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Chapter-5: Transmission Lines

In previous chapter, the wave propagation considered was in unbounded media, media of infinite extent. Such wave propagation is said to be unguided in that the uniform plane wave exists throughout all space and EM energy associated with the wave spreads over a wide area.

Another means of transmitting power or information is by guided structures. Guided structures serve to guide (or direct) the propagation of energy from the source to the load. Typical examples of such structures are transmission lines and wave-guides.

Transmission lines are used in power distribution (usually at low frequencies) and in communications (at high frequencies). Various types of transmission lines such as the twisted pair and coaxial cables (thinnet and thicknet) are used in computer networks such as Ethernet & internet.

A transmission line basically consists of two or more parallel conductors used to connect a source to a load. The source may be generator, transmitter or oscillator & the load may be the equipments, antenna or an oscilloscope respectively.

Transmission line problems are usually solved using EM field theory and electric circuit theory, the two major theories on which electrical engineering is based.

Transmission Line Parameters

It is convenient to describe a transmission line in terms of its line parameters, which are its resistance per unit length R, inductance per unit length L, conductance per unit length G, and capacitance per unit length C.

The following points should be noted:

① The line parameters R, L, G & C are not discrete or lumped but distributed, i.e the parameters are uniformly distributed along the entire length of the line.

② For each line, the conductors are characterized by σ_c, μ_c; $\varepsilon_c = \varepsilon_0$, and the homogeneous dielectric separating the conductors is characterized by σ, μ, ε.

③ $G \neq \frac{1}{R}$; R is the ac resistance per unit length of the conductors comprising the line and G is the conductance per unit length due to the dielectric medium separating the conductors.

④ The value of L is the external inductance per unit length; that is, $L = L_{ext}$. The effects of internal inductance $L_{in} \{= (R/\omega)\}$ are negligible as high frequencies at which most communication systems operate.

⑤ For each line,

$$LC = \mu\varepsilon \quad \text{and} \quad \frac{G}{C} = \frac{\sigma}{\varepsilon}$$

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Transmission Line Equations

A two conductor transmission line supports a TEM wave; that is the electric and magnetic fields on the line are transverse to the direction of wave propagation. An important property of TEM waves is that the fields $\vec{E}$ & $\vec{H}$ are uniquely related to voltage V and current I, respectively.

$$V = -\int \vec{E} \cdot \vec{dl} \quad \& \quad I = \oint_L \vec{H} \cdot \vec{dl}$$

So, the use of circuit quantities V & I in solving the transmission line problem instead of solving field quantities $\vec{E}$ & $\vec{H}$ will be simpler and more convenient.

Let us consider an incremental portion of length Δz of a two conductor transmission line. The portion of length Δz contains a resistance $R\Delta z$, an inductance $L\Delta z$, a conductance $G\Delta z$ and a capacitance $C\Delta z$ shown in figure, which is an equivalent circuit for the transmission line with R, L, G & C line parameters.

Let us assume that the wave propagates in +ve z-direction, from the source to the load.

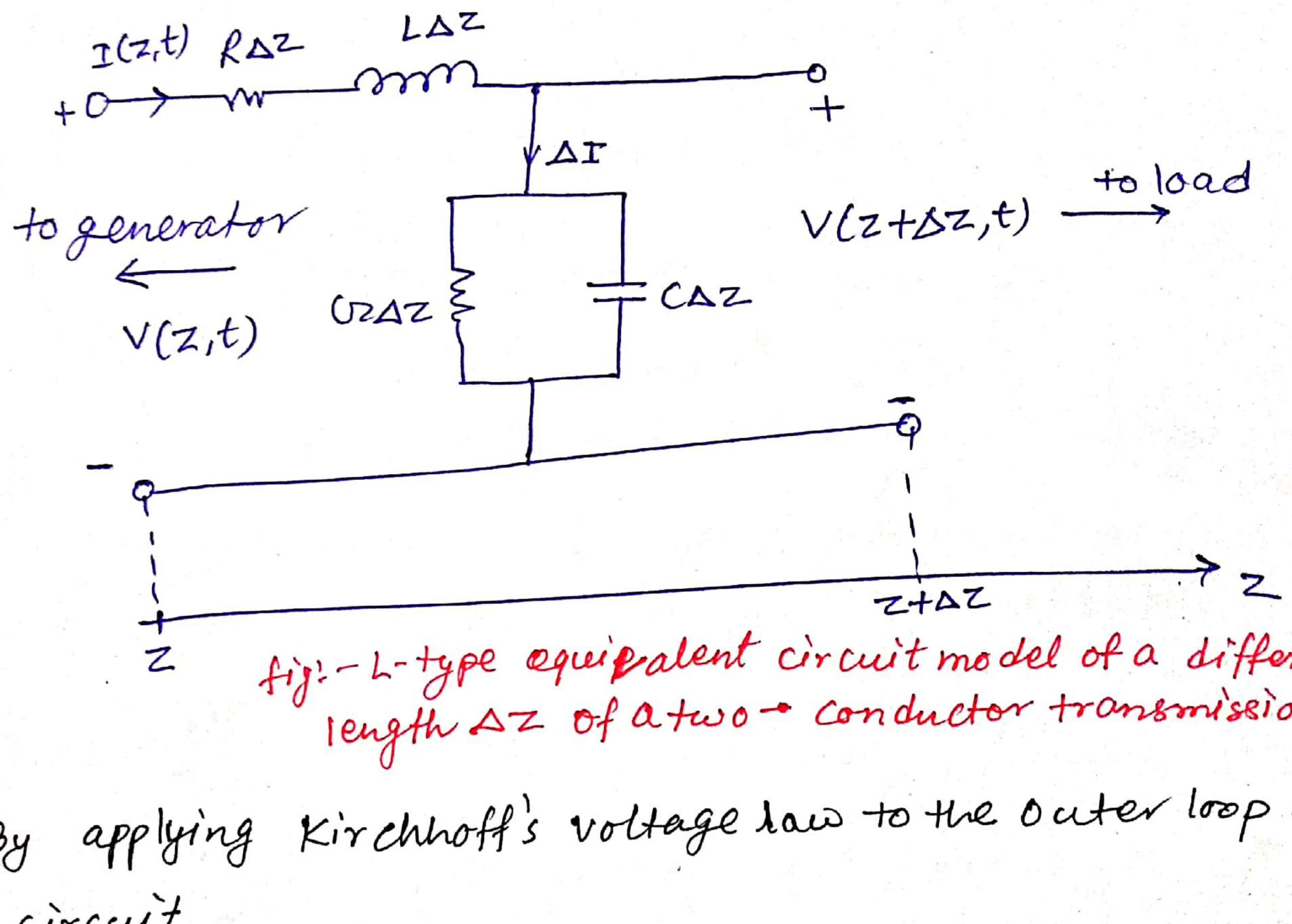


fig:- L-type equivalent circuit model of a differential length Δz of a two-conductor transmission line

By applying Kirchhoff's voltage law to the outer loop of the circuit

$$V(z,t) = R\Delta z\, I(z,t) + L\Delta z \frac{\partial I(z,t)}{\partial t} + V(z+\Delta z, t)$$

or, $$-\frac{V(z+\Delta z,t) - V(z,t)}{\Delta z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}$$

Taking limit as $\Delta z \to 0$ leads to

$$\boxed{-\frac{\partial V(z,t)}{\partial z} = RI(z,t) + L\frac{\partial I(z,t)}{\partial t}}$$

