Chapter-6 Rectangular Wave Guides

A transmission line can be used to guide EM energy from one point (generator) to another (load). A waveguide is another means of achieving the same goal. However, a waveguide differs from a transmission lines in some respects, although we may regard the waveguide as special case of a transmission line. A transmission line can support only transverse wave (EM wave), whereas a waveguide can support many possible field configurations. Second, at microwave frequencies (roughly 3 – 300 GHz), transmission lines become inefficient due to skin effect and dielectric losses; waveguides are used at that range of freq^n. to obtain larger bandwidth and lower signal attenuation. Moreover, a transmission line may operate from dc ($f \simeq 0$) to a very high freq^n; a waveguide can operate only above a certain freq^n called the cutoff frequency and therefore acts as a high pass filter. Thus, waveguides cannot transmit dc, and they become excessively large at frequencies below microwave frequencies.

Although, a waveguide may assume any arbitrary but uniform cross section, common waveguides either rectangular or circular. We will consider only rectangular waveguides in this chapter. By assuming lossless waveguides ($\sigma_c \simeq \infty, \sigma \simeq 0$), we shall apply Maxwell's equations with the appropriate boundary conditions to obtain different modes of wave propagation and the corresponding $\vec{E}$ and $\vec{H}$ fields.

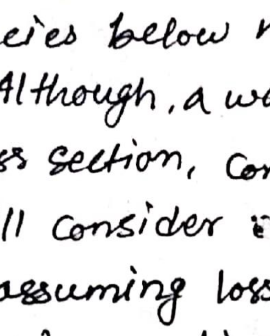


fig:- circular waveguide

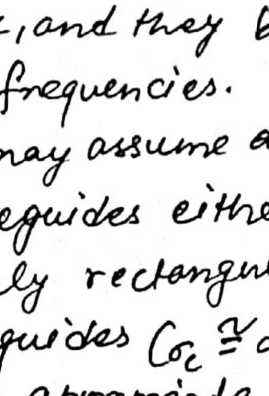
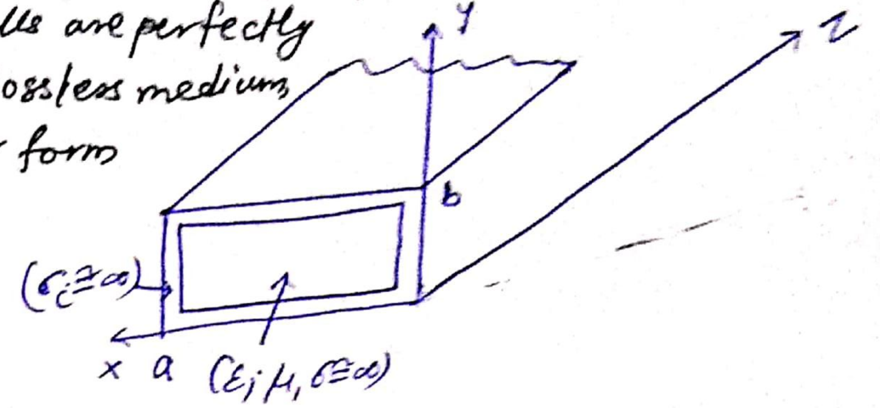


fig:- Rectangular waveguide

Rectangular Waveguides

Consider a rectangular waveguide shown in figure. Let us assume that the waveguide is filled with a source free ($\rho_v = 0$, $\vec{J} = 0$) lossless dielectric material ($\sigma \cong 0$) and its walls are perfectly conducting ($\sigma_c \cong \infty$). So, for a lossless medium Maxwell's equations in phasor form become



$$\nabla^2 \vec{E_s} + k^2 \vec{E_s} = 0$$

$$\nabla^2 \vec{H_s} + k^2 \vec{H_s} = 0$$

where, $k = \omega\sqrt{\mu\varepsilon}$

and the time factor $e^{j\omega t}$ is assumed. If we let $\vec{E_s} = (E_{xs}, E_{ys}, E_{zs})$ and $\vec{H_s} = (H_{xs}, H_{ys}, H_{zs})$ then above equations comprise three scalar Helmholtz equations. In other words, to obtain $\vec{E}$ & $\vec{H}$ fields, six equations need to solve. For example; for z-component

$$\frac{\partial^2 E_{zs}}{\partial x^2} + \frac{\partial^2 E_{zs}}{\partial y^2} + \frac{\partial^2 E_{zs}}{\partial z^2} + k^2 E_{zs} = 0 \quad \text{——} \quad (1)$$

which is a partial differential equation. The equation can be solved using separation of variables (product solution). So we let

$$E_{zs}(x, y, z, t) = X(x)\, Y(y)\, Z(z) \quad \text{——} \quad (2)$$

where $X(x)$, $Y(y)$ & $Z(z)$ are functions of x, y & z, respectively. Substituting (2) in (1) and dividing by XYZ gives.

$$\frac{X''}{X} + \frac{Y''}{Y} + \frac{Z''}{Z} = -k^2 \quad \text{——} \quad (3)$$

Since, the variables are independent, each term in (3) must be constant, so we can write

$$-k_x^2 - k_y^2 + \gamma^2 = -k^2$$

where, $-k_x^2$, $-k_y^2$ and γ^2 are separation constants.

