The Effect of Differentiated Emission Taxes: Does an Emission Tax Favor Industry?

Shiro Takeda*

Graduate School of Economics, Hitotsubashi University, Kunitachi, Tokyo, 186-8601, Japan.

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Abstract

Extending a standard 2×2 Heckscher-Ohlin model to include emissions, this paper investigates the effect of differentiated emission taxes on output. Our finding is that raising the emission tax imposed on an industry may increase output of the industry. This result is quite surprising in the sense that such a paradoxical result can occur in a simple and standard model under fairly plausible values of parameters. By numerical examples and using a graphical method, it is also shown that the mechanism behind the result is the factor market adjustment effect which works through two different channels.

Keywords: Emission; the Heckscher-Ohlin model; differentiated emission taxes.

JEL Classification: Q28; F11; F18; H20.

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1 Introduction

Emissions from production activities are one of the major causes of various environmental problems and regulations on emissions are regarded as an important policy subject (see, for example, UNFCCC, 1997). In this paper, we analyze the effects of differentiated emission taxes in a general equilibrium setting. Although such differentiated emission taxes are often observed in actual policies (see OECD, 1994), there are few previous studies on this subject. Using the standard 2×2 Heckscher-Ohlin (HO) model with emissions, we focus on how the differentiated emission taxes affect output. Since we assume good prices as a given constant, our model represents the production side of an economy or a small open economy. Our finding is that raising the emission tax on an industry may *increase* output of the industry. This result is counter-intuitive because the rise in an emission tax should have the cost-push effect and thus lead to the downward pressure on the output of the industry. In addition, it is surprising in the sense that such paradoxical result can arise in a simple and standard 2×2 model under fairly plausible values of parameters. It is also shown that the mechanism behind the result is the factor market adjustment effect which works through two different channels.

2 The Model

We employ the standard 2×2 model and, as in previous literature on the subject, incorporate emissions as the third production factor.[1] Thus, the model has a similar structure to the standard 2×3 HO model employed in Batra and Casas (1976) and Jones and Easton (1983). However, there is one important difference from their model: while all factor prices are endogenously determined in the standard 2×3 model, the factor prices corresponding to emissions in our model are policy instruments determined exogenously.[2]

Let v^i_j denote the amount of factor $j = K, L$ employed in sector $i = 1, 2$ and v^i_Z denote the level of emission from sector i. The production function of sector i is given by $Q_i = f^i(v^i)$ where Q_i is the output of sector i and $v^i = (v^i_K, v^i_L, v^i_Z)$. We assume that $f^i(\cdot)$ is concave and homogeneous of degree one in v^i.

[1] This approach is commonly used in general equilibrium analyses, for example, Yohe (1979), Copeland and Taylor (1995) and Ishikawa and Kiyono (2000).

[2] The model more similar to ours is the 2×3 model with capital mobility like Wong (1995) chapter 4 because one of the factor prices in his model (the rental rate) is also constant.

The cost function of sector i is

$$c^i(w_K, w_L, w_Z^i) \equiv \min_{\{a_{Ki}, a_{Li}, a_{Zi}\}} \{w_K a_{Ki} + w_L a_{Li} + w_Z^i a_{Zi} \mid f^i(a_{Ki}, a_{Li}, a_{Zi}) \geq 1\}$$

where w_j $(j = K, L)$ denotes the price of factor j and w_Z^i denotes the specific emission tax imposed on sector i. From Shephard's lemma, $a_{ji}(w_K, w_L, w_Z^i) = \partial c^i(w_K, w_L, w_Z^i)/\partial w_j$ $(j = K, L, Z)$.

