CMPLXSYS 530 Lab3

shirlynw

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All codes and original figures are in Github:
`https://github.com/ShirlynWY/CMPLXSYS_LABS.git`
Project specs: `https://epimath.github.io/cscs-530-materials/Labs/Lab3.html`

1 Exploring the SIR mean-field model

(a) For an Erdos-Renyi network with N nodes and edge probability p_e, the average degree is

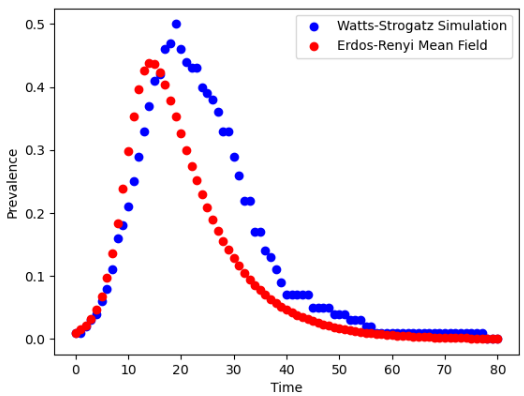
$$\binom{N}{2}\frac{p_e * 2}{N}$$

$\binom{N}{2}$ is the number of edges in the network. We multiply the number of edges by 2 because we now see them as "studs" so that each stud has one node and "half" en edge. The studs can then connect to form edges.

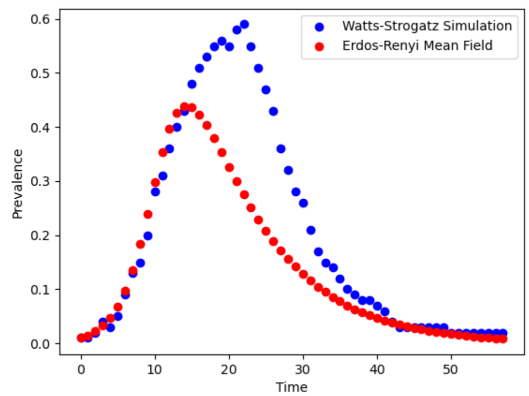
(b) Fig1 shows the prevalence trajectory of Erdos-Renyi mean-field model vs simulations in Watts-Strogatz network. In general, as β, the rewiring probability, increase from 0 to 1, the Erdos-Renyi mean field can approximate the simulated prevalence in a Watts-Strogatz network better and more consistently.

(c) Because the Watts-Strogatz model is stochastic, one or two simulations is not representative of what the behavior of model. Therefore, we simulated the SIR model in 100 Watts-Strogatz networks for 100 timesteps and aim to find the overall trend of prevalence trajectory compared to the mean-field model. In Fig 2 The black line is the prevalence trajectory in the Erdos-Renyi mean-field model and the colorful dots are from 100 SIR model simulations in the Watts-Strogatz network. We can clearly see that the colorful dots get more congregated near the black line as β increases. We will discuss if the mean-field model is a good approximation of the SIR model in Watts-Strogatz networks in (d).

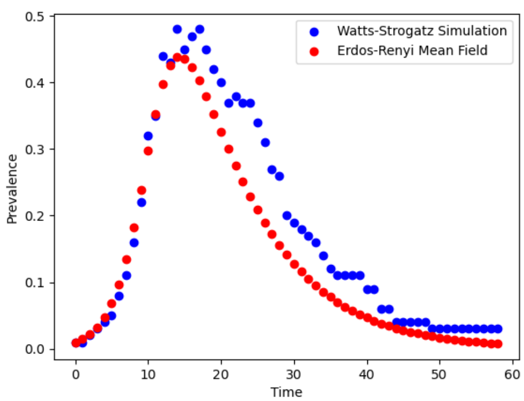
(d) Fig3 shows the quantile plots of the same simulations in 1(c). We can see that as β increases, the interquantile range, as represented by the light blue region, narrows. Both the median prevalence trajectory and interquntile range are fairly close to the mean-field prevalence trajectory when $\beta \geq 0.5$ (subfigures d,e,f). This means that when $\beta \geq 0.5$, most of the simulations can be predicted by the mean-field model resaonably well. Thus, I would feel comfortable using the mean field model for this system when $\beta \geq 0.5$.