Thus, eqⁿ ③ is separated as:

$$X'' + k_x^2 X = 0 \quad — ④$$

$$Y'' + k_y^2 Y = 0 \quad — ⑤$$

$$Z'' + k_z^2 Z = 0 \quad — ⑥$$

So, we obtain the solution of these equations as:

$$X(x) = C_1 \cos k_x x + C_2 \sin k_x x$$

$$Y(y) = C_3 \cos k_y y + C_4 \sin k_y y$$

$$Z(z) = C_5 e^{\gamma z} + C_6 e^{-\gamma z}$$

Substituting these values in ②, we get

$$E_{zs}(x,y,z) = (C_1 \cos k_x x + C_2 \sin k_x x)(C_3 \cos k_y y + C_4 \sin k_y y).(C_5 e^{\gamma z} + C_6 e^{-\gamma z})$$

If we assume the propagation along +ve z-direction, the value of C_5 will be zero ~~because~~ because the wave has to be finite at infinity. [i.e $E_{zs}(x,y,z=\infty) = 0$]. Hence,

$$E_{zs}(x,y,z) = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y) e^{-\gamma z}$$

where, $A_1 = C_6 C_1$, $A_2 = C_2 C_6$, $A_3 = C_3 C_6$, $A_4 = C_4 C_6$.

So, using similar steps. we get

$$H_{zs}(x,y,z) = (B_1 \cos k_x x + B_2 \sin k_x x)(B_3 \cos k_y y + B_4 \sin k_y y) \cdot e^{-\gamma z}$$

Instead of solving for other components $E_{xs}, E_{ys}, H_{xs}, H_{ys}$ in Helmholtz equations, in same manner, we simply use the Maxwell's equations to determine them from E_{zs} and H_{zs}. From

$$\nabla \times \vec{E_s} = -j\omega\mu \vec{H_s} \quad \text{and} \quad \nabla \times \vec{H_s} = j\omega\epsilon \vec{E_s}$$

we obtain,

$$\frac{\partial E_{zs}}{\partial y} - \frac{\partial E_{ys}}{\partial z} = -j\omega\mu H_{xs}$$

$$\frac{\partial H_{zs}}{\partial y} - \frac{\partial H_{ys}}{\partial z} = j\omega\varepsilon E_{xs}$$

$$\frac{\partial E_{xs}}{\partial z} - \frac{\partial E_{zs}}{\partial x} = j\omega\mu H_{ys}$$

$$\frac{\partial H_{xs}}{\partial z} - \frac{\partial H_{zs}}{\partial x} = j\omega\varepsilon E_{ys}$$

$$\frac{\partial E_{ys}}{\partial x} - \frac{\partial E_{xs}}{\partial y} = -j\omega\mu H_{zs}$$

$$\frac{\partial H_{ys}}{\partial x} - \frac{\partial H_{xs}}{\partial y} = j\omega\varepsilon E_{zs}$$

Now, expressing E_{xs}, E_{ys}, H_{xs} & H_{ys} interms of E_{zs} & H_{zs} we obtain,

$$j\omega\varepsilon E_{xs} = \frac{\partial H_{zs}}{\partial y} + \frac{1}{j\omega\mu}\left(\frac{\partial^2 E_{xs}}{\partial z^2} - \frac{\partial^2 E_{zs}}{\partial x \partial z}\right)$$

Since, all field components vary with z-according to $e^{-\gamma z}$

i.e $E_{zs} \sim e^{-\gamma z}$, $E_{xs} \sim e^{-\gamma z}$

Hence, $\frac{\partial E_{zs}}{\partial z} = -\gamma E_{zs}$, $\frac{\partial^2 E_{xs}}{\partial z^2} = \gamma^2 E_{xs}$

and

$$j\omega\varepsilon E_{xs} = \frac{\partial H_{zs}}{\partial y} + \frac{1}{j\omega\mu}\left(\gamma^2 E_{xs} + \gamma \frac{\partial E_{zs}}{\partial x}\right)$$

or, $$-\frac{1}{j\omega\mu}(\gamma^2 + \omega^2\mu\varepsilon) E_{xs} = \frac{\gamma}{j\omega\mu}\frac{\partial E_{zs}}{\partial x} + \frac{\partial H_{zs}}{\partial y}$$

Thus, if we let $h^2 = \gamma^2 + \omega^2\mu\varepsilon = \gamma^2 + k^2$

$$E_{xs} = -\frac{\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial y}$$

Similarly, we obtain,

$$\boxed{\begin{aligned} E_{xs} &= \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial x} - \frac{j\omega\mu}{h^2}\cdot\frac{\partial H_{zs}}{\partial y} \quad &(7)\\ E_{ys} &= \frac{-\gamma}{h^2}\frac{\partial E_{zs}}{\partial y} - \frac{j\omega\mu}{h^2}\frac{\partial H_{zs}}{\partial x} \quad &(8)\\ H_{xs} &= \frac{j\omega\varepsilon}{h^2}\cdot\frac{\partial E_{zs}}{\partial y} - \frac{\gamma}{h^2}\cdot\frac{\partial H_{zs}}{\partial x} \quad &(9)\\ H_{ys} &= \frac{-j\omega\varepsilon}{h^2}\cdot\frac{\partial E_{zs}}{\partial x} - \frac{\gamma}{h^2}\cdot\frac{\partial H_{zs}}{\partial y}. \quad &(10)\end{aligned}}$$

where, $h^2 = \gamma^2 + k^2 = k_x^2 + k_y^2$

From above equations it is seen that there are different types of field patterns or configurations. Each of these distinct field patterns is called a mode. Four different mode categories can exist, namely:

① $E_{zs} = 0 = H_{zs}$ (TEM mode): This is Transverse electromagnetic (TEM) mode, in which both $\vec{E}$ & $\vec{H}$ fields are transverse to the direction of wave propagation. From (7), (8), (9) & (10) all field components vanishes for $E_{zs} = 0 = H_{zs}$. Hence, rectangular waveguide cannot support TEM wave.

