Problem1:

Time to maturity is 0.038356164383561646 using the 365 calendar days

For this problem, we set the volatility to increment by 1% during each loop within the range of 10% - 80%. We utilized the black-scholes model formulas in the class notes:

$$d_1 = \frac{ln\left(\frac{S}{X}\right)+\left(b+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

$$Call = Se^{(b-r)T}\Phi\left(d_1\right) - Xe^{-rT}\Phi\left(d_2\right)$$

$$Put = Xe^{-rT}\Phi\left(-d_2\right) - Se^{(b-r)T}\Phi\left(-d_1\right)$$

And get the call and put prices for the range of volatilities. After those, we plotted value of call and put into one chart like below:

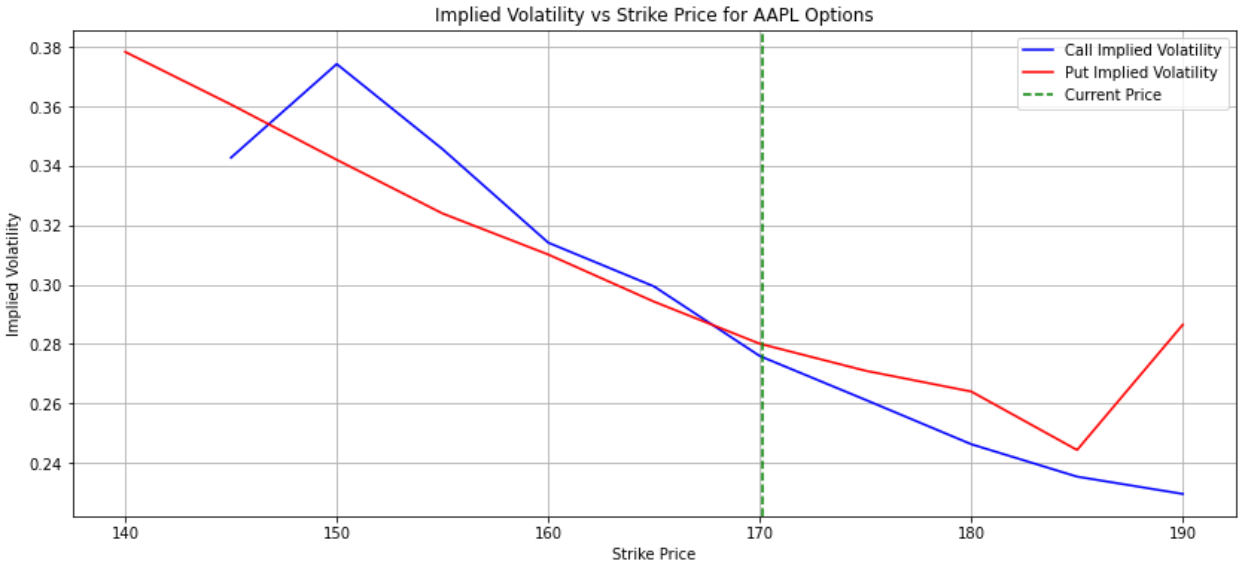
As we can see from the graph, we can see there is a positive relationship between the implied volatility and option prices for both put and call options, which is as the implied volatility increases, the option value also increases. It makes sense because as we have higher implied volatility, it means that the market expects larger price changes in the future,

also the range of future prices widens, which means that there's larger probability for the option value to be in-the-money to increase the pay-off. At the same time, higher implied volatility meaning that the price hits/surpasses the strike price is higher, so options become more valuable because of the larger price movements.

Supply and demand impact on the implied volatility: when demand for call option increases, which can be due to the increase in the anticipation for the price, call prices go up; increase demand for put option can also cause put prices to go up. Increase in both put and call demand can cause implied volatility to go up, meaning that market expects larger changes in price. On the opposite side, higher supply for both options can reduce option prices and implied volatility because lower market expectations on price volatility.

Problem2:

For this problem, we also first applied the black-scholes model into python functions, then we utilized the root-finding method to get the implied volatility as shown in class. Then we used the given sheet to loop through the data to either get the items or calculated the data to get the strike prices and implied volatility and do the plots. The plot looks like below:



As we can observe from the graph, in general, the implied volatility falls as the strike price increases for the most part meaning that out-of-money options have higher implied volatility; there's also a noticeable reversal for both two options that there's a increase in the initial of call option and end of the put option, whereas strike price increases, the implied volatility also rises, which might be caused by different demands or uncertainties for the price changes.