Similarly, applying Kirchhoff's current law to the main node of the circuit

$$I(z,t) = I(z+\Delta z, t) + \Delta I$$

$$= I(z+\Delta z, t) + G\,\Delta z\, V(z+\Delta z, t) + C\,\Delta z \frac{\partial V(z+\Delta z, t)}{\partial t}$$

or, $$-\frac{I(z+\Delta z,t) - I(z,t)}{\Delta z} = G\,V(z+\Delta z, t) + C\frac{\partial V(z+\Delta z,t)}{\partial t}$$

Taking limit as $\Delta z \to 0$

$$\boxed{-\frac{\partial I(z,t)}{\partial z} = G\,V(z,t) + C\frac{\partial V(z,t)}{\partial t}}$$

If we assume harmonic time dependance, so that

$$V(z,t) = Re\left[V_s(z)\, e^{j\omega t}\right]$$

$$I(z,t) = Re\left[I_s(z)\, e^{j\omega t}\right]$$

where, $V_s(z)$ and $I_s(z)$ are the phasor forms of $V(z,t)$ and $I(z,t)$, respectively, then above equations become

$$-\frac{dV_s}{dz} = (R + j\omega L)\, I_s \quad \text{--- (1)}$$

& $$-\frac{dI_s}{dz} = (G + j\omega C)\, V_s \quad \text{--- (2)}$$

Taking second derivative of (1) & Replacing value of (2) in (1), we get

$$\frac{d^2 V_s}{dz^2} = (R + j\omega L)\cdot(G + j\omega C)\, V_s$$

or, $$\frac{d^2 V_s}{dz^2} - \gamma^2 V_s = 0 \quad \text{--- (3)}$$

where, $\boxed{\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)\cdot(G + j\omega C)}}$

Taking second derivative of 2 & Replacing value of 1 in 2, we get

$$\frac{d^2 I_s}{dz^2} - \gamma^2 I_s = 0 \quad \text{——} (4)$$

Here, 3 & 4 are the wave equations for voltage and current similar to the equations obtained for plane waves

$$\frac{d^2 \vec{E}_s}{dz^2} - \gamma^2 \vec{E}_s = 0 \quad [\nabla^2 \vec{E}_s - \gamma^2 \vec{E}_s = 0]$$

$$\& \quad \frac{d^2 \vec{H}_s}{dz^2} - \gamma^2 \vec{H}_s = 0 \quad [\nabla^2 \vec{H}_s - \gamma^2 \vec{H}_s = 0]$$

As previous, γ is the propagation constant (in per meter), α is the attenuation constant (in nepers per meter or decibels per meter) and β is the phase constant (in radians per meter)

The wavelength & velocity v are as:

$$\boxed{\lambda = \frac{2\pi}{\beta} \quad \& \quad v = \frac{\omega}{\beta} = f\lambda}$$

The solutions of the linear homogeneous differential equations 3 & 4 are:

$$V_s(z) = \underbrace{V_0^+ e^{-\gamma z}}_{\rightarrow +z} + \underbrace{V_0^- e^{\gamma z}}_{-z \leftarrow} \quad \text{——} (5)$$

$$\text{and } I_s(z) = \underbrace{I_0^+ e^{-\gamma z}}_{\rightarrow +z} + \underbrace{I_0^- e^{\gamma z}}_{-z \leftarrow} \quad \text{——} (6)$$

where, V_0^+, V_0^-, I_0^+ and I_0^- are wave amplitudes; the +ve and −ve signs; respectively denote wave travelling along +ve z-axis and −ve z axis directions. Thus we obtain the instantaneous expression for voltage as:

$$v(z,t) = Re\{V_s(z)\, e^{j\omega t}\}$$

$$= V_0^+ e^{-\alpha z} \cos(\omega t - \beta z) + V_0^- e^{\alpha z} \cos(\omega t + \beta z)$$

Now,

the characteristic impedance, Z_0 of the line is the ratio of positively traveling voltage wave to current wave at any point on the line.

where, Z_0 is analogous to η, the intrinsic impedance of the medium of wave propagation. By substituting eq^{n}s. (5) & (6) into (1) & (2) and equating co-efficients of $e^{\gamma z}$ and $e^{-\gamma z}$, we get

$$Z_0 = \frac{V_0^+}{I_0^+} = -\frac{V_0^-}{I_0^-} = \frac{R+j\omega L}{\gamma} = \frac{\gamma}{G+j\omega C}$$

or, $$\boxed{Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = R_0 + jX_0}$$

where, R_0 and X_0 are the real and imaginary parts of Z_0. Here R_0 is in ohms and R is in ohms per meter. The propagation constant γ and the characteristic impedance Z_0 are important properties of the line because they both depend on the line parameters R, L, G & C and the freq^n of operation. The reciprocal of Z_0 is the characteristic admittance Y_0, that is $Y_0 = 1/Z_0$.

The transmission line considered in this section is the lossy type in that the conductors comprising the line are imperfect ($\sigma_c \neq \infty$) and the dielectric in which the conductors are embedded is lossy ($\sigma \neq 0$). Having this general case we can consider two special cases lossless & distortionless lines.

(A) Lossless Line ($R = 0 = G$)

A transmission line is said to be lossless if the conductors of the line are perfect ($\sigma_c \approx \infty$) and the dielectric medium separating them is lossless ($\sigma = 0$)

For such line, $\sigma_c = \infty$ & $\sigma = 0$

So, $\boxed{R = 0 = G}$

This is the necessary condition for a line to be lossless.

Again, from $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$

$$\boxed{\alpha = 0, \ \gamma = j\beta = j\omega\sqrt{LC}}$$

$$\boxed{v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f \times \lambda}$$

& from $Z_0 = \sqrt{\dfrac{R + j\omega L}{G + j\omega C}} = R_0 + jX_0$

$$\boxed{X_0 = 0, \ Z_0 = R_0 = \sqrt{\frac{L}{C}}}$$

(B) Distortionless line ($R/L = \frac{G}{C}$)

A signal normally consists of a band of frequencies; wave amplitudes of different frequency components will be attenuated differently in a lossy line as α is frequency dependent. This results in distortion.

A distortionless line is one in which the attenuation constant α is frequency independent while the phase constant β is linearly dependent on frequency.

So, in $\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$, a distortionless line results if the line parameters are as: $\boxed{\frac{R}{L} = \frac{G}{C}}$

So, $$\gamma = \sqrt{RG\left(1+\frac{j\omega L}{R}\right)\left(1+\frac{j\omega C}{G}\right)}$$

$$= \sqrt{RG}\left(1+\frac{j\omega C}{G}\right) = \alpha + j\beta$$

or, $$\boxed{\alpha = \sqrt{RG} \quad \& \quad \beta = \omega\sqrt{LC}}$$

showing that α does not depend on frequency whereas β is a linear function of freq^n.

Also,

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{R\left(1+\frac{j\omega L}{R}\right)}{G\left(1+\frac{j\omega C}{G}\right)}} = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}$$

$$= R_0 + jX_0.$$

or, $$\boxed{R_0 = \sqrt{\frac{R}{G}} = \sqrt{\frac{L}{C}}, \quad X_0 = 0.}$$

and $$\boxed{v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}} = f\times\lambda}$$

<u>Note:</u>

(1) The phase velocity is independent of frequency because the phase constant β linearly depends on freq^n. We have shape distortion of signals unless α and v are independent of freq^n.

(2) v & Z_0 remain the same for lossless lines.

(3) A lossless line is also distortionless line, but distortionless line is not necessary to be lossless. Although lossless lines are desirable in power transmission, telephone lines are required to be distortionless.

