Assignment 1

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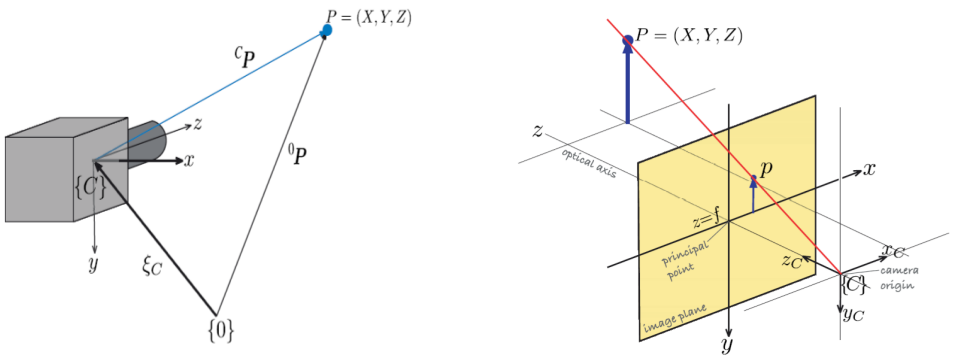
Instructions:

- The assignment deliverables include a single `.pdf` report containing:
  - Detailed solutions for Questions 1 and 2.
  - Results for Question 3: Submit well-commented and readable code (in either `Python` or `MATLAB`). Include the generated plots/figures and your comments as mentioned.
  - Note that there will be a demo where you will be called individually to discuss your code and solutions. The schedule will be released shortly.
- Penalties for plagiarism will be severe. In other words, submissions with substantially similar reports or programs will be marked at a much lower level than they would otherwise and/or will be reported to the Academic Section/DOAA.

Question 1.

Let $\{O\}$ and $\{C\}$ be the world frame and camera frame, respectively. The coordinate of point P are given by $[X,Y,Z]^T$ and $[X_c,Y_c,Z_c]^T$ w.r.t. $\{O\}$ and $\{C\}$ respectively. Given a 3-D point P expressed in the camera frame, the corresponding 2-D image plane coordinates are given as follows:



(a) (b)

Figure 1: Geometric model of pinhole camera (a) Camera coordinate frames (b) The perspective transform model.

$$p = \begin{bmatrix} u \\ v \end{bmatrix} = \frac{f}{Z_c} \begin{bmatrix} X_c \\ Y_c \end{bmatrix}, \tag{1}$$

where f is the focal length of the camera. Consider a camera which is located at the center of the world frame and perfectly aligned with the world frame as shown in Fig. 1b with a focal length of 15mm. Answer the following:

- (a) Find the image plane coordinates of a world frame point $(0.3, 0.4, 3.0)$. **(2 marks)**

- (b) If the camera frame is translated by $(0.1, 0, 0)$, then find the image plane coordinates of a world frame point $(0.3, 0.4, 3.0)$. **(2 marks)**
- (c) If the camera frame is translated by $(0.1, 0.1, 0)$ and the camera orientation is changed to $(0.2, 0, 0)$ then find the image plane coordinates of a world frame point $(0.3, 0.4, 3.0)$. **(3 marks)**

Question 2.

Consider the following operation with respect to world frame: Rotation by an angle of 60^0 about an axis in the yz-plane that is inclined at an angle of 60^0 to the positive y-axis. Answer the following

- (a) Calculate the quaternion q associated with the given orientation. **(3 marks)**
- (b) Determine the orientation matrix R that represents the given orientation. **(3 marks)**
- (c) Determine the coordinates of a world point (1, 1,2) in the new coordinate frame using both the quaternion and the orientation matrix. Also, compare the results. **(2 marks)**

Question 3.

Consider a configuration space given in Figure 2. The start and goal positions are $(1, 1)$ and $(20, 20)$ respectively. The obstacles are considered as the circular objects represented by center position and radius (r) as given below:

- (1) Obstacle 1 $\rightarrow$ Center- $(4.5, 3)$, r = 2.
- (2) Obstacle 2 $\rightarrow$ Center- $(3, 12)$, r = 2.
- (3) Obstacle 3 $\rightarrow$ Center- $(15, 15)$, r = 3.

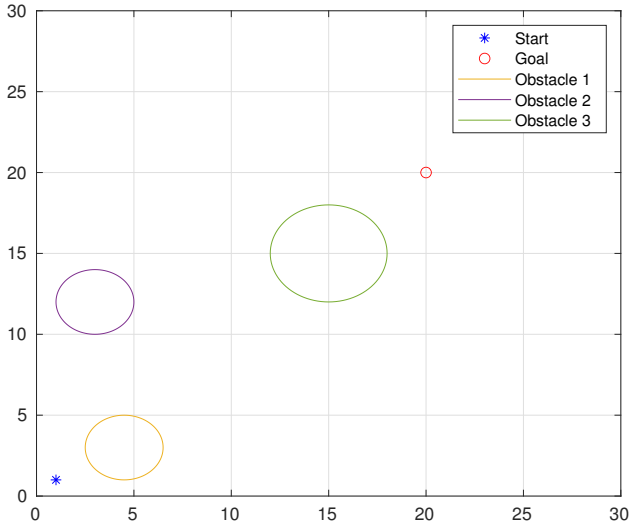


Figure 2: Configuration Space

- (a) Implement and simulate the following path planning algorithms from scratch (i.e., without using built-in planning libraries) in the given environment:
  - Bug Algorithm. **(10 marks)**
  - Probabilistic Roadmap (PRM). **(10 marks)**
  - Rapidly-exploring Random Tree (RRT). **(10 marks)**

  Specifically, for each method, generate and plot the path from the start to the goal while avoiding the obstacles.
- (b) Compare the performance of the three methods in terms of: (i) Path length (total distance traveled by the robot), (ii) Computational time (average runtime for finding the path). Also, provide a short discussion on the advantages and disadvantages of each algorithm based on your results. **(5 marks)**