② $E_{zs} = 0$, $H_{zs} \neq 0$ (TE modes): For this case, the remaining components (E_{xs} and E_{ys}) of the electric field are transverse to the dirn of propagation $\hat{a}_z$. Under this condition fields are said to be in transverse electric (TE) modes.

③ $E_{zs} \neq 0$, $H_{zs} = 0$ (TM modes): In this case, the $\vec{H}$ field is transverse to the dirn of wave propagation. Thus we have transverse magnetic (TM) modes.

④ $E_{zs} \neq 0$, $H_{zs} \neq 0$ (HE modes): This is the case when neither $\vec{E}$ nor $\vec{H}$ field is transverse to the dirn of wave propagation. They are sometimes referred to as hybrid modes.

Transverse Magnetic (TM) Modes

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For this case, the magnetic field has its components transverse (normal to the dirⁿ of propagation of wave. This implies that we set $H_z = 0$ and determine E_x, E_y, E_z, H_x, and H_y using the boundary conditions. At the walls of waveguide, the tangential components of the $\vec{E}$ field must be continuous; i.e

$$E_{zs} = 0 \text{ at } y = 0, \quad E_{zs} = 0 \text{ at } y = b, \quad E_{zs} = 0 \text{ at } x = 0, \quad E_{zs} = 0 \text{ at } x = a.$$

So, from for these equations, $A_1 = 0 = A_3$ must be satisfied in

$$E_{zs}(x,y,z) = (A_1 \cos k_x x + A_2 \sin k_x x)(A_3 \cos k_y y + A_4 \sin k_y y) e^{-\gamma z}$$

So, it becomes.

$$E_{zs} = E_0 \sin k_x x \sin k_y y \, e^{-\gamma z} \quad \text{where, } E_0 = A_2 A_4$$

using $E_{zs} = 0$ at $y = b$ & $x = a$ we get

$$\sin k_x a = 0 \quad \& \quad \sin k_y b = 0$$

This implies that

$$k_x a = m\pi, \quad m = 1, 2, 3, \ldots$$

$$k_y b = n\pi, \quad n = 1, 2, 3, \ldots$$

or, $$\boxed{k_x = \frac{m\pi}{a} \quad \& \quad k_y = \frac{n\pi}{b}}$$

The −ve integers are not chosen for m & n.

Substituting these values into $E_{zs} = E_0 \sin k_x x \sin k_y y \, e^{-\gamma z}$

$$\boxed{E_{zs} = E_0 \sin\left(\frac{m\pi x}{a}\right) \cdot \sin\left(\frac{n\pi y}{b}\right) \cdot e^{-\gamma z}} \quad \text{——— (a)}$$

So, we can obtain other field components setting $H_{zs} = 0$.

Thus,
$$\left.\begin{aligned} E_{xs} &= \frac{-\gamma}{h^2}\left(\frac{m\pi}{a}\right) E_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \\ E_{ys} &= \frac{-\gamma}{h^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi x}{a}\right) \cdot \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \\ H_{xs} &= \frac{j\omega\varepsilon}{h^2}\left(\frac{n\pi}{b}\right) E_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z} \end{aligned}\right\} \text{(b)}$$

$$H_{ys} = \frac{-j\omega\varepsilon}{h^2}\left(\frac{m\pi}{a}\right) \cancel{E_o}\, E_o \cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

where,

$$\boxed{h^2 = k_x^2 + k_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$$

So, each pair of m & n gives a different field pattern or mode, referred to as TM_{mn} mode, in the waveguide. Integer m equals the number of half cycle variations in x-dirⁿ & integer n is the number of half cycle variations in y-dirⁿ. From ⓐ & ⓑ that if (m,n) is (0,0), (0,n) or (m,0), all field components vanish. Thus neither m nor n can be zero. Consequently, TM_{11} is the lowest order mode of all the TM_{mn} modes.

From previous equations we obtain propagation constant

$$\gamma = \sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 - k^2}$$

where, $k = \omega\sqrt{\mu\varepsilon}$ and $\gamma = \alpha + j\beta$.

So, here, we have three possibilities depending on k (or ω), m and n.

<u>CASE-A (cutoff)</u>

If $k^2 = \omega^2\mu\varepsilon = \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$

$\gamma = 0$ or $\alpha = 0 = \beta$.

The value of ω that causes that is called the cutoff angular frequency, ω_c; i.e. $\boxed{\omega_c = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2}}$

<u>CASE-B (Evanescent)</u>

If $k^2 = \omega^2\mu\varepsilon < \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$

$\gamma = \alpha,\ \beta = 0$

In this case, we have no wave propagation at all. These non propagating or attenuating modes are said to be evanescent.

