Q1

$$\begin{bmatrix}3&4&3\\1&5&-1\\6&3&7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}10\\7\\15\end{bmatrix}\begin{matrix}\\-①/3\\-①\times 2\end{matrix}$$

$$\begin{bmatrix}3&4&3\\0&3.6667&-2\\0&-5&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}10\\3.6667\\-5\end{bmatrix}\begin{matrix}\\\\+②\times\frac{5}{3.6667}\end{matrix}$$

$$\begin{bmatrix}3&4&3\\0&3.6667&-2\\0&0&-1.7272\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}10\\3.6667\\0\end{bmatrix}$$

$x_3=0$
$x_2=1$
$3x_1+4=10$
$x_1=2$

$$\Rightarrow\begin{bmatrix}2\\1\\0\end{bmatrix}$$

$$\begin{bmatrix}3&4&3\\1&5&-1\\6&3&7\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}10\\7\\15\end{bmatrix}$$

$$\begin{bmatrix}6&3&7\\1&5&-1\\3&4&3\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}15\\7\\10\end{bmatrix}\begin{matrix}\\+-\frac{1}{6}①\\+-\frac{1}{2}①\end{matrix}$$

$$\begin{bmatrix}6&3&7\\0&4.5&-2.1667\\0&2.5&-0.5\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}15\\4.5\\2.5\end{bmatrix}\begin{matrix}\\\\-\frac{2.5}{4.5}②\end{matrix}$$

$$\begin{bmatrix}6&3&7\\0&4.5&-2.1667\\0&0&0.7037\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}15\\4.5\\0\end{bmatrix}$$

$\Rightarrow$ $x_3=0$
$x_2=1$
$x_1=2$

$$\Rightarrow\begin{bmatrix}2\\1\\0\end{bmatrix}$$

Q2

```
eps   abs(x1-1)   abs(x2-eps)/eps   cond(A)
 6.00e-08  0.00e+00  1.04e+06  1.22e+15
 5.00e-07  4.88e-04  9.78e+02  1.60e+13
 9.00e-07  2.44e-04  2.72e+02  4.94e+12
 9.50e-07  2.44e-04  2.58e+02  4.43e+12
>>
```

Sqrt(eps_much)=10e-8

Conclusion: when eps is really close to sqrt(eps_much), the error for x1 is almost zero, but for x2, the relative error is large than other eps since the eps is too small. Than when eps grow, the error for x1 and relative error for x2 are smaller. From the cond(A) we can see error for x1 and relative error for x2 will grow when conditional number grow.

Matlab code

```
function [ex1,ex2,condA]=test(eps)

    A=[1,1+eps;1-eps,1];
    B=[1+(1+eps)*eps;1];
    sol=inv(A)*B;
```

```
    ex1=abs(sol(1,1)-1);
    ex2=abs((sol(2,1)-eps)/eps);
    condA=cond(A);
end
[e1a,e2a,condAa]=test(6e-8);
[e1b,e2b,condAb]=test(5e-7);
[e1c,e2c,condAc]=test(9e-7);
[e1d,e2d,condAd]=test(9.5e-7);
fprintf("eps  abs(x1-1)  abs(x2-eps)/eps
cond(A)\n %.2e  %.2e  %.2e  %.2e\n %.2e  %.2e  %.2e  %.2e\n
%.2e  %.2e  %.2e  %.2e\n %.2e  %.2e  %.2e  %.2e\n ",6e-
8,e1a,e2a,condAa,5e-7,e1b,e2b,condAb,9e-
7,e1c,e2c,condAc,9.5e-7,e1d,e2d,condAd)
```

Q3

a.

matlab code

```
function B=GE(A)
    S=size(A);
    n=S(1,1);
    B=eye(n);
    for i=1:n-1
        for j=i+1:n
            B(j,i)=A(j,i)/A(i,i);
            for k=i:n
                A(j,k)=A(j,k)-B(j,i)*A(i,k);
            end
        end
    end
    for p=1:n
        for q=p:n
            B(p,q)=A(p,q);
        end
    end

end
function [B,ipivot]=GEPP(A)
    S=size(A);
    n=S(1,1);
    P=eye(n);
    ipivot=zeros(1,n);
    for i=1:n-1
        [x,y]=max(abs(A(i:n,i)));
        %ipivot(1,i)=y+i-1;
```

```
        if((y+i-1)~=i)
            A([i y+i-1],:)=A([y+i-1 i],:);
            P([i y+i-1],:)=P([y+i-1 i],:);
        end
    end
    for i=1:n
        for j=1:n
            if(P(j,i) ==1)
                ipivot(1,i)=j;
            end
        end
    end
    B=GE(A);
end
function x=backward(B,b,ipivot)
    S=size(B);
    n=S(1,1);
    P=zeros(n,n);
    for j=1:n
        P(ipivot(j),j)=1;
    end
    U1=triu(B);
    for i=1:n
        B(i,i)=1;
    end
    L1=tril(B);
    Newb=P*b;
    y=L1\Newb;
    x=U1\y;
end
```

