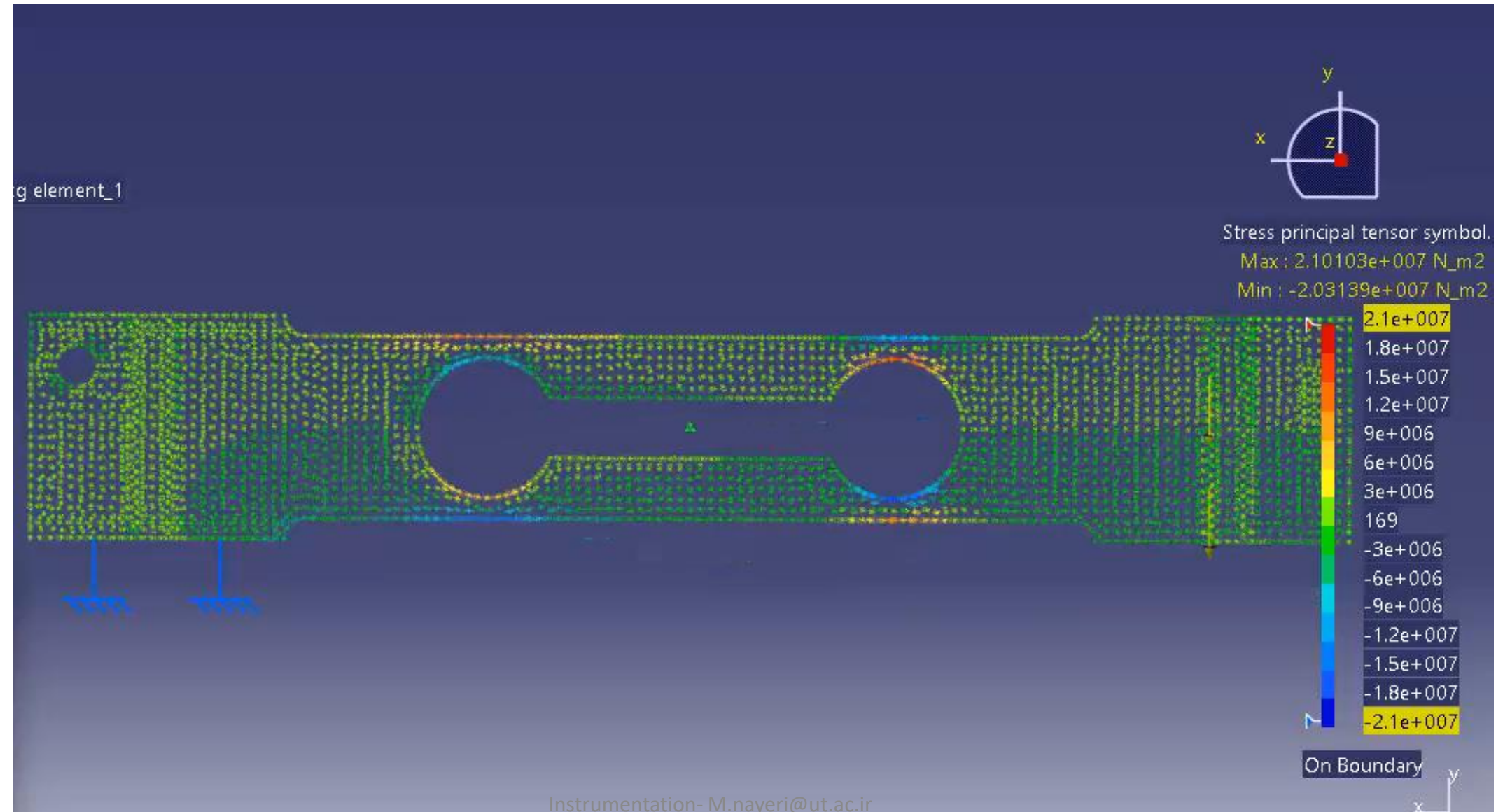
Load cell



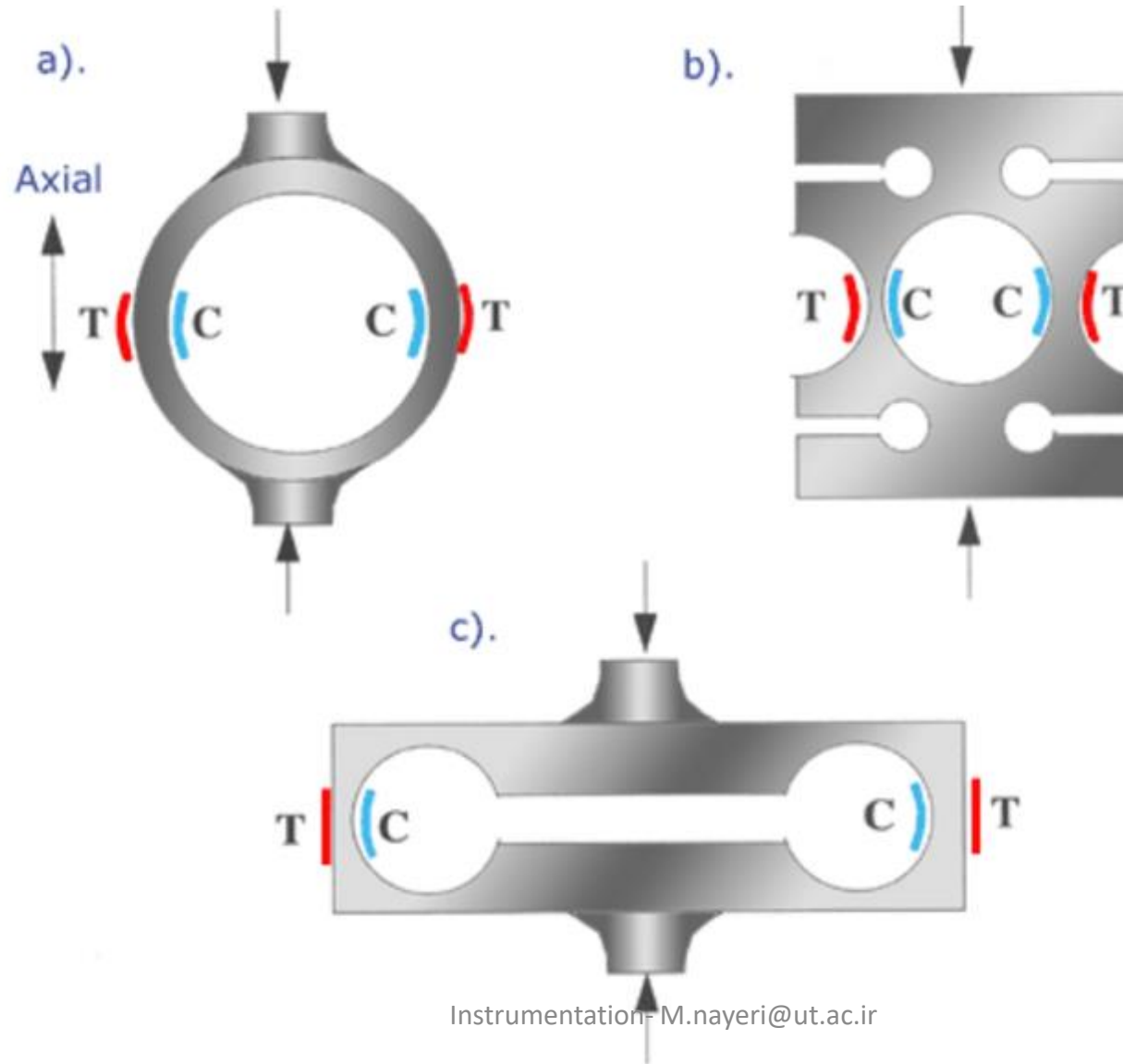
Load cell



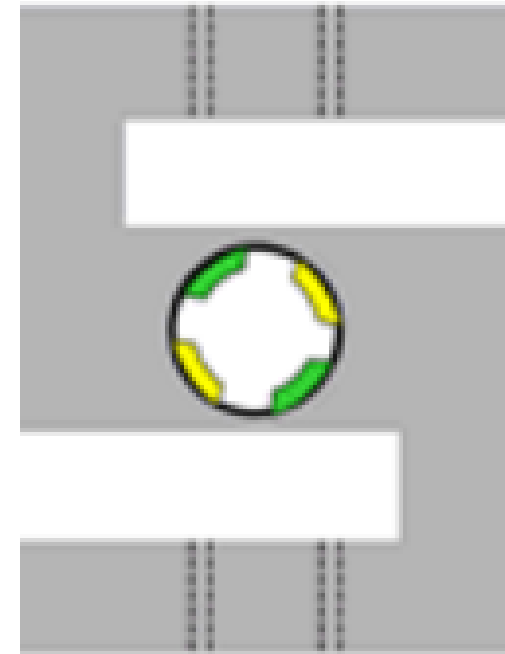