Market Dynamics:

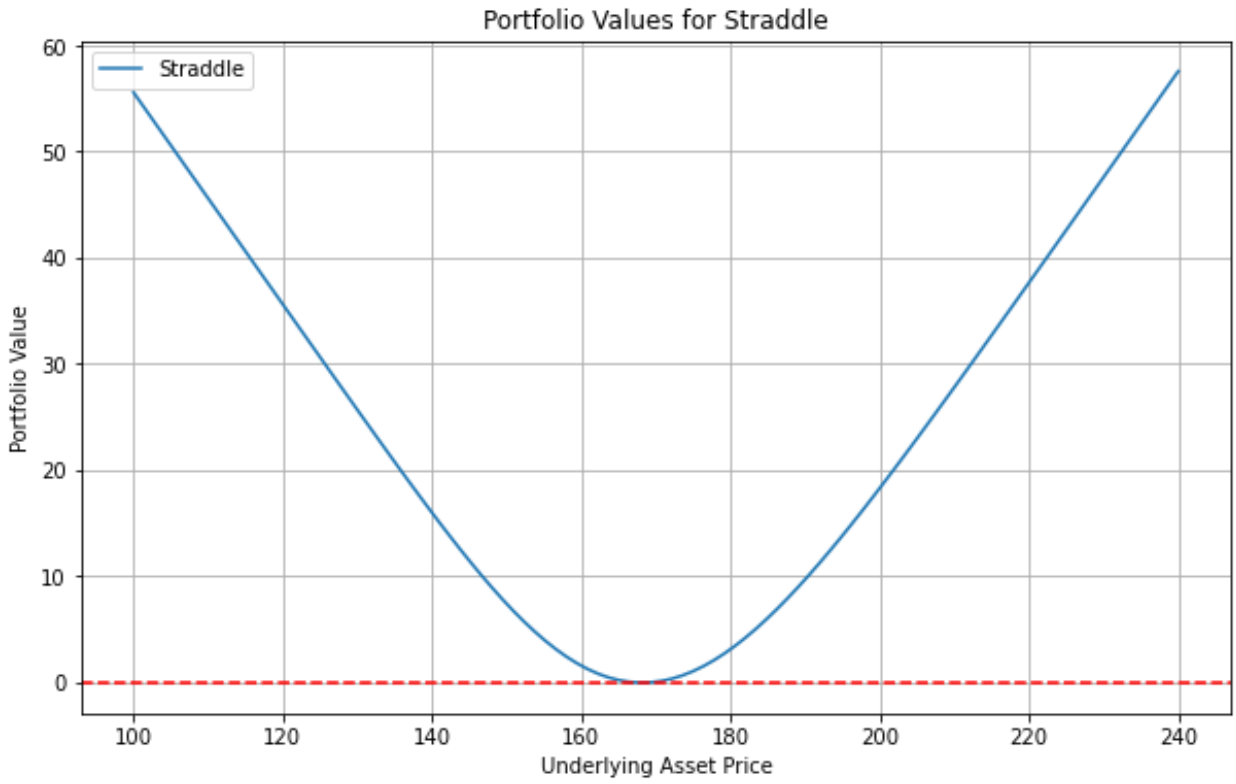
- Higher demand for both put and call options at certain strike prices might increase the implied volatility, this action to hedge specific risks might cause some irregularities in the graph.
- If there're any upcoming events or news that can have large impacts on the investors' expectation on the prices, the implied volatility can also increase largely.
- When investors are more worried about the large price moves, they're willing to pay more to protect themselves thus the implied volatility will also go up. For example, people will buy more puts with lower strike price if they worried the asset price will go down, this extra demand makes put options' implied volatility to go up.

Problem3:

Plotting and Shaping: using the problem3.csv sheet to group different portfolios based on type. We're going to use this formula from notes to explain each chart:

