University of Tehran

College of Engineering

School of Electrical and Computer Engineering (ECE)

School of Mechanical Engineering (ME)

Mechatronics & Robotics

Homework 4:
Dynamics of Serial Robotic Manipulators

Teaching Assistant:
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Deadline: May 17, 2024 (Ordibehesht 28), 23:59

List of Problems

Problem 1: IKP (25 points)

Given the position of point P represented by its coordinates (x,y) and the orientation ϕ of the last link of a serial robot, on which point P is located, accomplish the following tasks:

a) Utilize the closed-form solution of the Inverse Kinematic Problem (IKP) to determine the angle between the links as a function of time.

b) Employ MATLAB to numerically solve the IKP for a desired time step and validate the obtained results by comparing them with the solution derived in section a. For section b, you may assume $L_1 = 17$ cm, $L_2 = 19$ cm, $L_3 = 23$ cm.

$$
\begin{aligned}
x = L_1 \cos(\frac{\pi}{4} + \frac{\pi}{9}\sin(\frac{\pi}{5}t)) + L_2 \cos(\frac{5\pi}{12} + \frac{\pi}{9}\sin(\frac{\pi}{5}t) + \frac{\pi}{18}\cos(\frac{\pi}{10}t)) \\
+ L_3 \cos(\frac{11\pi}{36} + \frac{\pi}{9}\sin(\frac{\pi}{5}t) + \frac{\pi}{18}\cos(\frac{\pi}{10}t) - \frac{\pi}{36}\sin(\frac{\pi}{15}t))
\end{aligned}
$$

$$
\begin{aligned}
y = L_1 \sin(\frac{\pi}{4} + \frac{\pi}{9}\sin(\frac{\pi}{5}t)) + L_2 \sin(\frac{5\pi}{12} + \frac{\pi}{9}\sin(\frac{\pi}{5}t) + \frac{\pi}{18}\cos(\frac{\pi}{10}t)) \\
+ L_3 \sin(\frac{11\pi}{36} + \frac{\pi}{9}\sin(\frac{\pi}{5}t) + \frac{\pi}{18}\cos(\frac{\pi}{10}t) - \frac{\pi}{36}\sin(\frac{\pi}{15}t))
\end{aligned}
$$

$$
\phi = \frac{\pi}{4} + \frac{\pi}{9}\sin(\frac{\pi}{5}t) + \frac{\pi}{6} + \frac{\pi}{18}\cos(\frac{\pi}{10}t) - \frac{\pi}{9} - \frac{\pi}{36}\sin(\frac{\pi}{15}t)
$$

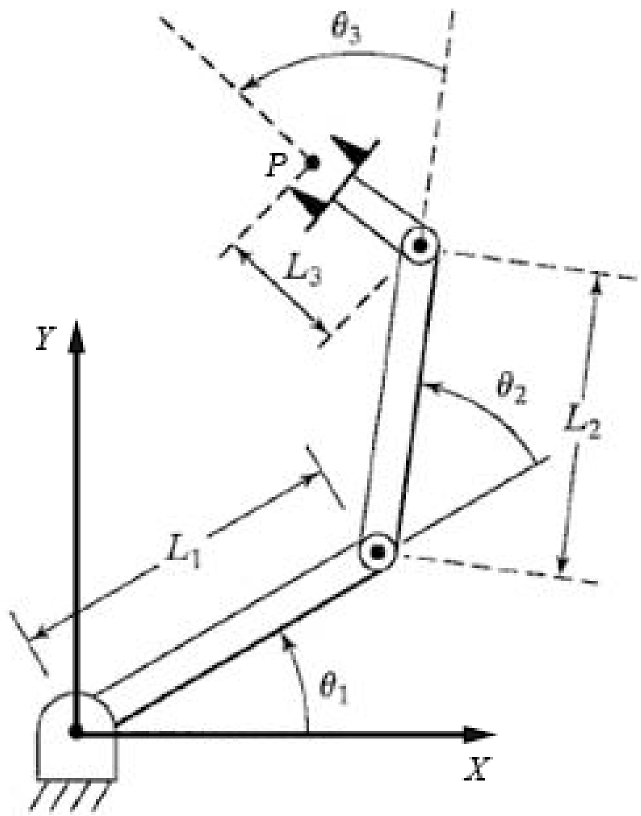

Figure 1: Point P is located at the end of the last link

Problem 2: A Walk in the Park (15 points)