b.

```
n=input("pls");
A=rand(n,n);
x=ones(n,1);
b=A*x;
ipivotGER=1:n;
ipivotGE=ipivotGER.';
output1=GE(A)
[output2,l]=GEPP(A)
GEx=backward(output1,b,ipivotGE);
GEPPx=backward(output2,b,l);
```

```
error1=abs(x-A\b)./x;
error2=abs(x-GEx)./x;
error3=abs(x-GEPPx)./x;
con=cond(A);
fprintf("inv(A)*b  NO PIVOTING  PIVOTING
cond(A)\n %.2e  %.2e  %.2e  %.2e\n
",error1(1,1),error2(1,1),error3(1,1),con)
```

Here are some sample output

```
output1 =

    0.8875    0.0730    0.5092    0.4944
    0.5925    0.5366    0.5371    0.0439
    0.8924    0.9872   -0.2963   -0.3258
    0.2290    0.8880   -0.6141    0.4003


output2 =

    0.8875    0.0730    0.5092    0.4944
    0.8924    0.5297    0.2339   -0.2825
    0.5925    1.0130    0.3002    0.3300
    0.2290    0.8996    1.4943    0.4003


l =

     1     3     2     4

inv(A)*b  NO PIVOTING  PIVOTING  cond(A)
 8.88e-16  1.22e-15  9.99e-16  2.93e+01
```

```
output1 =

    0.1804    0.4667    0.4304    0.0502    0.3641    0.9277    0.5117
    2.6731   -1.0644   -0.2753    0.4310   -0.8552   -1.9377   -0.5155
    4.5353    1.8323   -1.1963   -0.2287    0.6053    0.0799   -0.5565
    3.3596    0.8123    0.2989    0.2504    0.0746   -1.5575   -0.7283
    2.2393    0.3001    0.5762    0.9764   -0.4924    0.0694    0.5125
    3.4076    0.7053    0.3266    1.9569    1.8701    1.6393   -0.1195
    1.7041    0.5005    0.4880    2.3595    0.2331    2.3592    1.6668


output2 =

    0.8180    0.1664    0.2514    0.7888    0.6897    0.7368    0.8196
    0.7514    0.7146    0.6930    0.2977   -0.4578   -0.0119   -0.0040
    0.5894    0.1191    0.6446    0.0648   -0.2338    0.1093    0.3697
    0.3757    0.2800   -0.4291    0.4285    0.3178    0.6319   -0.0202
    0.7408    0.8118    0.1800   -0.3193    0.7885   -0.3450   -0.2713
    0.2205    0.6017   -0.0652   -0.6968    0.8798    1.5234    0.5821
    0.4937    0.9005   -0.8629   -0.5766    0.6862    0.2841    0.3989


l =

     6     3     1     5     7     2     4

inv(A)*b  NO PIVOTING  PIVOTING  cond(A)
 3.11e-15  3.11e-15  8.88e-16  3.33e+01
```

```
   1.2026    2.0610    0.7381    0.0101    0.3446    1.2402   -1.5883   -1.1807   -0.2411    0.3213
   1.5348    1.0616    0.2578   -0.2647   -0.3008   -0.2468   -0.4215   -0.3997   -0.3819   -0.2000
   0.0678    0.7663    0.1580   -0.7830    0.2476    0.4756    0.0642    0.0295    0.6053    0.9134
   0.7786    0.0824    0.9075   -2.0759   -3.9370    0.1887    0.9935    0.9673    1.4877    3.0822
   0.2527    2.3524    0.4097   -3.3927   -3.7838   12.8782  -14.2449  -14.1180  -16.9034  -35.8306
   1.6954   -1.6243   -0.0221   -1.5084   -3.9663    0.6596   -0.0542    0.0284   -2.1864   -2.5839
   1.7957   -2.2320   -0.0450    2.3099    1.7484  -14.0219   -1.1892   -0.8775   -3.0569   -4.1002
   1.3851   -0.5675    0.7203   -1.7372   -4.8107    1.2077   -0.0257   -3.8071    2.8387   -0.3147


output2 =

   0.9683    0.0656    0.3497    0.0595    0.4068    0.2353    0.7204    0.9202    0.8870    0.9515
   0.6697    0.7717    0.5680    0.1558    0.7217    0.6233   -0.4152   -0.2244   -0.5472    0.1503
   0.4336    0.2954    0.4597    0.7000   -0.0739    0.1633    0.1199    0.3962    0.1532    0.4094
   0.9442    0.1359   -0.1920    1.0542   -0.3601    0.5282    0.1882    0.1471   -0.5874   -0.7760
   0.0377    0.2392   -0.1392    0.2172    0.4891    0.1773    0.7708    0.7532    0.8634    1.1077
   0.5569    0.3496   -0.5380    0.5413    0.2541    0.1113   -0.3946   -0.4142    0.8172    0.6727
   0.1407    0.7499   -0.5724    1.0189    0.3903   -3.1439   -0.5855   -0.6732    3.6430    3.2060
   0.7713    0.3246    0.6754    0.3131   -0.4410    0.1263   -0.6498   -0.1790    3.2051    1.7902
   0.8547    0.8351   -0.9057    0.5162   -0.6109   -2.3191    1.0134   -0.2377    0.1969    0.1894
   0.5184    0.6066   -0.9659    0.7007    0.0062   -6.2203    3.0074   -0.9612   -9.6479   -1.0241


l =

    6    10     2     9     5     3     7     4     1     8

inv(A)*b  NO PIVOTING  PIVOTING  cond(A)
 1.95e-14  1.98e-14  8.55e-15  2.81e+03
```