Load cell



Load cell



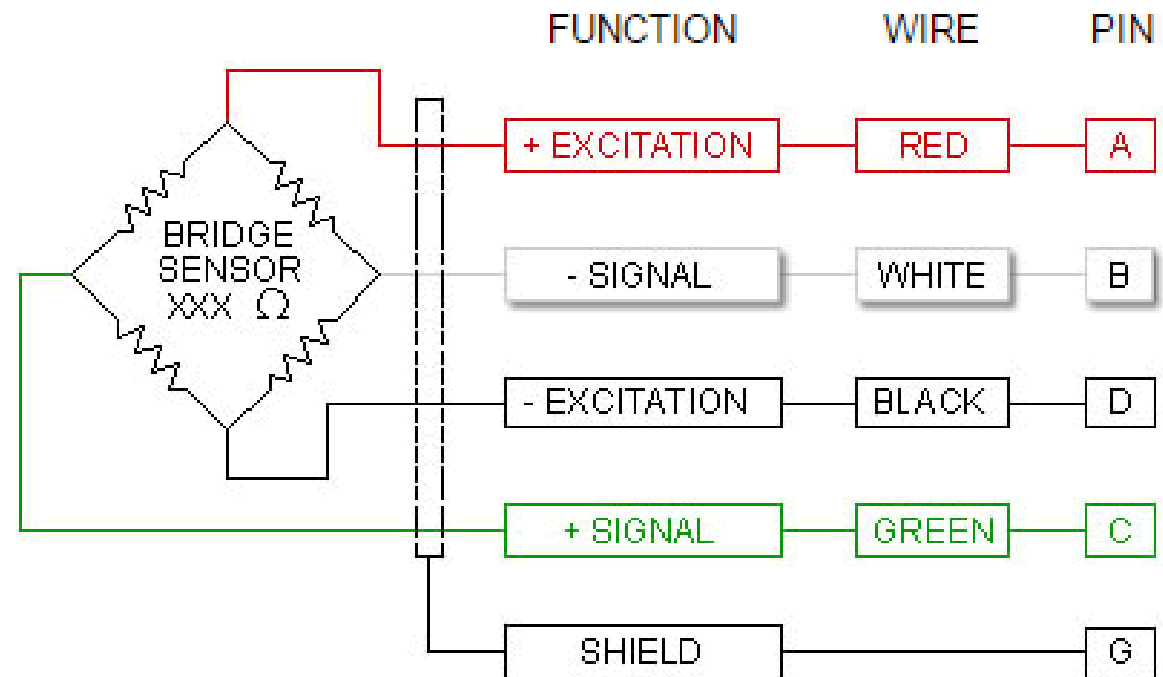
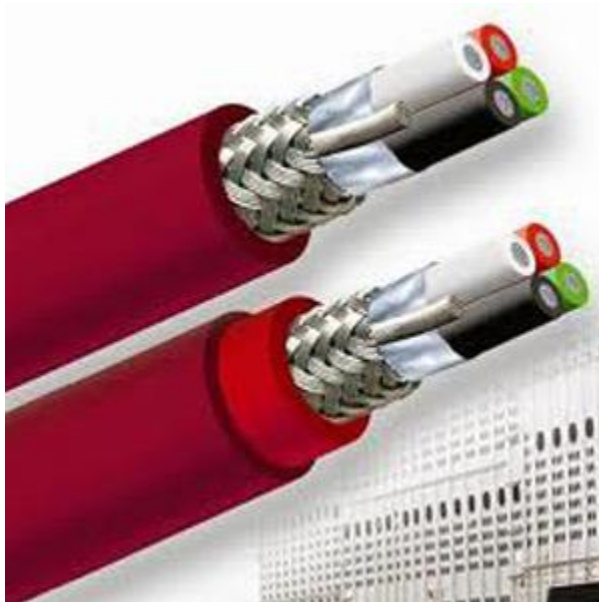
Load cell