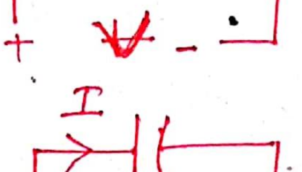
Distributed Line Parameters at high freq.

Parameters	Co-axial Line	Two wire line	Planar line
$R\ (\Omega/m)$	$\frac{1}{2\pi\delta\sigma_c}\left[\frac{1}{a}+\frac{1}{b}\right]$ $(\delta \ll a, c-b)$	$\frac{1}{\pi a\delta\sigma_c}$ $(\delta \ll a)$	$\frac{2}{w\delta\sigma_c}$ $(\delta \ll t)$
$L\ (H/m)$	$\frac{\mu}{2\pi}\ln\left(\frac{b}{a}\right)$	$\frac{\mu}{\pi}\cosh^{-1}\frac{d}{2a}$	$\frac{\mu d}{\omega}$
$G\ (S/m)$	$\frac{2\pi\sigma}{\ln b/a}$	$\frac{\pi\sigma}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\sigma\omega}{d}$
$C\ (F/m)$	$\frac{2\pi\epsilon}{\ln\frac{b}{a}}$	$\frac{\pi\epsilon}{\cosh^{-1}\frac{d}{2a}}$	$\frac{\epsilon\omega}{d}$ $(\omega \gg d)$

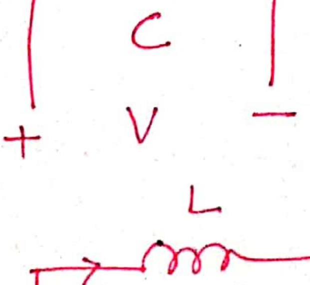
$\delta = \frac{1}{\sqrt{\pi f\mu_c\sigma_c}}$ = skin depth of the conductor;

$\cosh^{-1}\frac{d}{2a} = \ln\frac{d}{a}$ if $\left[\frac{d}{2a}\right]^2 \gg 1$.

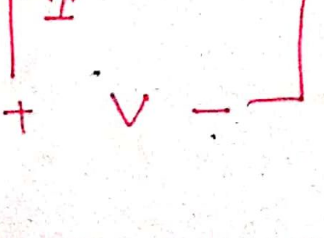
* $R = \frac{V}{I}$



* $C\frac{dv}{dt} = I$



* $V = -L\frac{dI}{dt}$



Transmission Line characteristics

Case	Propagation Constant $\gamma = \alpha + j\beta$	Characteristic impedance $Z_0 = R_0 + jX_0$
General	$\sqrt{(R+j\omega L)(G+j\omega C)}$	$\sqrt{\frac{R+j\omega L}{G+j\omega C}}$
Lossless	$0 + j\omega\sqrt{LC}$	$\sqrt{\frac{L}{C}} + j0$
Distortionless	$\sqrt{RG} + j\omega\sqrt{LC}$	$\sqrt{\frac{L}{C}} + j0$.

Input impedance, SWR and Power

Consider a transmission line of length l, characterized by γ and Z_0, connected to a load Z_L as shown in figure. Looking into the line, the generator sees the line with the load as an input impedance Z_{in}. The major aim is to determine the input impedance, the standing wave ratio (SWR), and power flow on the line.

Let the transmission line extend from $z=0$ at the generator to $z=l$ at the load. So, firstly we need the voltage and current waves as:

$$V_s(z) = V_0^+ e^{-\gamma z} + V_0^- e^{\gamma z} \quad \text{—— (a)}$$

$$I_s(z) = I_0^+ e^{-\gamma z} + I_0^- e^{\gamma z}$$

$$\text{or, } I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z} \quad \text{—— (b)}$$

Now, using the terminal conditions to find V_0^+ & V_0^-, we can write

$$V_0 = V(z=0), \quad I_0 = I(z=0)$$

i.e the conditions at input.

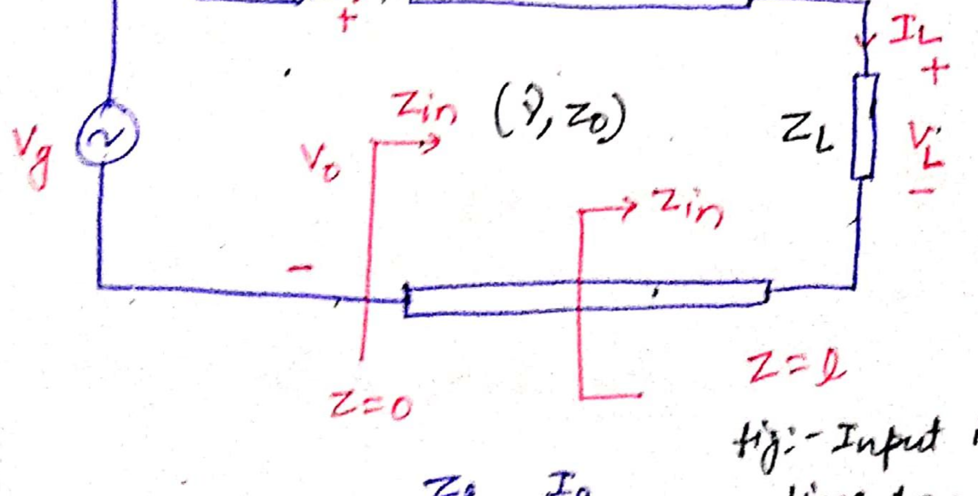


fig:- Input impedance due to a line terminated by a load

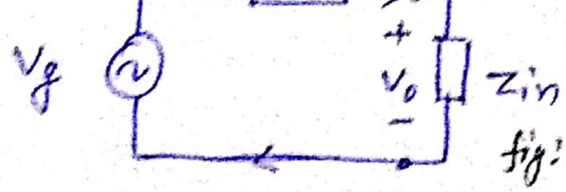


fig:- equivalent circuit for finding V_0 and I_0 in terms of Z_{in} at input.

Substituting in (a) & (b)

$$V_0^+ = \frac{1}{2}(V_0 + Z_0 I_0) \quad \text{—— (c)}$$

$$V_0^- = \frac{1}{2}(V_0 - Z_0 I_0) \quad \text{—— (d)}$$

If the input impedance at the input terminals is Z_{in}, the input voltage V_0 and the input current I_0 are easily obtained as:

$$\boxed{V_0 = \frac{Z_{in}}{Z_{in}+Z_g} V_g \,, \quad I_0 = \frac{V_g}{Z_{in}+Z_g}}$$

On the other hand, if we are given the conditions at the load

$$V_L = V(z=l) \,, \quad I_L = I(z=l)$$

then substituting in (a) & (b) we get

$$V_0^+ = \frac{1}{2}(V_L + Z_0 I_L) e^{\gamma l} \quad \text{—— (e)}$$

$$V_0^- = \frac{1}{2}(V_L - Z_0 I_L) e^{-\gamma l} \quad \text{—— (f)}$$

Then, the input impedance $Z_{in} = \frac{V_s(z)}{I_s(z)}$ at any point on the line. At the generator, Z_{in} will be:

$$\boxed{Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{Z_0(V_0^+ + V_0^-)}{V_0^+ - V_0^-}}$$

Now,

From eq's. (e) & (f) & using $\frac{e^{\gamma l}+e^{-\gamma l}}{2} = \cosh \gamma l$, $\frac{e^{\gamma l}-e^{-\gamma l}}{2} = \sinh \gamma l$

$$\text{or, } \tanh \gamma l = \frac{\sinh \gamma l}{\cosh \gamma l} = \frac{e^{\gamma l} - e^{-\gamma l}}{e^{\gamma l} + e^{-\gamma l}}$$

we get, $\boxed{Z_{in} = Z_0 \left[\frac{Z_L + Z_0 \tanh \gamma l}{Z_0 + Z_L \tanh \gamma l} \right]}$ (lossy) —— (g)

Although eq^n (8) is derived for the input impedance Z_{in} at the generation end, it is a general expression for finding Z_{in} at any point on the line. To find Z_{in} at a distance l' from the load as shown in figure. we replace l by l'.

