1. What is stochastic calculus?

→ Main deep idea of calculus that we can find values of a function knowing the rate of change of the function.

for example given $\frac{df}{dt} = f'(t) = C(t, f(t))$,

$f(0) = x_0$: $f(t) = x_0 + \int_0^t C(s, f(s))\,ds$

2. In stochastic calculus, we add randomness to the change

∴ (does this make the output of fn a random variable with a probability distribution instead of $f: \mathbb{R} \to \mathbb{R}$?)

→ we will have eqn like $dX_t = m(t, X_t)\,dt + \sigma(t, X_t)\,dB_t$

B_t : std Brownian Motion. This is an example of stochastic differential equation.

Read eqn as X_t is evolving (at t) like a Brownian motion with drift $m(t, X_t)$ & variance $\sigma(t, X_t)^2$.

→ different numerical techniques to solve like the stochastic Euler method through which we do Monte Carlo Simulatn.

$$X((k+1)\Delta t) = X(k\Delta t) + \Delta t\, m(k\Delta t, X(k\Delta t)) + \sqrt{\Delta t}\,\sigma(k\Delta t, X(k\Delta t))N_k$$

$N_k \sim N(0,1)$

→ We define integral : X_t is a soln to SDE above if

$$X_t = X_0 + \int_0^t m(s, X_s)\,ds + \int_0^t \sigma(s, X_s)\,dB_s.$$

. How to define $\int_0^t A_s\,dB_s$?

3. Stochastic Integral:

To define the process $Z_t = \int_0^t A_s\,dB_s$, think of Z_t as brownian motion with variance A_s^2 at time s.

To fit in with the betting analogy, if A_s is negative, then the process is going down.

4. Integration of simple processes:

Allows one to change bets at prescribed finite number of times. → A_t is simple, if $\exists\ 0 = t_0 < t_1 < \dots < t_n < \infty$ & random variables $Y_j, j = 0, 1, \dots n$ that are F_{t_j} measurable s.t

$A_t = Y_j$, $t_j \le t < t_{j+1}$ → A_t is F_t measurable. Also $E[Y_j^2] < \infty$.

$Z_{t_j} = \sum_{i=0}^{j-1} Y_i(B_{t_{i+1}} - B_{t_i})$, $Z_t = Z_{t_j} + Y_j(B_t - B_{t_j})$ s.t $t_j \le t < t_{j+1}$

$$\int_r^t A_s \, dB_s = Z_t - Z_r .$$