Atividade 5

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Exercício 1

1. $f(x,y) = 2x^4y^3 - xy^2 + 3y + 1$

$$\begin{aligned}\frac{df(x,y)}{dx} =& 8x^3y^3 - y^2 \\ \frac{df(x,y)}{dy} =& 6x^4y^2 - 2xy + 3\end{aligned}$$

2. $f(x,y) = (x^3 - y^2)^5$

$$\begin{aligned}\Rightarrow f(x,y) =& \sum_{i=0}^{5} \binom{5}{i} (x^3)^{5-i} (y^2)^i \\ \Rightarrow f(x,y) =& \frac{5!}{0!(5-0)!}(x^3)^5(-y^2)^0 + \\ & \frac{5!}{1!(5-1)!}(x^3)^4(-y^2)^1 + \\ & \frac{5!}{2!(5-2)!}(x^3)^3(-y^2)^2 + \\ & \frac{5!}{3!(5-3)!}(x^3)^2(-y^2)^3 + \\ & \frac{5!}{4!(5-4)!}(x^3)^1(-y^2)^4 + \\ & \frac{5!}{5!(5-5)!}(x^3)^0(-y^2)^5 \\ \Rightarrow f(x,y) =& x^{15} - 5x^{12}y^2 + 10x^9y^4 - 10x^6y^6 + 5x^3y^8 - y^{10} \\ \frac{df(x,y)}{dx} =& 15x^{14} - 60x^{11}y^2 + 90x^8y^4 - 60x^5y^6 + 15x^2y^8 \\ \frac{df(x,y)}{dy} =& 10x^{12}y - 40x^9y^3 + 60x^6y^5 - 40x^3y^7 + 10y^9\end{aligned}$$

Exercício 3

$f(x,y) = ln\sqrt{x^2 + y^2}$

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\begin{aligned}
\frac{df(u)}{dx} =& \frac{df}{du} \cdot \frac{du}{dx} \\
f =& ln(u) \\
u =& \sqrt{x^2 + y^2} \\
\frac{df}{du} =& \frac{1}{u} \\
\frac{dg(v)}{dx} =& \frac{dg}{dv} \cdot \frac{dv}{dx} \\
g =& \sqrt{v} \\
v =& x^2 + y^2 \\
\frac{dg}{dv} =& \frac{1}{2\sqrt{v}} \\
\frac{dv}{dx} =& 2x \\
\frac{dg(v)}{dx} =& \frac{1}{2\sqrt{v}} \cdot 2x \\
\frac{du}{dx} =& \frac{x}{\sqrt{x^2 + y^2}} \\
\frac{df(u)}{dx} =& \frac{1}{u} \cdot \frac{x}{\sqrt{x^2 + y^2}} \\
\frac{df(x,y)}{dx} =& \frac{x}{x^2 + y^2} \\
\frac{d^2 f(x,y)}{dx^2} =& \frac{df(x,y)}{dx} (\frac{x}{x^2 + y^2}) \\
\frac{d\frac{f}{g}}{dx} =& \frac{\frac{df}{dx} \cdot g - \frac{dg}{dx} \cdot f}{g^2} \\
f =& x \\
\frac{df}{dx} =& 1 \\
g =& x^2 + y^2 \\
\frac{dg}{dx} =& 2x \\
\frac{d\frac{f}{g}}{dx} =& \frac{1(x^2 + y^2) - 2xx}{(x^2 + y^2)^2} \\
\frac{d^2 f(x,y)}{dx^2} =& \frac{-x^2 + y^2}{(x^2 + y^2)^2}
\end{aligned}
$$

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\begin{aligned}
\frac{df(u)}{dy} =& \frac{df}{du} \cdot \frac{du}{dy} \\
f =& ln(u) \\
u =& \sqrt{x^2 + y^2} \\
\frac{df}{du} =& \frac{1}{u} \\
\frac{dg(v)}{dy} =& \frac{dg}{dv} \cdot \frac{dv}{dy} \\
g =& \sqrt{v} \\
v =& x^2 + y^2 \\
\frac{dg}{dv} =& \frac{1}{2\sqrt{v}} \\
\frac{dv}{dy} =& 2y \\
\frac{dg(v)}{dx} =& \frac{1}{2\sqrt{v}} \cdot 2y \\
\frac{du}{dy} =& \frac{y}{\sqrt{x^2 + y^2}} \\
\frac{df(u)}{dy} =& \frac{1}{u} \cdot \frac{y}{\sqrt{x^2 + y^2}} \\
\frac{df(x,y)}{dy} =& \frac{y}{x^2 + y^2} \\
\frac{d^2 f(x,y)}{dy^2} =& \frac{df(x,y)}{dy}(\frac{y}{x^2 + y^2}) \\
\frac{d\frac{f}{g}}{dy} =& \frac{\frac{df}{dy} \cdot g - \frac{dg}{dy} \cdot f}{g^2} \\
f =& y \\
\frac{df}{dy} =& 1 \\
g =& x^2 + y^2 \\
\frac{dg}{dy} =& 2y \\
\frac{d\frac{f}{g}}{dy} =& \frac{1(x^2 + y^2) - 2yy}{(x^2 + y^2)^2} \\
\frac{d^2 f(x,y)}{dy^2} =& \frac{x^2 - y^2}{(x^2 + y^2)^2}
\end{aligned}
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\begin{aligned}
\frac{d^2 f(x,y)}{dx^2} + \frac{d^2 f(x,y)}{dy^2} =& 0 \\
\frac{-x^2 + y^2}{(x^2+y^2)^2} + \frac{x^2 - y^2}{(x^2+y^2)^2} =& 0
\end{aligned}
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