CASE-C (propagation)

If $k^2 = \omega^2 \mu \varepsilon > \left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2$

$\gamma = j\beta, \quad \alpha = 0$

i.e $\beta = \sqrt{k^2 - \left[\frac{m\pi}{a}\right]^2 - \left[\frac{n\pi}{b}\right]^2}$ = phase constant

This is the only case when propagation takes place because all field components will have the factor $e^{-\gamma z} = e^{-j\beta z}$. Thus for each mode, characterized by a set of integers m & n, there is a corresponding cutoff frequency f_c.

The cutoff freq^n is the operating freq^n below which attenuation occurs and above which propagation takes place.

The waveguides therefore operates as highpass filter. The cutoff freq^n is obtained as:

$$\boxed{f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \sqrt{\left[\frac{m\pi}{a}\right]^2 + \left[\frac{n\pi}{b}\right]^2}}$$

or $\boxed{f_c = \frac{u'}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}$; where $u' = \frac{1}{\sqrt{\mu\varepsilon}}$ = phase velocity of uniform plane wave in the lossless dielectric medium ($\sigma = 0, \mu, \varepsilon$) filling the waveguide

The cutoff wavelength, λ_c is given by

$$\boxed{\lambda_c = \frac{u'}{f_c}}$$

or, $\boxed{\lambda_c = \frac{2}{\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}}}$

TM_{11} has the lowest cutoff freq^n (or the longest cutoff wavelength) of all the TM modes. The phase constant β can be written in terms of f_c as:

$$\beta = \omega\sqrt{\mu\varepsilon}\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \quad \text{or,} \quad \beta = \beta'\sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

where, $\beta' = \frac{\omega}{u'} = \omega\sqrt{\mu\varepsilon}$ phase constant in dielectric.

γ for evanescent mode is

$$\gamma = \alpha = \beta' \sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$$

Phase velocity and the wavelength in guide are

$$u_p = \frac{\omega}{\beta}, \quad \lambda = \frac{2\pi}{\beta} = \frac{u_p}{f}$$

The intrinsic impedance of the mode is

$$\eta_{TM} = \frac{E_x}{H_y} = -\frac{E_y}{H_x}$$

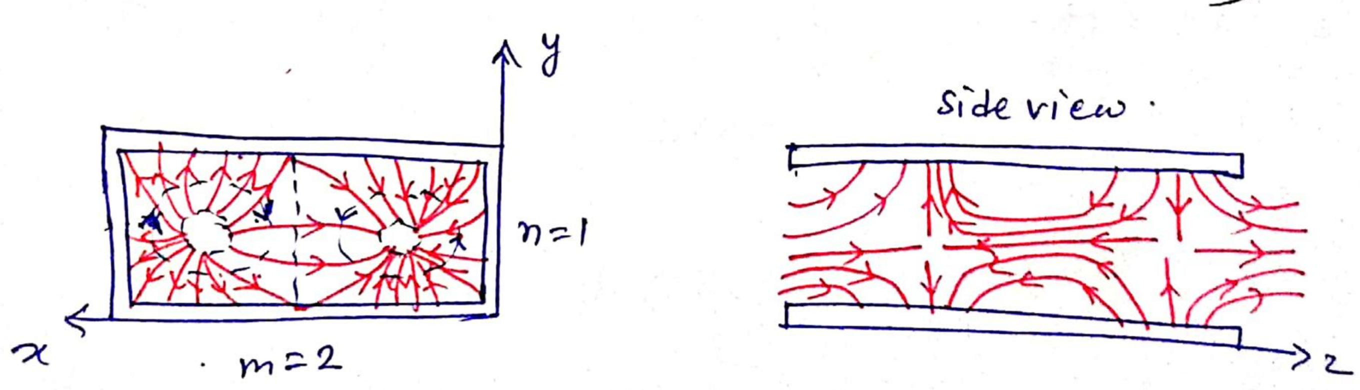
$$= \frac{\beta}{\omega\varepsilon} = \sqrt{\frac{\mu}{\varepsilon}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

or,

$$\eta_{TM} = \eta' \sqrt{1 - \left(\frac{f_c}{f}\right)^2}$$

where, $\eta' = \sqrt{\mu/\varepsilon}$ = intrinsic impedance of uniform plane wave in the medium.

<u>Example</u> : TM_{21} mode (field configuration for fixed time)



- - - - - H field

——— E field

fig:- field configuration for TM_{21} mode.