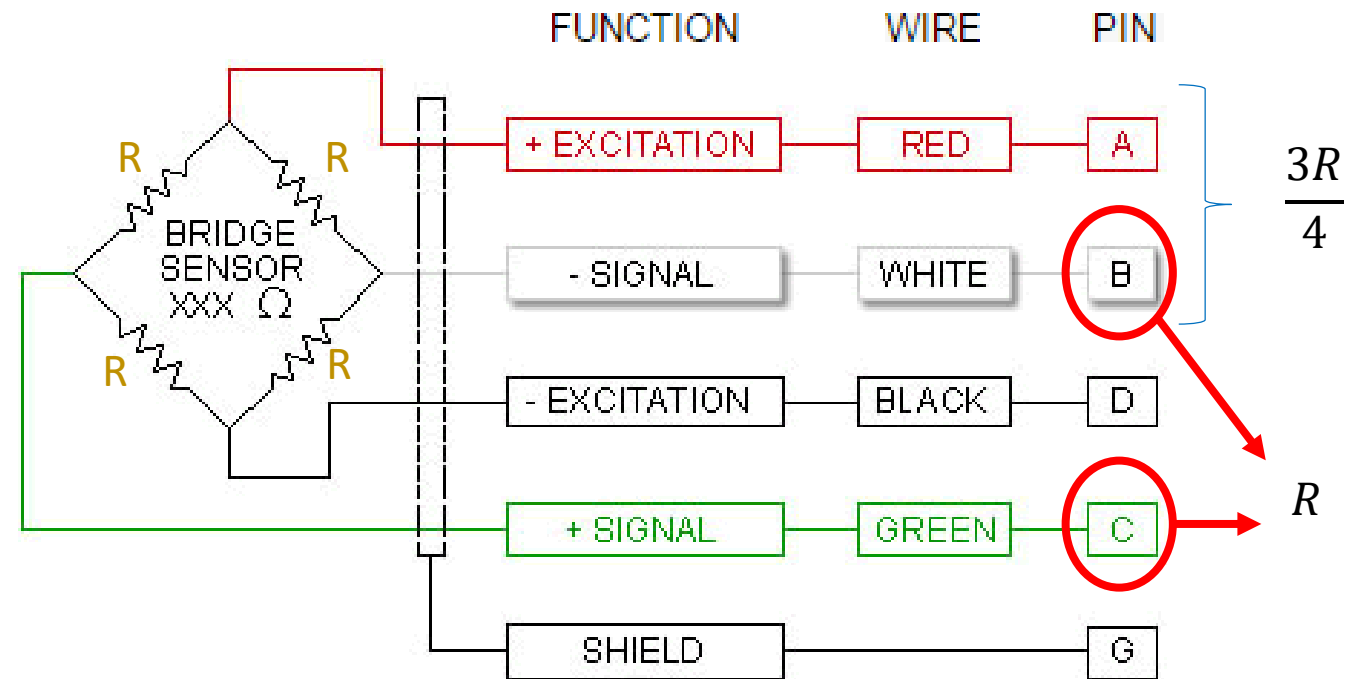
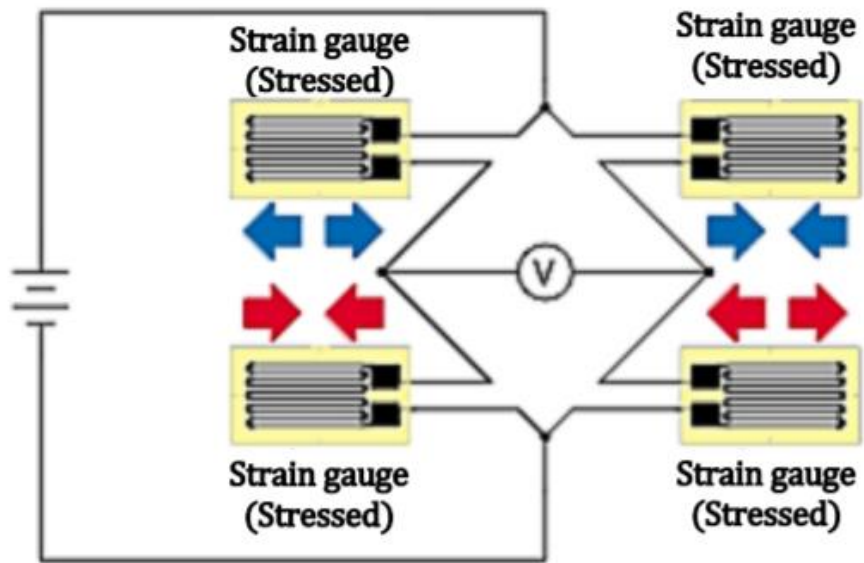
Load cell