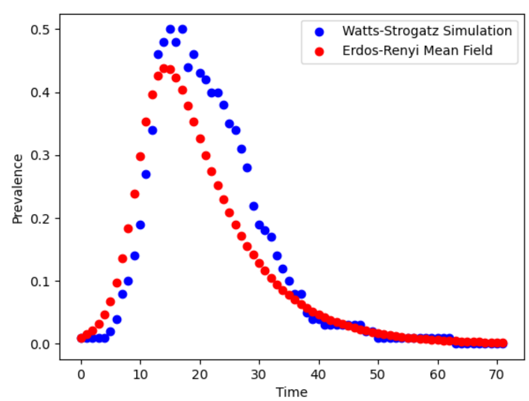
(a) $\beta = 0.2$



(b) $\beta = 0.5$

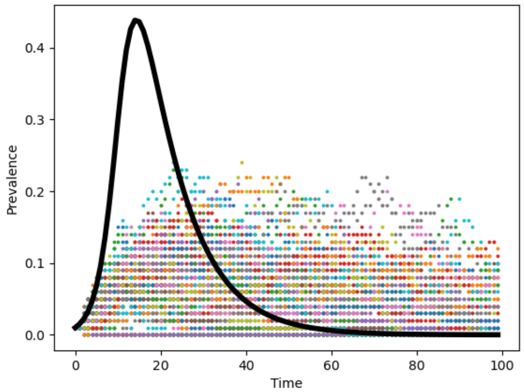


(c) $\beta = 0.8$

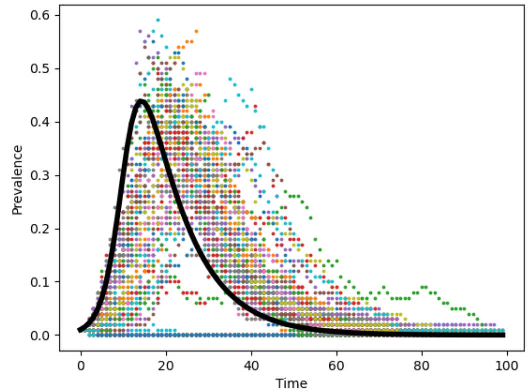


(d) $\beta = 1$

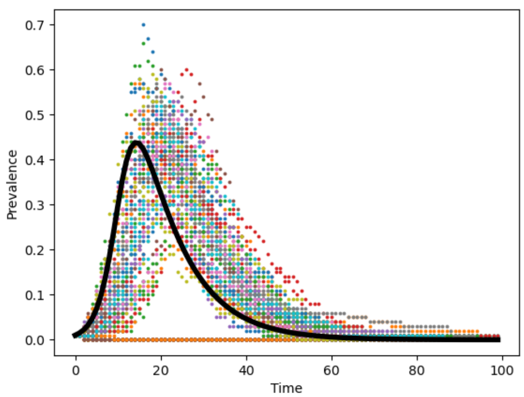
Figure 1: Prevalence trajectory in Watts-Strogatz simulations vs in Erdos-Renyi mean-field model for varying β