For lossless line, $\gamma = j\beta$ and $\tanh j\beta l = j\tan\beta l$

and $Z_0 = R_0$.

So eq^n (8) becomes.

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] \text{(lossless)}}$$

Showing that the input impedance varies periodically with distance l from the load. The quantity βl is refered to as the electrical length of the line and can be expressed in degrees or radians.

Now, let us define Γ_L as the voltage reflection Co-efficient (at the load). Γ_L is the ratio of the voltage reflection wave to the incident wave at load,

i.e $$\boxed{\Gamma_L = \frac{V_0^- e^{\gamma l}}{V_0^+ e^{-\gamma l}}} \quad \text{—— (h)}$$

Substituting values of V_0^+ & V_0^- from (e) & (f) to (h), we get

$$\boxed{\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}} \quad \text{—— (i)}$$

where, $Z_L = \dfrac{V_L}{I_L}$

The voltage reflection coefficient at any point on the line is the ratio of the magnitude of the reflected wave to that of the incident wave.

i.e $\Gamma(z) = \dfrac{V_0^- e^{\gamma z}}{V_0^+ e^{-\gamma z}} = \dfrac{V_0^-}{V_0^+} e^{2\gamma z}$

But $z = l - l'$, substituting & combining with (h), we get

$$\boxed{\Gamma(z) = \frac{V_0^-}{V_0^+} e^{2\gamma l} \cdot e^{-2\gamma l'} = \Gamma_L e^{-2\gamma l'}} \quad \text{— (j)}$$

The current reflection co-efficient at any point on the line is negative of the voltage reflection co-efficient at that point.

Thus, the current reflection co-efficient at load is:

$I_0^- e^{\gamma l} / I_0^+ e^{-\gamma l} = -\Gamma_L$.

At load transmission co-efficient

$\tau = 1 + \Gamma_L = 1 + \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$= \dfrac{2Z_L}{Z_L + Z_0}$

So, the standing wave ratio, s (SWR) as:

$$\boxed{s = \frac{V_{max}}{V_{min}} = \frac{I_{max}}{I_{min}} = \frac{1 + |\Gamma_L|}{1 - |\Gamma_L|}} \quad \text{— (P)}$$

It is easy to show that, $I_{max} = \dfrac{V_{max}}{Z_0}$ and $I_{min} = \dfrac{V_{min}}{Z_0}$.

The input impedance Z_{in} in lossless line has maxima & minima that occur respectively, at maxima & minima of the voltage & current standing wave. It can be seen that

$$\boxed{|Z_{in}|_{max} = \frac{V_{max}}{I_{min}} = s Z_0 \ \& \ |Z_{in}|_{min} = \frac{V_{min}}{I_{max}} = \frac{Z_0}{s}}$$

As the transmission line is used in transferring power from source to the load, the average input power at a distance l from the load is given by.

$$\boxed{P_{avg} = \frac{1}{2} Re\{V_s(l)\, I_s^*(l)\}}$$

$*$ - denotes Complex Conjugate.

Where, the factor $\frac{1}{2}$ is needed since we are dealing with the peak values instead of rms values. Assuming Lossless line, we substitute eqn ⓐ & ⓑ to obtain.

$$P_{avg} = \frac{1}{2} Re\left\{ V_0^+ (e^{j\beta l} + \Gamma e^{-j\beta l}) \frac{V^{+*}}{Z_0} (e^{-j\beta l} - \Gamma^* e^{j\beta l}) \right\}$$

$$= \frac{1}{2} Re\left[\frac{|V_0^+|^2}{Z_0} (1 - |\Gamma|^2 + \Gamma e^{-2j\beta l} - \Gamma^* e^{2j\beta l}) \right]$$

Since the last two terms are purely imaginary, we have

$$\boxed{P_{avg} = \frac{|V_0^+|^2}{2Z_0} (1 - |\Gamma|^2)}$$

i.e $P_t = P_i - P_r$

where,

$$P_i = \frac{|V_0^+|^2}{2Z_0}$$

$$\& \; P_r = \frac{|V_0^+|^2}{2Z_0} \cdot |\Gamma|^2$$

where, P_t = the input or transmitted power

and the negative sign is due to the negative going wave, as we take the reference of +ve dirn for the voltage/current traveling toward right. Since it is a lossless line power is constant & doesnot depend on l.

Maximum power is delivered to the load when $\Gamma = 0$.

Now, we consider the special cases when the line is connected to the load $Z_L = 0$, $Z_L = \infty$ & $Z_L = Z_0$.

Eg: Consider a lossless line with characteristic impedance of $Z_0 = 50\Omega$. Let the line is terminated with a pure ~~resistive~~ resistive load $Z_L = 100\,\Omega$ and the voltage at the load is 100V (rms).

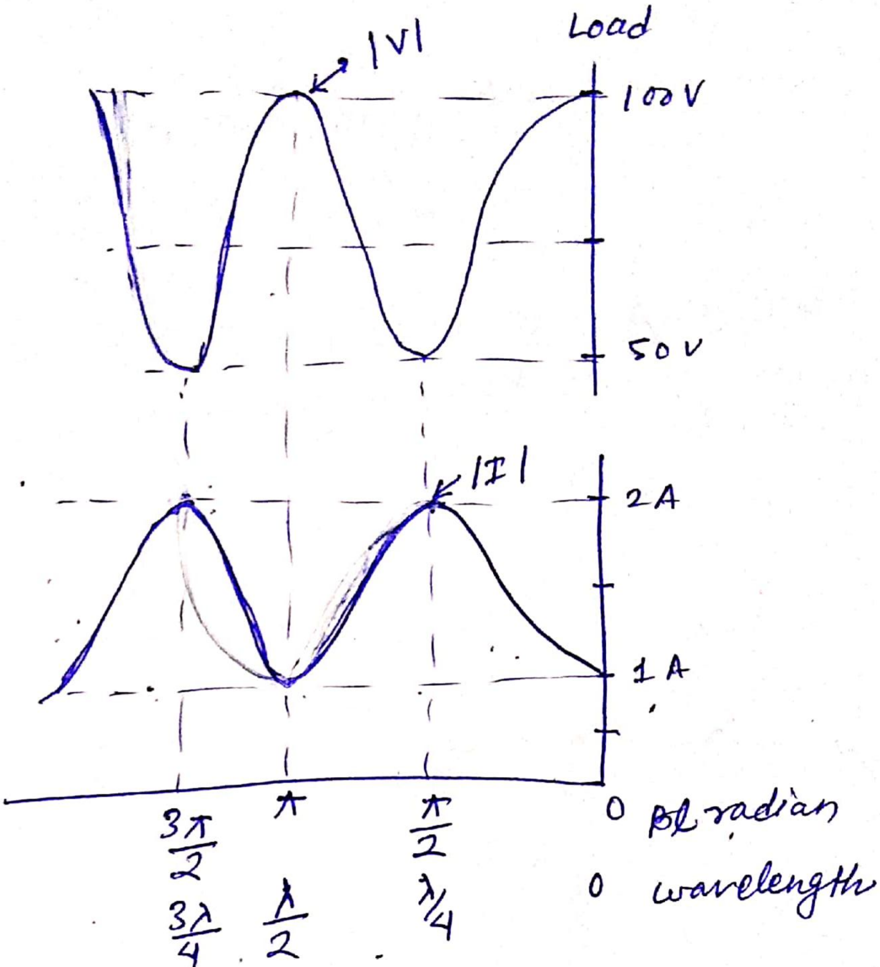


fig:- voltage and current wave patterns on a lossless line terminated by a resistive Load.

