University of Tehran
College of Engineering
School of Electrical and Computer Engineering (ECE)
School of Mechanical Engineering (ME)

Mechatronics & Robotics

Homework 3

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Deadline: 23 April (4 Ordibehesht), 23:59

This assignment is designed to help you develop a better understanding of trajectory generation and jacobian concepts and use cases. About the Conceptual/Analysis questions (shown with a star (*) next to their indicative question number) the use of LLM Models is not permitted. References should be provided if necessary.

List of Problems

Problem 1: Wrist Robot Jacobian - 15 points

Consider the following 6-degree-of-freedom robot.The third joint is a prismatic joint. According to the problem in the previous homework:

- For the configuration shown, find the velocity of point P with respect to the joint velocity as follows: $\overrightarrow{\theta} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ (rad/s)

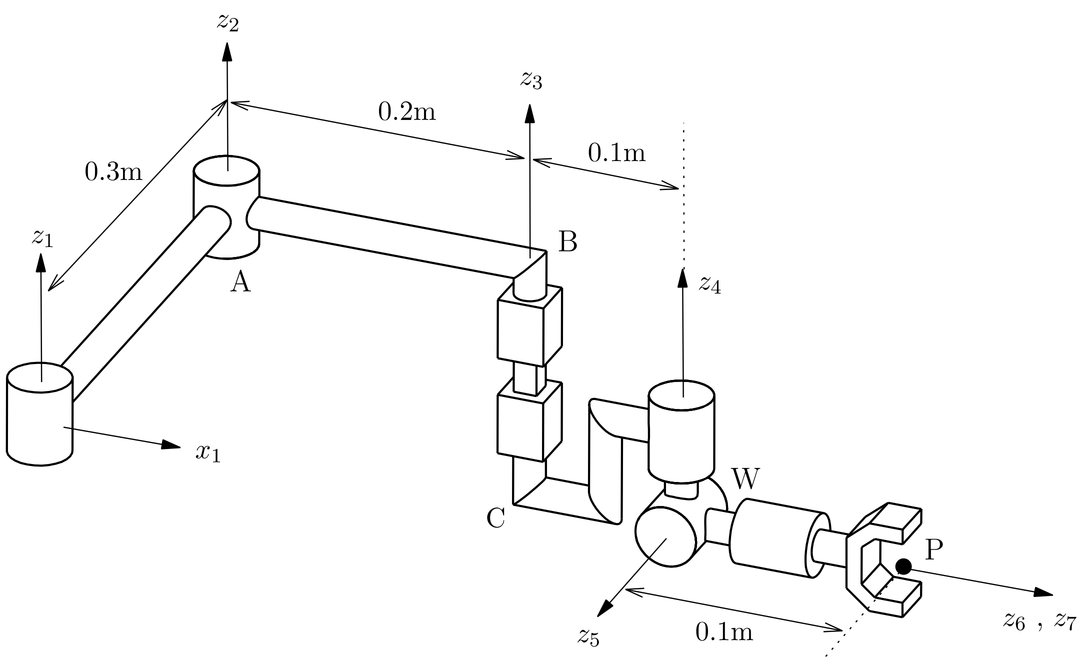

Figure 1: 6-DOF robot

Problem 2: Jacobian and Singularity - 20 points

Consider the following three-degree-of-freedom robot, where all axes of rotation are at an angle of 90 degrees. According to the problem in the previous homework:

- Find the Jacobian matrix of the robot for $l_1 = l_2 = 1$.
- Obtain the expression that gives the singularities of the robot (Use Matlab or Maple software)

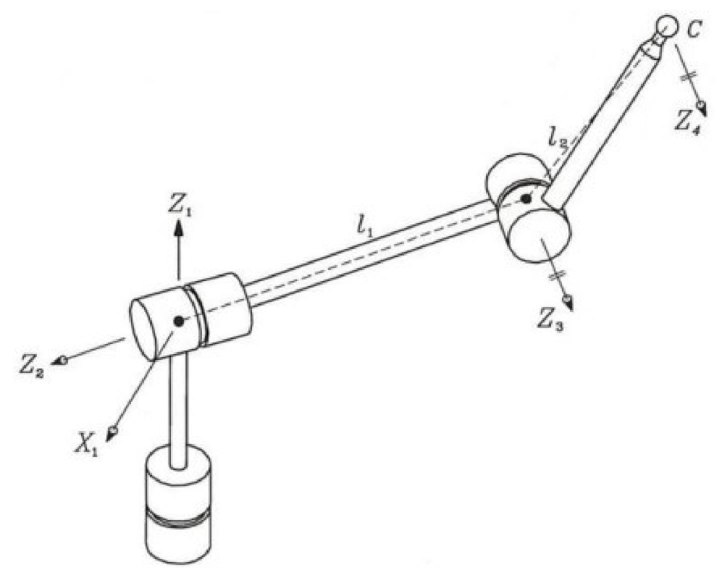

Figure 2: 3-DOF Robot

Problem 3: Static Analysis - 15 points

Consider the wrist three-degree-of-freedom robot with the following C wrist point. In a special case, when $\theta_2 = 90$, the Jacobian matrix of this robot is as follows:

$$[J]_2 = \begin{bmatrix} 0 & 0 & \sqrt{3} \\ \sqrt{3} & 0 & 0 \\ -1 & 2 & -1 \end{bmatrix}$$

