Atividade 7

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Exercício 1

1. $f(x, y) = \sqrt{x^2 + y^2},\ p(-4, 3)$

$$
\begin{aligned}
\frac{df(x,y)}{dx} =& \frac{df(u)}{du} \cdot \frac{du}{dx} \\
u = x^2 + y^2,\ f(u) =& \sqrt{u} \\
\frac{df(u)}{du} =& (\sqrt{u})' \\
=& (u^{\frac{1}{2}})' \\
=& \frac{u^{-\frac{1}{2}}}{2} \\
=& \frac{1}{2u^{\frac{1}{2}}} \\
\Rightarrow \frac{df(u)}{du} =& \frac{1}{2\sqrt{u}} \\
\frac{du}{dx} =& (x^2 + y^2)' \\
\Rightarrow \frac{du}{dx} =& 2x \\
\frac{df(x,y)}{dx} =& \frac{1}{2\sqrt{u}} \cdot 2x \\
=& \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2x \\
\Rightarrow \frac{df(x,y)}{dx} =& \frac{x}{\sqrt{x^2 + y^2}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{df(x,y)}{dy} =& \frac{df(u)}{du} \cdot \frac{du}{dy} \\
u = x^2 + y^2,\ f(u) =& \sqrt{u} \\
\frac{df(u)}{du} =& (\sqrt{u})' \\
=& (u^{\frac{1}{2}})' \\
=& \frac{u^{-\frac{1}{2}}}{2} \\
=& \frac{1}{2u^{\frac{1}{2}}} \\
\Rightarrow \frac{df(u)}{du} =& \frac{1}{2\sqrt{u}} \\
\frac{du}{dy} =& (x^2 + y^2)' \\
\Rightarrow \frac{du}{dy} =& 2y \\
\frac{df(x,y)}{dy} =& \frac{1}{2\sqrt{u}} \cdot 2y \\
=& \frac{1}{2\sqrt{x^2 + y^2}} \cdot 2y \\
\Rightarrow \frac{df(x,y)}{dy} =& \frac{y}{\sqrt{x^2 + y^2}}
\end{aligned}
$$

$$
\begin{aligned}
\frac{df(-4,3)}{dx} =& \frac{(-4)}{\sqrt{(-4)^2 + (3)^2}} \\
=& \frac{-4}{\sqrt{16 + 9}} \\
=& -\frac{4}{\sqrt{25}} \\
=& -\frac{4}{5} \\
\Rightarrow \frac{df(-4,3)}{dx} =& -\frac{4}{5}
\end{aligned}
$$

$$
\begin{aligned}
\frac{df(-4,3)}{dy} &= \frac{(3)}{\sqrt{(-4)^2+(3)^2}} \\
&= \frac{3}{\sqrt{16+9}} \\
&= \frac{3}{\sqrt{25}} \\
&= \frac{3}{5} \\
\Rightarrow \frac{df(-4,3)}{dy} &= \frac{3}{5}
\end{aligned}
$$

$$
\nabla f(x,y) = -\frac{4}{5}\vec{i} + \frac{3}{5}\vec{j}
$$

Exercício 2

2. $f(x,y,z) = xy^2z^2,\ p(2,-1,4),\ \vec{u} = \vec{i} + 2\vec{j} - 3\vec{k}$

$$
\begin{aligned}
\|\vec{u}\| &= \sqrt{(1)^2+(2)^2+(-3)^2} \\
&= \sqrt{1+4+9} \\
&= \sqrt{14} \\
\vec{v} &= \frac{\vec{u}}{\|\vec{u}\|} \\
\Rightarrow \vec{v} &= \frac{1}{\sqrt{14}}\vec{i} + \frac{2}{\sqrt{14}}\vec{j} - \frac{3}{\sqrt{14}}\vec{k}
\end{aligned}
$$

$$
\begin{aligned}
\frac{df(x,y,z)}{dx} &= y^2z^2 \\
\frac{df(2,-1,4)}{dx} &= (-1)^2(4)^2 \\
&= 1 \cdot 16 \\
\Rightarrow \frac{df(2,-1,4)}{dx} &= 16 \\
\frac{df(x,y,z)}{dy} &= 2xyz^2 \\
\frac{df(2,-1,4)}{dy} &= 2(2)(-1)(4)^2 \\
&= 4 \cdot -1 \cdot 16 \\
\Rightarrow \frac{df(2,-1,4)}{dy} &= -64 \\
\frac{df(x,y,z)}{dz} &= 2xy^2z \\
\frac{df(2,-1,4)}{dz} &= 2(2)(-1)^2(4) \\
&= 4 \cdot 1 \cdot 4 \\
\Rightarrow \frac{df(2,-1,4)}{dz} &= 16
\end{aligned}
$$

$$
\begin{aligned}
D_{\vec{v}}f(x,y,z) = \frac{df(x,y,z)}{dx} \cdot v_1 + \frac{df(x,y,z)}{dy} \cdot v_2 + \frac{df(x,y,z)}{dz} \cdot v_3 \\
D_{\vec{v}}f(2,-1,4) = \frac{df(2,-1,4)}{dx} \cdot v_1 + \frac{df(2,-1,4)}{dy} \cdot v_2 + \frac{df(2,-1,4)}{dz} \cdot v_3 \\
&= 16 \cdot \frac{1}{\sqrt{14}} - 64 \cdot \frac{2}{\sqrt{14}} - 16 \cdot \frac{3}{\sqrt{14}} \\
&= \frac{16}{\sqrt{14}} - \frac{128}{\sqrt{14}} - \frac{48}{\sqrt{14}} \\
\Rightarrow D_{\vec{v}}f(2,-1,4) &= \frac{160}{\sqrt{14}}
\end{aligned}
$$