Imagine a non-rotating frame $\{xyz\}$ with unit vectors **i**, **j**, and **k**, and a rotating frame $\{x'y'z'\}$ with unit vectors **i'**, **j'**, and **k'**, where the angular velocity of the rotating frame $\omega(t)$ is defined as

$$\omega(t) = t\mathbf{i} - t^2\mathbf{j} + \frac{1}{t+1}\mathbf{k}$$

and the position of the origin of the rotating frame $O'(t)$ is defined as

$$O'(t) = (1+t)\mathbf{i} + t\mathbf{j} + t\mathbf{k}.$$

A point P, where is the standing point of Parsa is defined in the rotating frame with its position as

$$P(t) = 2t^2\mathbf{i'} + t\mathbf{j'}$$

Determine the velocity and acceleration of Parsa in the non-rotating frame as functions of time. Assume that at $t = 0$, the axes of the rotating and the non-rotating frame are parallel.

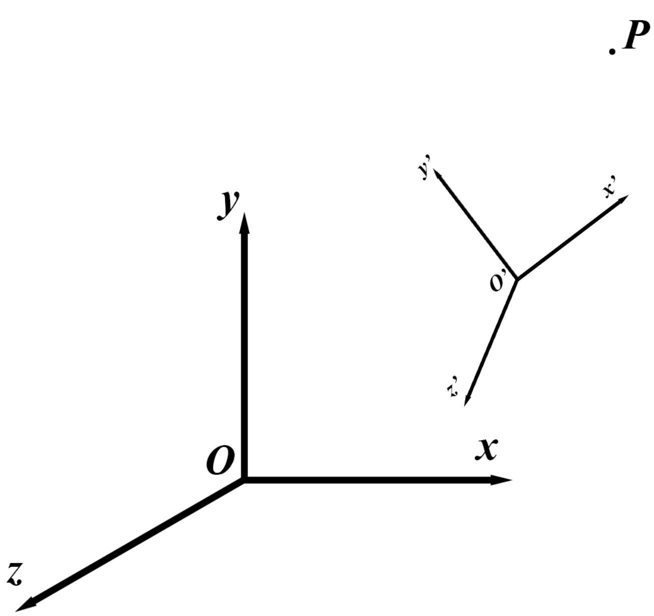

Figure 2: The frames and point P

Problem 3: Isaac vs Joseph (30 points)

Use the Newtonian and Lagrangian methods to determine the dynamic equations of motion for the system shown. Assume the force $F_1(t)$ and $F_2(t)$ are applied on the upper and lower cart and the moment $M_1(t)$ and $M_2(t)$ are applied on the upper and lower pendulum respectively.