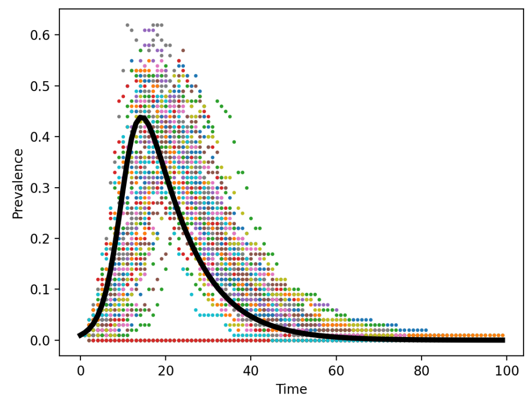
(a) $\beta = 0$



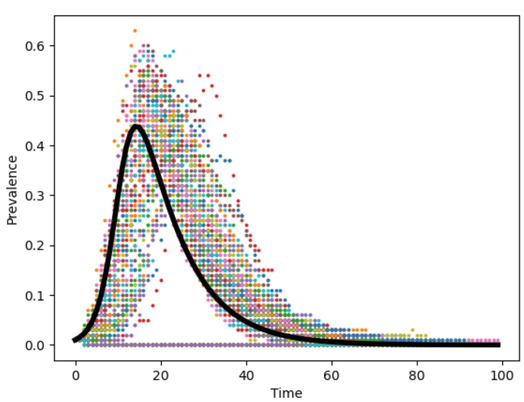
(b) $\beta = 0.2$



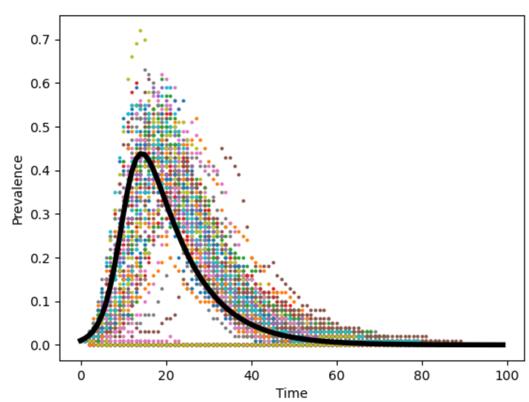
(c) $\beta = 0.4$



(d) $\beta = 0.5$

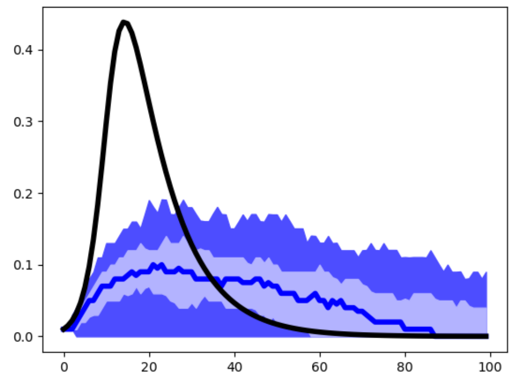


(e) $\beta = 0.8$

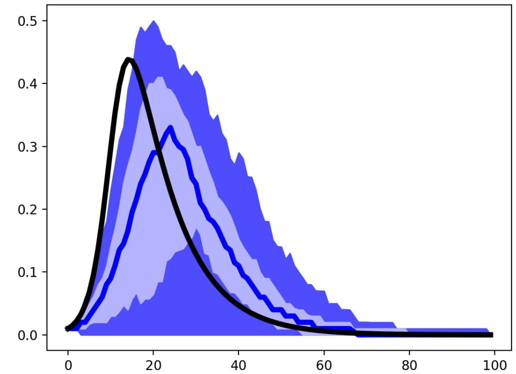


(f) $\beta = 1$

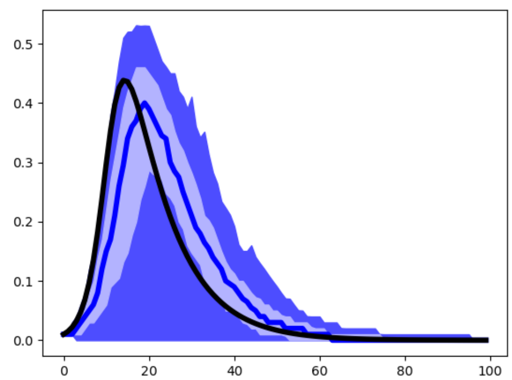
Figure 2: 100 SIR simulations vs Erdos-Renyi mean-field