Q4

Problem 4 [4 points] You have to interpolate e^x by a polynomial of degree five using equally spaced points in $[0, 1]$. What error would you expect if you use this polynomial?

Using equally spaced points, what degree polynomial would you use to achieve a maximum error of 10^{-8}?

$n = 5 \quad h = \frac{1}{5} \quad M = \max_{0 \le x \le 1} |e^x| = e$

$$|f(x) - e^x| \le \frac{e}{4 \times 6} \cdot \left(\frac{1}{5}\right)^6 = 7.2487 \times 10^{-6}$$

$$E(n) = \frac{M}{4(n+1)} \left(\frac{1}{n}\right)^{n+1} = \frac{e}{4(n+1)\, n^{(n+1)}} \le 10^{-8}$$

$E(6) = 3.4680 \times 10^{-7} > 10^{-8}$

$E(7) = 1.4735 \times 10^{-8} > 10^{-8}$

$E(8) = 5.6258 \times 10^{-10} < 10^{-8}$

We can attach error 10^{-8} by degree 8.

Problem 5 [5 points] You are given the data points

Q5

```
approx =

    1.0247    1.0724


M =

   -0.9375


BoundError =

   5.8594e-06


actualerror1 =

   2.4158e-06


actualerror2 =

   1.3607e-06
```

a. From the matlab output, we can see approximation for x=0.05 is 1.0247 and for x=0.15 is 1.0724.
b. The bound error is 5.8594e-6
c. The bound error is the max error between [0,0.3]. so it is larger than the actual error. From the matlab output, we can see actual error for x=0.05 is 2.4158e-6<bounderror
   X=0.15 is 1.3607e-6<bounderror

Matlab code

```
x=[0,0.1,0.2,0.3];
f = @(x) sqrt(x+1);
y=f(x);
p=polyfit(x,y,3)
data=[0.05,0.15];
approx=polyval(p,data)
h=0.1;
n=3;

M=-15/16*1^(-7/2)
BoundError=abs(M/(4*(n+1))*h^(n+1))
actualerror1=abs(f(0.05)-approx(1))
actualerror2=abs(f(0.15)-approx(2))
```

c.2000*2000

```
ommand Window
    0.0999    0.9149    0.9680    0.6164    0.8884    0.6698    0.4455    0.2193    0.5525    0.7201
    0.8536    0.9295    0.1613    0.4429    0.3019    0.0062    0.9204    0.1953    0.7575    0.4362
    0.3321    0.0975    0.6454    0.2794    0.9183    0.5133    0.3930    0.6047    0.5101    0.4162
    0.2786    0.6502    0.5039    0.3973    0.8741    0.7023    0.4308    0.9809    0.7380    0.2407
    0.5941    0.1951    0.2294    0.0483    0.0144    0.7324    0.7999    0.4006    0.3181    0.4700
    0.7463    0.6068    0.8575    0.5008    0.1767    0.6212    0.5705    0.2348    0.5328    0.3458
    0.0193    0.6422    0.0638    0.0107    0.8721    0.9370    0.7193    0.5145    0.5006    0.3225
    0.7642    0.2706    0.2591    0.6607    0.0504    0.1197    0.1057    0.9719    0.5371    0.9811
    0.3677    0.1886    0.4315    0.5825    0.7596    0.8520    0.5731    0.4985    0.9194    0.2987
    0.9191    0.5026    0.6528    0.8936    0.9340    0.6713    0.2630    0.3641    0.1616    0.5509
    0.5547    0.9971    0.4025    0.9597    0.8768    0.2417    0.3208    0.2558    0.1421    0.2401
    0.8616    0.1558    0.1148    0.4605    0.4859    0.0629    0.2630    0.1993    0.7779    0.7369
    0.6661    0.9438    0.7818    0.8672    0.9263    0.1896    0.2748    0.8277    0.1313    0.9105
    0.8531    0.4882    0.6358    0.6106    0.7580    0.2374    0.9739    0.0375    0.8882    0.9569
    0.6956    0.9759    0.0737    0.4147    0.3656    0.1121    0.1334    0.3769    0.0553    0.1221
    0.1047    0.1541    0.9924    0.7140    0.1078    0.8395    0.6564    0.5897    0.8794    0.0662

inv(A)*b  NO PIVOTING  PIVOTING  cond(A)
 5.46e-13  1.74e-11  1.71e-10  1.59e+05
```