Ⓐ <u>Shorted line</u> $(Z_L = 0)$

$$\boxed{Z_{sc} = Z_{in}\Big|_{Z_L=0} = jZ_0 \tan\beta l.}$$

$$\left[Z_{in} = Z_0 \left(\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right) \text{ where } Z_L = 0 \right]$$

Also, $\boxed{\Gamma_L = -1,\ s = \infty}$

We see that Z_{in} is a pure reactance, which could be capacitive or inductive depending on the value of l.

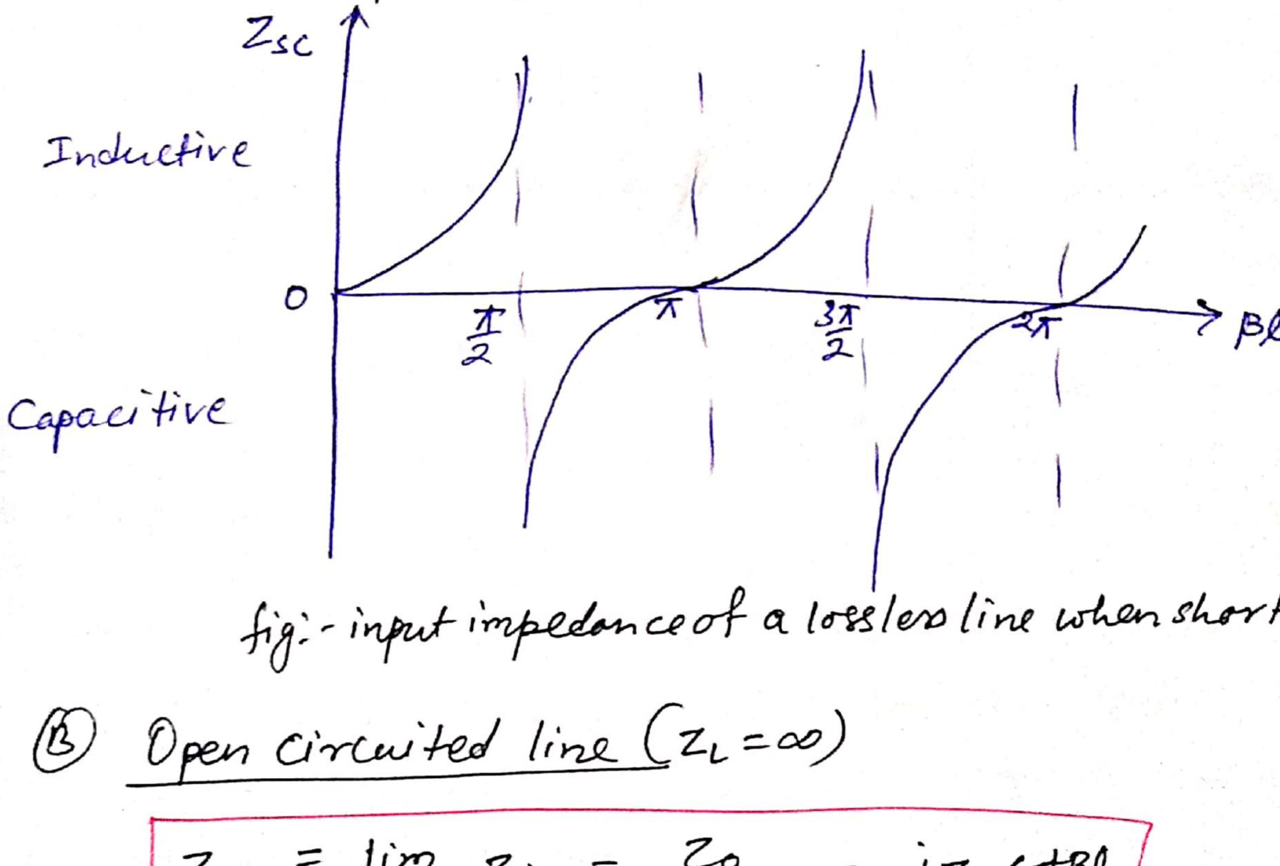


fig:- input impedance of a lossless line when shorted.

Ⓑ <u>Open circuited line</u> $(Z_L = \infty)$

$$\boxed{Z_{oc} = \lim_{Z_L \to \infty} Z_{in} = \frac{Z_0}{j\tan\beta l} = -jZ_0 \cot\beta l}$$

and $\boxed{\Gamma_L = 1,\ s = \infty}$

We see from above conditions that

$$\boxed{Z_{sc} \cdot Z_{oc} = Z_0^2}$$

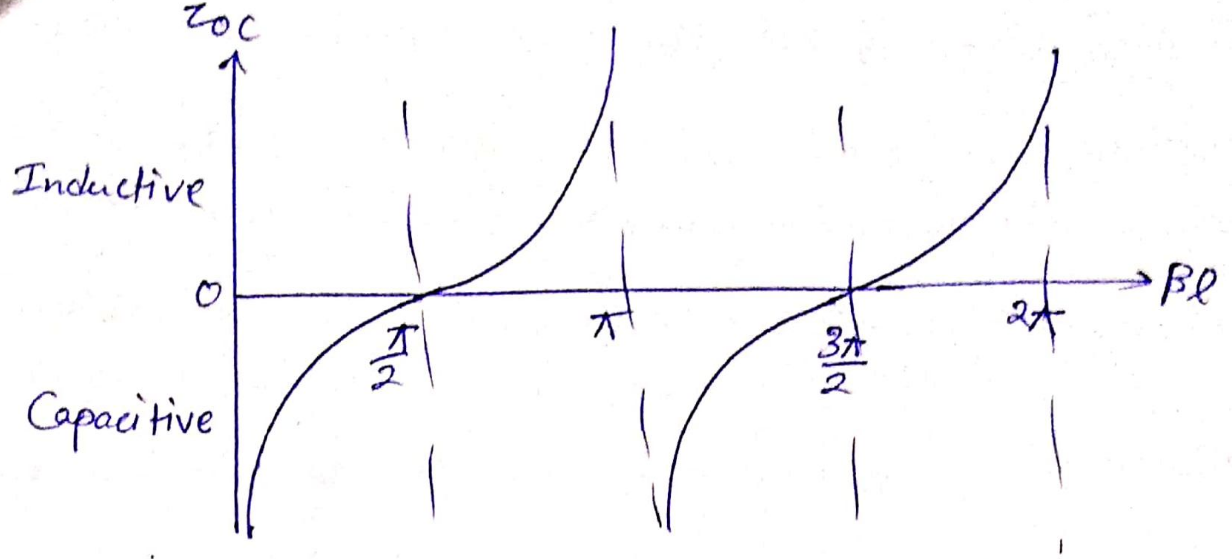


fig:- Input impedance of a lossless line when open.

Ⓒ <u>Matched line</u> $(Z_L = Z_0)$

It is the most desired case for the practical point of view, where $Z_L = Z_0$ and hence

$$\boxed{Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right] = Z_0}$$

and $\Gamma_L = 0$, $s = 1$

i.e $V_0^- = 0$, the whole wave is transmitted and there is no reflection. The incident power is fully absorbed by the load. Thus the maximum power transfer is possible when a transmission line is matched to the load.

