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تمرین تحلیل سیستم

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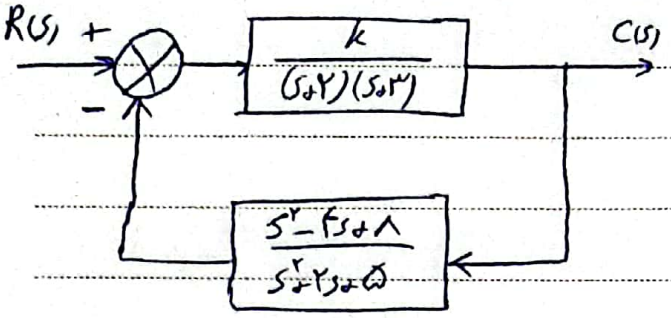
کلاس هشتم کنترل خطی



الف) تابع تبدیل: $\frac{C(s)}{R(s)} = \frac{\frac{k}{s^2+5s+6}}{1+\frac{k}{s^2+5s+6}\cdot\frac{s^2-4s+8}{s^2+2s+5}} \Rightarrow G(s) = \frac{s^2-4s+8}{(s+2)(s+3)(s^2+2s+5)}$

$= \frac{s^2-4s+8}{s^4+7s^3+25s^2+45s+30}$

قطب‌ها: $s^2+2s+5=0 \Rightarrow s = -1 \pm \sqrt{1-5} = -1 \pm j2 \Rightarrow \begin{cases} s_1 = -2 \\ s_2 = -3 \\ s_3 = -1-j2 \\ s_4 = -1+j2 \end{cases}$

صفرها: $s^2-4s+8=0 \Rightarrow s = 2 \pm \sqrt{4-8} = 2 \pm j2 \Rightarrow \begin{cases} s_1 = 2-j2 \\ s_2 = 2+j2 \end{cases}$

محل برخورد مجانب‌ها: $\frac{(2+2+1+1)-(-4)}{2} = 5/0$

تعداد مجانب‌ها $= |2-4| = 2$

زاویه مجانب‌ها $= \frac{(2l+1)\pi}{2} = \frac{\pi}{4}, -\frac{\pi}{4}$

نقاط شکست: $ND' - DN' = 0 \Rightarrow (2s-4)(s+2)(s+3)(s^2+2s+5) - (s^2-4s+8)\times$

$(4s^3+21s^2+42s+37) = 0 \Rightarrow \begin{cases} s_1 = -2,014 & \rightarrow \text{تنها نقطه شکست} \\ s_{2,3} = -1,501 \pm 1,479j & \times \\ s_{4,5} = 4,908 \pm 1,902j & \times \end{cases}$

زاویه ورود صفر حقیقی: $\varphi_1 = \pi - \frac{\pi}{2} + 0 + \tan^{-1}\left(\frac{4}{2}\right) + \tan^{-1}\left(\frac{2}{4}\right) + \tan^{-1}\left(\frac{2}{5}\right)$

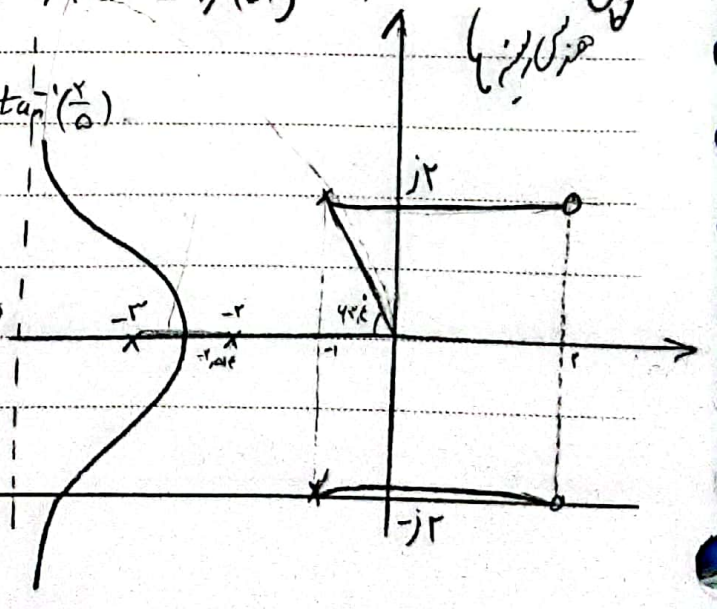
$= 3,34 \text{ radian} \simeq 191°$

زاویه خروج از قطب: $\theta_1 = -\pi + 0 - 0 = -\pi$

$\theta_2 = -\pi + 0 - \pi = -2\pi$

$\theta_3 = -\pi + \left(\pi + \frac{\pi}{4} + \arctan\left(\frac{2}{4}\right)\right) - \left(\frac{\pi}{2} + \tan^{-1}(2) + \tan^{-1}(1)\right)$