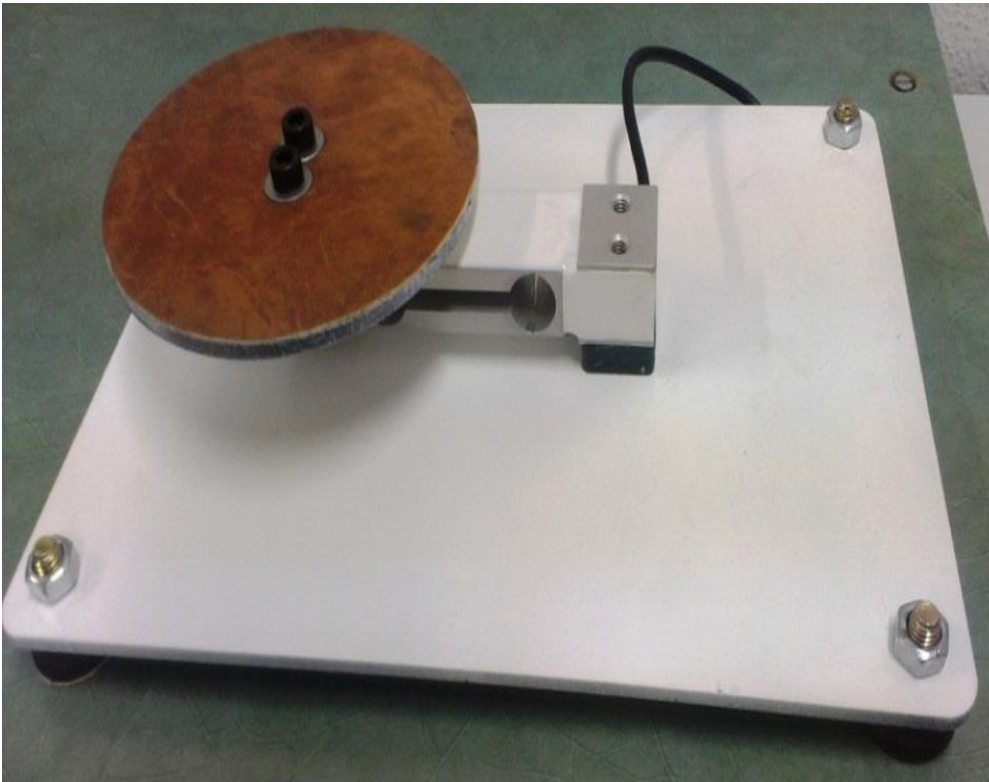
Load cell



Load cell

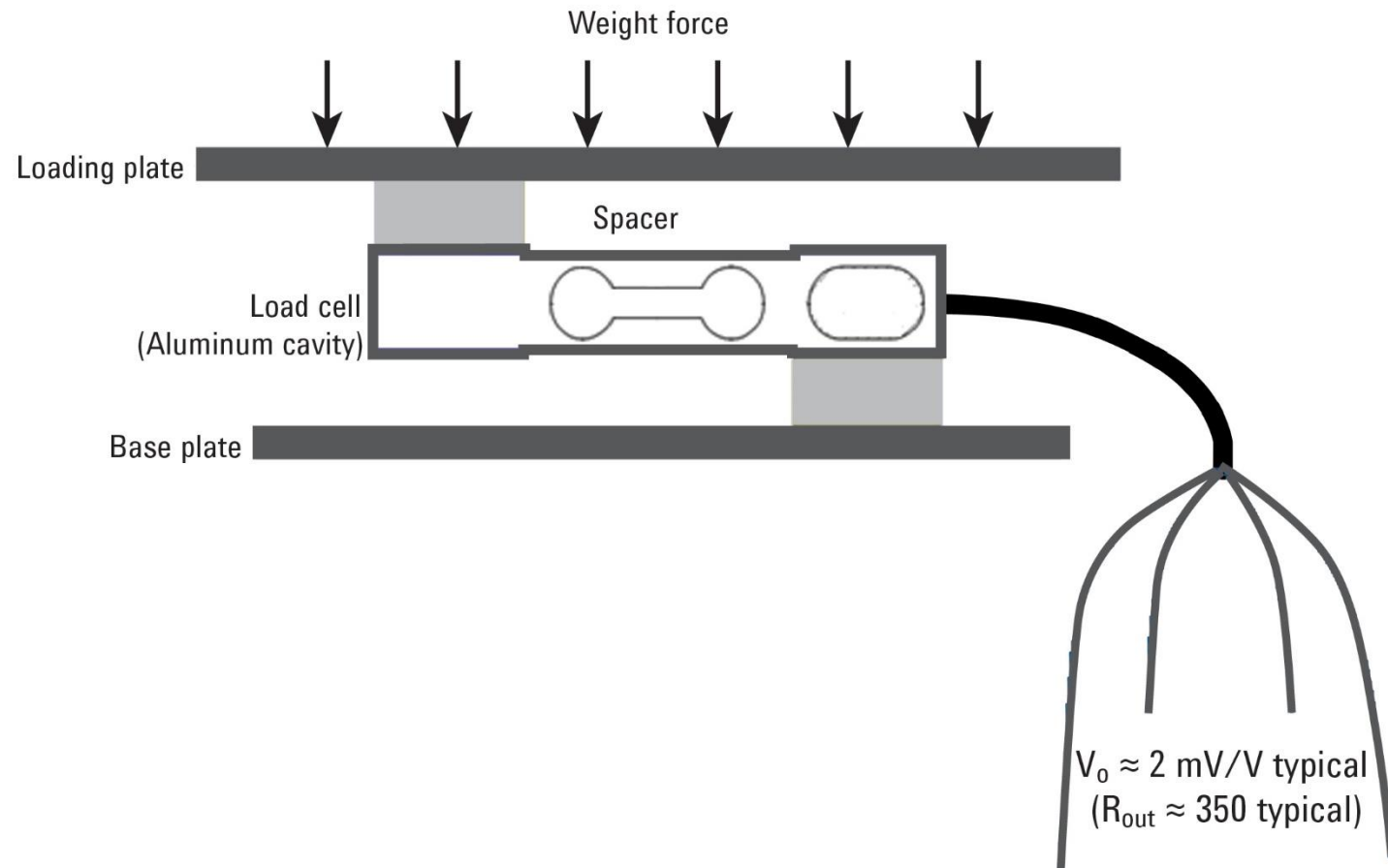


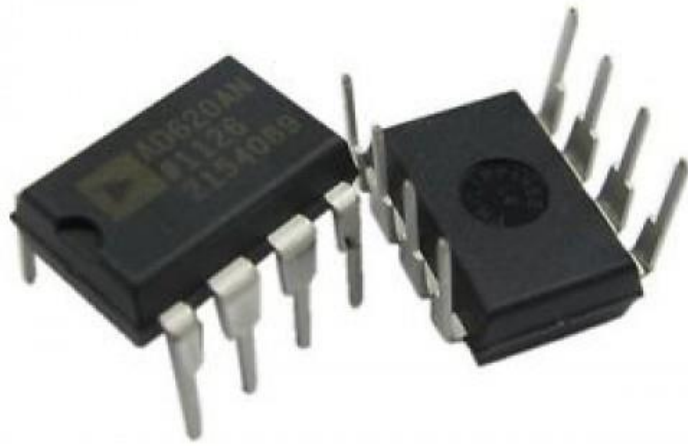
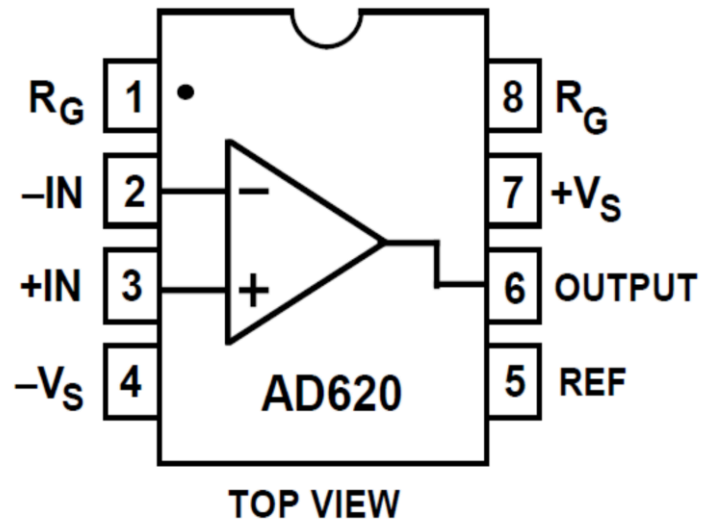
Load cell



Output Sensitivity	$1.8 \pm 0.2 \text{ mV/V}$
Safe Overload	150%
Ultimate Overload	300%
Excitation Voltage	5 ~ 12 V
Excitation Maximum	18 V