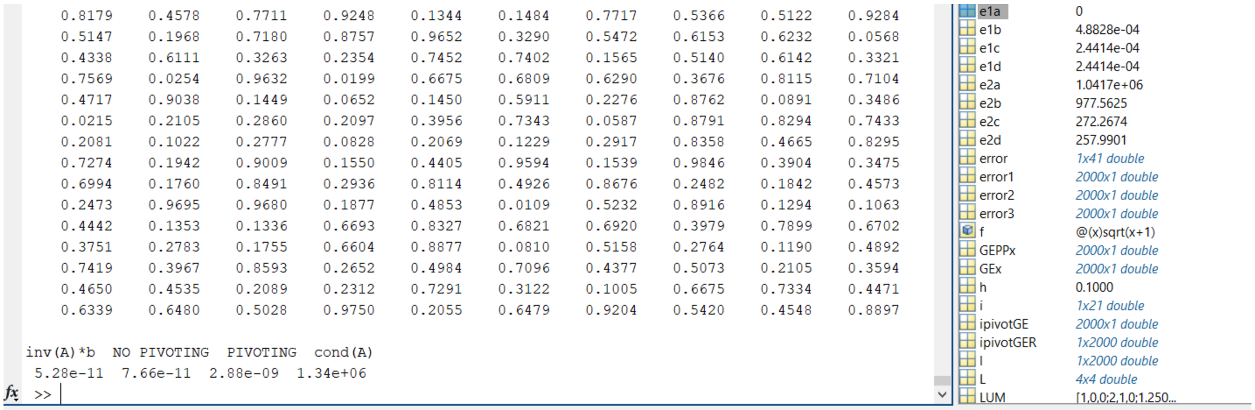
data	[0.0500,0.1500]
e1a	0
e1b	4.8828e-04
e1c	2.4414e-04
e1d	2.4414e-04
e2a	1.0417e+06
e2b	977.5625
e2c	272.2674
e2d	257.9901
error	1x41 double
error1	2000x1 double
error2	2000x1 double
error3	2000x1 double
f	@(x)sqrt(x+1)
GEPPx	2000x1 double
GEx	2000x1 double
h	0.1000
i	1x21 double
ipivotGE	2000x1 double
ipivotGER	1x2000 double
l	1x2000 double
L	4x4 double

```
    0.8272    0.5791    0.0645    0.3362    0.6241    0.8615    0.5747    0.9540    0.1601    0.5288
    0.5319    0.2283    0.1950    0.9973    0.5671    0.2502    0.1264    0.4234    0.1719    0.5350
    0.2665    0.6329    0.0823    0.5934    0.7850    0.2514    0.6052    0.5103    0.2067    0.4471
    0.9422    0.0414    0.1276    0.6590    0.2065    0.9228    0.3921    0.7052    0.3057    0.7284
    0.7451    0.7870    0.9213    0.4056    0.3248    0.3956    0.5969    0.8574    0.7825    0.4514
    0.8033    0.9405    0.1391    0.1342    0.5959    0.7765    0.0848    0.8716    0.4492    0.8162
    0.8426    0.9150    0.0831    0.9670    0.9673    0.4791    0.5127    0.8988    0.7508    0.3624
    0.3038    0.9388    0.4491    0.2868    0.6685    0.9344    0.9872    0.0132    0.2562    0.2993
    0.4424    0.4039    0.0163    0.8567    0.8254    0.7789    0.7824    0.5533    0.5840    0.2831
    0.3441    0.8673    0.2988    0.8494    0.5955    0.2262    0.6322    0.1958    0.0136    0.3896
    0.7298    0.4202    0.1050    0.0342    0.4971    0.1744    0.6130    0.9507    0.0573    0.5883
    0.0698    0.3896    0.5540    0.4350    0.9455    0.5469    0.8629    0.6156    0.9853    0.8775
    0.8756    0.1129    0.2698    0.4075    0.7828    0.0846    0.7007    0.5352    0.8405    0.3826
    0.4216    0.4629    0.5734    0.8554    0.6604    0.3036    0.6801    0.7077    0.8465    0.4251
    0.2009    0.3340    0.6402    0.1386    0.3861    0.5958    0.6220    0.4447    0.2689    0.0438
    0.4136    0.4210    0.3815    0.2486    0.5710    0.4944    0.5144    0.7582    0.1459    0.8069

inv(A)*b  NO PIVOTING  PIVOTING  cond(A)
 1.60e-12  3.64e-10  4.36e-10  1.63e+05
```