Transverse Electric (TE) Modes

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In the TE modes, the electric field is transverse (or normal) to the direction of wave propagation. We set $E_z = 0$ and determine other field components, E_{xs}, E_y, H_x, H_y, and H_z from previous eqⁿs. and boundary conditions. The tangential components of the electric field must be continuous at the walls of the waveguide; i.e

$$E_{xs} = 0 \text{ at } y = 0,\quad E_{xs} = 0 \text{ at } y = b,\quad E_{ys} = 0 \text{ at } x = 0,$$

$$E_{ys} = 0 \text{ at } x = a.$$

So, the boundary conditions can be written as

$$\frac{\partial H_{zs}}{\partial y} = 0 \text{ at } y = 0,\quad \frac{\partial H_{zs}}{\partial y} = 0 \text{ at } y = b$$

$$\frac{\partial H_{zs}}{\partial x} = 0 \text{ at } x = 0 \quad \frac{\partial H_{zs}}{\partial x} = 0 \text{ at } x = a.$$

Then, from previous equation.

$$H_{zs} = H_0 \cos\left(\frac{m\pi x}{b}\right) \cdot \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

where, $H_0 = B_1 B_3$. Other field components are:

$$E_{xs} = \frac{j\omega\mu}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$E_{ys} = -\frac{j\omega\mu}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_{xs} = \frac{\gamma}{h^2}\left(\frac{m\pi}{a}\right) H_0 \sin\left(\frac{m\pi x}{a}\right) \cdot \cos\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

$$H_{ys} = \frac{\gamma}{h^2}\left(\frac{n\pi}{b}\right) H_0 \cos\left(\frac{m\pi x}{a}\right) \cdot \sin\left(\frac{n\pi y}{b}\right) e^{-\gamma z}$$

where, $m = 0, 1, 2, 3, \ldots$ and $n = 0, 1, 2, 3, \ldots$; h & γ remains same as for TM modes. Again, m & n denote the number of half cycle variations in the x-y cross section of the guide.

For TE modes, (m, n) may be $(0,1)$ or $(1,0)$ but not $(0,0)$. Both m & n cannot be zero at the same time because this will force the field components to vanish.

This implies that the lowest mode can be TE_{10} or TE_{01}, dependi[ng] for the values of a & b, the dimensions of the guide. It is standa-rd practice to have $a > b$ so that $\frac{1}{a^2} < \frac{1}{b^2}$ in cutoff freq^n eq^n. Thus TE_{10} is the lowest mode because $f_{cTE_{10}} = \frac{u'}{2a} < f_{cTE_{01}} = \frac{u'}{2b}$

This mode is called dominant mode of the waveguide and is of practical importance. The cutoff freq^n for the TE_{10} mode is

$f_{c_{10}} = \frac{u'}{2a}$ and the cutoff wavelength for TE_{10} mode is $\lambda_{c_{10}} = 2a$.

Here, cut off freq^n of TM_{11} is $\frac{u'(a^2+b^2)^{1/2}}{2ab}$; which is greater than the cutoff freq^n for TE_{10}. Hence, TM_{11} cannot be regarded as the dominant mode.

The dominant mode is the mode with the lowest cutoff freq^n (or highest cut off wavelength).

Also, note that any EM wave with frequency $f < f_{c_{10}}$ (or $\lambda > \lambda_{c_{10}}$) will not be propagated in the guide.

The intrinsic impedance for TE mode is not the same as for TM modes.

$$\eta_{TE} = \frac{E_x}{H_y} = -\frac{E_y}{H_x} = \frac{\omega\mu}{\beta}$$

$$= \sqrt{\frac{\mu}{\varepsilon}} \cdot \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

Cutoff freq^n

$$\boxed{f_c = \frac{1}{2\pi\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}}$$

or, $$\eta_{TE} = \frac{\eta'}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$$