Load cell

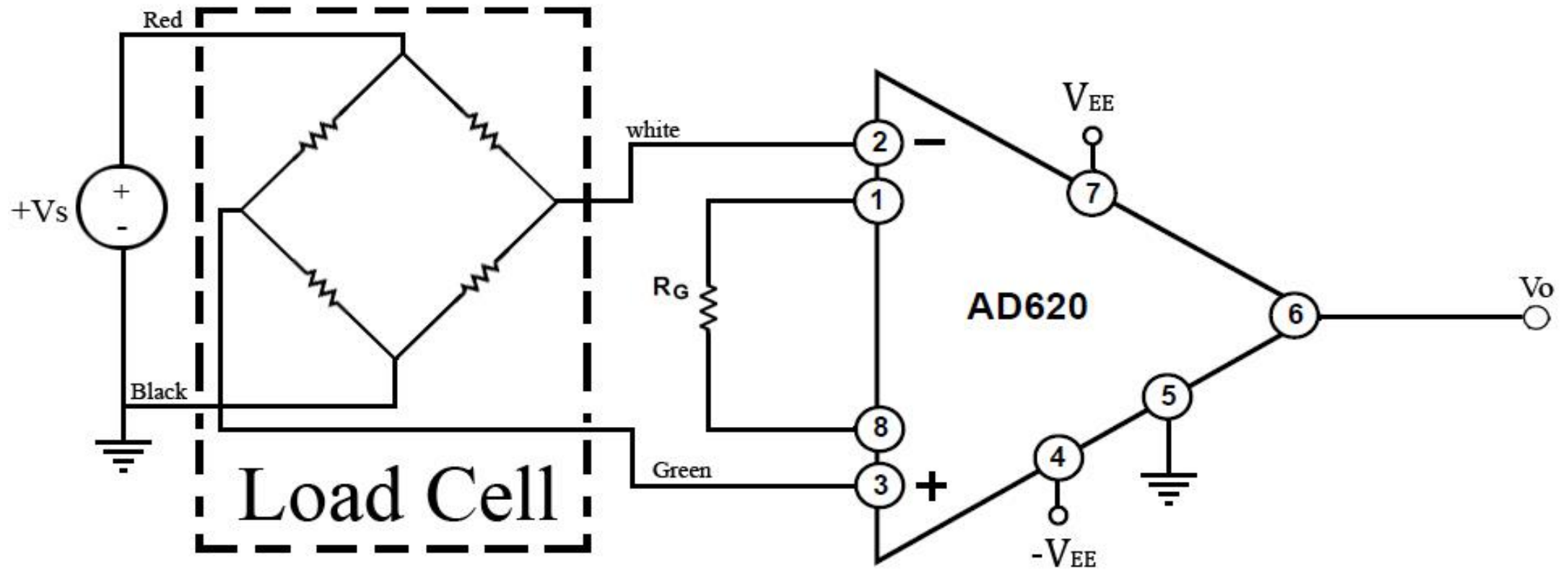




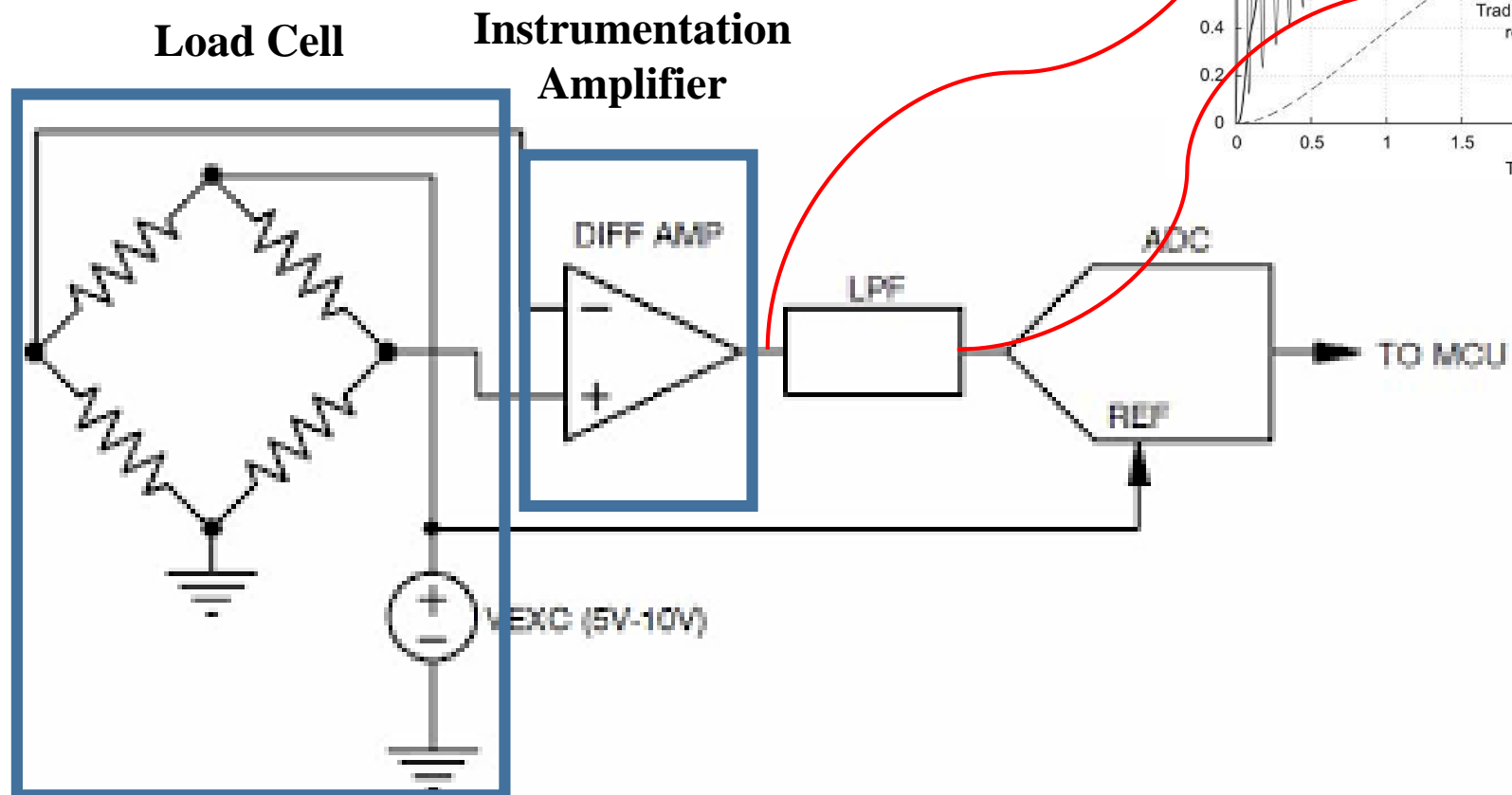
Load cell

مقاومت‌های استاندارد 1%	بهره	مقاومت‌های استاندارد 0.1%	بهره
49.9 K Ω	1.990	49.3 K Ω	2.002
12.4 K Ω	4.984	12.4 K Ω	4.984
5.49 K Ω	9.998	5.49 K Ω	9.998
2.61 K Ω	19.93	2.61 K Ω	19.93
1.00 K Ω	50.40	1.01 K Ω	49.91
499 Ω	100.0	499 Ω	100.0
249 Ω	199.4	249 Ω	199.4
100 Ω	495.0	98.8 Ω	501.0
49.9 Ω	991.0	49.3 Ω	1003