data	[0.0500,0.1500]
e1a	0
e1b	4.8828e-04
e1c	2.4414e-04
e1d	2.4414e-04
e2a	1.0417e+06
e2b	977.5625
e2c	272.2674
e2d	257.9901
error	1x41 double
error1	2000x1 double
error2	2000x1 double
error3	2000x1 double
f	@(x)sqrt(x+1)
GEPPx	2000x1 double
GEx	2000x1 double
h	0.1000
i	1x21 double
ipivotGE	2000x1 double
ipivotGER	1x2000 double
l	1x2000 double
L	4x4 double

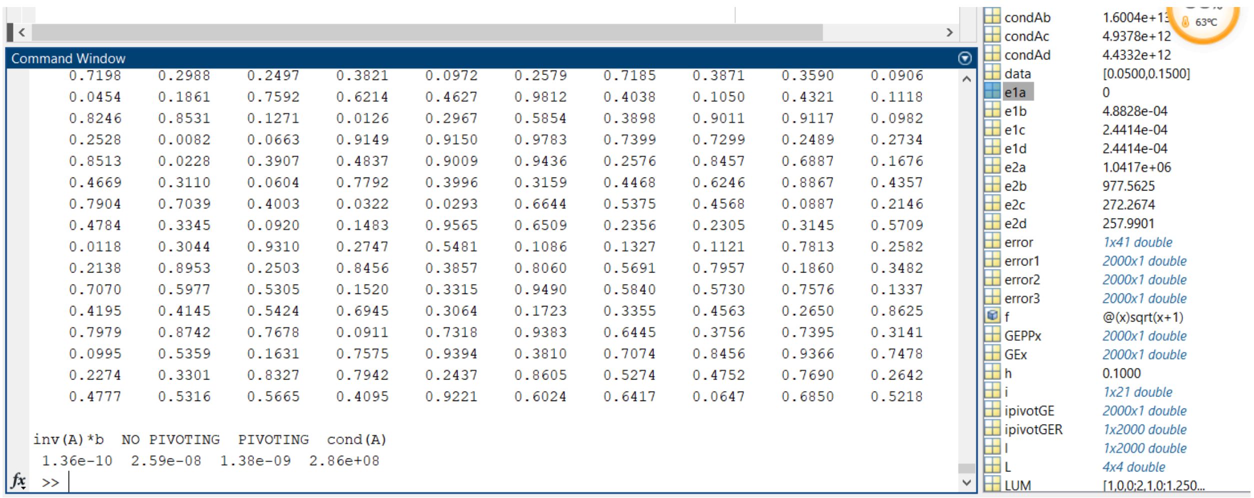
```
ommand Window
    0.2168    0.7534    0.3779    0.5966    0.4757    0.3929    0.3573    0.0557    0.6555    0.5642
    0.0629    0.2350    0.1159    0.7535    0.0112    0.5609    0.6375    0.0556    0.6681    0.4291
    0.8331    0.5357    0.1519    0.8989    0.2967    0.9439    0.7352    0.8540    0.4775    0.4676
    0.0046    0.4709    0.9311    0.9557    0.3461    0.7755    0.4830    0.7089    0.1432    0.3812
    0.7437    0.1943    0.2010    0.1391    0.3943    0.9129    0.4017    0.4057    0.6574    0.6686
    0.6616    0.8366    0.7151    0.1230    0.9981    0.2777    0.4837    0.4353    0.6362    0.9937
    0.9268    0.4402    0.6785    0.8743    0.1410    0.5178    0.0343    0.0288    0.3610    0.1429
    0.8403    0.9431    0.3008    0.3656    0.7433    0.9150    0.6439    0.7666    0.6353    0.1344
    0.0012    0.7677    0.7264    0.4500    0.2658    0.8871    0.6279    0.3683    0.5124    0.9033
    0.9900    0.0106    0.2318    0.0075    0.1660    0.0629    0.3318    0.6095    0.3859    0.2035
    0.5210    0.1837    0.4123    0.7027    0.2887    0.8124    0.6534    0.3510    0.4158    0.0612
    0.7070    0.6447    0.5907    0.2658    0.1133    0.7274    0.1103    0.8997    0.1962    0.3508
    0.4102    0.2214    0.8424    0.6215    0.8775    0.2448    0.7846    0.3366    0.8893    0.5960
    0.9595    0.4117    0.0760    0.7337    0.2175    0.8904    0.3128    0.9233    0.6859    0.1519
    0.2528    0.2430    0.8446    0.7491    0.7065    0.7384    0.2178    0.6163    0.2612    0.0725
    0.7376    0.1890    0.5762    0.9611    0.6976    0.5280    0.6181    0.2859    0.0684    0.3454

inv(A)*b  NO PIVOTING  PIVOTING  cond(A)
 7.31e-13  4.10e-10  2.10e-09  1.55e+05
>> |
```