η_{TE} and η_{TM} are purely resistive and they vary with freq^n

Also, $\eta_{TE} \cdot \eta_{TM} = \eta'^2$

Example TE_{32}

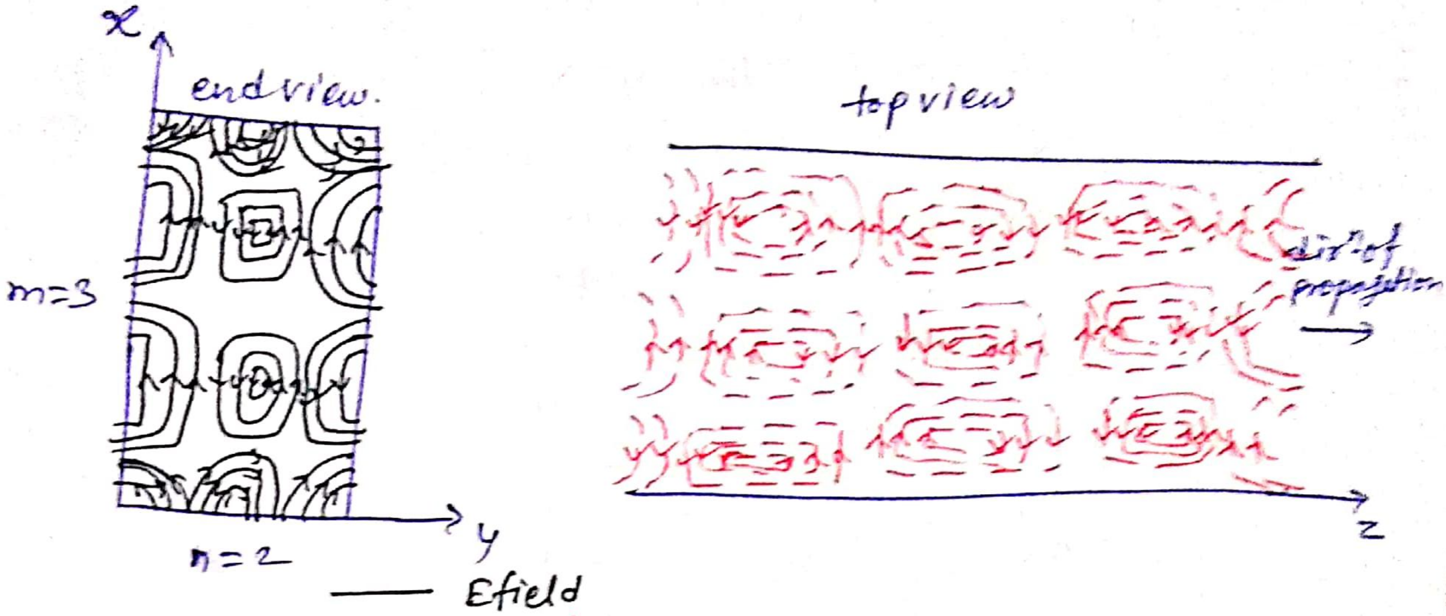

fig: Field Configuration for TE_{32} mode.

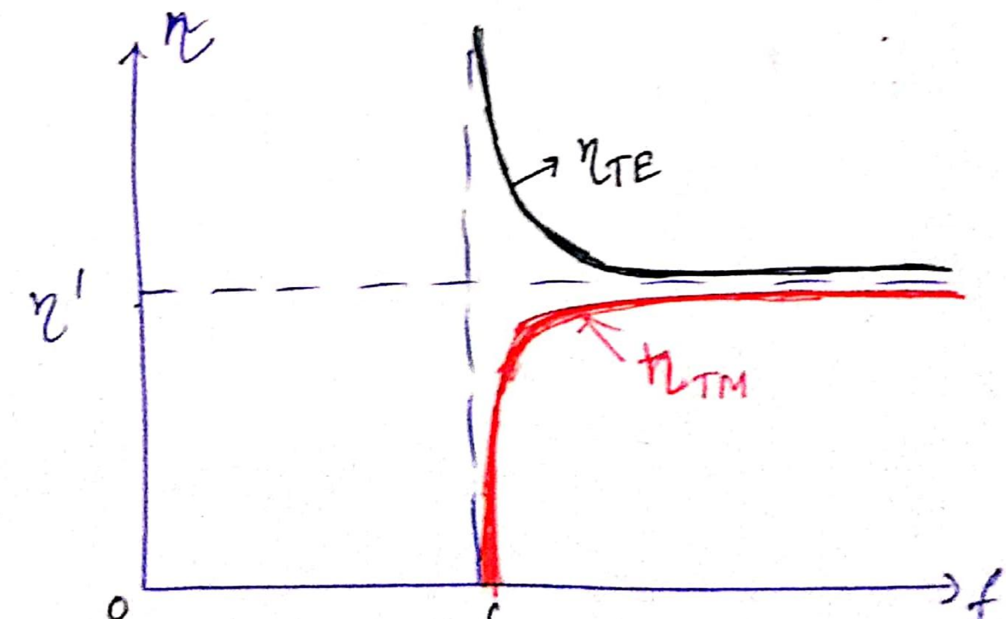

fig:- Variation of wave impedance with freq^n for TE & TM modes.

Possible values of m, n & modes of TE, TM

m	n	TE	TM
0	0	X	X
0	1	TE_{01}	X
1	0	TE_{10} (dominant)	X
1	1	TE_{11}	TM_{11} ⇒ degenerate modes.
1	2	TE_{12}	TM_{12}
2	1	TE_{21}	TM_{21}
1	3	TE_{13}	TM_{13}
3	1	TE_{31}	TM_{31}
3	2	TE_{32}	TM_{32}
3	3	TE_{33}	TM_{33}
2	2	TE_{22}	TM_{22}
0	2	TE_{02}	X
⋮	⋮	⋮	⋮
m	n	TE_{mn}	TM_{mn}.

Q: Calculate the cut off frequencies of the first four propagating modes for an air filled Copper waveguide with dimensions $a = 2.5\,cm$, $b = 1.2\,cm$. [2073 Shrawan]

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Sol.ⁿ Given, An air filled Copper waveguide with dimensions

$a = 2.5\,cm$ & $b = 1.2\,cm$, $\mu = \mu_0$ & $\epsilon = \epsilon_0$

① The cutoff freq.ⁿ for TE_{10} is

$$f_{c_{10}} = \frac{1}{2\sqrt{\mu_0\epsilon_0}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{c}{2} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{3\times10^8}{2} \sqrt{\left(\frac{100}{2.5}\right)^2 + \left(\frac{0\times100}{1.2}\right)^2}$$

$$= 1.5\times10^8 \sqrt{(40)^2}$$

$$= 6\times10^9\,Hz$$

$$= 6\,GHz$$

② The cutoff freq.ⁿ for $TE_{01} \Rightarrow f_{c_{01}} = 1.5\times10^8 \sqrt{\left(\frac{100}{1.2}\right)^2}$

$= 12.5\,GHz$

③ The cutoff freq.ⁿ of $TE_{11} \Rightarrow f_{c_{11}} = 1.5\times10^8 \times \sqrt{\left(\frac{100}{2.5}\right)^2 + \left(\frac{100}{1.2}\right)^2}$