At a competitive equilibrium, unit cost must be equal to price if the commodity is actually produced and, capital and labor must be fully employed. Thus, for $\forall i = 1, 2$, and $\forall j = K, L$:

$$c^i(w_K, w_L, w_Z^i) = p_i \qquad\qquad \sum_{i=1,2} a_{ji} Q_i = v_j \tag{1}$$

where p_i is the price of good i and v_j is the endowment of factor j. Given commodity prices, factor endowments, and emission taxes, equilibrium factor prices and outputs are determined by (1). The level of emission from sector i is given by $v_Z^i = a_{Zi} Q_i$. Let θ_{ji} be the cost share of factor j in sector i and λ_{ji} the fraction of factor j employed in sector i. Then, equations of change are given by

$$\sum_{j=K,L} \theta_{ji} \hat{w}_j = \hat{p}_i - \theta_{Zi} \hat{w}_Z^i \qquad\qquad \sum_{i=1,2} \lambda_{ji} \hat{Q}_i = \hat{v}_j - \sum_{i=1,2} \lambda_{ji} \hat{a}_{ji} \tag{2}$$

where a hat over a variable denotes the rate of change (e.g. $\hat{w}_j = dw_j/w_j$). In addition, we define $|\theta_{KL}| \equiv \theta_{K1}\theta_{L2} - \theta_{L1}\theta_{K2}$, $|\lambda_{KL}| \equiv \lambda_{K1}\lambda_{L2} - \lambda_{L1}\lambda_{K2}$, $Y \equiv \sum_{i=1,2} p_i Q_i$, $\alpha_j \equiv w_j v_j / Y$, and $\gamma_i \equiv p_i Q_i / Y$. α_j and γ_i represents the factor and sector share in GDP respectively. By definition, we have $\lambda_{ji} = \gamma_i \theta_{ji} / \alpha_j$. In the remainder of the paper, we will focus on the effects of the change in the emission taxes and set $\hat{p}_i = \hat{v}_j = 0$. From (2) and the above notations, the following relations are derived.

$$\hat{w}_K = |\theta_{KL}|^{-1}(-\theta_{L2}\theta_{Z1}\hat{w}_Z^1 + \theta_{L1}\theta_{Z2}\hat{w}_Z^2) \qquad \hat{w}_L = |\theta_{KL}|^{-1}(\theta_{K2}\theta_{Z1}\hat{w}_Z^1 - \theta_{K1}\theta_{Z2}\hat{w}_Z^2) \tag{3}$$

$$\hat{Q}_1 = |\lambda_{KL}|^{-1}(\lambda_{L2}\hat{\beta}_K - \lambda_{K2}\hat{\beta}_L) \qquad \hat{Q}_2 = |\lambda_{KL}|^{-1}(-\lambda_{L1}\hat{\beta}_K + \lambda_{K1}\hat{\beta}_L) \tag{4}$$

where $\hat{\beta}_j \equiv -\sum_{i=1,2} \lambda_{ji} \hat{a}_{ji}$.

To derive the expression of $\hat{Q}_i / \hat{w}_Z^i$, we define further notations.

$$\varepsilon_{jl}^i \equiv \frac{w_l}{a_{ji}} \frac{\partial a_{ji}}{\partial w_l} \qquad i = 1, 2 \quad j, l = K, L, Z$$

ε_{jl}^i is the price elasticity of factor demand per output in sector i. If $\varepsilon_{jl}^i > (<)\ 0$, factor j and l are called *substitutes* (*complements*) in sector i.[3] ε_{jl}^i has the following three properties: (i) $\varepsilon_{lj}^i = \theta_{ji}\varepsilon_{jl}^i / \theta_{li}$, (ii) From

[3] $\varepsilon_{jl}^i / \theta_{li}$ is the well-known Allen's partial elasticity of substitution (see Chambers, 1988, p. 95). While most papers including Batra and Casas (1976), Yohe (1979), and Siebert, Eichverger, Gronych and Pethig (1980) use this Allen's measure of elasticity, we use ε_{jl}^i as Jones and Easton (1983).

