Prova 1 - Cálculo diferencial e integral II

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Questão 1. Calcule as seguintes integrais:

a) $\int \frac{\sqrt{x}+3}{\sqrt{2x}} dx$

$$
\begin{aligned}
&\int \frac{\sqrt{x}+3}{\sqrt{2x}} dx \\
\Rightarrow &\int \frac{1}{\sqrt{2}} + \frac{3}{\sqrt{2}\sqrt{x}} dx \\
\Rightarrow &\int \frac{1}{\sqrt{2}} dx + \int \frac{3}{\sqrt{2}\sqrt{x}} dx \\
\Rightarrow &\frac{1}{\sqrt{2}} x + 3\sqrt{2}\sqrt{x} + C
\end{aligned}
$$

b) $\int x^2 e^{-x} dx$

$$
\begin{aligned}
&\int x^2 e^{-x} dx \\
\Rightarrow &- e^{-x}x^2 - \int -2e^{-x}x dx \\
\Rightarrow &- e^{-x}x^2 - (-2(-e^{-x}x - e^{-x})) \\
\Rightarrow &- e^{-x}x^2 + 2(-e^{-x}x - e^{-x}) + C
\end{aligned}
$$

Questão 2. Calcule o comprimento do arco da curva:

$$y = x^{\frac{2}{3}},\ x \in [1, 2]$$

Solução

$$\begin{aligned}
f(x) &= x^{\frac{2}{3}} \\
f'(x) &= \frac{2}{3x^{\frac{1}{3}}} \\
L &= \int_1^2 \sqrt{1 + [\frac{2}{3x^{\frac{1}{3}}}]^2} dx \\
&= \int_1^2 \frac{(9x^{\frac{2}{3}} + 4)^{\frac{1}{2}}}{3(x^{\frac{2}{3}})^{\frac{1}{2}}} dx \\
&= \frac{1}{3} \cdot \int_1^2 \frac{(9x^{\frac{2}{3}} + 4)^{\frac{1}{2}}}{(x^{\frac{2}{3}})^{\frac{1}{2}}} dx \\
&= \frac{1}{3} \cdot \int_{13}^{9 \cdot 2^{\frac{2}{3}} + 4} \frac{u^{\frac{1}{2}}}{6} du \\
&= \frac{1}{3} \cdot \frac{1}{6} \cdot \int_{13}^{9 \cdot 2^{\frac{2}{3}} + 4} u^{\frac{1}{2}} du \\
&= \frac{1}{3} \cdot \frac{1}{6} [\frac{2}{3} u^{\frac{3}{2}}]_{13}^{9 \cdot 2^{\frac{2}{3}} + 4} \\
&= \frac{1}{18} [\frac{2}{3} u^{\frac{3}{2}}]_{13}^{9 \cdot 2^{\frac{2}{3}} + 4} \\
&= \frac{1}{18} \cdot \frac{2(9 \cdot 2^{\frac{2}{3}} + 4)^{\frac{3}{2}} - 26\sqrt{13}}{3} \\
L &= \frac{(4 + 9 \cdot 2^{\frac{2}{3}})^{\frac{3}{2}} - 13\sqrt{13}}{27}
\end{aligned}$$

Questão 3. Calcule o volume do sólido, obtido pela rotação da região limitada pela curva da Questao 2 e retas $x = 1$, $x = 2$ em torno do eixo OX.

$$\begin{aligned}
V &= \pi \int_1^2 [x^{\frac{2}{3}}]^2 dx \\
&= \pi \frac{12 \cdot 2^{\frac{1}{3}} - 3}{7} \\
V &= \frac{\pi (12 \cdot 2^{\frac{1}{3}} - 3)}{7}
\end{aligned}$$

Questão 4. Calcule a área da região limitada pelos gráficos das funções:

$$f(x) = \sqrt{1 - x^2}$$
$$g(x) = 1 - x$$

Solução

$$\begin{aligned}
A(f(x)) &= \int_0^1 \sqrt{1-x^2}dx \\
&= \int_0^{\frac{\pi}{2}} \cos^2(u)du \\
&= \int_0^{\frac{\pi}{2}} \frac{1+\cos(2u)}{2}du \\
&= \frac{1}{2} \cdot \int_0^{\frac{\pi}{2}} 1 + \cos(2u)du \\
&= \frac{1}{2}(\int_0^{\frac{\pi}{2}} 1du + \int_0^{\frac{\pi}{2}} \cos(2u)du) \\
&= \frac{1}{2}(\frac{\pi}{2} + 0) \\
A(f(x)) &= \frac{\pi}{4} \\
A(g(x)) &= \int_0^1 1 - xdx \\
&= \int_0^1 1dx - \int_0^1 xdx \\
&= [x]_0^1 - [\frac{x^2}{2}]_0^1 \\
&= (1-0) - (\frac{1}{2} - \frac{0}{2}) \\
&= 1 - \frac{1}{2} \\
A(g(x)) &= \frac{1}{2} \\
A(f(x) - g(x)) &= \frac{\pi}{4} - \frac{1}{2} \\
&= \frac{\pi}{4} - \frac{2}{4} \\
A(f(x) - g(x)) &= \frac{\pi - 2}{4}
\end{aligned}$$