$= 13.865\,GHz$

④ The cut off freq.ⁿ of $TM_{11} \Rightarrow f_{c_{11}} = 13.865\,GHz$

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⑤ The cut off freq.ⁿ of $TE_{20} \Rightarrow f_{c_{TE_{20}}} = 1.5\times10^8 \sqrt{\left(\frac{200}{2.5}\right)^2}$

$= 12\,GHz$

Qn: A rectangular waveguide having cross-section of 2cm X 1cm is filled with a lossless medium characterized by $\varepsilon = 4\varepsilon_0$ and $\mu_r = 1$. Calculate the cutoff freq^n of dominant mode. [2074 Ashwin]

Sol^n:

$$f_{c(TE_{10})} = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$a = 2cm$
$b = 1cm$.
$\mu_r = 1$
$\varepsilon = 4\varepsilon_0$.

$$= \frac{1}{2\sqrt{4\times1}\cdot\sqrt{\mu_0\varepsilon_0}} \sqrt{\left(\frac{1}{2\times10^{-2}}\right)^2 + \left(\frac{0}{1\times10^{-2}}\right)^2}$$

$$= \frac{3\times10^8}{4} \times \frac{1}{2\times10^{-2}}$$

$$= \frac{3}{8} \times 10^{10}$$

$$= 3.75 \times 10^9 \text{ Hz}$$

$$= 3.75 \text{ GHz}$$

Qn: Consider a rectangular waveguide with $\varepsilon_r = 2.25$ and $\mu_r = 1$ with dimensions $a = 1.07$, $b = 0.43$. Find the cutoff freq^n for TM_{11} mode and dominant mode. [2073 Chaitra].

Sol^n:

$$f_c = \frac{1}{2\sqrt{\mu\varepsilon}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} \quad \text{for } TM_{11} \text{ \& } TE_{10}$$

Qn: A rectangular air filled waveguide has a cross section of 45x90 mm. Find the cutoff frequencies of the first four propagating modes. [2072 Chaitra]

Sol.^{n} A rectangular waveguide has a cross section of 45x90mm and filled with air then

$\varepsilon = \varepsilon_0$, $\mu = \mu_0$, $a = 45\,mm = 0.045\,m$ & $b = 90\,mm = 0.09\,m$

The first four propagating modes are, TE_{10}, TE_{01}, TE_{11} & TM_{11}

(2)

$$\therefore f_{c_{TE_{10}}} = \frac{1}{2\sqrt{\mu_0\varepsilon_0}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$= 1.5\times10^8\sqrt{\left(\frac{1000}{45}\right)^2}$$

$$= 3.33\ GHz$$

$$f_{c_{TE_{01}}} = 1.5\times10^8\sqrt{\left(\frac{1000}{90}\right)^2} = 1.66\ GHz$$

$$f_{c_{TE_{11}}} = 1.5\times10^8\sqrt{\left(\frac{1000}{45}\right)^2+\left(\frac{1000}{90}\right)^2} = 3.726\ GHz$$

$$f_{c\,TM_{11}} = 1.5\times10^8\sqrt{\left(\frac{1000}{45}\right)^2+\left(\frac{1000}{90}\right)^2} = 3.726\ GHz$$

Qn: Determine the cutoff freq^n for an air filled rectangular waveguide with a = 2.5 cm and b = 1.25 cm for TE_{11} mode. [2072 Kartik]

Sol.^{n} Given, rectangural waveguide is filled with air.

i.e $\mu = \mu_0$, $\varepsilon = \varepsilon_0$, $a = 2.5\,cm = 0.025\,m$ & $b = 1.25\,cm = 0.0125\,m$.

$$\therefore f_{c_{TE_{11}}} = \frac{1}{2\sqrt{\mu_0\varepsilon_0}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2}$$

$$= \frac{c}{2}\sqrt{\left(\frac{100}{2.5}\right)^2+\left(\frac{100}{1.25}\right)^2} = 13.416\ GHz$$

Q: Consider a rectangular waveguide with $\varepsilon_r = 2$, $\mu = \mu_0$ with dimensions $a = 1.07\,cm$, $b = 0.43\,cm$. Find the cut off freq^n for TM_{11} mode and the dominant mode. [2068 chaitra]

Sol.^{n} Given, $\varepsilon_r = 2$, $\mu = \mu_0$, $a = 1.07\,cm$, $b = 0.43\,cm$.

then, Cutoff freq^n for TM_{11} mode is

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$$f_{c_{TM_{11}}} = \frac{1}{2\sqrt{\mu_0\varepsilon_0\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{c}{2\times\sqrt{2}}\sqrt{\left(\frac{100}{1.07}\right)^2 + \left(\frac{100}{0.43}\right)^2}$$

$$= 1.0606\times10^8 \times 250.634$$

$$= 26.58\,GH$$

cut off freq^n for dominant mode (TE_{10}) is

$$f_{c_{TE_{10}}} = \frac{1}{2\sqrt{\mu_0\varepsilon_0\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = 1.0606\times10^8 \times \sqrt{\left(\frac{100}{1.07}\right)^2}$$

$$= 9.912\,GHZ.$$

Q: An air filled rectangular waveguide has dimensions $a = 2\,cm$ and $b = 1\,cm$. Determine the range of frequencies over which the guide will operate single mode TE_{10}.