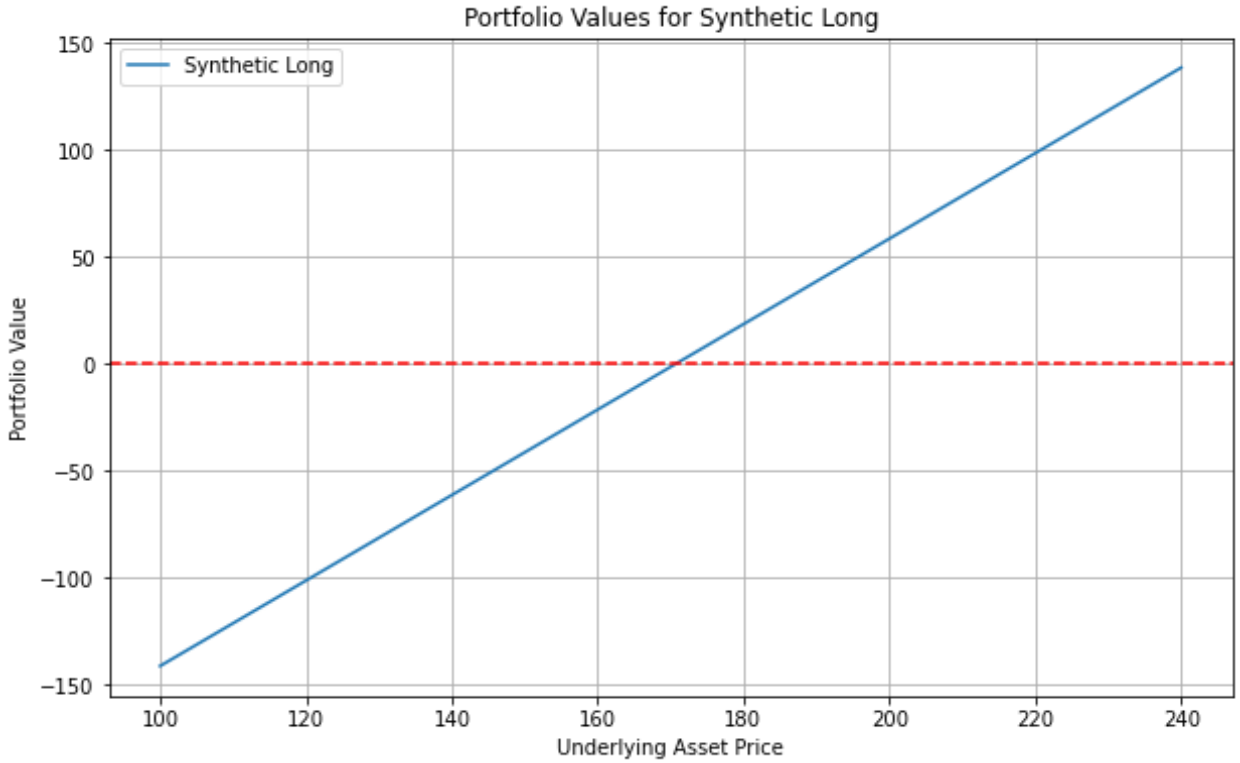
$$C + Xe^{-rT} = P + S$$

1. Straddle



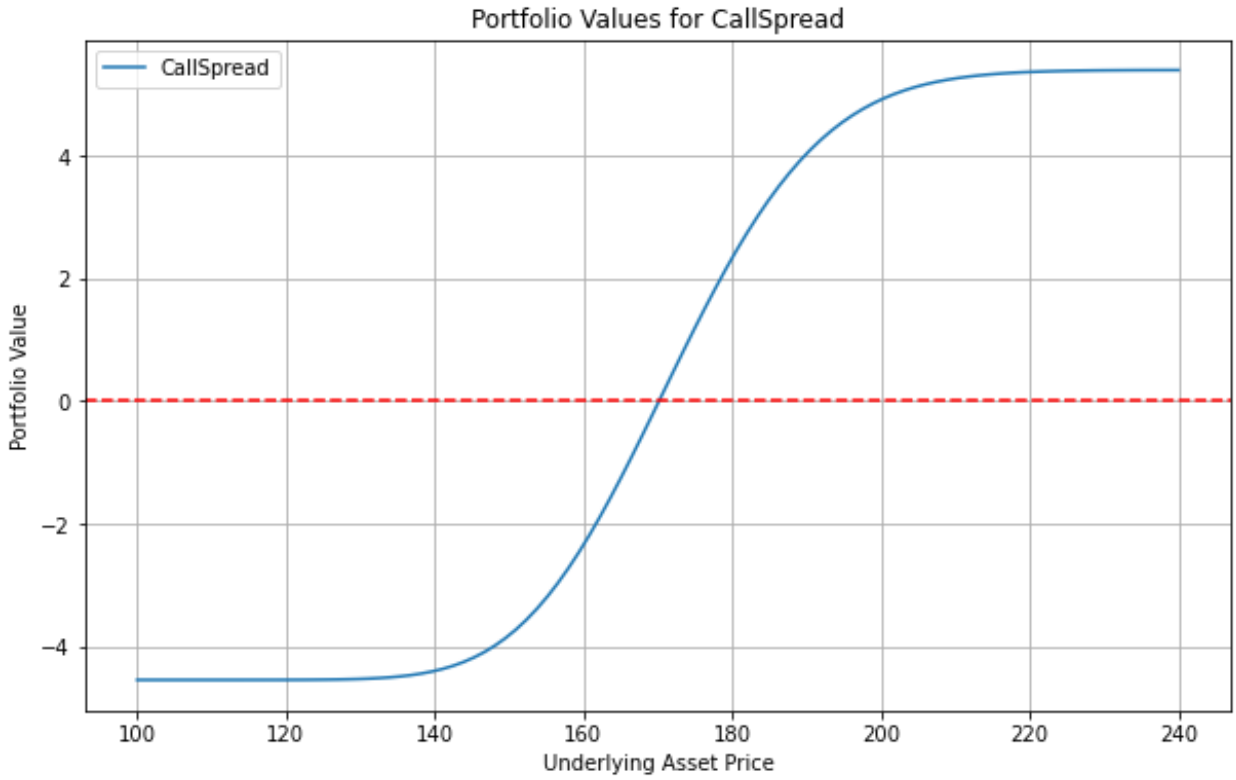
Straddle Strategy is to buy a call and put option at the same asset price and expiration date. As you can see from the graph, it is symmetric and has a min value at the strike price. This strategy makes profit whenever the asset price moves away from the strike price (either larger or lower) because people are gaining money from higher volatility.