condAd	4.4332e+12
data	[0.0500,0.1500]
e1a	0
e1b	4.8828e-04
e1c	2.4414e-04
e1d	2.4414e-04
e2a	1.0417e+06
e2b	977.5625
e2c	272.2674
e2d	257.9901
error	1x41 double
error1	2000x1 double
error2	2000x1 double
error3	2000x1 double
f	@(x)sqrt(x+1)
GEPPx	2000x1 double
GEx	2000x1 double
h	0.1000
i	1x21 double
ipivotGE	2000x1 double
ipivotGER	1x2000 double
l	1x2000 double
L	4x4 double
LUM	[1,0,0;2,1,0;1.250...



0.8179	0.4578	0.7711	0.9248	0.1344	0.1484	0.7717	0.5366	0.5122	0.9284
0.5147	0.1968	0.7180	0.8757	0.9652	0.3290	0.5472	0.6153	0.6232	0.0568
0.4338	0.6111	0.3263	0.2354	0.7452	0.7402	0.1565	0.5140	0.6142	0.3321
0.7569	0.0254	0.9632	0.0199	0.6675	0.6809	0.6290	0.3676	0.8115	0.7104
0.4717	0.9038	0.1449	0.0652	0.1450	0.5911	0.2276	0.8762	0.0891	0.3486
0.0215	0.2105	0.2860	0.2097	0.3956	0.7343	0.0587	0.8791	0.8294	0.7433
0.2081	0.1022	0.2777	0.0828	0.2069	0.1229	0.2917	0.8358	0.4665	0.8295
0.7274	0.1942	0.9009	0.1550	0.4405	0.9594	0.1539	0.9846	0.3904	0.3475
0.6994	0.1760	0.8491	0.2936	0.8114	0.4926	0.8676	0.2482	0.1842	0.4573
0.2473	0.9695	0.9680	0.1877	0.4853	0.0109	0.5232	0.8916	0.1294	0.1063
0.4442	0.1353	0.1336	0.6693	0.8327	0.6821	0.6920	0.3979	0.7899	0.6702
0.3751	0.2783	0.1755	0.6604	0.8877	0.0810	0.5158	0.2764	0.1190	0.4892
0.7419	0.3967	0.8593	0.2652	0.4984	0.7096	0.4377	0.5073	0.2105	0.3594
0.4650	0.4535	0.2089	0.2312	0.7291	0.3122	0.1005	0.6675	0.7334	0.4471
0.6339	0.6480	0.5028	0.9750	0.2055	0.6479	0.9204	0.5420	0.4548	0.8897