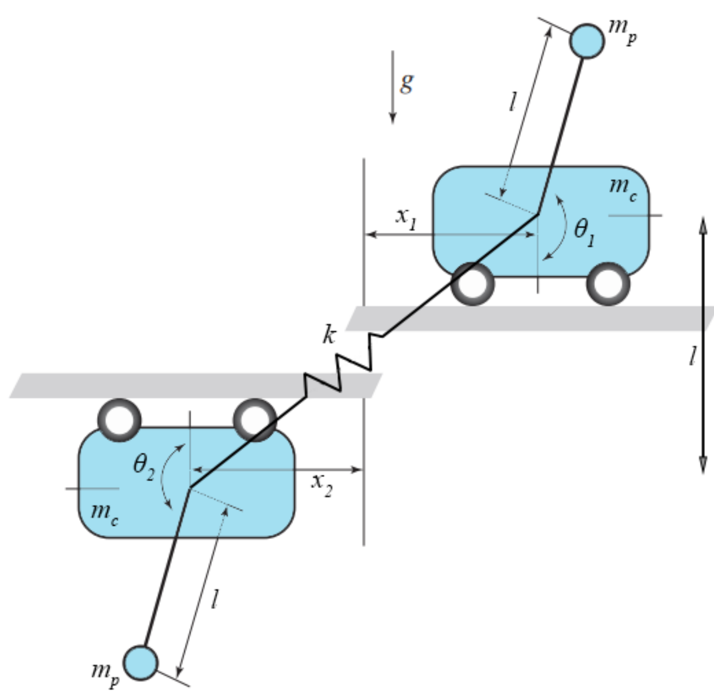

Figure 3: The dynamic system

Problem 4: Dynamics of Serial Robots (30 points)

A two-revolute pointing manipulator is shown in the figure. The centroidal inertia matrices of the links are denoted by I_1 and I_2. These are given, in link-fixed coordinates, by:

$$I_1 = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{21} & I_{22} & I_{23} \\ I_{31} & I_{32} & I_{33} \end{bmatrix}, I_2 = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}$$

Moreover, the mass centers of the links are denoted by C_1 and C_2, respectively, and are shown in the same figure, the masses being denoted by m_1 and m_2. Determine the dynamic equations of motion for the system.

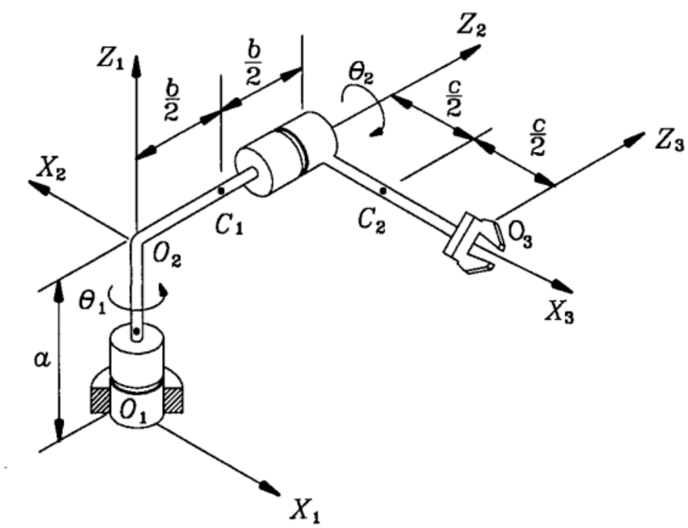

Figure 4: The serial robot

Homework Guidelines and Instructions

- The deadline for submitting this homework will be the end of Friday, May 17.
- This time cannot be extended and you can use time grace if needed.
- The implementation must be in Python programming language and your codes must be executable and uploaded along with the report.
- This homework should be done individually.
- If any similarity is observed in the work report or implementation codes, this will be considered fraud for the parties.
- Using ready-made codes without mentioning the source and without changing them will constitute cheating and your practice score will be considered zero.
- If you do not follow the format of the work report, you will not be awarded the grade of the report.
- Handwritten exercise delivery is not acceptable.
- All pictures and tables used in the work report must have captions and numbers.
- A large part of your grade is related to the work report and problem-solving process.
- Please upload the report, code file and other required attachments in the following format in the system: `HW3_[Lastname]_[StudentNumber].zip`
  For example the: `HW3_Rahmati_810699209.zip`
- If you have questions or doubts, you can contact the corresponding teaching assistant via the following Telegram link.
  - `https://t.me/erfunbsarmadi` (Erfun B. Sarmadi)
- Stay happy and healthy