2. Synthetic Long



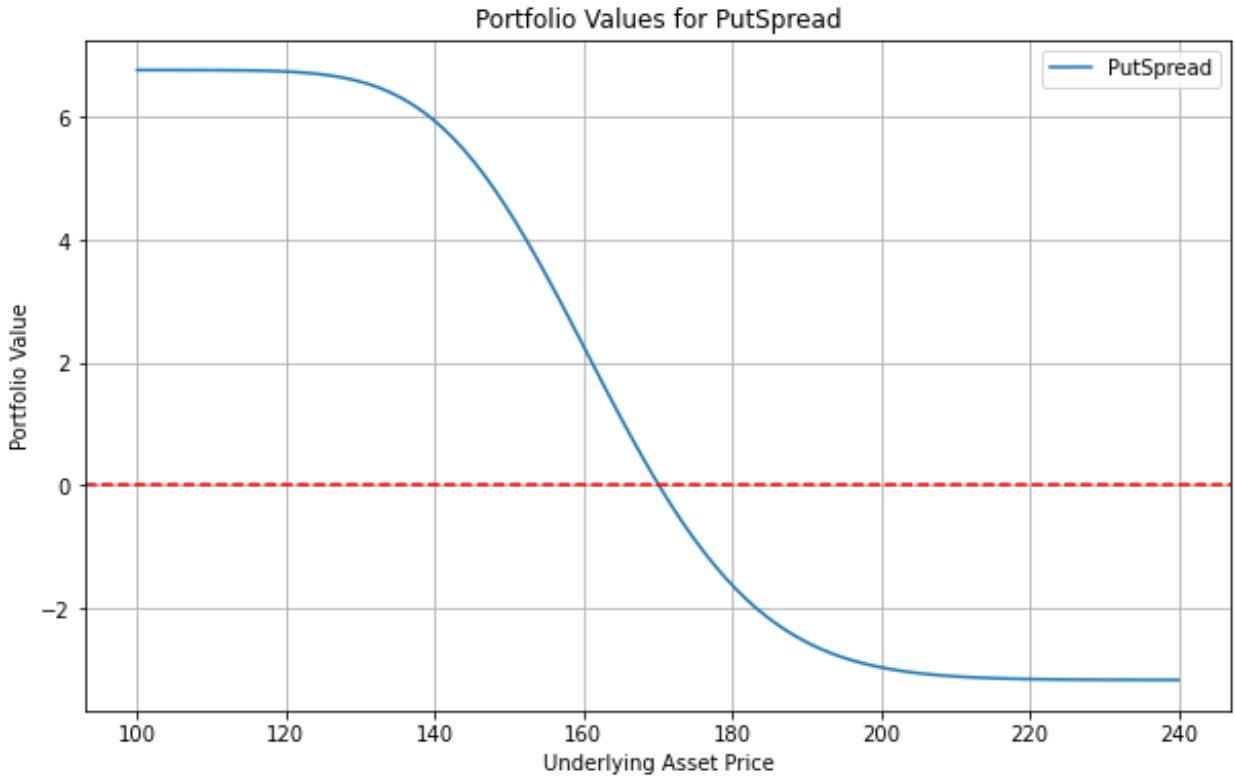
This strategy is to long call and short put at the same strike, as you can see from the chart, the payoff has positive relationship with asset price, as price increases, more profits. According to the put-call parity formula, it is like to hold the underlying stock.

