## 1.

To find the maximum sharpe ratio and the optimal portfolio, we are going to use what we learned last week:

$$SR = \frac{E(r_p) - r_{rf}}{\sigma_p}$$

First, we are going to calculate the mean and covariance matrix of the assets, then we used the formula above to set up the function for the sharpe ratio. We then defined the constraints and bounds, which are the sum of the weights that should be 1 and allow short positions but bounded between -1 and 1. Then we performed the optimization to find the maximum sharpe ratio using the SLSQP algorithm. Then we can get the results below:

```
Optimal Sharpe Ratio: 0.34413588
Optimal Portfolio Return: 0.18205896
Optimal Portfolio Standard Deviation (Volatility): 0.39100532
Weight for Asset 1: 0.15202905
Weight for Asset 2: -0.13302032
Weight for Asset 3: 0.98099127
```

Here we can find that by using sharpe ratio to find the optimal portfolio, we shorted asset2.

## 2.

This part we are going to replace the sharpe ratio with risk-adjusted return and find the optimal portfolio based on that.

$$RR_p = \frac{E(r_p) - RF}{ES(r_p - RF;\ \alpha = 2.5\%)}$$

From the observation of the formulas, the only difference between RRp and Sharpe Ratio is on the denominator, here we are going to use ES to replace sigma.

First we are going to calculate the ES, which measures the average of worst losses, then we will setup the function for the risk-adjusted return, for the optimization process, we are going to have the same boundaries and constraints as question1. We got the result as below:

```
Optimal Risk-Adjusted Return: 0.29460224
Optimal Portfolio Return: 0.16038810
Optimal Portfolio Volatility: 0.41777086
Weight for Asset 1: 0.33005598
Weight for Asset 2: 0.04734918
Weight for Asset 3: 0.62259484
```

# 3.

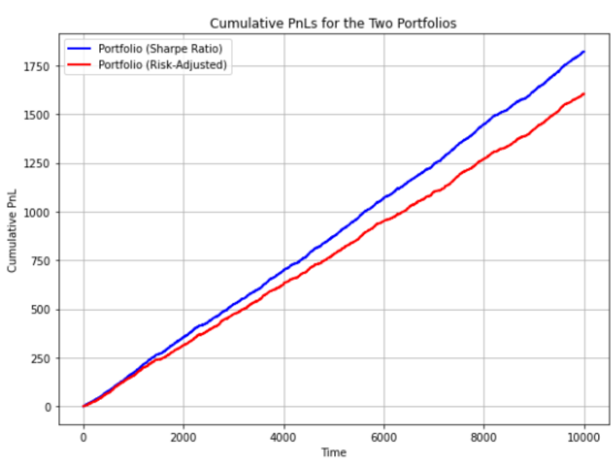
As we can see from the first two questions about different ways to optimize the portfolios,

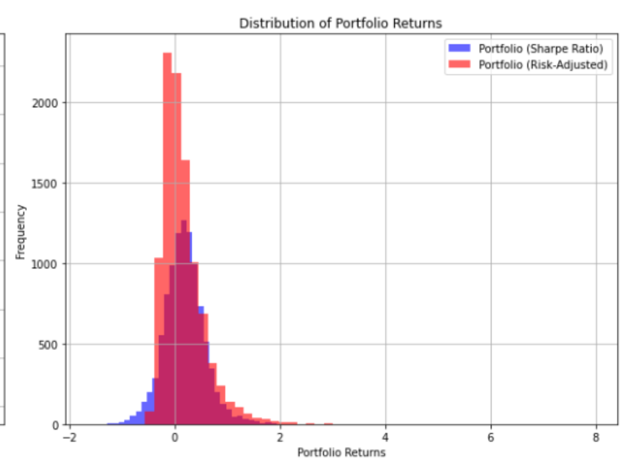
```
Portfolio Optimization Comparison Table
                          Metric  Sharpe Ratio Portfolio  \
0      Optimal Portfolio Return                0.182059
1  Optimal Portfolio Volatility                0.391005
2          Optimal Sharpe Ratio                0.344136
3  Optimal Risk-Adjusted Return                     NaN
4            Weight for Asset 1                0.152029
5            Weight for Asset 2               -0.133020
6            Weight for Asset 3                0.980991

   Risk-Adjusted Portfolio
0                 0.160388
1                 0.417771
2                      NaN
3                 0.294602
4                 0.330056
5                 0.047349
6                 0.622595
```