the zero homogeneity of a_{ji} with respect to (w_K, w_L, w^i_Z), for $\forall j = K, L, Z$, $\varepsilon^i_{jK} + \varepsilon^i_{jL} + \varepsilon^i_{jZ} = 0$, (iii) From the concavity of cost function, $\varepsilon^i_{jj} \leq 0$ and $\varepsilon^i_{jj}\varepsilon^i_{ll} - \varepsilon^i_{jl}\varepsilon^i_{lj} \geq 0$. From these properties, if ε^i_{jl} is negative, both ε^i_{jk} and ε^i_{lk} must be positive, that is, there is at most one pair of factors that are complements. Moreover, property (ii) and (iii) imply that the following inequality holds for $\forall i = 1, 2$, $\forall j, l, k = K, L, Z$, $j \neq l$, $l \neq k$, $k \neq j$:

$$\varepsilon^i_{jl} \geq -\frac{\theta_{li}\varepsilon^i_{jk}\varepsilon^i_{lk}}{\theta_{ji}\varepsilon^i_{jk} + \theta_{li}\varepsilon^i_{lk}} \tag{5}$$

This means that even if $\varepsilon^i_{jl} < 0$ (i.e. factor j and l are complements), the degree of complementarity is limited by some bound.

In addition, we define $\varepsilon_{jl} \equiv \lambda_{j1}\varepsilon^1_{jl} + \lambda_{j2}\varepsilon^2_{jl}$. ε_{jl} expresses the price elasticity of *total* factor demand and has the similar properties to ε^i_{jl}: (i) $\varepsilon_{lj} = \alpha_j\varepsilon_{jl}/\alpha_l$, (ii) $\varepsilon_{jK} + \varepsilon_{jL} + \varepsilon_{jZ} = 0$, (iii) $\varepsilon_{jj} \leq 0$. Using above notations, we can rewrite $\hat{\beta}_j$ as follows

$$-\hat{\beta}_j = \varepsilon_{jK}\hat{w}_K + \varepsilon_{jL}\hat{w}_L + \lambda_{j1}\varepsilon^1_{jZ}\hat{w}^1_Z + \lambda_{j2}\varepsilon^2_{jZ}\hat{w}^2_Z \tag{6}$$

In the rest of the paper, we consider only the change in the emission tax on sector 1 without loss of generality. The same argument is applicable to the other case. Combining (3)-(4), and (6), we can derive the expression of $\hat{Q}_1$:

$$\hat{Q}_1 = (|\theta_{KL}||\lambda_{KL}|)^{-1}A_1\hat{w}^1_Z \tag{7}$$

where
$$A_1 = -B_1\theta_{Z1} + |\theta_{KL}|(\lambda_{K2}\lambda_{L1}\varepsilon^1_{LZ} - \lambda_{L2}\lambda_{K1}\varepsilon^1_{KZ})$$
$$B_1 = \frac{\gamma_2}{\alpha_L}(\theta_{K2} + \theta_{L2})^2\varepsilon_{KL} + \frac{\gamma_2}{\alpha_L}(\theta_{L2})^2\varepsilon_{KZ} + \frac{\gamma_2}{\alpha_K}(\theta_{K2})^2\varepsilon_{LZ}$$

Since both $|\theta_{KL}|$ and $|\lambda_{KL}|$ have the same signs, $|\theta_{KL}||\lambda_{KL}| > 0$.

3 Analysis

Since (7) includes a lot of parameters, we cannot derive analytical insights from it except for extreme cases.[4] However, we can show the following paradoxical proposition by numerical examples.