Sol.^{n} Since waveguide is air filled

$$\mu = \mu_0,\ \varepsilon = \varepsilon_0,\ a = 2\,cm,\ b = 1\,cm$$

then,
$$f_{c_{TE_{10}}} = \frac{c}{2}\sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2} = 1.5\times10^8\sqrt{\left(\frac{100}{2}\right)^2 + \left(\frac{0\times100}{1}\right)^2}$$

$$= 7.5\,GHZ$$

The next higher mode will be either TE_{20} or TE_{01}

where, $f_{c_{TE_{20}}} = 15\,GHz$ & $f_{c_{TE_{01}}} = 15\,GHZ$

So, the operating range over which the guide will be single mode is $7.5\,GHZ < f < 15\,GHZ$.

Q. A rectangular waveguide has dimensions $a = 3.5\,cm$, $b = 2\,cm$ and is to be operated below 15 GHz. The medium in the waveguide is air. Determine (a) cutoff freq^n (b) Number of $TE_{m,n}$ and $TM_{m,n}$ modes that waveguide can support? [2068 Shrawan]

Sol.^{n} Given, $a = 3.5\,cm$, $b = 2\,cm$, the medium is air.

i.e $\mu = \mu_0$, $\varepsilon = \varepsilon_0$ and the highest freq^n i.e. to be operated is 15 GHz.

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Then, The modes will be

① TE_{01}, $f_{c\,TE_{01}} = \frac{1}{2\sqrt{\mu_0\varepsilon_0}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$

$= \frac{3\times10^8}{2} \sqrt{\left(\frac{0}{3.5}\right)^2 + \left(\frac{100}{2}\right)^2}$

$= 7.5\,GHz$

⑪ $TE_{02} = 1.5\times10^8 \sqrt{\left(\frac{200}{2}\right)^2}$

$= 15\,GHz$

② TE_{10}, $f_{c\,TE_{10}} = 1.5\times10^8 \sqrt{\left(\frac{100}{3.5}\right)^2} = 4.286\,GHz$

③ TE_{11}, $f_{c\,TE_{11}} = 1.5\times10^8 \sqrt{\left(\frac{100}{3.5}\right)^2 + \left(\frac{100}{2}\right)^2} = 8.638\,GHz$

④ TM_{11}, $f_{c\,TM_{11}} = 8.638\,GHz$

⑤ $TE_{2,0}$, $f_{c\,TE_{20}} = 1.5\times10^8 \sqrt{\left(\frac{2\times100}{3.5}\right)^2} = 8.57\,GHz$

⑥ $TE_{21} \Rightarrow f_{c\,TE_{21}} = 1.5\times10^8 \sqrt{\left(\frac{2\times100}{3.5}\right)^2 + \left(\frac{100}{2}\right)^2} = 11.389\,GHz$

⑦ $TM_{21} \Rightarrow f_{c\,TM_{21}} = 1.5\times10^8 \times \sqrt{\left(\frac{2\times100}{3.5}\right)^2 + \left(\frac{100}{2}\right)^2} = 11.389\,GHz$

⑧ $TE_{22} \Rightarrow f_{c\,TE_{22}} = 1.5\times10^8 \times \sqrt{\left(\frac{200}{3.5}\right)^2 + \left(\frac{200}{2}\right)^2} = 17.27\,GHz$

⑨ $TE_{12} \Rightarrow f_{c\,TE_{12}} = 1.5\times10^8 \times \sqrt{\left(\frac{100}{3.5}\right)^2 + \left(\frac{200}{2}\right)^2} = 15.6\,GHz$

⑩ $TM_{12} \Rightarrow f_{c\,TM_{12}} = 15.6\,GHz$.

The modes with cutoff freq^n's. below or equal to 15 GHz are transmitted so, the supported TE & TM numbers are:

TE ⇒ 5 numbers and TM ⇒ 2 numbers.

ans. n: A rectangular waveguide has dimensions $a = 4.5\,cm$, $b = 2.5\,cm$. The medium within waveguide has relative permitivity $\varepsilon_r = 1$, relative permeability $\mu_r = 1$, conductivity $\sigma = 0$ and conducting walls of waveguide are perfect conductors. Determine the cutoff freq^n for the modes $TE_{(1,0)}$ and $TM_{(1,1)}$. [2069 Ashad]

(5)

Sol.^{n} Given, $a = 4.5\,cm$, $b = 2.5\,cm$, $\mu_r = 1$, $\sigma = 0$, $\varepsilon_r = 1$

For a waveguide,

Then,

cutoff freq^n for $TE_{(1,0)}$ mode is

$$f_{c_{TE_{10}}} = \frac{1}{2\sqrt{\mu_0 \varepsilon_0}} \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

$$= \frac{c}{2} \sqrt{\left(\frac{100}{4.5}\right)^2 + \left(\frac{0 \times 100}{2.5}\right)^2}$$

$$= \frac{3 \times 10^8}{2} \times 22.22$$

$$= 3.33\ GHz$$

Also, cutoff freq^n for $TM_{(1,1)}$ mode is

$$f_{c_{TM_{11}}} = 1.5 \times 10^8 \sqrt{\left(\frac{100}{4.5}\right)^2 + \left(\frac{100}{2.5}\right)^2}$$

$$= 6.86\ GHz$$