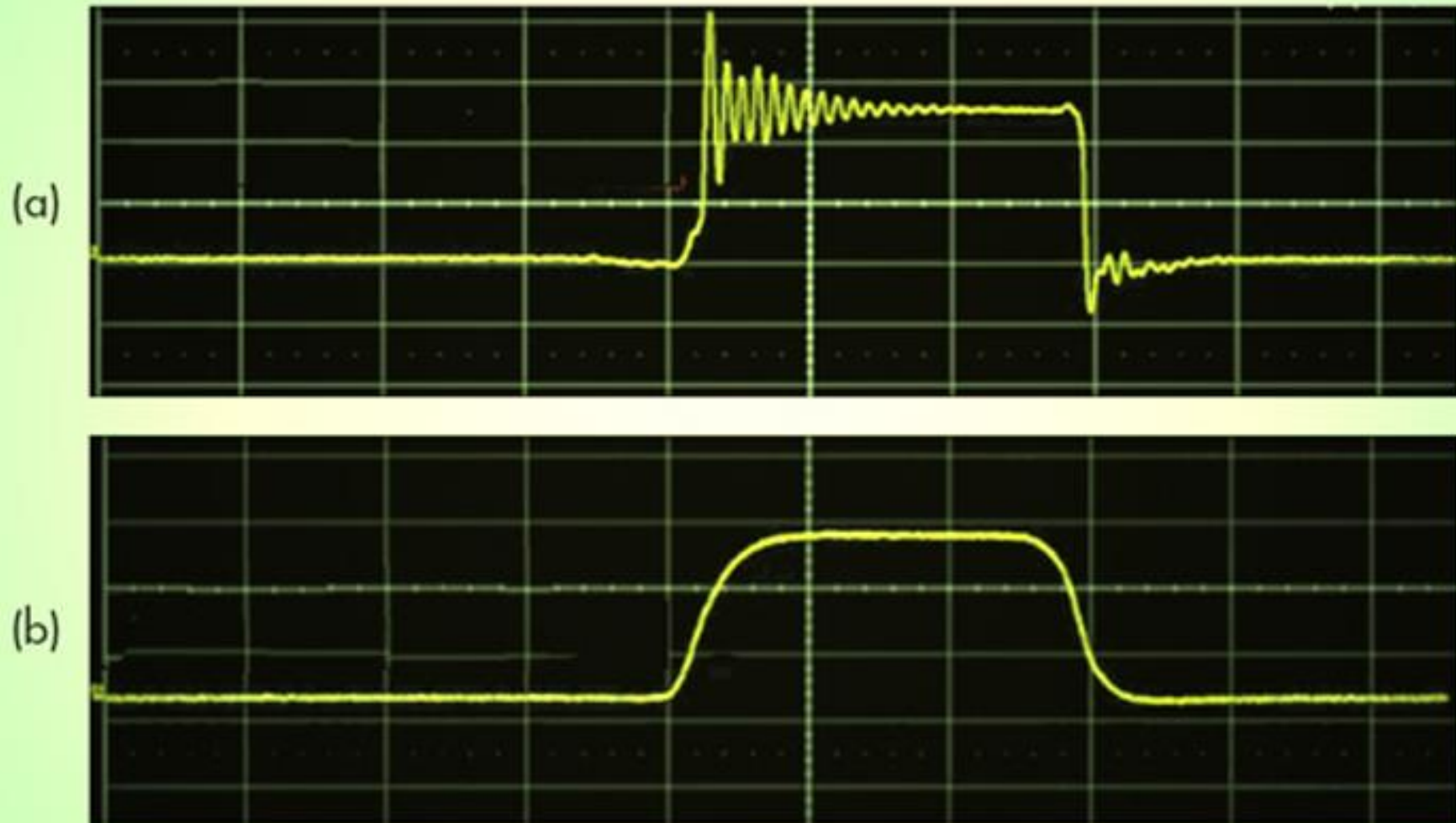
Load cell



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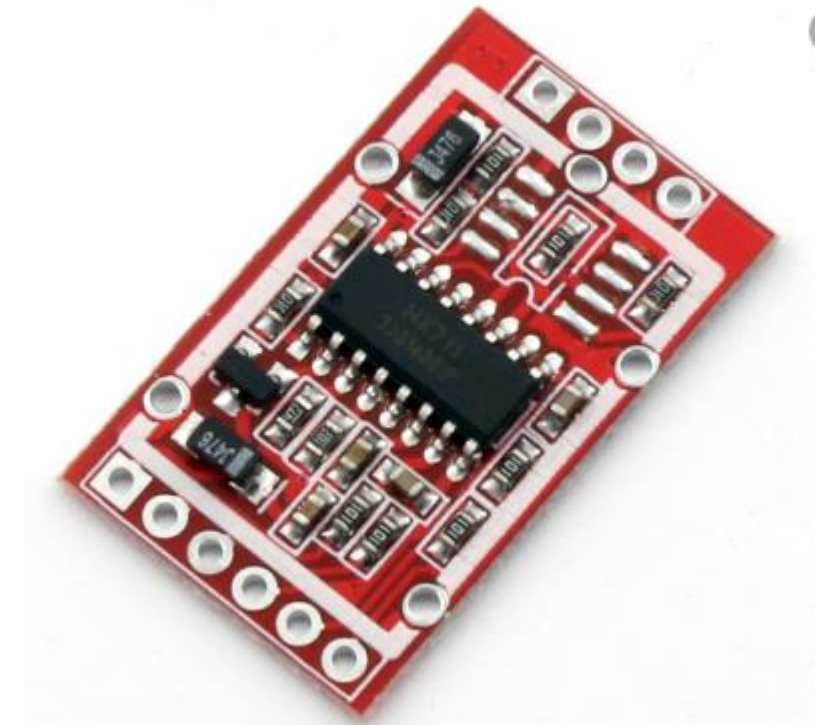
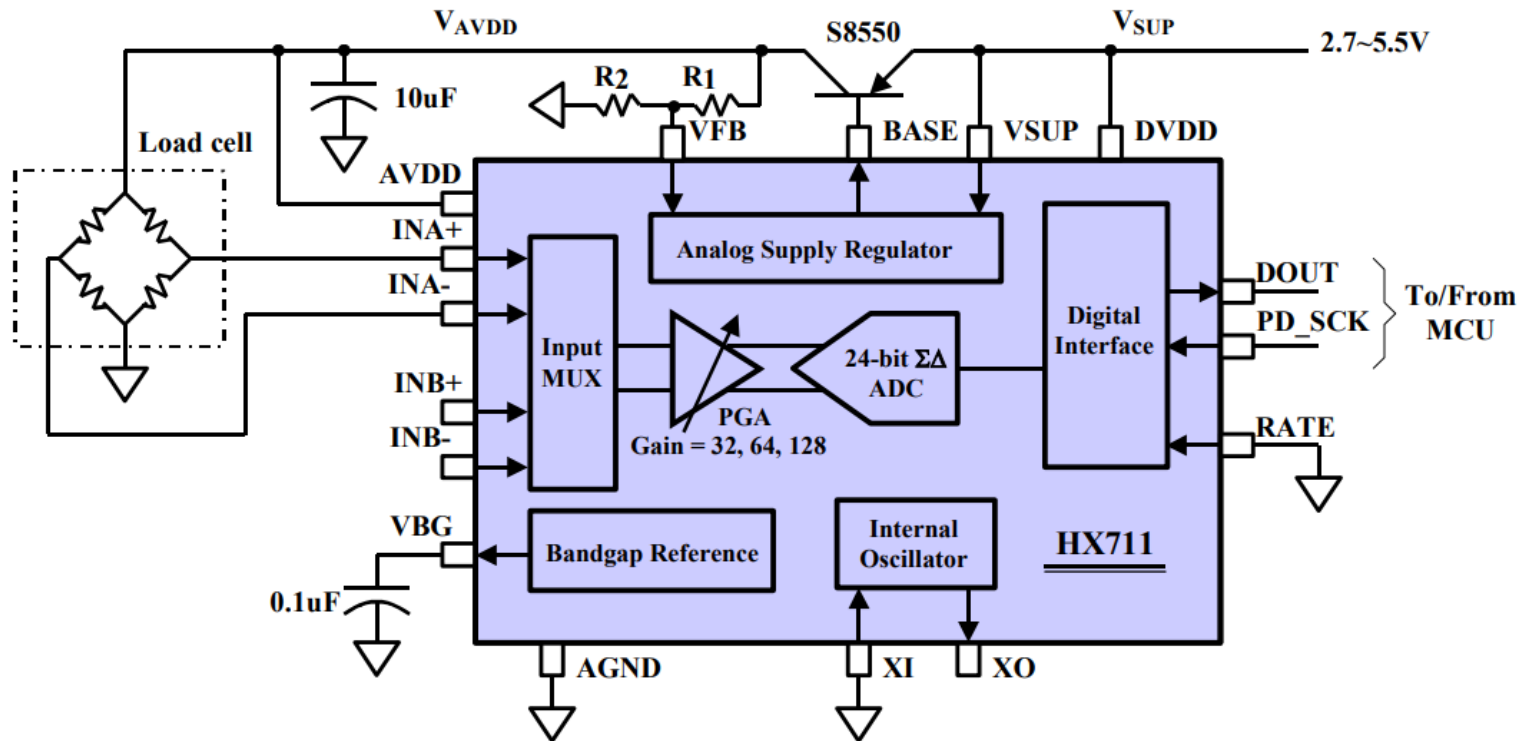
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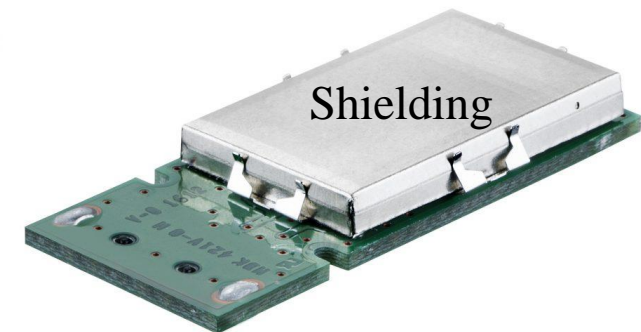
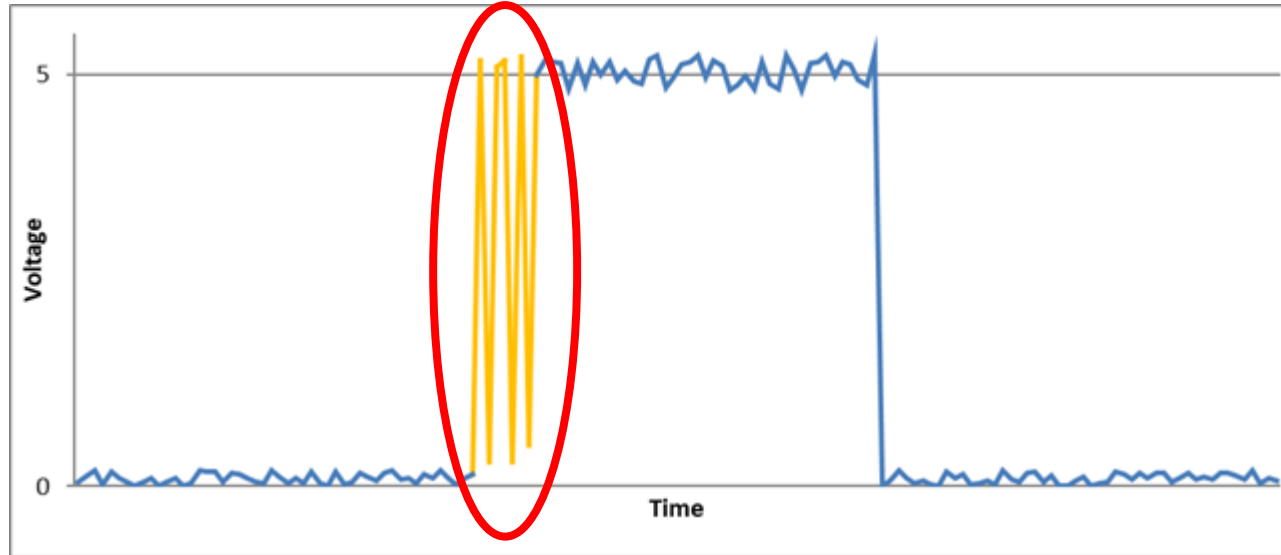
HX711

