From Static to Dynamic: Implementation of Long-range Entanglement GHZ States for Dynamic Circuit-Based Quantum Teleportation

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Github Link

Quantum teleportation is the process of sending an arbitrary quantum state from one qubit to another without using any channel. This is feasible because Entangled pairs which exploit correlation. This may appear to violate the theory of relativity by allowing communication to occur faster than light, but this is not the case, as we will explore in more detail. However, as the number of qubits rises, so does the amount of entanglement. This increases the circuit depth, resulting in additional errors and increase in time complexity. In this blog, we propose dynamic circuits for the entanglement circuit and post-teleportation measurements at the destination qubit. This decreases error and keeps circuit depth constant. Prior to getting too technical, let's review how teleportation operates and why dynamic circuits are better than Unitary quantum circuits.

1 Implementation of Quantum Teleportation :

As previously mentioned, quantum teleportation transmits unknown quantum states from source to destination by employing entanglement states. Let say Alice has an arbitrary quantum state $|\psi\rangle$ and wants to send it to Bob. Now the protocol works such that Alice will transfer $|\psi\rangle$ from her qubit to bob's qubit by just measuring at Alice qubit and the shared entangled state qubits. However one needs to send classical outcome of measurement to bob so that he can apply unitary gates accordingly and measure to get the teleported state. In this way quantum teleportation still obeys theory of relativity and it is bound to the speed of light.

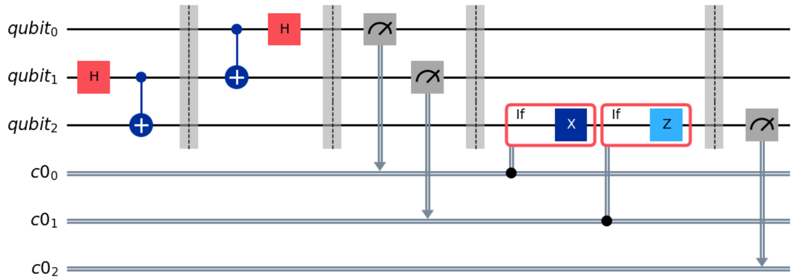

Figure 1.1: Single qubit teleportation circuit

Above is the actual teleportation circuit designed using IBM-Qiskit, where we teleport $|0\rangle$ from $qubit_0$ to $qubit_2$ by just measuring $qubit_0$ and $qubit_1$. After Alice performs measurement on her two qubits and based on the outcomes Bob apply's unitary gates on $qubit_2$, to retrieve back the teleported state.

2 Single Qubit Teleportation Using 3 Qubits

Let $|\phi\rangle$ be our unknown state to be teleported:

$$|\phi\rangle = a|0\rangle + b|1\rangle$$

The GHZ state is defined as:

$$|GHZ\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)$$

The combined state ψ is formed by tensor product:

$$|\psi\rangle = |\phi\rangle|GHZ\rangle$$

$$|\psi\rangle = a\frac{1}{\sqrt{2}}(|0000\rangle + |0111\rangle) + b\frac{1}{\sqrt{2}}(|1000\rangle + |1111\rangle)$$

By further simplification:

$$\begin{aligned}
|\psi\rangle =& \frac{a}{2}[|00\rangle(|\phi_+\rangle + |\phi_-\rangle)|11\rangle] \\
&+ \frac{b}{2}[|00\rangle(|\psi_+\rangle - |\psi_-\rangle)|11\rangle] \\
&+ \frac{a}{2}[|\phi_+\rangle|00\rangle|11\rangle] \\
&+ \frac{b}{2}[|\psi_+\rangle|00\rangle|11\rangle] \\
&+ \frac{a}{2}[|\phi_-\rangle|00\rangle|11\rangle] \\
&+ \frac{b}{2}[|\psi_-\rangle|00\rangle|11\rangle]
\end{aligned}$$

Let's focus on the first part alone:

$$|\psi\rangle = a|0\rangle(|+\rangle + |-\rangle)\frac{1}{\sqrt{2}} + b|1\rangle(|+\rangle - |-\rangle)\frac{1}{\sqrt{2}}$$

In the last equation, we have successfully teleported the qubit. And we have performed bell state measurement basis on bob's qubit.

3 Dynamic Circuits :

One of the biggest challenges that the current quantum computers of the NISQ era faces is a noisy environment. This can be overcome to some extent by the introduction of non-unitary dynamic circuits by employing mid-circuit measurement and feed-forward operations.
However we are taking on the major application of dynamic circuit which is efficient implementation of long range entanglement using dynamic circuit. We use this in our teleportation circuits with an increase in qubits. This helps us to reduce the circuit depth to constant.