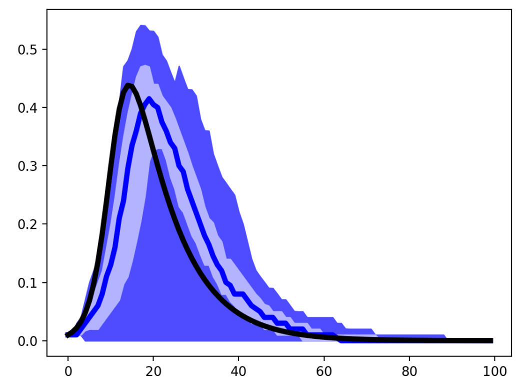
(a) $\beta = 0$



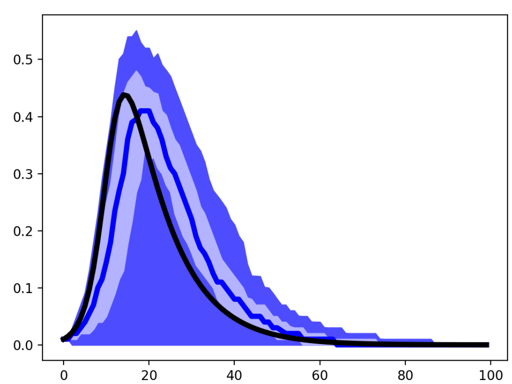
(b) $\beta = 0.2$



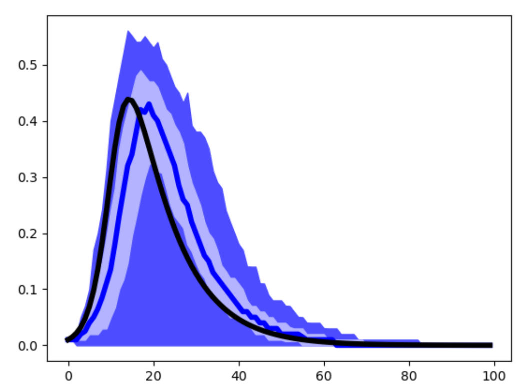
(c) $\beta = 0.4$



(d) $\beta = 0.5$

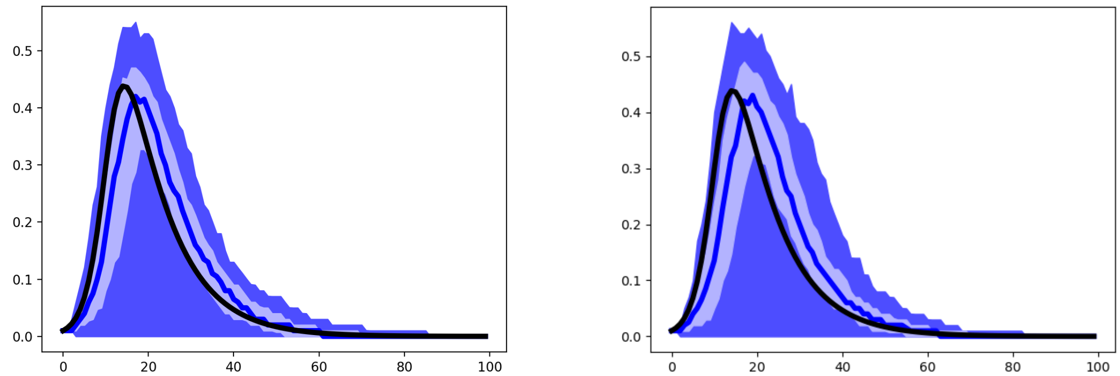


(e) $\beta = 0.8$



(f) $\beta = 1$

Figure 3: Quantile plot of the SIR model in Watts-Strogatz networks vs Erdos-Renyi mean-field model. The dark blue filled area is the 5% and 95% quantiles, the light blue filled area is the 25% and 75% quantiles, the blue line is the median of the network simulations, and the black line is the mean-field model.



(a) Erdos-Renyi meanfield model

(b) Watts-Strogatz network with $\beta = 1$

Figure 4: Quantile plot of the Erdos-Renyi mean-field model vs the Watts-Strogatz network with $\beta = 1$

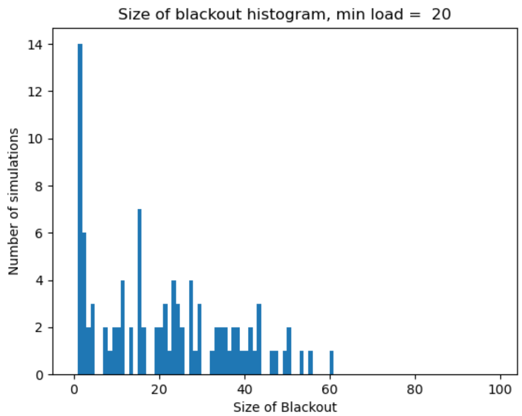
(e) Fig4 compares the quntile plots of the simulations in a random Erdos-Renyi network vs the simulations in the most random Watts-Strogatz network ($\beta = 1$). The interquantile range in subfigure(a) is clearly narrower than that in subfigure(b), meaning that the Erdos-Renyi mean-field model offers predictions in a random Erdos-Renyi network better and more consistently than a Watts-Strogatz model.