Propostion *The sign of $\hat{Q}_i/\hat{w}^i_Z$ may be positive, that is, raising emission tax imposed on an industry may increase the outputs of the industry.*

Table 1:

	θ_{K1}	θ_{L1}	θ_{K2}	θ_{L2}	γ_1	ε^1_{KL}	ε^1_{LZ}	ε^1_{KZ}	ε^2_{KL}	ε^2_{LZ}	ε^2_{KZ}
Case 1	0.5	0.45	0.3	0.65	0.6	0.5	0.5	−0.118	0.5	0.5	−0.171
Case 2	0.7	0.25	0.3	0.65		2.5	1.5	−0.309			
Case 3	0.3	0.65	0.5	0.45	0.7	0.5	0.5	2.5	1	0.5	1

We show and explain this proposition by giving three numerical examples.[5] The values of the parameters in the three cases are shown in Table 1. In all three cases, we have $\hat{Q}_1/\hat{w}^1_Z > 0$ (see Appendix 1). From the table, one can see that it occurs at fairly plausible values of parameters. In Case 1, I assume that K and Z are complements in both sectors. I assume this situation because the empirical result in Chambers (1988), p. 98 indicates such a situation.

The result of the proposition is quite counter-intuitive because the rise in emission tax should have the cost-push effect and thus lead to the downward pressure on the output of the industry. However, a close look at A_1 in (7) reveals that, in addition to the cost-push effect, the rise in the emission tax has another effect. Two effects can be explained as follows. First, the sector specific rise in emission tax alters factor prices in the same way as the fall in the commodity price (see the RHS of (2)). These changes in factor prices lead to the changes in factor demand and the output is adjusted so as to clear factor markets. This effect, which we call *the cost-push* or *indirect effect*, is represented by the first term in A_1. In addition, the change in emission tax directly affects factor demand through substitution (or complementarity) between factors. This *substitution* or *direct effect* is represented by the second term.

For example, suppose that w^1_Z rises by one percent. This has the same impact on factor prices as a θ_{Z1} percent fall of p_1, and its impact on Q_1 is represented by $-B_1\theta_{Z1}$. We can show that this indirect effect of the rise in w^1_Z through factor price adjustment always decreases the output, i.e. $B_1 \geq 0$, (see Appendix 2). On the other hand, one percent rise in w^1_Z raises the demands for capital and labor by $\lambda_{K1}\varepsilon^1_{KZ}$ and $\lambda_{L1}\varepsilon^1_{LZ}$ respectively (or reduces them if they are complements). If, for example, sector 1 has a higher capital-labor ratio than sector 2 (i.e. $|\theta_{KL}| > 0$), the increased demand for capital gives rise to a downward pressure on the output of sector 1 and the increased demand for labor gives rise to a upward pressure. This effect is represented by the second term. It follows that if the direct (substitution) effect works strongly in the opposite direction to the indirect (cost-push) one, the rise in the emission tax on an industry may

[4] For example, we can show that when K and L are perfect complements in both sectors, $\hat{Q}_1/\hat{w}^1_Z < 0$ always holds.

[5] Of course, one can easily find other various cases in which the paradoxical result happens.

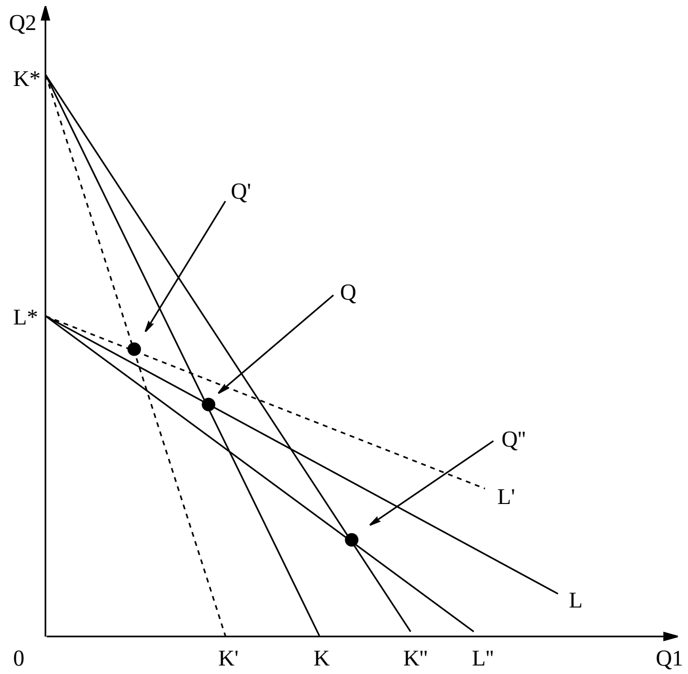

Figure 1:

raise the output of the industry.