$\simeq -1,28 \text{ radian} \simeq -71,45°$



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ب) محل برخورد با محور ساز:

$$s^4 + 7s^3 + (21+k)s^2 + (37-4k)s + 8k + 30 = 0$$

s^4	1	$21+k$	$8k+30$
s^3	7	$37-4k$	0
s^2	$(110+11k)\frac{1}{7}$	$8k+30$	0
s^1	A	0	
s^0	$8k+30$		

$$A = \frac{(37-4k)\frac{11}{7}(10+k) - 56k - 210}{\frac{11}{7}(10+k)}$$

$-3k$

$37k + 370 - 4k^2$

$A = 0 \Rightarrow \frac{11}{7}(37-4k)(k+10) = 56k + 210 \Rightarrow -33k + 4070 - 44k^2$

$= 392k + 1470 \Rightarrow 44k^2 + 425k - 2600 = 0 \Rightarrow$ $\boxed{k = 4.2487}$

$k = -13.91$ ✗ غ‌ق‌ق

$\Rightarrow 22.391 s^2 + 63.99 = 0 \Rightarrow$ $\boxed{s = \pm 1.69j}$ محل های برخورد با محور ساز

$\boxed{\varphi_1 = 191^\circ}$

ب) در قسمت الف بدست آمده است.

ت) k

$$M_p = 0.3 \Rightarrow e^{\frac{-\zeta}{\sqrt{1-\zeta^2}}\pi} = 0.3 \overset{\ln}{\Rightarrow} \frac{-\zeta\pi}{\sqrt{1-\zeta^2}} = \ln(0.3) \simeq -1.204$$

$$\Rightarrow \frac{\zeta}{\sqrt{1-\zeta^2}} = 0.383 \overset{^2}{\Rightarrow} \frac{\zeta^2}{1-\zeta^2} \simeq 0.147 \Rightarrow \zeta^2 = \frac{0.147}{1.147} = 0.128$$

$$\Rightarrow \zeta \simeq 0.358 \Rightarrow \beta = \cos^{-1}(0.358) \simeq 69^\circ \Rightarrow \beta = 69^\circ$$

نقطه تقاطع ← ۴ محل تقاطع دارد ، مکان ریشه با ، $\zeta = 0.358$ خط

$\tan\beta \simeq 2.6$; $\bar{s} = -\zeta\omega_n + \omega_n\sqrt{1-\zeta^2}\,j \Rightarrow \frac{\sqrt{1-\zeta^2}}{-\zeta} = 2.6$ یعنی قسمت موهومی $\bar{s}$ باید ۲.۶ برابر قسمت حقیقی آن باشد

$\Rightarrow \bar{s} = -a + j2.6a$

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$$1+k\,\frac{\bar{s}^2-4\bar{s}+8}{\bar{s}^4+7\bar{s}^3+21\bar{s}^2+37\bar{s}+30}=0 \xrightarrow{\bar{s}=-a+j2{,}6a}$$

$$1+k\left(\frac{a^2-2{,}6^2a^2+j5{,}2a^2+4a-j10{,}4a+8}{A}\right)=0$$

$$\Rightarrow 1+k\left(\frac{-5{,}76a^2+4a+8+j(5{,}2a^2-10{,}4a)}{A}\right)=0$$

$$\Rightarrow (6{,}1376+59{,}904j)a^4+(134{,}96-68{,}432j)a^3+(-120{,}96-109{,}2j-5{,}76k+j5{,}2k)$$
$$\times a^2+(-37+96{,}2j+4k-j10{,}4k)a+30+8k=0+0j$$

$$\Rightarrow \begin{cases} 6{,}1376a^4+134{,}96a^3+a^2(-120{,}96-5{,}76k)+a(-37+4k)+30+8k=0 \\ 59{,}904a^4-68{,}432a^3+a^2(-109{,}2+5{,}2k)+a(96{,}2-10{,}4k)=0 \end{cases}$$