4 Constant circuit depth :

In this section we show that with the help of a dynamic circuit we can reduce the circuit depth to constant whereas it will be O(n) when designing using unitary dynamics. Let's say we are having 120 qubit and we have attached our code here, which generates 120 qubits and CNOT is applied on 118 qubits linearly for entanglement. When we compute circuit depth it results as 119. Hence we conclude that when we have 'n' qubits we will be having circuit depth as 'n+1'. This proves that when we have more qubits we will be more prone to errors like bit flip and phase flip. Below plot shows that increase in qubits increase in circuit depth drastically. Now let's design the same using dynamic circuits, and we could see that the circuit depth results as '6', which is most significant when compared to the previous one. This is achieved by applying the CNOT gate dynamically, which helps in dynamically reducing the circuit depth. The plot below illustrates the comparative circuit depth for GHZ-state generation using both the unitary circuit and the dynamic circuit.

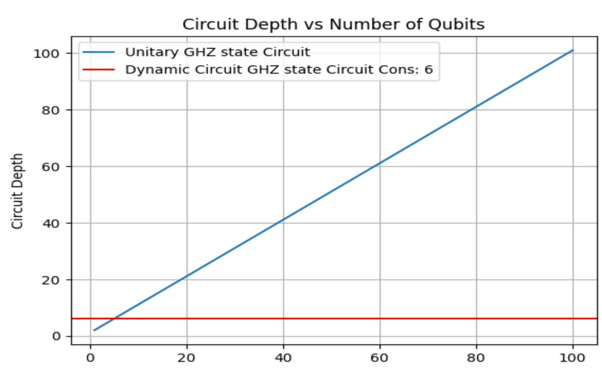

Figure 4.1: Depth comparison

5 Feed forward operation :

Feed forward operations are also a part of dynamic circuits. To understand it much better let us take our 2 qubit teleportation, thereafter teleportation protocol we will be getting classical values by measuring alice qubit. Now these classical values will be sent to bob and bob will apply unitary gates based on the measurement value and he performs measurement to get teleported qubit information. Please find the derivation below to get it completely.

Let $|\phi\rangle$ be our unknown state to be teleported and one of bell state.

$$|\phi\rangle = a|0\rangle + b|1\rangle$$

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}|00 + 11\rangle$$

$$|\phi\rangle = |\phi\phi^+\rangle$$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(a|0\rangle|00 + 11\rangle + b|1\rangle|00 + 11\rangle\right)$$

Now with reference to Figure 1.1 let's apply a CNOT gate with control as first qubit and second qubit as target.

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(a|0\rangle|00 + 11\rangle + b|1\rangle|10 + 01\rangle\right)$$

Then hadamard on the first qubit results as,

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(a|0 + 1\rangle|00 + 11\rangle + b|0 - 1\rangle|10 + 01\rangle\right)$$

We can rewrite the above $|\phi\rangle$ as,

$$|\phi\rangle = \frac{1}{\sqrt{2}}\left(|00\rangle a|0\rangle + b|1\rangle) + |01\rangle(a|1\rangle + b|0\rangle) + |10\rangle(a|0\rangle - b|1\rangle) + |11\rangle(a|1\rangle - b|0\rangle)\right)$$

Now we perform the measurement on the first qubit and with respect to the outcome we will apply respective gate's on the bob circuit. The protocol goes as below in table 1.1.
The above protocol can be used to teleport any single qubit teleportation, instead of $|0 + 1\rangle$ we can teleport $|0\rangle, |1\rangle$ or superposition too. And in this blog we are trying to generalize the protocol for long range entanglement-teleportation with the help of GHZ state with a dynamic circuit. Before diving into that, one must have an understanding of how to generate long range entanglement of the GHZ state with a dynamic circuit.

Measurement value	Gate to be applied on Bob's qubit
$\|00\rangle$	I
$\|10\rangle$	X
$\|01\rangle$	Z
$\|11\rangle$	XZ

Table 5.1: Measurement reference table

6 Creating long range entangled state - GHZ state :

As we mentioned in the previous section dynamic circuits can be used to implement efficient long range entanglement circuits. Here we are trying to generate 6 qubit entanglement using a dynamic circuit then we use that for teleportation. Please find below the circuit and code here.