2 Cascading failure on a power grid

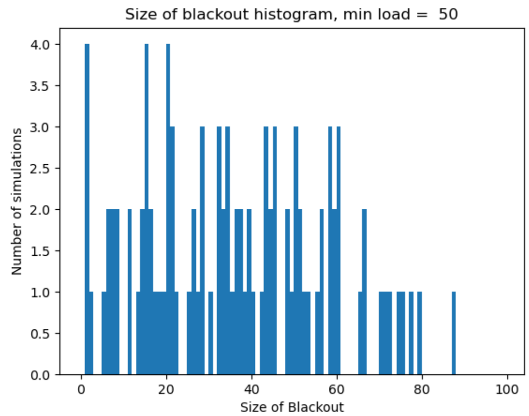
(a) In the simulations, I used a Barabasi-Albert preferential attachment network with 100 nodes , where each node attaches to 2 existing nodes. I then added attributes "state", "max", "load" to each node to represent its state of running, maximum capacity and current workload. A node is chosen at random and its load is set to 1.1 times its maximum capacity. Then the simulations run for 100 time steps. In my code, when a failed node has no living neighbors, the workload simply disappears. Although this is not the most practical, it is easy to implement. (Theoretically this reduces the load in the network, but we will see later that cascading failures are still extremely frequent when the power plants initially run at even moderate capacities.)

(b) I plotted histograms for the size of blackout (number of failed nodes) with different different starting load ranges. As the minimum initial load increases from 20% to 80% of the maximum capacity, the bars of the histogram shift to the right, meaning that blackouts become larger and more frequent.

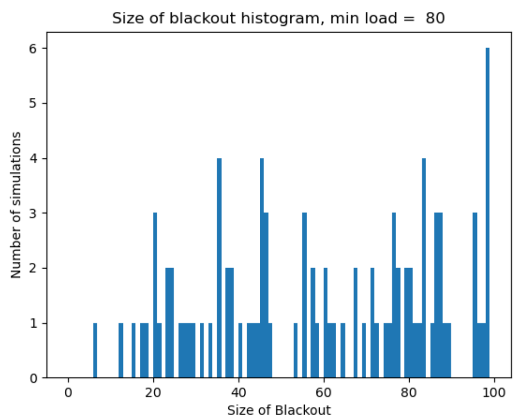
(c) Fig6(a) and (b) show the spectral layout of the preferential attachment network and the power grid network. The power grid network doesn't look similar to the preferential attachment network. In the power grid network, the paths between nodes look very limited considering the large number of nodes in this network. And they are not tangled. But there are more paths in the preferential attachment network and they are more tangled.



(a) Minimum initial load = 20% maximum capacity



(b) Minimum initial load = 50% maximum capacity

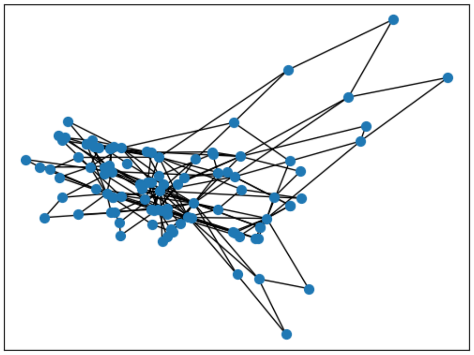


(c) Minimum initial load = 80% maximum capacity

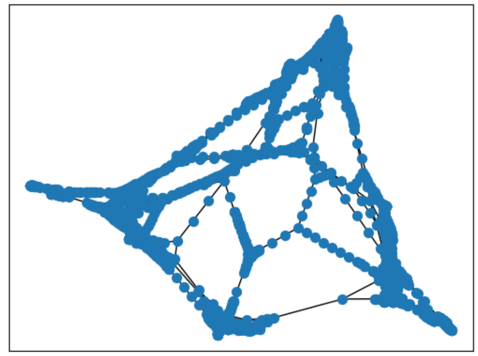
Figure 5: Histogram of blackout size for varying minimum initial load

The power grid network is scale-free. Fig6(c) shows the histogram of the degree of nodes. Subfigures(d) leaves out the bars in the histogram and only plots the number of nodes in each bin. Subfigure(e) then takes ln of the number of nodes. We can see that the relationship between ln(number of nodes) vs degree is fairly linear.