- Show that the above matrix can be the Jacobian matrix of the robot (Hint: pay attention to the general form of the Jacobin matrix of the wrist robot that was presented in the class and pay attention to the characteristics of each row)
- A force equal to f at point P at a distance r from point C produces a torque n around point C which is sensed by the sensors of the torque-meter in the joints. The torque values sensed by each joint are as follows:

  $$\tau_1 = 20, \tau_2 = 20, \tau_3 = -10$$

  Calculate n in the device F_2(The device connected to joint 2 according to the D-H instruction)

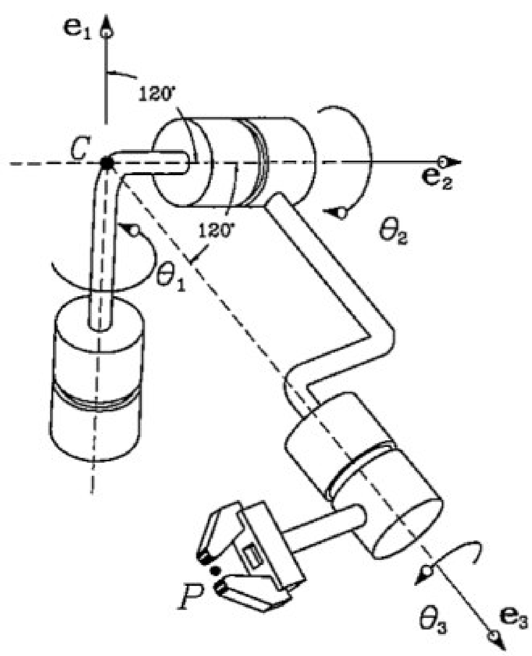

Figure 3: Wrist 3-DOF robot

Problem 4: Conceptual Analysis - 17.5 points

Problem 4.1*

Name three **real-world applications** of trajectory planning in robotics and analyze and discuss the following for each of those applications.

- The **role and purpose** of trajectory planning in these applications.
- The **importance and impact** of (not) using trajectory planning.
- The **challenges of real-time** trajectory planning in a dynamic environment.

Problem 4.2*

Choose two trajectory planning methods (i.e. 3-4-5 interpolating polynomial) and **predict their performance** in the 3 applications from the previous question and analyze their advantages and disadvantages in each scenario.

Problem 4.3*

Discuss one method to **optimize** an established trajectory and explain the impacts of the optimization. What are your **metrics** for evaluating good performance of a trajectory?

Problem 5: Trajectory Generation - 20 points

For a 3-DOF planar robot (as shown in Fig. 4), the goal is to generate a trajectories using various methods in the **joint space** in an overall time period of 2 seconds for each operation. The trajectory will be generated using the methods outlined below:

1. EE From Point A to B using a **3-4-5 Interpolating Polynomial**
   (initial and final velocity and acceleration set to zero)
2. EE From Point A to B using a **4-5-6-7 Interpolating Polynomial**
   (initial and final velocity, acceleration, and jerk set to zero)
3. EE From Point A to B using **Trapezoidal Method**
   (initial and final velocity set to zero)
4. EE From Point A to B to C with the help of a **Polynomial**-based trajectory
   (initial and final velocity and acceleration set to zero — The robot should not stop at point B)

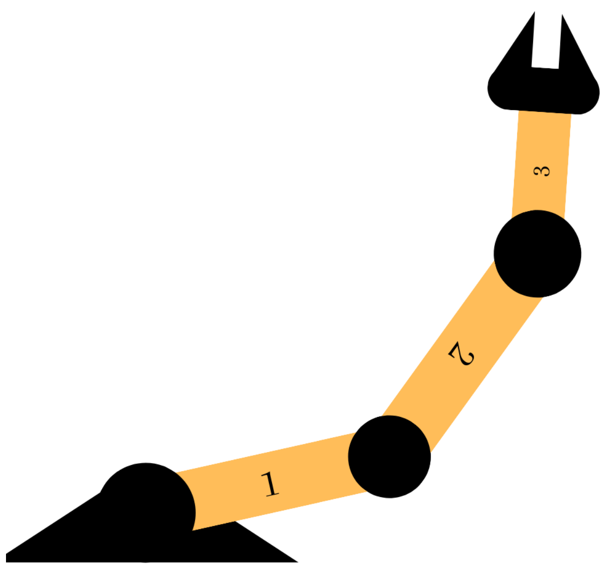

Figure 4: 3-DOF planar robot; $l_1 = l_2 = \frac{3}{2}l_3 = L$

Configuration of the robot:

$$\text{Point } A \equiv x_{EE} = 1.3660L, \quad y_{EE} = 2.0327L, \quad \Phi_{EE} = 90\,\text{deg} \tag{1}$$
$$\text{Point } B \equiv x_{EE} = 1.0774L, \quad y_{EE} = 0.4673L, \quad \Phi_{EE} = 30\,\text{deg} \tag{2}$$
$$\text{Point } C \equiv x_{EE} = -0.6994L, \quad y_{EE} = 1.9434L, \quad \Phi_{EE} = 120\,\text{deg} \tag{3}$$

Problem 5.0

Derive the equations for **Forward and Inverse Kinematics** of the robot.