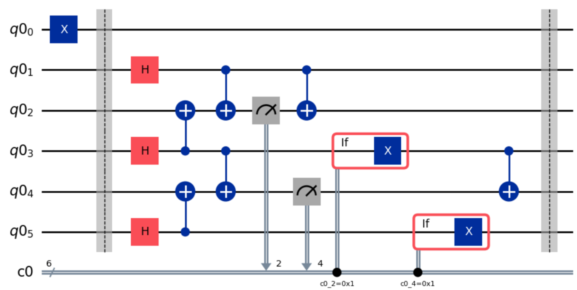

Figure 6.1: Qubit GHZ state using dynamic circuit

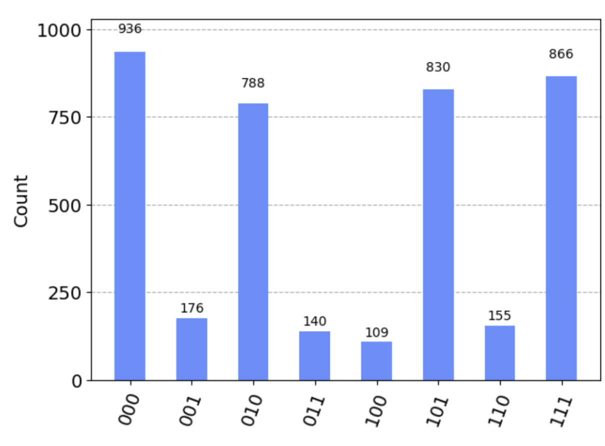

Figure 6.2: Probability distribution obtained from hardware execution

7 Teleportation with 6 qubit :

After generating a 6 qubit GHZ state using a dynamic circuit, now our model is ready to teleport single qubit from one qubit to another. This is possible by applying a series of hadamard and CNOT gate as shown in below circuit.

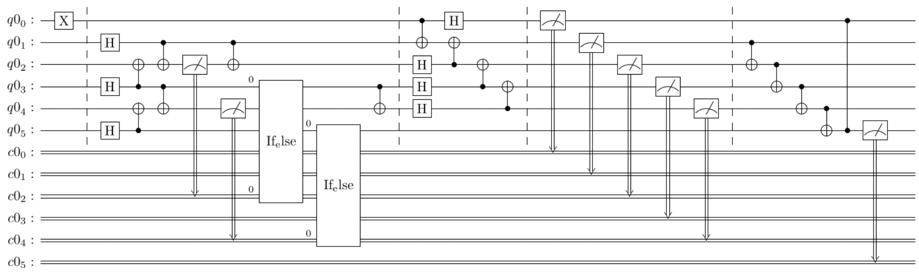

Figure 7.1: Teleportation using 6-qubit

After teleportation we apply a CNOT gate as shown in figure 6.1 to get the teleported state. We have executed the circuit in the real hardware and got the probability of teleportation. In figure 6.2 the probability distribution is un uniform which is due to noise experienced by real hardware.

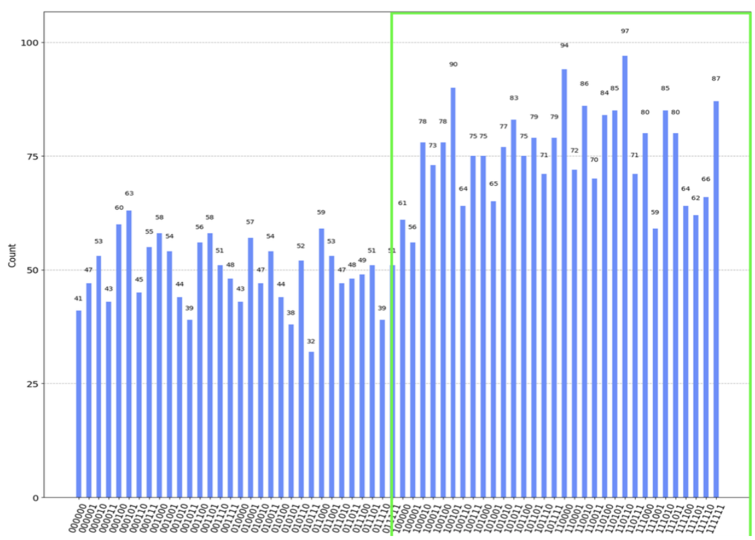

Figure 7.2: Probability distribution obtained from hardware execution

8 Future work :

As we can see that we have implemented feed forward operation in 3 qubit circuit but it can also be implemented on 6 qubit circuit. If we use feed forward operation or dynamic circuit post teleportation circuit the error rate would be reduced significantly. Also we would like to increase the qubit for teleportation and work towards multi qubit teleportation protocols.

9 Conclusion :

We have implemented teleportation in a 6 qubit circuit which has a long range entangled GHZ state, which is generated with a dynamic circuit. And we have successfully teleported a single qubit. With the help of a dynamic circuit we are able to reduce the circuit depth and reduce error rate as well. Thus we conclude that dynamic circuits play a major role in teleportation with an increase in the number of qubits.