Next, using Case 2 and the figure 1, I explain the above two effects in detail. In Case 2, I assume, for graphical exposition, that capital and labor are perfect complements in sector 2 (i.e. $\varepsilon^2_{Kj} = \varepsilon^2_{Lj}$ for $\forall j = K, L, Z$). Figure 1 depicts the equilibrium in the output space. The horizontal and vertical axes represent the outputs of sector 1 and 2 respectively. Let the full employment lines for capital and labor at the initial equilibrium be denoted by line K*K and L*L whose slopes are given by a_{K1}/a_{K2} and a_{L1}/a_{L2}. Since in Case 2, the capital-labor ratio in sector 1 is higher than that in sector 2, line K*K is steeper than line L*L. The outputs at the initial equilibrium is given by the point Q where both factor markets are cleared.

Now suppose that the emission tax on sector 1 is raised by 1%. First, let us consider the indirect effect. From (3), 1% rise in w^1_Z leads to $-0.0235/0.38\%$ fall in w_K and $0.015/0.38\%$ rise in w_L because the capital-labor ratio in sector 1 is higher than that in sector 2 (the Stolper-Samuelson effect). Since capital and labor are substitutes in sector 1, these changes in factor prices lead to the rise in a_{K1} and the fall in a_{L1}: $\hat{a}^{ID}_{K1} = 0.28609$ and $\hat{a}^{ID}_{L1} = -0.93421$ (the superscript ID means indirect effect). On the other hand, from the perfect complementarity between K and L in sector 2, both $\hat{a}^{ID}_{K2}$ and $\hat{a}^{ID}_{L2}$ are zero.

Therefore, by the indirect effect, the full employment lines shift to K*K' and L*L', and outputs shift to Q' where the output of sector 1 decreases. As has already been pointed out, the indirect effect always works in this direction.

Next, consider the direct effect. The direct effect of a tax on input coefficients in sector 1 is given by $\hat{a}^D_{K1} = \varepsilon^1_{KZ} = -0.309$ and $\hat{a}^D_{L1} = \varepsilon^1_{LZ} = 1.5$. Thus, the direct effect works in the opposite direction to the indirect one. Moreover, since the size of the direct effects is larger than that of the indirect effects (i.e. $|\hat{a}^D_{K1}| > |\hat{a}^{ID}_{K1}|$ and $|\hat{a}^D_{L1}| > |\hat{a}^{ID}_{L1}|$), the direct effects dominate the indirect one. Taking account of the two effects, the full employment lines shift to K*K" and L*L" and the new equilibrium output shifts to Q". Therefore, in the example above, the rise in the emission tax on sector 1 increases the output of sector 1.

Both Case 1 and 2 include complementary factors. However, it does not mean that complementary factors are necessary for the paradoxical result to occur. This is shown by Case 3 in which all factors are substitutes in both sectors.

4 Concluding remarks

In this paper, we have considered the two sector economy with two primary factors and emissions and have explored the effects of differentiated emission taxes on output. Our finding is surprising: raising the emission tax imposed on an industry may increase the output of the industry. The result is all the more surprising because it can arise in the simple standard 2×2 model under fairly plausible values of parameters. Finally, we present two additional comments. First, our result depends on the multi-factor nature of the model and cannot be derived in a one primary factor model because, as Proposition 7 in Ishikawa and Kiyono (2000) shows, production is always specialized to one sector in such a model. Second, our result is reminiscent of the well-known 'Porter hypothesis' which suggests that environmental regulation is beneficial to firms because it promotes technological innovation and reduces production cost. Although at first sight our result seems to support the hypothesis, it is not the case because the unit cost of firms are kept constant through our analysis and the mechanism behind our paradoxical result is factor market adjustment effects and not cost reduction effects.