<u>Impedance matching</u> (or tuning)

It is the practice of designing the input impedance of an electrical load or the output impedance of its corresponding signal source to maximize the power transfer or minimize signal reflection from the load. The matching network is ideally lossless, to avoid unnecessary loss of power.

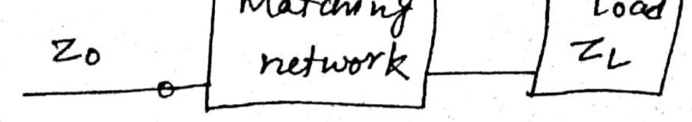


fig:- Matching load impedance to line.

Impedance Matching are important for following reasons:

1. Maximum power is delivered when the load is matched with line and power loss in the transmission line is minimized.
2. Impedance matching in a power distribution network will reduce the amplitude and phase errors.

Factors that may be important in selection of a particular matching network are:

1. Complexity 2. Bandwidth 3. Implementation
4. Adjustability.

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(A) Impedance Matching with Quarter wave Transformer

When $Z_0 \neq Z_L$; we say that the load is mismatched and a reflected wave exists on the line. However, for maximum power transfer, it is desired that the load be matched to the transmission line ($Z_0 = Z_L$) so that there is no reflection ($|\Gamma| = 0$ or $s = 1$). The matching is achieved by using shorted sections of transmission lines.

From previous relations, when $l = \lambda/4$ or $\beta l = \left(\frac{2\pi}{\lambda}\right) \cdot \left(\lambda/4\right) = \frac{\pi}{2}$,

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right] = \frac{Z_0^2}{Z_L}$$

that is $\boxed{\frac{Z_{in}}{Z_0} = \frac{Z_0}{Z_L}}$

or, $Z_{in} = \frac{1}{Z_L} \Rightarrow \underbrace{Y_{in}}_{\text{admittance}} = Z_L$

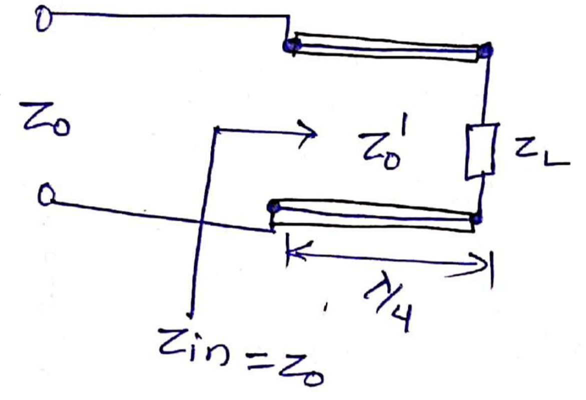
By adding a $\lambda/4$ line on the transmission line we see the impedance matching in mismatched Load Z_L.

i.e A mismatched load Z_L can be properly matched to a line (with characteristic impedance Z_0) by inserting prior to the load a transmission line $\lambda/4$ long (with characteristic impedance Z_0') as shown in figure. The ~~section~~ $\lambda/4$ section of the transmission line is called a quarter wave transformer because it is used for impedance matching like an ordinary transformer.

So, Z_0' is selected in such a way that $Z_{in} = Z_0$.

$$\therefore \boxed{Z_0' = \sqrt{Z_0 Z_L}}$$

where, Z_0', Z_0 & Z_L are all real.



The main disadvantage of quarterwave transformer is that it is a narrowband or freq^n sensitive device.

fig:- Load matching using $\lambda/4$ transformer

(B) Single Stub Matching or Tuner

The major drawback of using a quarter wave transformer as a line matching device is eliminated by using a single stub tuner. The tuner consists of an open or shorted section of transmission line of length d connected in parallel with the main line at some distance l from the load as shown in figure. Here the stub has same characteristic impedance as the main line. It is more difficult to use a series stub although it is theoretically feasible. An open circuited stub radiates some energy at high frequencies. Consequently, shunt short circuited parallel stubs are preferred.

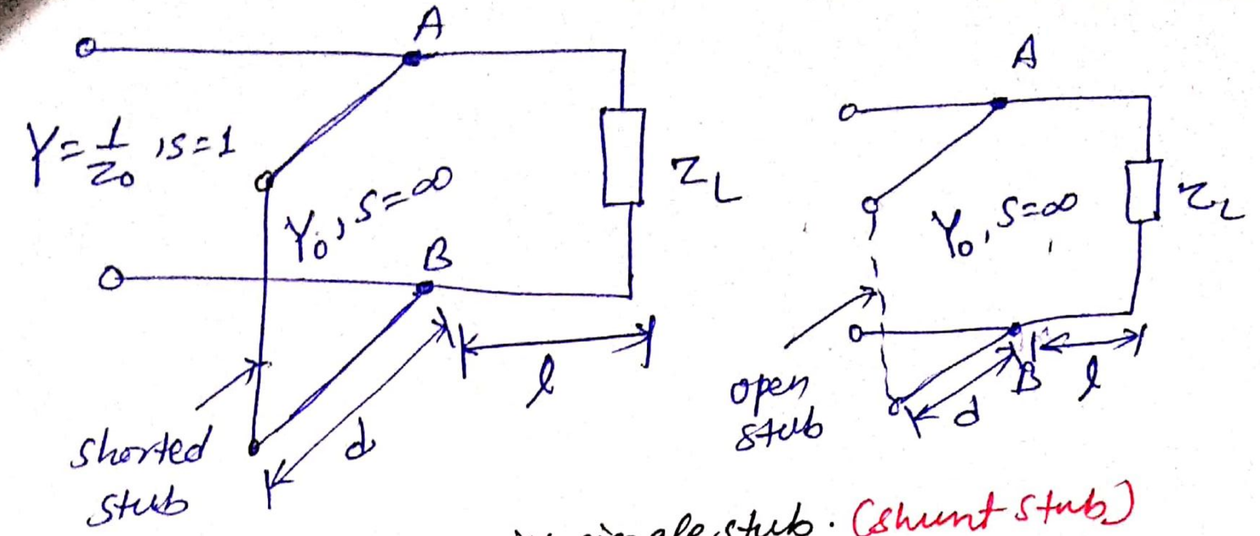


fig:- Matching with single stub. (shunt stub)

As we intend that $Z_{in} = Z_0$, the distance l is selected so that the impedance Z seen looking into the line at a distance l from the load is the form of $Z_0 + jX$. Then stub reactance is chosen as $-jX$, resulting the matched condition.

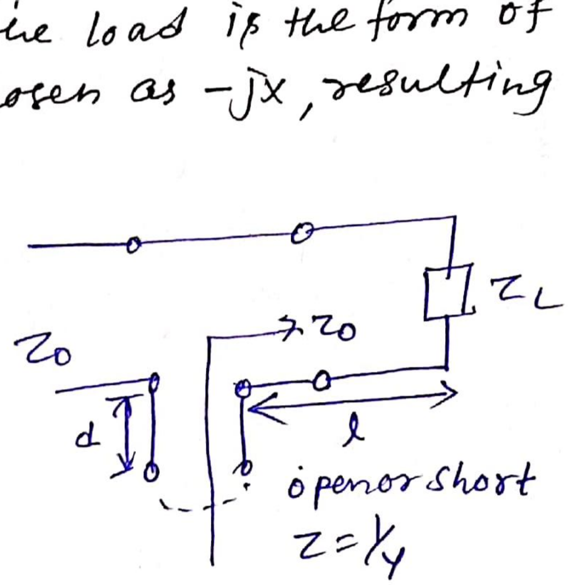


fig:- series stub.