[illegible] $\Rightarrow a=0{,}7 \xrightarrow{\text{[illegible]}} \bar{s}=-0{,}7+j1{,}82$

$$\boxed{k=0{,}938}$$

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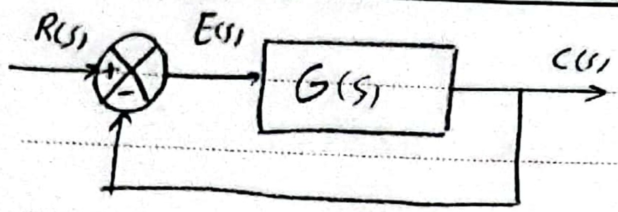


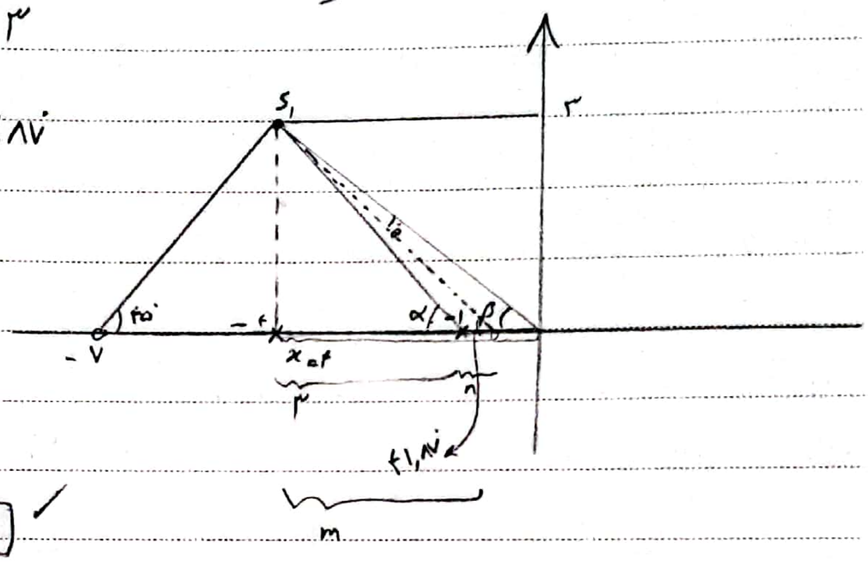
سوال ۶

$$G(s) = \frac{k}{(s+4)(s+1)}$$

$$G_c(s) = k_p + k_D s + \frac{k_I}{s} = k_c (s+Z_1)\left(\frac{s+Z_2}{s}\right)$$

$$\begin{cases} t_p = \frac{\pi}{\omega_d} = 1.047 \Rightarrow \omega_d = 3 \\ \zeta = 0.8 \Rightarrow \cos\beta = 0.8 \Rightarrow \beta = 36.87^\circ \end{cases}$$

$$\tan\beta = \frac{3}{x} \Rightarrow x = \frac{3}{\tan\beta} = \frac{3}{\frac{0.6}{0.8}} = 4$$



$\checkmark\ \alpha = 45^\circ$

$$\angle G(s_1) = -90 - (135) = -225^\circ$$

$\Rightarrow$ نقص زاویه $= 45^\circ \Rightarrow \boxed{Z_1 = 7}$ ✓

خطای ماندگار $= \frac{1}{k_p+1} = 0 \Rightarrow k_p \to \infty$

$$k_p = \lim_{s\to 0} G_c(s)G(s) = \lim_{s\to 0} \frac{k'}{(s+4)(s+1)}(s+7)\left(\frac{s+Z_2}{s}\right)$$ $\Rightarrow k_p$ اگر Z_2 مخالف صفر باشد به بی‌نهایت میل خواهد کرد

$$\tan(41.87^\circ) = \frac{3}{m} \Rightarrow m \cong 3.35 \Rightarrow n = 0.35 \Rightarrow Z_2 = -(-1+0.35) = 0.65$$

$$\Rightarrow \boxed{Z_2 = 0.65}\ \checkmark$$

$$1 + k'\frac{(s+7)}{(s+4)(s+1)}\cdot\frac{s+0.65}{s} = 0 \Rightarrow |G_c(s)G(s)| = 1$$

$$\Rightarrow k' \times \frac{3\sqrt{2}}{3 \times 3\sqrt{2}} \cdot \frac{\sqrt{9+3.35^2}}{5} = 1 \Rightarrow k' \cong \frac{15}{4.5} \Rightarrow k' = \frac{10}{3}$$

$$\Rightarrow G_c(s) = \frac{10}{3}(s+7)\left(\frac{s+0.65}{s}\right) = \frac{10}{3}\left[s + 0.65 + 7 + \frac{7\times 0.65}{s}\right]$$