24-Bit Analog-to-Digital Converter (ADC) for Weigh Scales

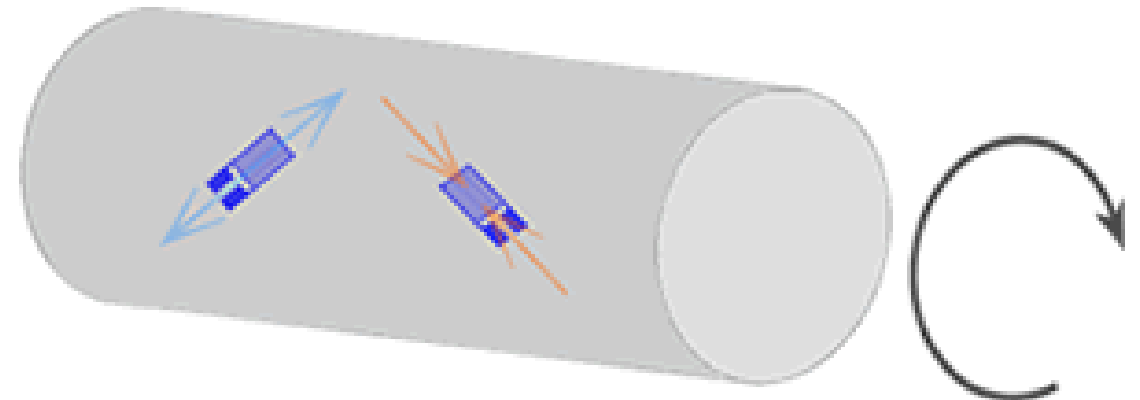
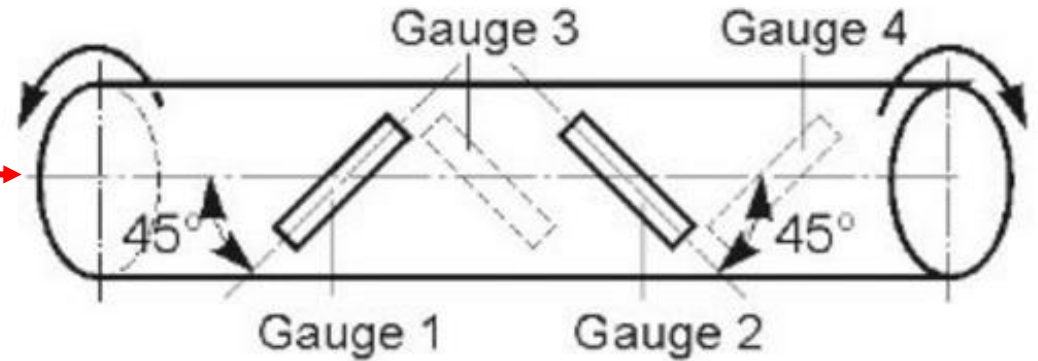
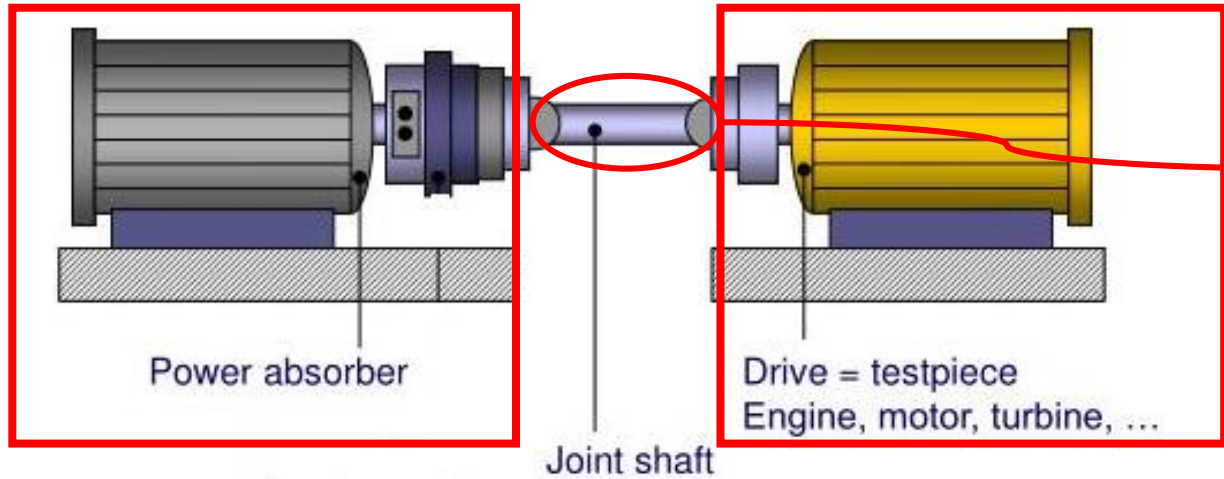