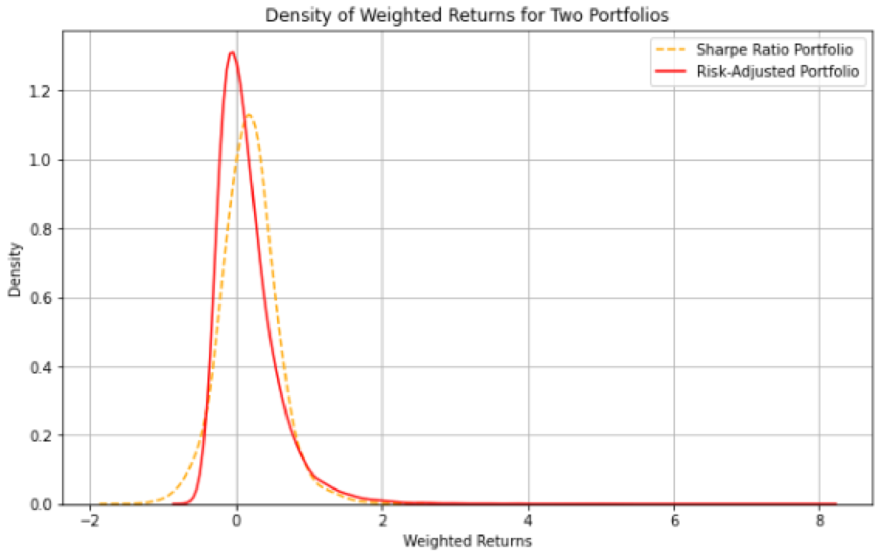
We can observe that sharpe ratio portfolio has slightly higher portfolio return but lower portfolio volatility, and for the weights for each assets within the portfolio, the one optimized under sharpe ratio shorted asset 2 and put much weights on asset 3; while the portfolio optimized using rrp didn't short any assets, and though still have majority weights on asset 3 but also include more on asset 1.

As we can see from the formula, the only difference between the two methods is in the denominator, where sharpe ratio used portfolio volatility while rrp used ES. And we learned from previous weeks that portfolio volatility assumes normal distribution of returns, meaning that this one captures the upside and downside equally; but ES captures the extreme losses (tail risks), and this method can mitigate the extreme losses.

To better explain the differences, I first plotted the PnL and distribution for the two portfolios and the graphs are shown as below:

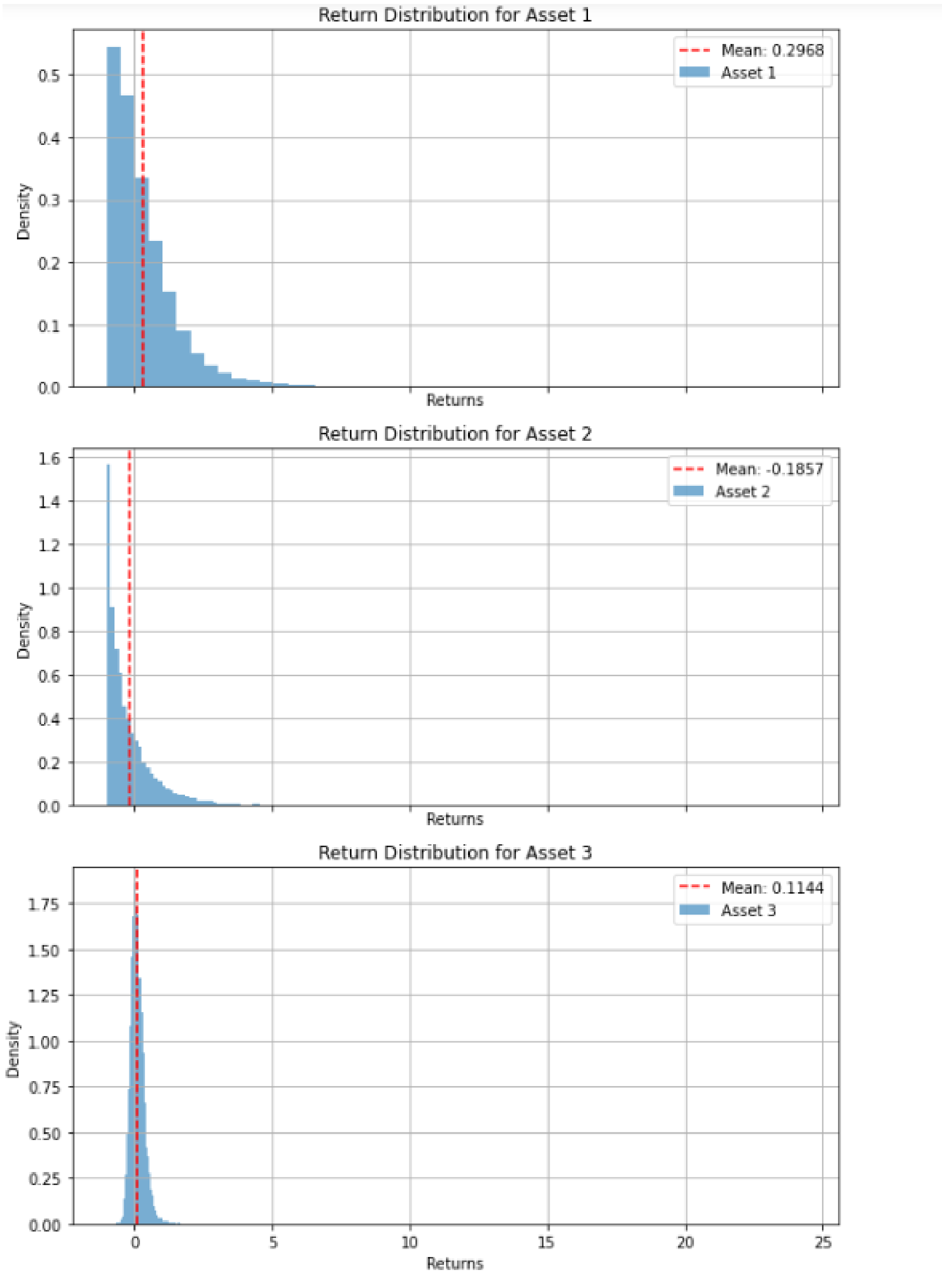





We can see that the cumulative PnL for RRp is slightly lower, which reflects on the point that the RRp method reducing the tail risks that might sacrifice some returns, thus is relatively more conservative compared to the sharpe ratio method, and sharpe ratio methods exposes to more tail risks. As for the distribution, we can find that sharpe ratio portfolio is more like normal distribution, which is the volatility of the portfolio, and can reflect the point that it treats upside and downside risks equally; and the rrp method is more skewed and more focus on the right tails and protect against the extreme losses.

We then can plot the distribution with mean annotations for each asset as below:

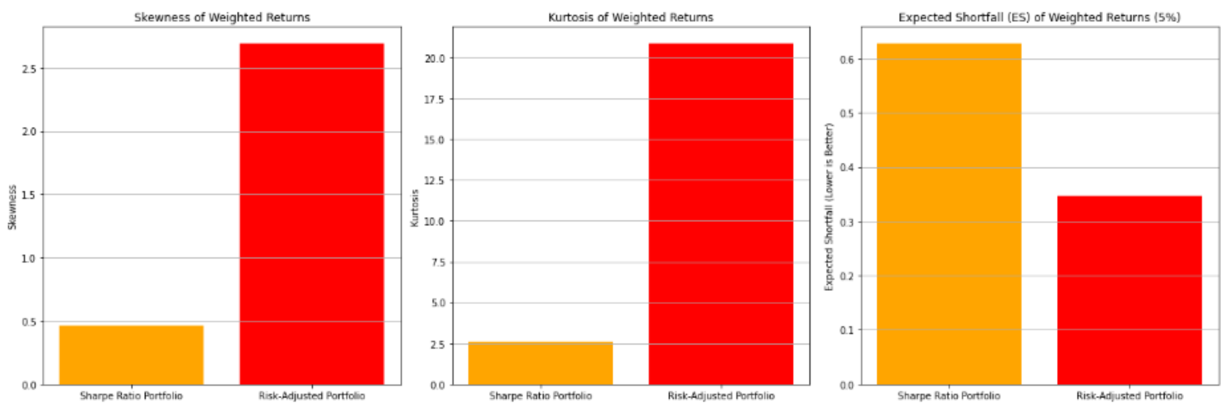



```
Standard Deviation (Volatility) for Asset 1: 1.22846120
Standard Deviation (Volatility) for Asset 2: 0.92593495
Standard Deviation (Volatility) for Asset 3: 0.25386977
```

We can see that asset 1 is positively skewed with a long right tail indicating potential for extreme positive returns and has mean of 0.29 and volatility of 1.22, asset 3 is right-skewed but with a higher concentration near the mean and fewer extreme values compared to asset 1 and has mean of 0.11 and volatility of 0.25 (makes it the most favorable because of the moderate return and stabilization), asset 2 is negatively skewed with a long left tail indicating higher downside risk and has mean of -0.18 and volatility of 0.92 (which makes it least favorable). Thus that’s why sharpe ratio portfolio short asset 2 because it wants to reduce the negative mean return, and since the methodology between this one is to maximize the return to risk ratio, asset 3 get the highest weight because of the moderate return and lower tail risks. For the rrp method, asset 3 still has the highest weight because of the concentrated distribution to provide the balance; the weight of asset 1 increases from sharpe ratio method because its more positive skewed and ES only focus on the downside so performs better; as for asset 2, the negative mean return and long left tail increase its contribution to ES and to minimize downside risk, the optimization assigns it the smallest weight.

We also can calculate the ES, skewness and kurtosis:

```
Sharpe Ratio Portfolio - Skewness: 0.4649, Kurtosis: 2.6224, ES (5%): 0.6272
Risk-Adjusted Portfolio - Skewness: 2.6904, Kurtosis: 20.8520, ES (5%): 0.3461
```




We can see that since rrp wants to minimize the downside risks, thus it generates lower ES, and it allows potential for the upside, contributed to higher skewness and kurtosis; and sharpe ratio balances return and risk more symmetrically, resulting in lower skewness and kurtosis, and since it suffers from higher downside risks, it generates higher ES.