$$\frac{C(s)}{R(s)} = \frac{16}{s^2 + (0.8+16k) + 16} \longrightarrow \omega_n = 4$$

$$\longrightarrow 2\eta\omega_n = 2 \times 0.5 \times 4 = 0.8 + 16k \longrightarrow k = 0.2$$

$$tr = \frac{\pi - \beta}{\omega_d}, \beta = \tan^{-1}\left(\frac{\omega_d}{\eta\omega_n}\right), \omega_d = \omega_n\sqrt{1-\eta^2} = 3.46$$

$$\longrightarrow \beta = 1.047 \longrightarrow tr = 0.605 \text{ sec}$$

$$tp = \frac{\pi}{\omega_d} = \frac{\pi}{3.46} = 0.907 \text{sec}$$

$$Mp = \exp\left(\frac{-\pi\eta}{\sqrt{1-\eta^2}}\right) \longrightarrow Mp = \exp\left(\frac{-0.5\times\pi}{\sqrt{1-0.25}}\right) = 0.163$$

$$ts = \frac{4}{\eta\omega_n} = \frac{4}{0.5\times 4} = 2 \text{ sec}$$

پاسخ به دومین پرسش:

$$1 - \frac{e^{-2t}\left(\sin(2\sqrt{3}\,t) + \sqrt{3}\cos(2\sqrt{3}\,t)\right)}{\sqrt{3}}$$

(5) الف

$$G(s)=\frac{16}{s^2+3s+16} \longrightarrow \omega_n=4\ ,\ 2\eta\times 4=3 \longrightarrow \eta=0.375$$

$$t_s=\frac{4}{\eta\omega_n}=2.667\ ,\qquad t_p=\frac{\pi}{\omega_d}=0.8472$$

$$\%OS=\exp\left(-\eta\pi/\left(\sqrt{1-\eta^2}\right)\right)=0.28\ ,\ t_r=\frac{\pi-\beta}{\omega_d}=0.356$$

ب)

$$G(s)=\frac{0.04}{s^2+0.02s+0.04} \longrightarrow \omega_n=0.2,\ \eta=0.05$$

$$t_s=400\ ,\ t_p=15.73\ ,\ \%OS=85.45\%\ ,\ t_r=5.26s$$

ج)

$$G(s)=\frac{1.05\times10^{+7}}{s^2+1.6\times10^{3}+1.05\times10^{7}} \longrightarrow \eta=0.247,\ \omega_n=3240$$

$$t_s=0.05,\ t_p=0.01\ ,\ \%OS=44.92\%\ ,\ t_r=3.88\times10^{-4}\ sec$$

(6)

$$t_p=\frac{\pi}{\omega_d}=0.01$$

$$MP=\exp\left(\frac{-\pi\eta}{\sqrt{1-\eta^2}}\right) \longrightarrow \eta=0.4$$

$$\left.\begin{array}{l}\omega_d=\omega_n\sqrt{1-\eta^2}=100\pi\\ \longrightarrow 0.91\omega_n=100\pi \longrightarrow \omega_n=\dfrac{100\pi}{0.91}\\ \longrightarrow \omega_n=345.22\end{array}\right\}$$

$$\longrightarrow G(s)=\frac{1.19\times10^{5}}{s^2+276.18s+1.19\times10^{5}}$$

(8)

$\ddot{y} + 3\dot{y} + 2y = 0$, $y(0) = 0.1$, $\dot{y}(0) = 0.05$

تبدیل لاپلاس $\longrightarrow$ $s^2 Y(s) - sy(0) - \dot{y}(0) + 3\left(sY(s) - y(0)\right) + 2Y(s) = 0$

$\longrightarrow Y(s)\left(s^2 + 3s + 2\right) = 0.1s + 0.35 \longrightarrow Y(s) = \dfrac{0.1s + 0.35}{s^2 + 3s + 2}$

$= \dfrac{0.25}{s+1} + \dfrac{-0.15}{s+2}$ $\xrightarrow{\text{inverse laplace}}$ $y(t) = 0.25e^{-t} - 0.15e^{-2t}$

a) $C(s) = \dfrac{5}{s(s+5)} = \dfrac{1}{s} - \dfrac{1}{s+5} \longrightarrow C(t) = 1 - e^{-5t}$ (1)

$T = \frac{1}{5}$, $tr = \frac{2.2}{5} = 0.44$, $ts = \frac{4}{5} = 0.8$

b) $C(s) = \dfrac{20}{s(s+20)} = \dfrac{1}{s} - \dfrac{1}{s+20} \longrightarrow C(t) = 1 - e^{-20t}$, $T = \frac{1}{20}$

$tr = \frac{2.2}{20} = 0.11$, $ts = \frac{4}{20} = 0.2$

(3)

$OS = 0.05 \longrightarrow \exp\left(\dfrac{-\pi\eta}{\sqrt{1-\eta^2}}\right) = 0.05 \longrightarrow \eta = 0.69$

$ts = 2 \longrightarrow \dfrac{4}{\eta\omega_n} = 2 \longrightarrow \eta\omega_n = 2 \longrightarrow \omega_n = \dfrac{2}{0.69} = 2.90$

ب) $\frac{5}{(s+3)(s+6)}$ → poles: $-3, -6$, $c(t) = A + Be^{-3t} + Ce^{-6t}$ (3)

ج) $\frac{10(s+7)}{(s+10)(s+20)}$ → poles: $-10, -20$, zero: -7, $c(t) = A + Be^{-10t} + Ce^{-20t}$

د) $\frac{20}{s^2+6s+144}$ → poles: $-3 \pm j3\sqrt{15}$, $c(t) = A + Be^{-3t}\cos(3\sqrt{15}t + \theta)$

ه) $\frac{s+5}{(s+10)^2}$ → poles: -10, zero: -5, $c(t) = A + Be^{-10t} + Cte^{-10t}$

(7)

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} -1 & -1 \\ 6.5 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}$$

$$\begin{cases} \dot{x}_1 = -x_1 - x_2 + u_1 + u_2 \\ \dot{x}_2 = 6.5x_1 + u_1 \\ y_1 = x_1 \\ y_2 = x_2 \end{cases} \longrightarrow \begin{cases} sX_1(s) = -X_1(s) - X_2(s) + U_1(s) + U_2(s) & (1) \\ sX_2(s) = 6.5X_1(s) + U_1(s) & (2) \\ Y_1(s) = X_1(s) \\ Y_2(s) = X_2(s) \end{cases}$$

$$\rightarrow (s+1)X_1(s) = \frac{-6.5}{s}X_1(s) - \frac{U_1(s)}{s} + U_1(s) + U_2(s)$$

$$\rightarrow X_1(s)\left(s^2+s+6.5\right) = (s-1)U_1(s) + U_2(s)$$

$$\rightarrow Y_1(s) = X_1(s) = \frac{s-1}{s^2+s+6.5}U_1(s) + \frac{5}{s^2+s+6.5}U_2(s)$$

$\xrightarrow{(2)}$ $Y_2(s) = X_2(s) = \dfrac{s+7.5}{s^2+s+6.5} U_1(s) + \dfrac{32.5}{s(s^2+s+6.5)}$

$\longrightarrow \omega_n = \sqrt{6.5}$, $\zeta = \dfrac{1}{2\sqrt{6.5}}$