3. CallSpread



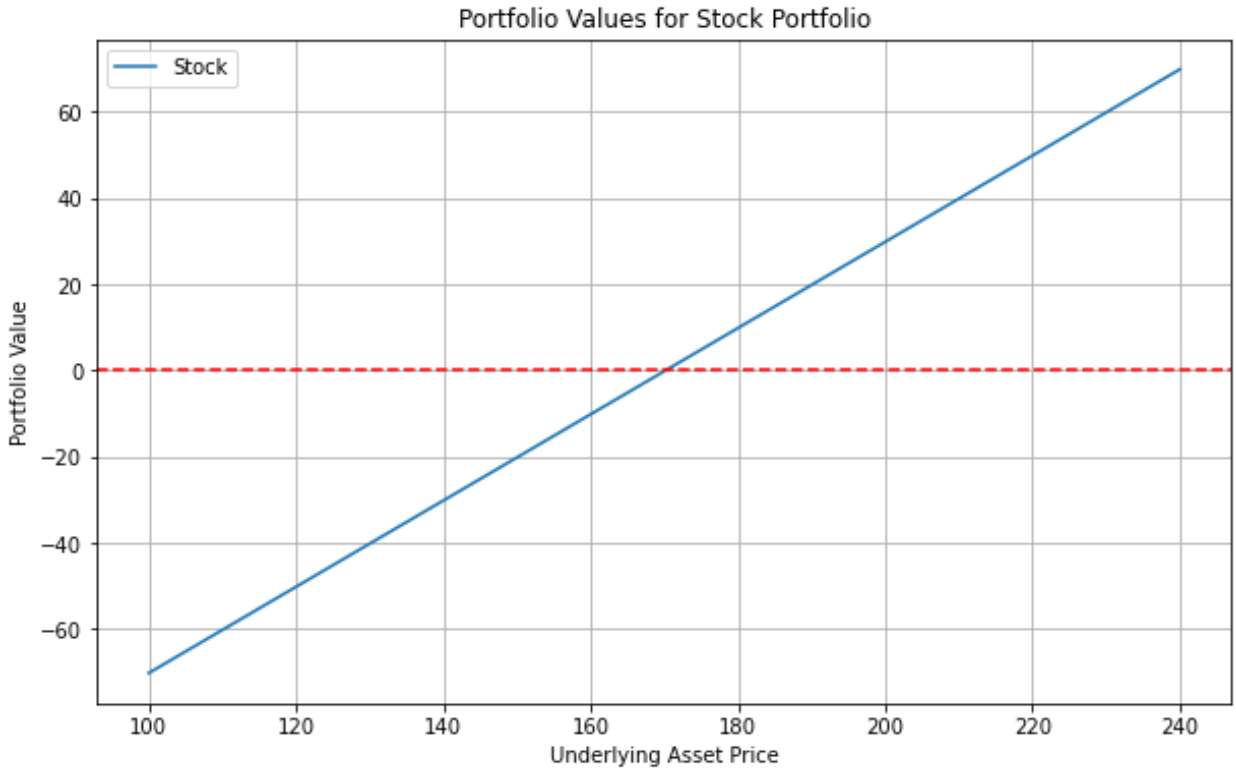
This strategy is to long call at lower strike and short call at higher strike. The payoff is capped, but there is a limitation at the higher strike price.