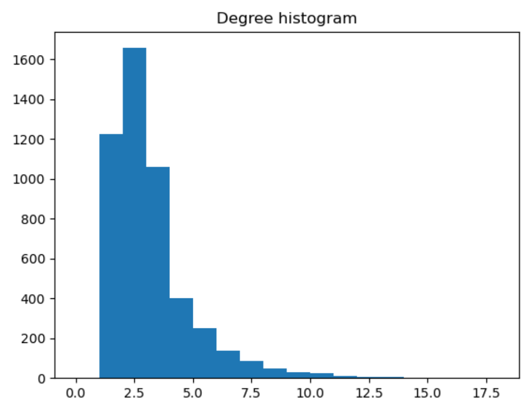
(d) To model power plant failures in the power grid netowrk, I again randomly select a node and set its load to 1.1 times its maximum capcity. I run 30 simulations on 2 network setups with minimum initial load 20% and 50% respectively. Fig6 shows the size of blackout in each simulation. When the minimum initial load is 20%, as shown in subfigure(a), about half of the simulations have massive blackout. When the minumum initial load is 50%, as shown in subfigure(b), in almost all the simulations, almost all the nodes in the network fail. This shows how vulnerable the network is when the initial minimum load is not low. To avoid massive cascading failure in the network when one node fails, it is necessary to keep the initial minimum load low for all nodes.



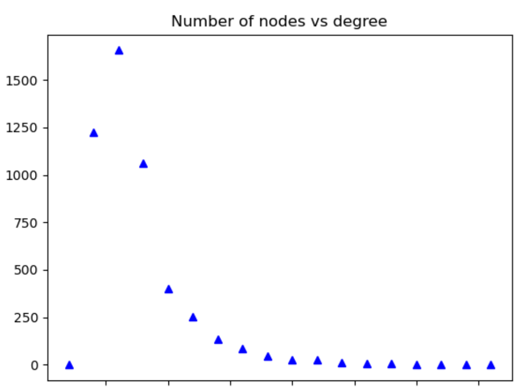
(a) Spectral layout of the 100-node Barabasi-Albrert preferential attachment network



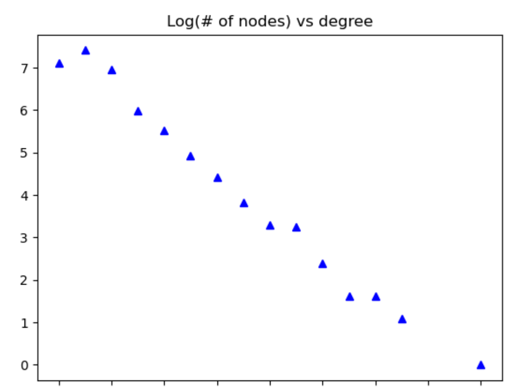
(b) Spectral layout of the power grid network



(c) Histogram of degree of nodes

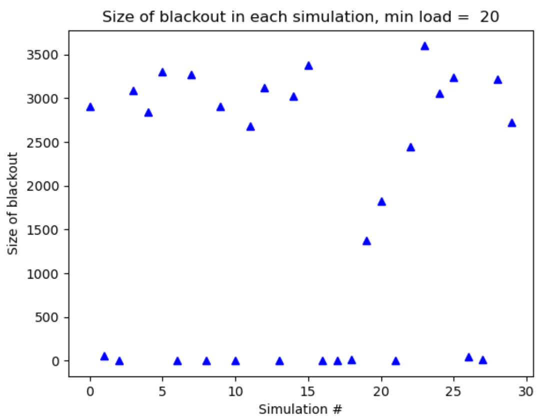


(d) number of nodes vs degree

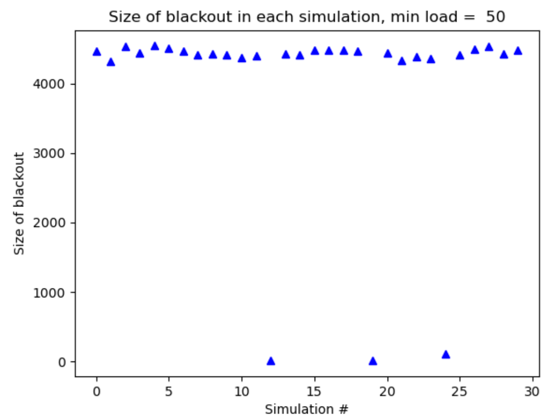


(e) ln(number of nodes) vs degree

Figure 6



(a) Minimum initial load = 20% maximum capacity



(b) Minimum initial load = 50% maximum capacity

Figure 7