inv(A)*b	NO PIVOTING	PIVOTING	cond(A)
5.28e-11	7.66e-11	2.88e-09	1.34e+06

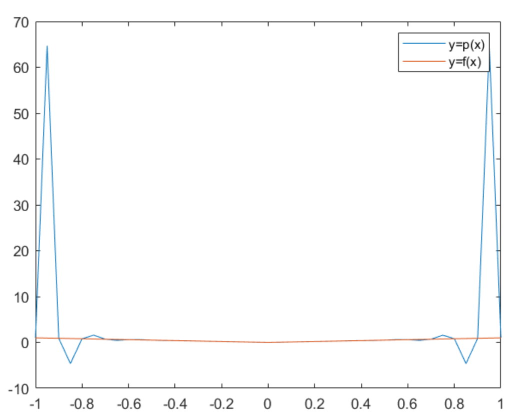


0.7198	0.2988	0.2497	0.3821	0.0972	0.2579	0.7185	0.3871	0.3590	0.0906
0.0454	0.1861	0.7592	0.6214	0.4627	0.9812	0.4038	0.1050	0.4321	0.1118
0.8246	0.8531	0.1271	0.0126	0.2967	0.5854	0.3898	0.9011	0.9117	0.0982
0.2528	0.0082	0.0663	0.9149	0.9150	0.9783	0.7399	0.7299	0.2489	0.2734
0.8513	0.0228	0.3907	0.4837	0.9009	0.9436	0.2576	0.8457	0.6887	0.1676
0.4669	0.3110	0.0604	0.7792	0.3996	0.3159	0.4468	0.6246	0.8867	0.4357
0.7904	0.7039	0.4003	0.0322	0.0293	0.6644	0.5375	0.4568	0.0887	0.2146
0.4784	0.3345	0.0920	0.1483	0.9565	0.6509	0.2356	0.2305	0.3145	0.5709
0.0118	0.3044	0.9310	0.2747	0.5481	0.1086	0.1327	0.1121	0.7813	0.2582
0.2138	0.8953	0.2503	0.8456	0.3857	0.8060	0.5691	0.7957	0.1860	0.3482
0.7070	0.5977	0.5305	0.1520	0.3315	0.9490	0.5840	0.5730	0.7576	0.1337
0.4195	0.4145	0.5424	0.6945	0.3064	0.1723	0.3355	0.4563	0.2650	0.8625
0.7979	0.8742	0.7678	0.0911	0.7318	0.9383	0.6445	0.3756	0.7395	0.3141
0.0995	0.5359	0.1631	0.7575	0.9394	0.3810	0.7074	0.8456	0.9366	0.7478
0.2274	0.3301	0.8327	0.7942	0.2437	0.8605	0.5274	0.4752	0.7690	0.2642
0.4777	0.5316	0.5665	0.4095	0.9221	0.6024	0.6417	0.0647	0.6850	0.5218

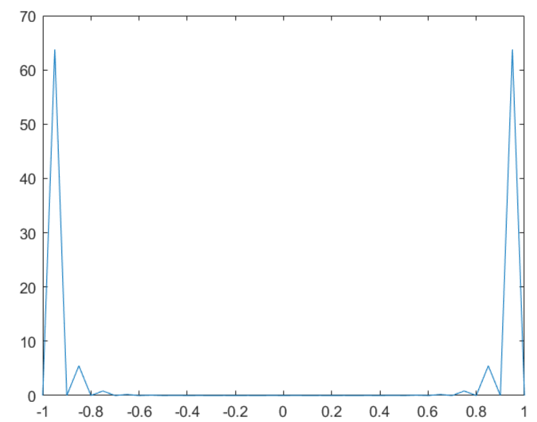
inv(A)*b	NO PIVOTING	PIVOTING	cond(A)
1.36e-10	2.59e-08	1.38e-09	2.86e+08

Q6

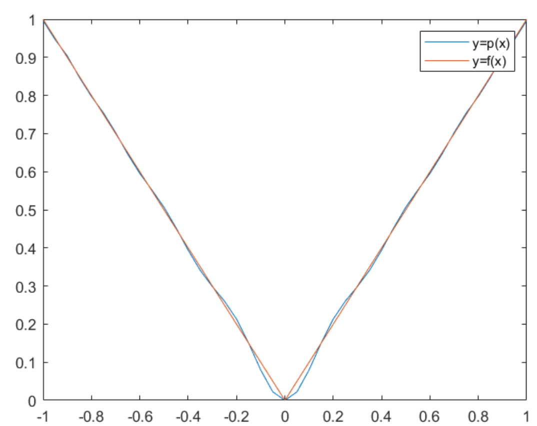
Plot 1: p(x) and f(x)



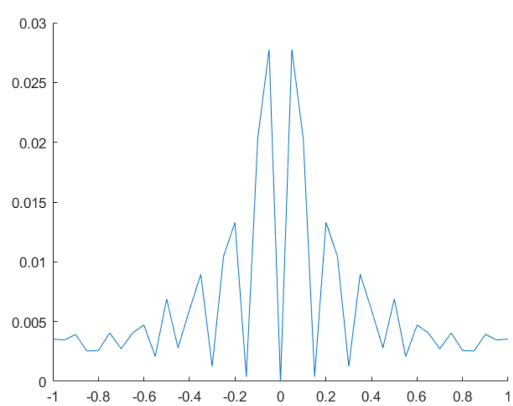
Plot 2: error for linespace



Plot 3: p(x) and y(x) for Chebyshev



Plot of error for Chebyshev



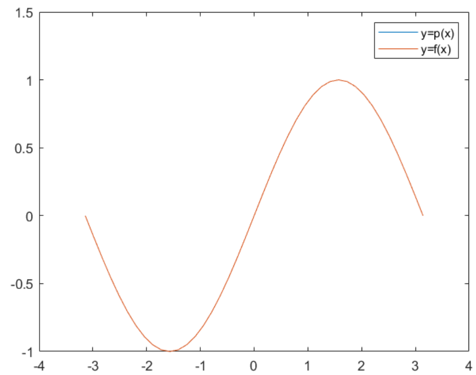
Matlab code

```
x=linspace(-1, 1, 21);
y=abs(x);
```