4. PutSpread



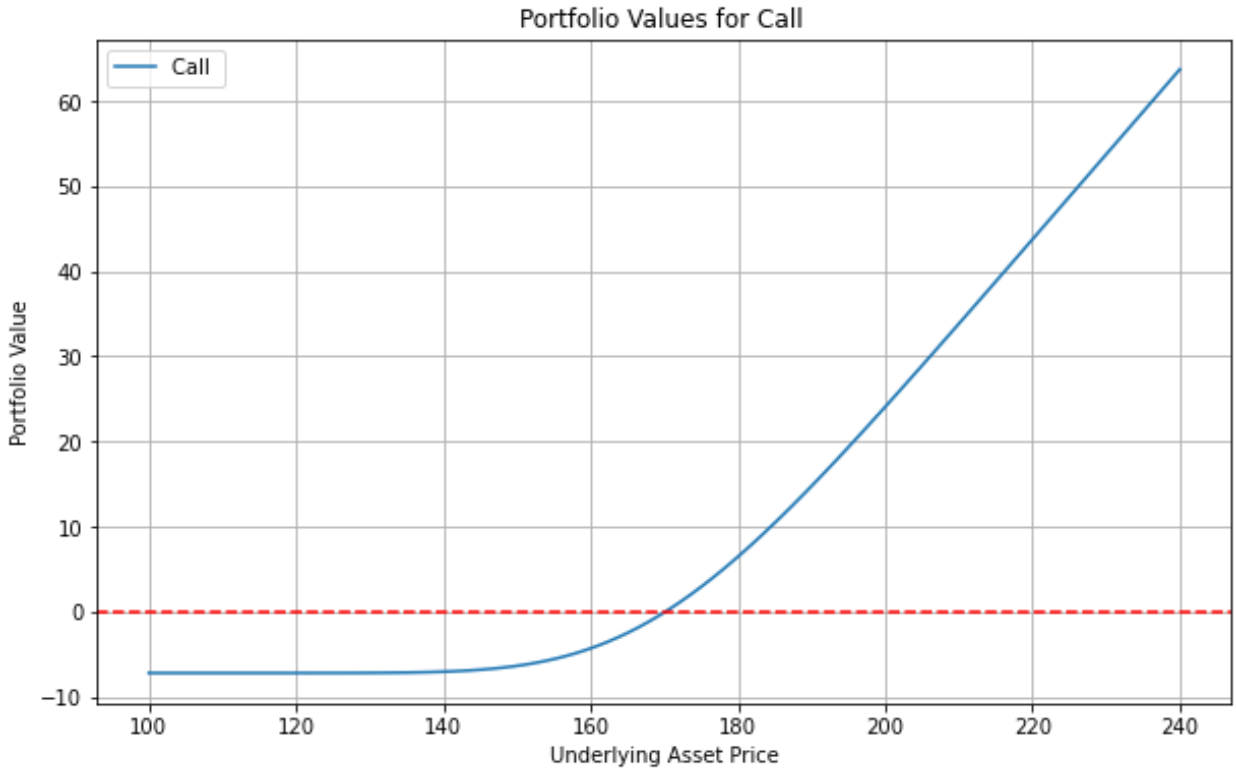
This strategy is to long put at higher strike and short put at lower strike, and the payoff declines as the asset price falls and is capped by the difference in strike prices. It is like the opposite side of the CallSpread. There's a limit on both upper and lower side. Lower strike price can achieve maximum profit.

5. Stock



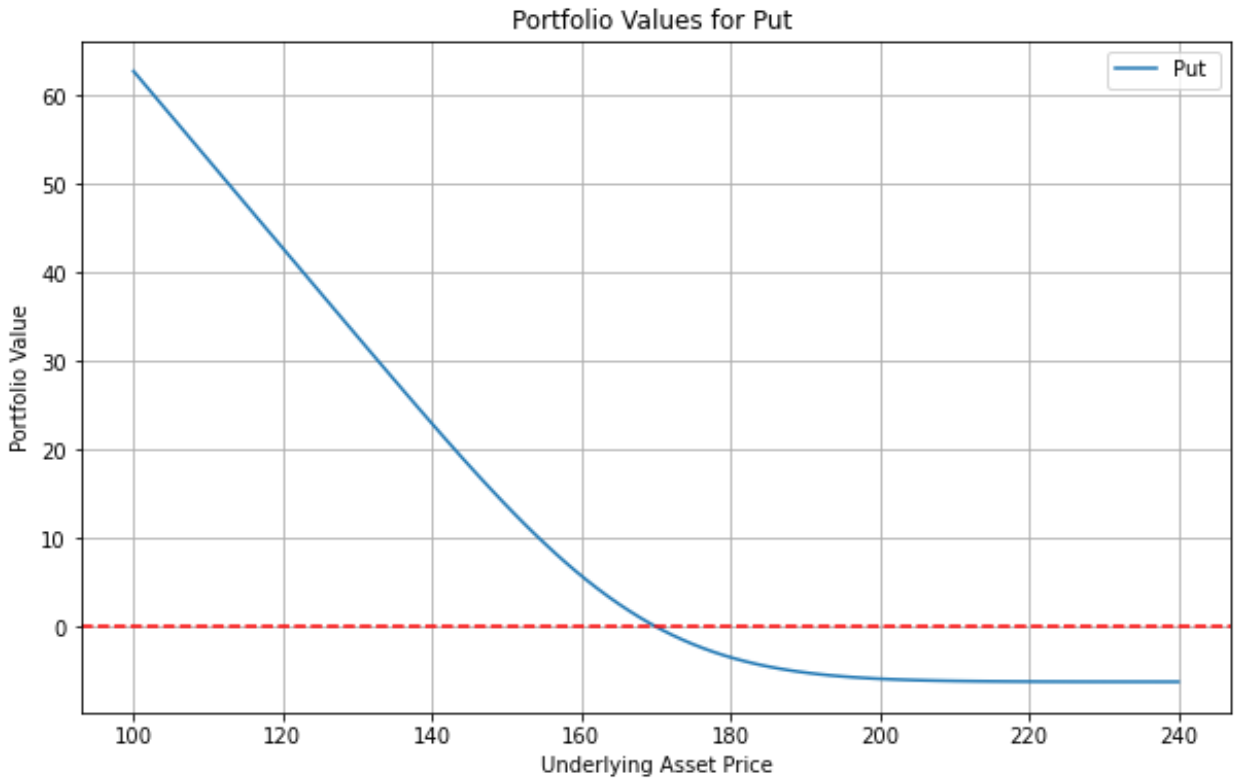
This one is straightforward that as price increase, the profit also increase.