Appendix 1

A_1 can be rewritten as $A_1 = -\gamma_2(\gamma_1 X_1 + \gamma_2\theta_{Z1}\theta_{K2}X_2)/\alpha_K\alpha_L$ where $X_1 = (\theta_{K2}+\theta_{L2})^2\theta_{Z1}\theta_{K1}\varepsilon^1_{KL} + \theta_{L2}\theta_{K1}[\theta_{L2}-\theta_{L1}(\theta_{K2}+\theta_{L2})]\varepsilon^1_{KZ} + \theta_{K2}\theta_{L1}[\theta_{K2}-\theta_{K1}(\theta_{K2}+\theta_{L2})]\varepsilon^1_{LZ}$, and $X_2 = (\theta_{K2}+\theta_{L2})^2\varepsilon^2_{KL} + (\theta_{L2})^2\varepsilon^2_{KZ} + \theta_{K2}\theta_{L2}\varepsilon^2_{LZ}$. Inserting numerical values of Table 1 into this equation leads to $\hat{Q}_1/\hat{w}^1_Z > 0$. In Case 2, we assume the perfect complementarity between K and L in sector 2 (i.e. $\varepsilon^2_{Kj} = \varepsilon^2_{Lj}$ for $\forall j = K, L, Z$). In this case, from the property (ii) of ε^i_{jl}, we have $\varepsilon^2_{KZ} = \varepsilon^2_{LZ} = -(1+\theta_{K2}/\theta_{L2})\varepsilon^2_{KL}$, thus $X_2 = 0$. When there is a pair of complementary factors, the constraint (5) must be satisfied. In both Case 1 and 2, this constraint is indeed satisfied.

Appendix 2

The proof of $B_1 \geq 0$. If all factors are substitutes, $B_1 \geq 0$ is clear. Thus, we have to prove $B_1 \geq 0$ when there is a pair of factors which are complements. We provide the proof in the case of $\varepsilon^i_{KL} < 0$ for $\forall i = 1, 2$, that is, the case where capital and labor are complements in both sectors. Similar arguments can be applied to the other cases.

We can rewrite B_1 as

$$B_1 = \frac{\gamma_2}{\alpha_K\alpha_L}\sum_{i=1,2}\gamma_i[(\theta_{K2}+\theta_{L2})^2\theta_{Ki}\varepsilon^i_{KL} + (\theta_{L2})^2\theta_{Ki}\varepsilon^i_{KZ} + (\theta_{K2})^2\theta_{Li}\varepsilon^i_{LZ}]$$

Then, from (5), we have

$$\begin{aligned} B_1 &\geq \frac{\gamma_2}{\alpha_K\alpha_L}\sum_{i=1,2}\gamma_i\left[-(\theta_{K2}+\theta_{L2})^2\theta_{Ki}\frac{\theta_{Li}\varepsilon^i_{KZ}\varepsilon^i_{LZ}}{\theta_{Ki}\varepsilon^i_{KZ}+\theta_{Li}\varepsilon^i_{LZ}} + (\theta_{L2})^2\theta_{Ki}\varepsilon^i_{KZ} + (\theta_{K2})^2\theta_{Li}\varepsilon^i_{LZ}\right] \\ &= \frac{\gamma_2}{\alpha_K\alpha_L}\sum_{i=1,2}\frac{\gamma_i}{\theta_{Ki}\varepsilon^i_{KZ}+\theta_{Li}\varepsilon^i_{LZ}}(\theta_{Ki}\theta_{L2}\varepsilon^i_{KZ} - \theta_{Li}\theta_{K2}\varepsilon^i_{LZ})^2 \geq 0 \end{aligned}$$

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