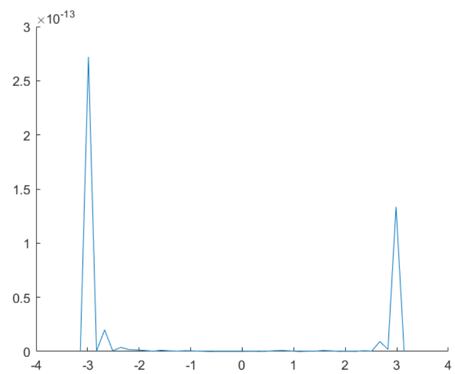
```
P=polyfit(x,y,20);
f = @(t) abs(t);
xx=linspace(-1,1,41);
yy=polyval(P,xx);
plot(xx,yy)
hold on
y2=f(xx);
plot(xx,y2)
legend('y=p(x)','y=f(x)')
%hold off
%error=abs(f(xx)-yy);
%plot(xx,error)


i=0:20;
n=20;
m= @(j)cos((2.*j+1).*pi./(2.*n+2));
x=m(i);
y=abs(x);
P=polyfit(x,y,20);
f = @(t) abs(t);
xx=linspace(-1,1,41);
yy=polyval(P,xx);
%plot(xx,yy)
hold on
y2=f(xx);
%plot(xx,y2)
%legend('y=p(x)','y=f(x)')
%hold off
error=abs(f(xx)-yy);
plot(xx,error)
```

Q7

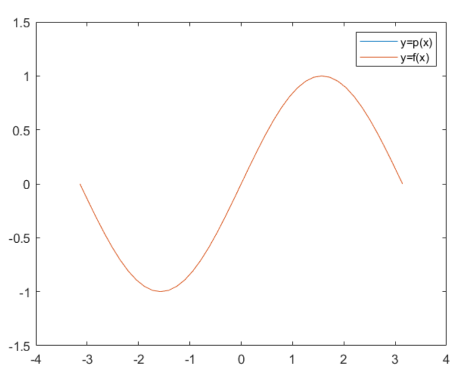
Plot 1: p(x) and f(x)



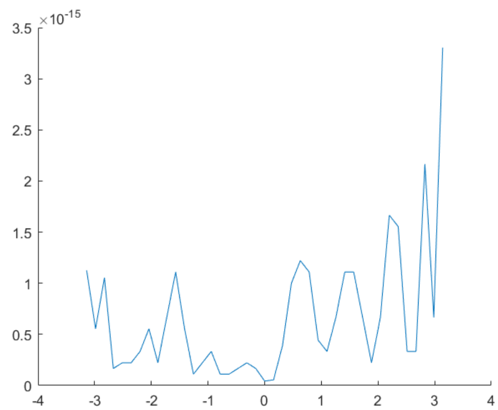
Plot 2: error for linespace



Plot 3: p(x) and y(x) for Chebyshev



Plot of error for Chebyshev



Discussion:

From the plot we can see the error between original function and interpolation for sin(x)(10e-15) are much smaller than abs(x)(10e-2). We can also see that Chebushev nodes are more accurate to present the interpolation. Also, the error for abs(x) are symmetric about y axis, and for sin(x) are not.

Matlab code

```
x=linspace(-pi, pi, 21);
y=sin(x);
P=polyfit(x,y,20);
f = @(t) sin(t);
xx=linspace(-pi,pi,41);
yy=polyval(P,xx);
%plot(xx,yy)
hold on
y2=f(xx);
%plot(xx,y2)
%legend('y=p(x)','y=f(x)')
%hold off
error=abs(f(xx)-yy);
plot(xx,error)


i=0:20;
n=20;
m= @(j) pi*cos((2.*j+1).*pi./(2.*n+2));
x=m(i);
y=sin(x);
P=polyfit(x,y,20);
f = @(t) sin(t);
xx=linspace(-pi,pi,41);
yy=polyval(P,xx);
%plot(xx,yy)
```

```
hold on
y2=f(xx);
%plot(xx,y2)
%legend('y=p(x)','y=f(x)')
%hold off
error=abs(f(xx)-yy);
plot(xx,error)
```

Q8

```
function [p,table]=newton(x,xx,yy,n)
    diff=zeros(n,n);
    for i=1:n
        diff(i,1) =yy(i);
    end
    for j=2:n
        for k=j:n
            diff(k,j)=(diff(k,j-1)-diff(k-1,j-1))/(xx(k)-
xx(k-j+1));
        end
    end
    p=diff(1,1);
    for i=2:n
        mult=1;
        for j=1:i-1
            mult=mult*(x-xx(j));
        end
        p=p+diff(i,i)*mult;
    end
    table=diff;
end
```