6. Call



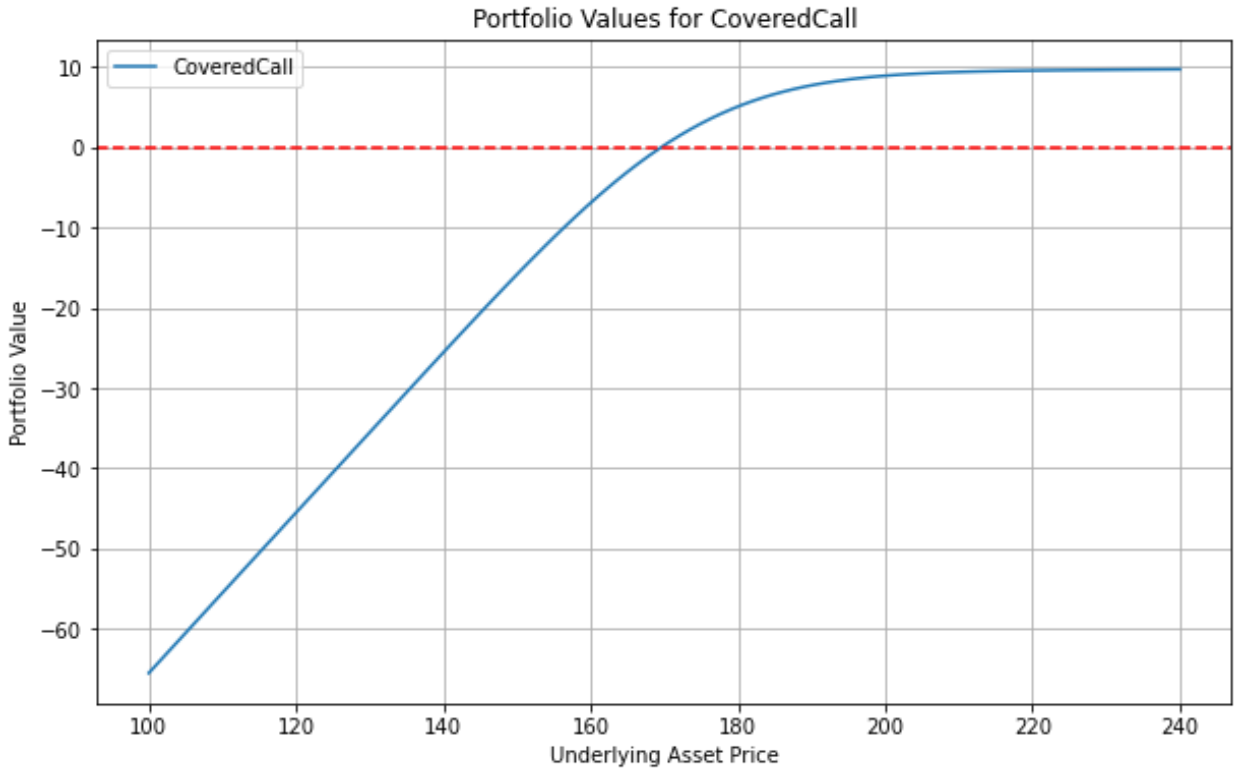
A long call position and the payoff is zero when the price exceeds the strike, where it then increases with the price increases.