Load cell

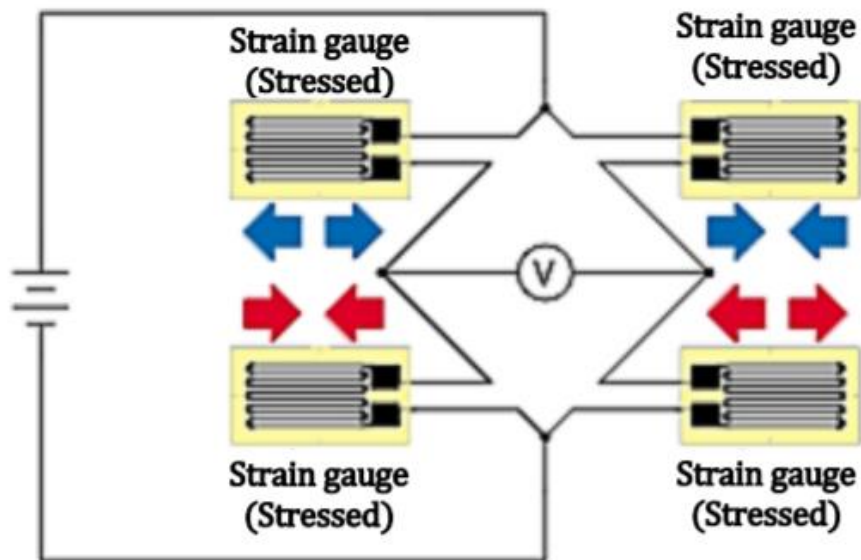
Ringing



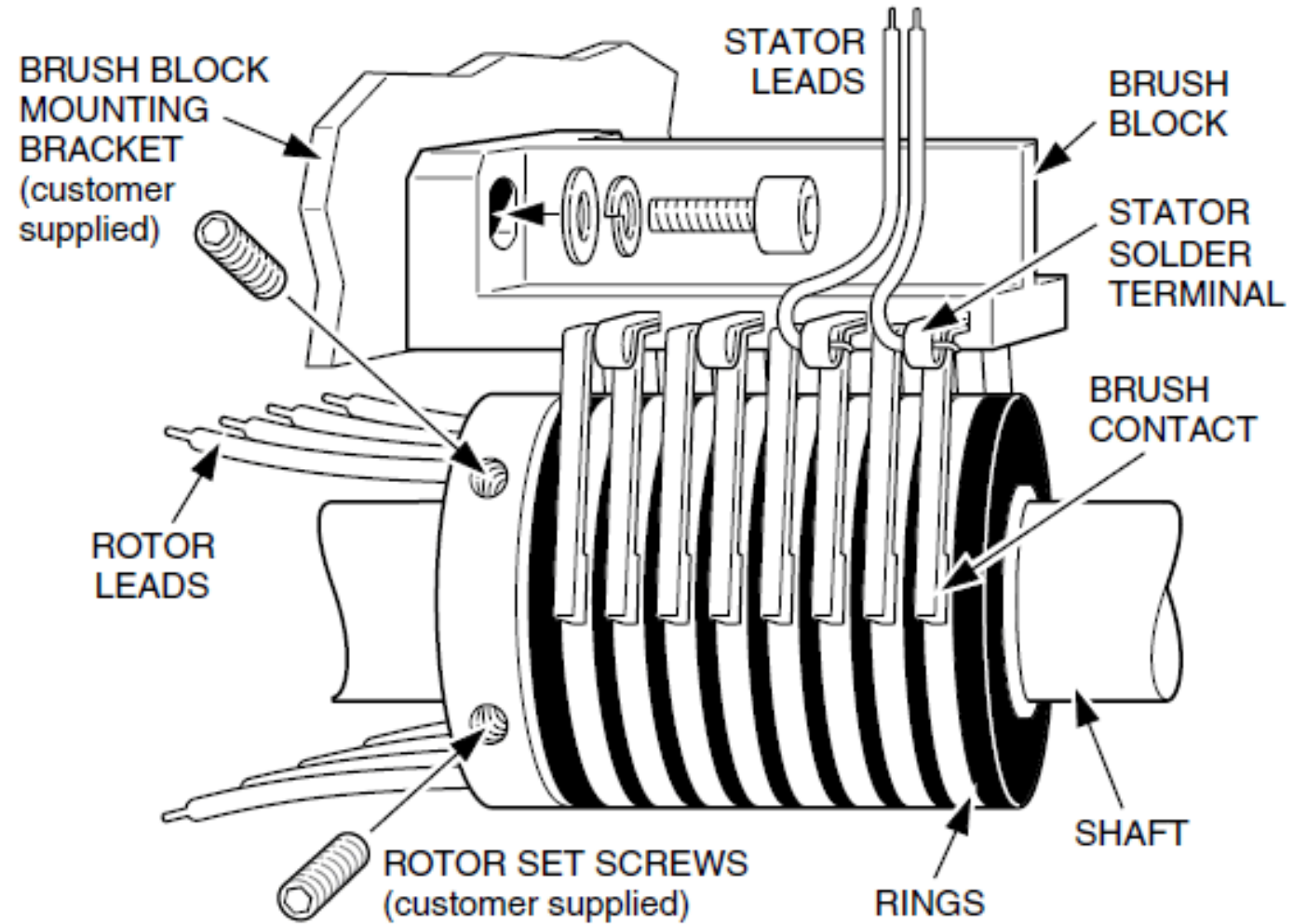
Torque Measurement



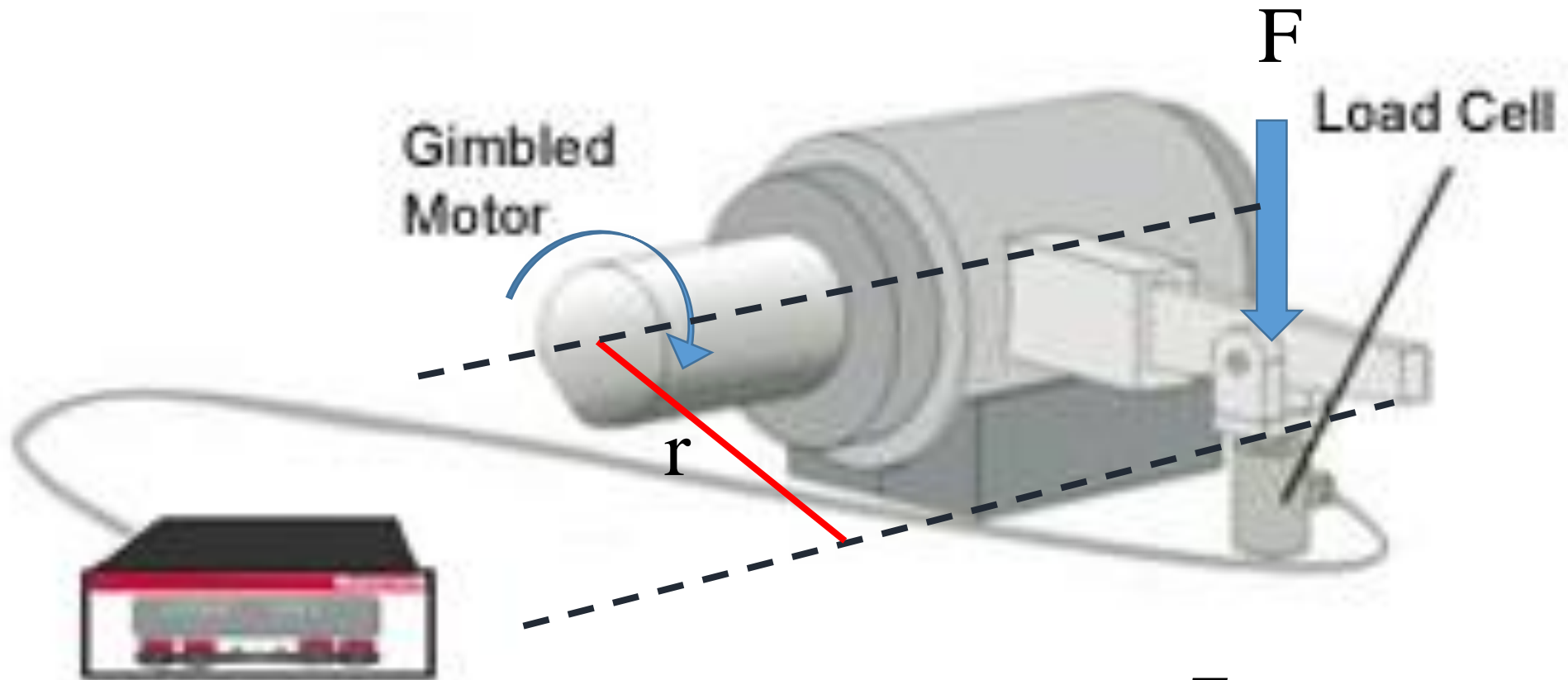
Strain Gauges Bonded to a Torque Shaft



Slip ring



Torque Measurement



$$\tau = F \times r$$