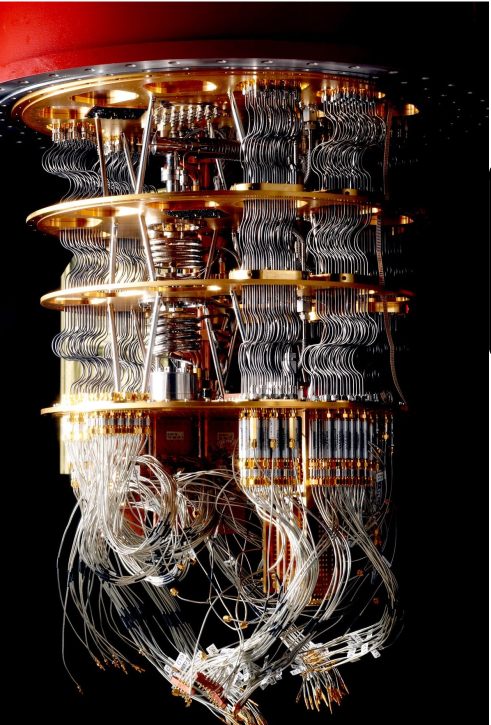

Quantum in Computational Fluid Dynamics

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Overview

Computational Fluid Dynamics

- Fluid mechanics deals with the motion of fluids (liquids and gases), induced by external forces.
- Fluid flow is modeled by partial differential equations (PDE), describing the conservation of mass, momentum, and energy
- Computational Fluid Dynamics (CFD) is the discipline of discretizing these PDE and solving them using computers.

Navier–Stokes equations.

- The governing equations of fluid dynamics are the conservation laws of mass, momentum, and energy.
- In CFD, this set of conservation laws are called the Navier–Stokes equations.

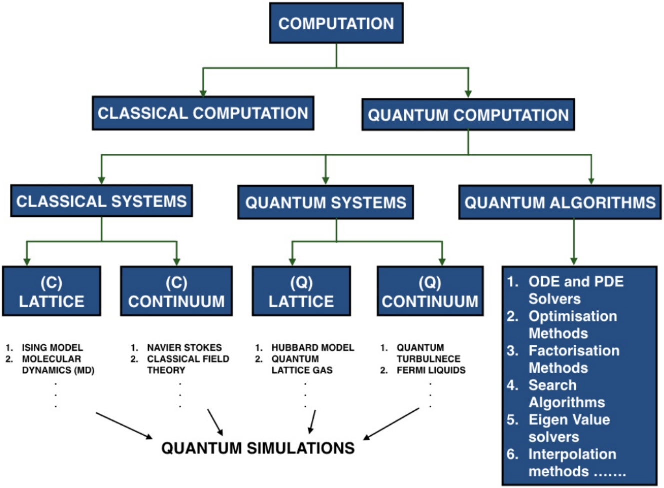
Incompressible Navier Stokes Equation

$$\nabla \cdot u = 0$$

$$\frac{\partial \rho}{\partial t} + (u.\nabla)u = \frac{-1}{\rho}\nabla p + \nu \nabla^2 u + f$$

where,$\nu = \frac{\mu}{\rho}$is the fluid kinematic viscosity.

Quantum Simulation



Classification of problems based on QC methods

Methods of Solving

Vortex in Cell

Vortex-in-cell method was used to solve Incompressible-Navier Stokes Equation in regular domain. This method is a well-studied hybrid particle-mesh method for incompressible flows and is particularly well suited for flows in regular domains such that efficient Poisson solvers can be used. For this, the fourier analysis approach(QFT) to solving the problem in fully periodic domain is used.

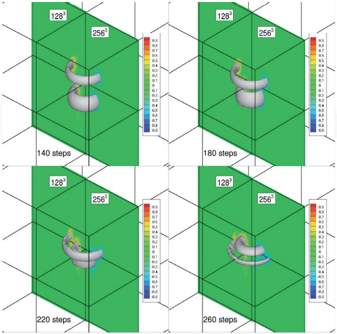
Discreet Velocity Method

The discrete velocity method (DVM) has been widely employed to solve the gas kinetic equations.It is the main deterministic method applied to solve the Boltzmann equation (BE) and its models. Its applications to the model equations is easier than to the BE.

Quantum Implementation of Solvers

Vortex-in-cell

1. The VIC method was implemented using QFT and Approximate QFT.
2. It was shown that despite the inevitable errors introduced by applying AQFT, the method produced meaningful results for three-dimensional example problems.

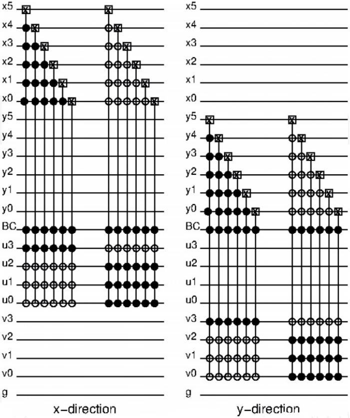


Vortex-in-cell simulation of leapfrogging vortex rings. For 1283 mesh, AQFT ("k" limit 5) is used. For 2563 mesh, "k" limit in AQFT is 6.

Quantum Implementation of Solvers

Discreet Velocity Method

1. For implementation of the method, the collisionless Boltzmann equation was considered, which was reduced to two kinetic equations for two reduced distribution functions.
2. 12-qubit register is used for a discretized function with 6 qubits defining the indices of 64 grid points in x- and y-coordinate directions.

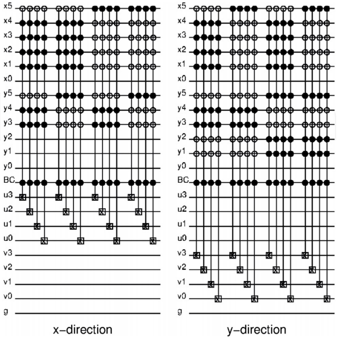


Quantum circuit implementation of streaming operations for discrete-velocity method (with 16 × 16 velocity mesh). Two-dimensional domain with 64 × 64 Cartesian mesh.

Quantum Implementation of Solvers

Specular Boundary Conditions

1. The specular-reflection boundary conditions gradually make the velocity vectors align with the solid wall such that there is no longer a flow into the solid body.
2. Control qubits were used to "select" cells in space for which to apply the boundary condition.
3. A control qubit involves the "BC" qubit representing the solid/fluid flag. X gate is applied to the qubits representing the velocity-mesh index to create the "change of sign" of the considered discrete-velocity data



Quantum circuit implementation of specular-reflection boundary conditions for rectangular body. 64 × 64 Cartesian mesh, 16 × 16 discrete-velocity mesh.

References

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- Steijl, Rene Barakos, George. (2018). Parallel Evaluation of Quantum Algorithms for Computational Fluid Dynamics. Computers Fluids. 173. 10.1016/j.compfluid.2018.03.080.
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