7. Put



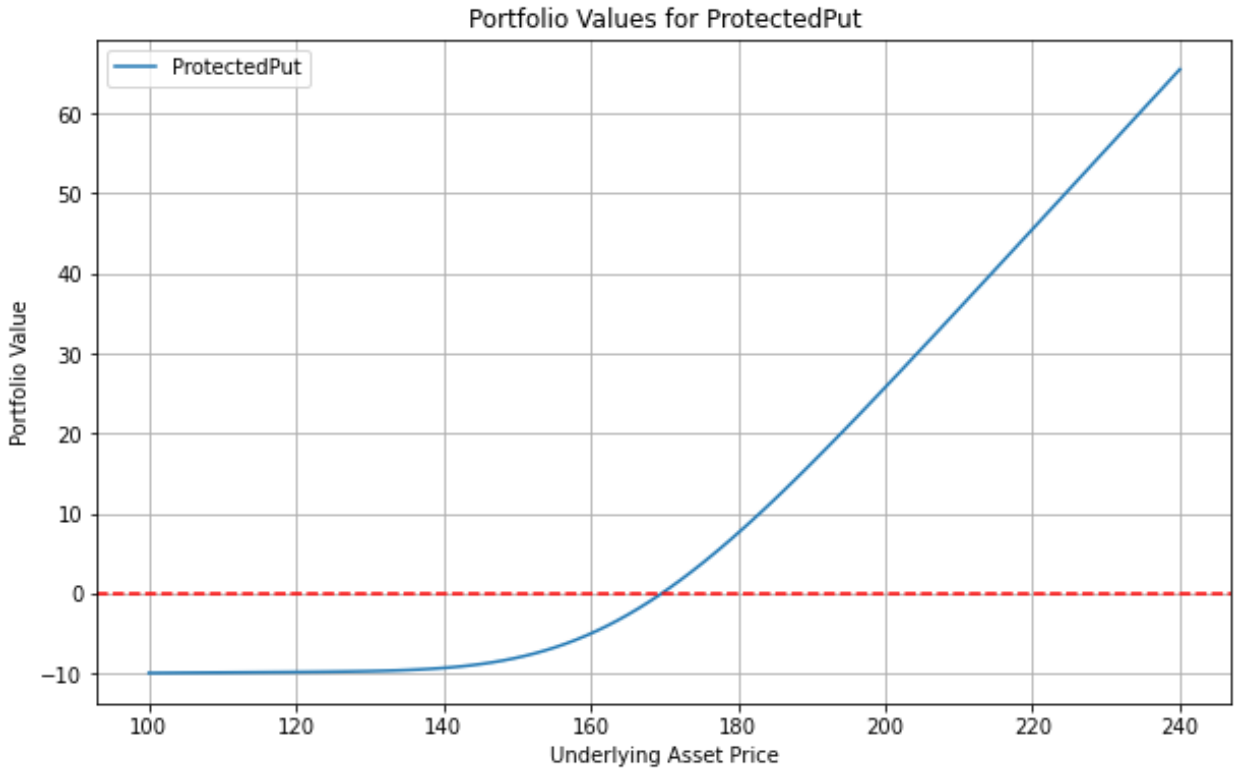
Long put position's payoff increases as the underlying price declines, protecting against downward moves. After exceeding the strike price payoff will be negative.

8. Covered Call



Long stock and short call. Payoff goes up as the stock price rises. However, if the stock price goes too high, the profits are limited by the short call option. Selling the call helps reduce your investment cost, but you give up some potential gains.

9. Protected Put



Long stock and long put. This portfolio acts like owning a stock, so people can benefit if the stock price goes up. However, if the stock price falls below the put's strike price, there will be a limit on the loss. (The put protects stock from big losses, so can get the upside potential of the stock with limited downside.)

For the second part:

Portfolio	Mean	VaR	ES
Call	0.528617	6.017239	6.473107
CallSpread	-0.069540	3.510103	3.887125
CoveredCall	-0.132371	10.703959	14.173460
ProtectedPut	0.636589	7.767884	8.590074
Put	0.263959	5.159674	5.530848
PutSpread	0.172062	2.518151	2.754629
Stock	0.455895	14.838503	18.521905
Straddle	0.792576	1.594029	1.600294
SynLong	0.264657	14.999575	18.667405

All portfolios' ES is greater than VaR. Straddle and Put Spread have lower VaR and ES, meaning lower risk because they focus on benefiting from stock price movements without needing the price to go up or down significantly, so less risk than other strategies. Stock and SynLong rely highly on the stock price show larger losses, meaning they are riskier. The CoveredCall strategy, which combines owning the stock with selling call options, shows a

slight average loss and relatively high risk, making it less favorable. Overall, strategies that profit from price movement tend to have less risk thus lower VaR and ES, while portfolios directly tied to the stock price carry more risk but also offer a chance for higher returns.