Double Stub Matching

The single stub tuners of the previous section are able to match any load impedance (as long as it has a nonzero real part) to a transmission line, but suffer from the disadvantage of requiring a variable length of line bet? the load and the stub. This may not be a problem for a fixed matching circuit, but would probably pose some difficulty if an adjustable tuner was desired. In this case, the double stub tuner, which uses two tuning stubs in fixed positions, can be used. Such tuners are often fabricated in coaxial line with adjustable stubs connected in parallel to the main coaxial line.

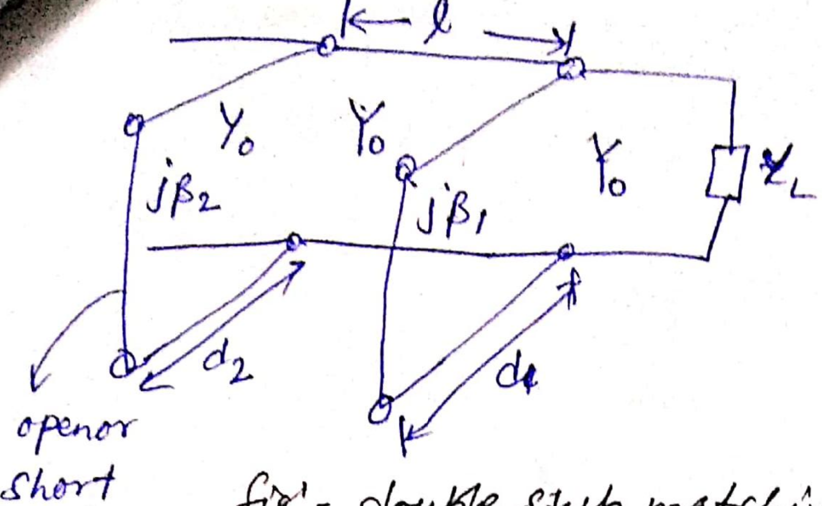


fig:- double stub matching (shunt stubs)

Smith Chart

It ~~is~~ graphically represents impedance along transmission line and is used for evaluation of reflection co-efficient, input impedance, standing wave ratio etc.

It is constructed within circles of unit radius.

$(|\Gamma| \leq 1)$

Given, $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \Gamma_r + j\Gamma_i$

$$\Gamma = \frac{Z_L/Z_0 - 1}{\frac{Z_L}{Z_0} + 1} = \frac{z_l - 1}{z_l + 1} \; ; \; z_l = r + jx \text{ (normalized)}$$

then, $$z_l = \frac{1+\Gamma}{1-\Gamma} = \frac{1+\Gamma_r + j\Gamma_i}{1-(\Gamma_r + j\Gamma_i)}$$

$$= \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} + j\,\frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2} = r + jx$$

Then comparing real to real and imaginary to imaginary part

$$r = \frac{1-\Gamma_r^2 - \Gamma_i^2}{(1-\Gamma_r)^2 + \Gamma_i^2} \Rightarrow \left(\Gamma_r - \frac{r}{1+r}\right)^2 + \Gamma_i^2 = \left(\frac{1}{1+r}\right)^2$$

Also, from $x = \dfrac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$

$$(\Gamma_r - 1)^2 + \left(\Gamma_i - \frac{1}{x}\right)^2 = \left(\frac{1}{x}\right)^2$$

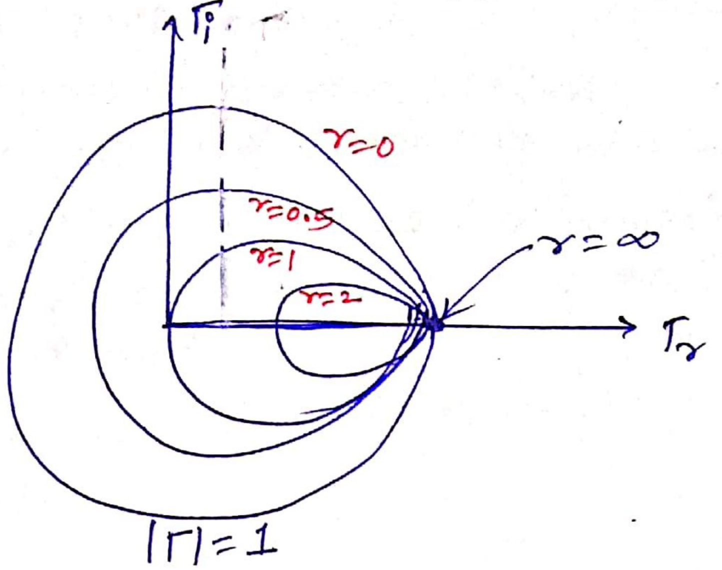


fig:- r circles (constant) are shown on Γ_r, Γ_i plane. The radius of any circle is $\frac{1}{1+r}$.

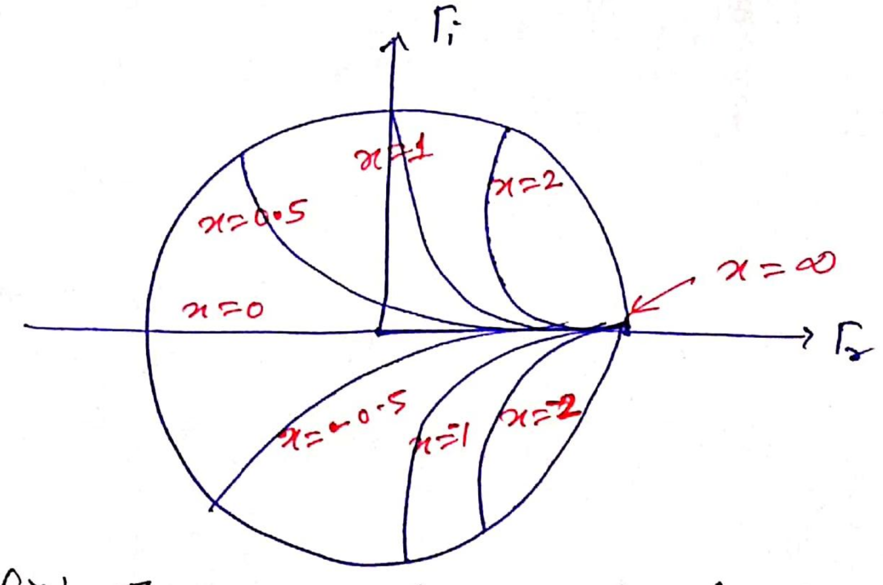


fig:- The portions of the circles of constant x lying within $|\Gamma|=1$ are shown on Γ_r, Γ_i axes. The radius of a circle is $\frac{1}{|x|}$.

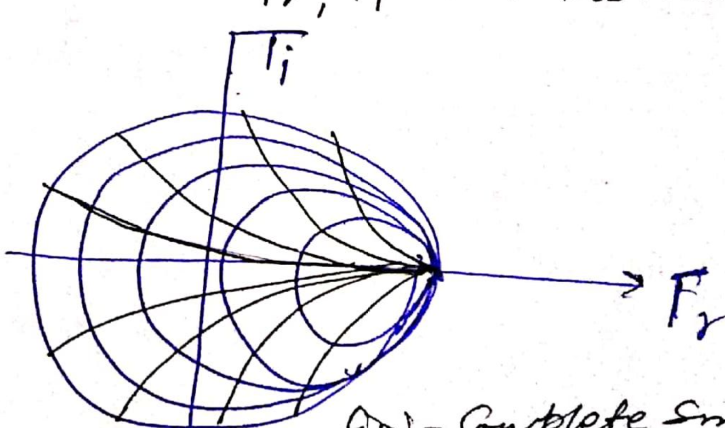


fig:- Complete Smith chart

Note:

(1) One revolution around the chart represents distance of $\lambda/2$ on the line.