$$\Rightarrow \boxed{G_c(s) = 25.5 + \frac{4.55}{s} + \frac{10}{3}s}$$

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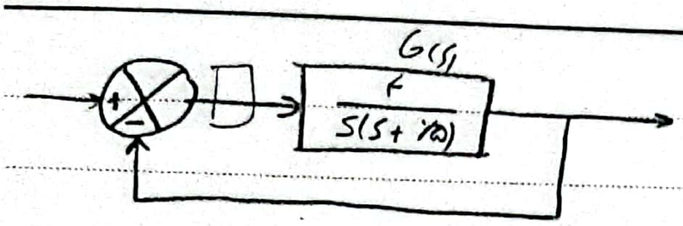
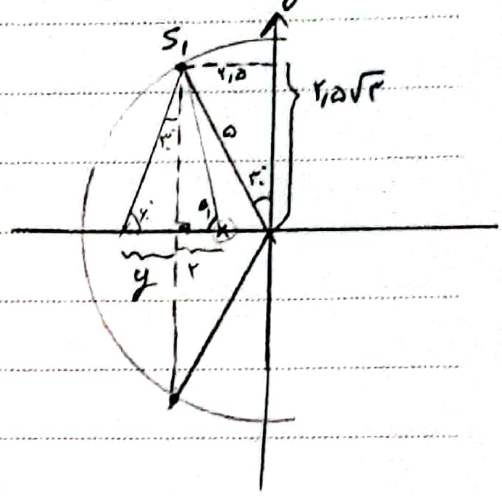


سوال ۸

$$G_c(s) = k_c \underbrace{\frac{s+z_1}{s+p_1}}_{lead} \; \underbrace{\frac{s+z_2}{s+p_2}}_{lag}$$



$\zeta = 0.5 \Rightarrow \cos\beta = 0.5 \Rightarrow \beta = \frac{\pi}{3}$

$\omega_n = 5 \ \frac{rad}{s}$

$s_{1,2} = -2.5 \pm j\, 2.5\sqrt{3}$

$\tan\theta_1 = \frac{2.5\sqrt{3}}{2} \Rightarrow \theta_1 = 65.21^\circ$

$\measuredangle G(s_1) = -120 - (180 - 65.21) = -234.8^\circ$

$\Rightarrow$ نقص زاویه $= 54.8^\circ$

صفر پیشفاز را روی قطب تابع تبدیل $G(s)$ قرار می‌دهیم:

$\boxed{z_1 = +0.5}$ → زاویه $= +(180 - 65.21) \cong 114.8$

$114.8 - x = 54.8 \Rightarrow x = 60$

$\tan 30^\circ = \frac{y}{2.5\sqrt{3}} \Rightarrow \frac{1}{\sqrt{3}} = \frac{y}{2.5\sqrt{3}} \Rightarrow y = 2.5 \Rightarrow \boxed{p_1 = +5}$

$\lim_{s\to 0} s G_c(s) G(s) = \lim_{s \to 0} \frac{1}{s+0.5} \times k_c \frac{s+0.5}{s+5} \frac{s+z_2}{s+p_2} = \frac{k_c}{5} \frac{z_2}{p_2} = 80$

$\Rightarrow k_c \frac{z_2}{p_2} = 400 \quad (I)$

$s = -2.5 + 2.5\sqrt{3} j$

شرط اندازه: $1 + k_c \frac{s+0.5}{s+5} \frac{1}{s(s+0.5)} = 0 \Rightarrow 1 + \frac{k_c}{s(s+5)} = 0 \Rightarrow k_c = -(2.5\sqrt{3}j - 2.5)$

$\times (2.5 + 2.5\sqrt{3} j) \Rightarrow k_c = -(-2.5^2 \times 3 - 2.5^2) = 4 \times 2.5^2 = 4 \times \frac{25}{4} = 25$

$\Rightarrow k_c = 25 \quad (II) \quad \overset{(I),(II)}{\Rightarrow} \frac{z_2}{p_2} = \frac{400}{25} = 4 \times 4 = 16 \Rightarrow \boxed{\frac{z_2}{p_2} = 16}$

فرض می‌کنیم $\begin{cases} z_2 = \frac{1}{2} \\ p_2 = \frac{1}{32} \end{cases} \Rightarrow \boxed{G_c(s) = 25 \frac{s+0.5}{s+5} \frac{s+\frac{1}{2}}{s+\frac{1}{32}}}$

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