Problem 5.1

Write the time dependant equations of **Angle, Angular Velocity, and Angular Acceleration** for all of the mentioned methods and actuator joints.

Problem 5.2

Plot the velocity profile for all of the mentioned methods. For each method the information of all actuator joints should be included in the plot representation during the overall time of operation.

Problem 5.3*

Compare velocity profiles between all methods and analyze what the data implies in terms of practicality.

Problem 5.4*

Discuss whether its a good idea to use the fourth method for a large number of points or not.

NOTE: Provide the used Python or MATLAB code in the report attachment.

Problem 6: MATLAB Coding - 17.5 points

In a generalized sense, each operation of robotic manipulation has four steps:

1. Task Planning
2. Path Planning
3. Trajectory Planning
4. Low-Level Control

The difference between a path and a trajectory is the following:

- Path: an array of discrete points that link a start and a goal.
- Trajectory: scheduled motion as a continuous function of time provided to the robot as a reference for following.

In other words, the path represents the desired points in 3D space that a robot should traverse based on the objectives of task planning, while the trajectory outlines a detailed schedule for this path. This trajectory may include information such as position, velocity, acceleration, and other relevant parameters at every instance of time during the movement. In this problem, you will treat the end-effector of the robot as a singular particle moving in 3D-space and will be provided with the desired paths of travel. Using MATLAB, you should generate the trajectory through different methods listed below.

$$\begin{aligned}\text{Path} = [&(-2.5, 1.8, 3.2), (0.7, -4.9, 2.3), (6.4, 2.1, -0.6), (-3.2, -0.5, 5.6),\\ &(1.9, 3.8, -2.7), (4.2, -5.2, 1.1), (-0.3, 6.7, -4.9),\\ &(5.6, -2.1, 3.4), (-4.8, 0.4, -1.9), (2.3, -3.6, 4.8)]\end{aligned}$$

The list of methods (hint: do not manually implement any of the methods):

1. Multi-Point Trapezoidal
2. Multi-Point Cubic Polynomials
3. Multi-Point Quintic Polynomials
4. B-spline

The list of the desired data for each path:

1. 3D Plot of the path points and the trajectory (highlight path points in red dots)
2. Position-Time Plot along each axes of x, y, z (one diagram for each method)
3. Velocity-Time Plot along each axes of x, y, z (one diagram for each method)
4. Acceleration-Time Plot along each axes of x, y, z (one diagram for each method)
5. Peak velocity and acceleration in each trajectory for all the axes in each method

Problem 6.1

Explain the last output element of these functions in MATLAB and its use.

Problem 6.2

Discuss the difference between the first three methods.

Problem 6.3

Explain the main different between the last method (B-spline) with the other methods and the discuss the reasons for that. How does this affect the performance of the robot?

Problem 6.4

In what robotic operation would you consider using method number 1 or number 4 (Mutli-Point Trapezoidal and B-Spline)? Explain the reasons for your choice.

Problem 6.5

Draw a general conclusion from the last three problems and explain what you understood from trajectory generation.

Homework Guidelines and Instructions

- The deadline for sending this exercise will be until 4 Ordibehesht.
- This time cannot be extended and you can use time grace if needed.
- The implementation must be in either Python or MATLAB programming language and your codes must be executable and uploaded along with the report.
- This exercise is done by one person.
- If any similarity is observed in the work report or implementation codes, this will be considered as fraud for the parties.
- Using ready-made codes without mentioning the source and without changing them will constitute cheating and your practice score will be considered zero.
- If you do not follow the format of the work report, you will not be awarded the grade of the report.
- Handwritten exercise delivery is not acceptable.
- All pictures and tables used in the work report must have captions and numbers.
- A large part of your grade is related to the work report and problem solving process.
- Please upload the report, code file and other required attachments in the following format in the system: `HW1_[Lastname]_[StudentNumber].zip`
  For example, the: `HW1_Ezati_12345678.zip`
- If you have questions or doubts, you can contact the assistants through the following e-mail with the subject 3HW_Mechatronics. Stay in touch educationally:
  - For questions one, two, and three: Navid.Razaghi@ut.ac.ir (Navid Razaghi)
  - For questions four, five, and six: arvin1844m@gmail.com (Arvin Mohammadi)
- Be happy and healthy.