(2) Clockwise movement on the chart is regarded as moving towards the generator and anticlockwise movemement on the chart is regarded as moving towards load.

(3) Two outermost scales are used to measure distance on the line in terms of wavelength (λ) and the innermost scale is used to measure angle.

(4) Smith chart can be used both as impedance chart & admittance chart.

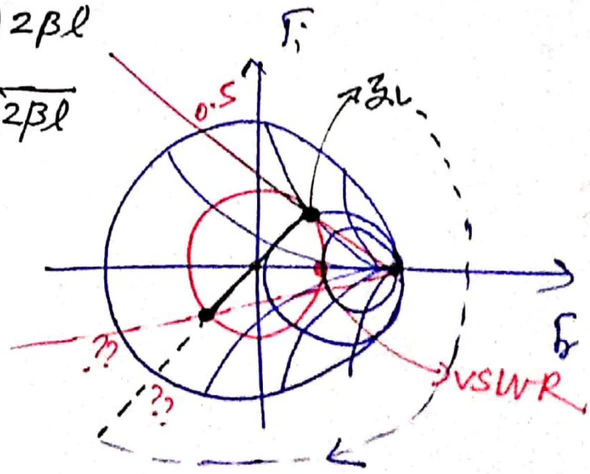
(5) Given normalized impedance

$$Z_{in} = \frac{V_s(z)}{I_s(z)} = \frac{V_i\left(e^{-j\beta z} + \Gamma e^{j\beta z}\right)}{\frac{V_i}{Z_0}\left(e^{-j\beta z} - \Gamma e^{j\beta z}\right)}$$

$$\frac{Z_{in}}{Z_0} = z_{in} = \frac{e^{-j\beta z} + \Gamma e^{j\beta z}}{e^{-j\beta z} - \Gamma e^{j\beta z}}$$

Considering $z = -l$

$$z_{in} = \frac{e^{j\beta l} + \Gamma e^{-j\beta l}}{e^{j\beta l} - \Gamma e^{-j\beta l}} = \frac{1 + \Gamma e^{-j2\beta l}}{1 - \Gamma e^{-j2\beta l}}$$



$$z_{in} = \frac{1 + \Gamma(l)}{1 - \Gamma(l)}$$

$$y_{in} = \frac{1}{z_{in}} = \frac{1 - \Gamma(l)}{1 + \Gamma(l)}$$

$$z_{in}(l + \lambda/4) = \frac{1 + \Gamma(l + \lambda/4)}{1 - \Gamma(l + \lambda/4)} = \frac{1 - \Gamma(l)}{1 + \Gamma(l)} = y_{in}$$

Q. Given $Z_0 = 100\,\Omega$ & $Z_L = 150 + j50$ then

$$z_L = \frac{Z_L}{Z_0} = 1.5 + j0.5$$

Qn: At an operating radian freq^n of 500 Mrad/s, typical circuit values for a certain transmission line are : $R = 0.2\,\Omega/m$, $L = 0.25\,\mu H/m$, $G = 10\,\mu S/m$ and $C = 100\,pF/m$. Find (a) α (b) β (c) λ (d) v_p (e) Z_0.

Sol^n Given,

$$\omega = 500\text{ Mrad/s} = 5\times10^{8}\text{ rad/s}.$$

$$R = 0.2\,\Omega/m,\quad L = 0.25\,\mu H/m = 25\times10^{-8}\text{ H/m}.$$

$$G = 10\,\mu S/m = 10\times10^{-6}\text{ S/m},\quad C = 100\,pF = 10^{-10}\text{ F}.$$

(a)
$$\alpha = \frac{1}{2}\left(R\sqrt{\frac{C}{L}} + G\sqrt{\frac{L}{C}}\right) = \frac{1}{2}\left(0.2\times\sqrt{\frac{100\times10^{-12}}{0.25\times10^{-6}}} + 10\times10^{-6}\sqrt{\frac{0.25\times10^{-6}}{100\times10^{-12}}}\right)$$

$$= \frac{4\times10^{-3} + 10\times10^{-6} * 5\times10^{1}}{2}$$

$$= 2.25\times10^{-3}\text{ Np/m}$$

$$= 2.25\text{ mNp/m}.$$

(b)
$$\beta = \omega\sqrt{LC}\left[1 + \frac{1}{8}\left(\frac{G}{\omega C} - \frac{R}{\omega L}\right)^{2}\right]$$

$$= 5\times10^{8}\sqrt{0.25\times10^{-6}\times10^{-10}}\left[1 + \frac{1}{8}\left(\frac{0.25\times10^{-6}}{5\times10^{8}\times10^{-10}} - \frac{0.2}{5\times10^{8}\times0.25\times10^{-6}}\right)^{2}\right]$$

$$= 25\times10^{8}\times10^{-9}\left[1 + \frac{1}{8}\left(5\times10^{-6} - 1.6\times10^{-3}\right)^{2}\right]$$

$$= 2.5\text{ rad/m}.$$

(c)
$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2.5} = 2.51\text{ m}.$$

(d)
$$v_p = \frac{\omega}{\beta} = \frac{5\times10^{8}}{2.5} = 2\times10^{8}\text{ m/s}$$

(e)
$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{0.2 + j5\times10^{8}\times0.25\times10^{-6}}{10\times10^{-6} + j5\times10^{8}\times10^{-10}}}$$

$$= \sqrt{\frac{0.2 + j125}{10^{-5} + j0.05}} = \sqrt{125\angle 89.908 / 0.5\angle 89.988^{\circ}}$$

$$= 50 - j0.035\,\Omega$$

The characteristic impedance of a certain lossless transmission line is 72 Ω. If $L = 0.5\,\mu H/m$, find (a) C (b) v_p (c) β if $f = 80\,MHz$. (d) The line is terminated with a load of 60 Ω. Find Γ and S.

Sol.^{n} Given, $Z_0 = 72\,\Omega$, $L = 0.5\,\mu H/m$, $f = 80\,MHz$, $Z_L = 60\,\Omega$

(a) then, for lossless ($R = 0 = G$) line

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\text{or, } 72 = \sqrt{\frac{0.5\times10^{-6}}{C}}$$

$$\text{or, } C = \frac{0.5\times10^{-6}}{5184} = 96.45\,pF/m.$$

(b) $$v_p = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{0.5\times10^{-6}\times96.45\times10^{-12}}} = 0.1455\times10^{9} = 1.455\times10^{8}\,m/s$$

(c) $$\beta = \frac{\omega}{v_p} = \frac{2\pi\times80\times10^{6}}{1.455\times10^{8}} = 3.455\,rad/m.$$

Again,
$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60-72}{60+72} = -0.091$$

$$\therefore\ S = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.091}{1-0.091} = 1.2$$

Q: A lossless transmission line having $Z_0 = 120\,\Omega$ is operating at $\omega = 5\times10^{8}\,rad/s$. If the velocity on the line is $2.4\times10^{8}\,m/s$, find (a) L (b) C (c) Let Z_L be represented by an inductance of $0.6\,\mu H$ in series with a 100 Ω resistance; find Γ & S.

Sol.^{n} Given, $Z_0 = 120\,\Omega$, $\omega = 5\times10^{8}\,rad/s$, $v = 2.4\times10^{8}\,m/s$

(a) with $Z_0 = \sqrt{\frac{L}{C}}$ & $v = \frac{1}{\sqrt{LC}}$, we have $L = \frac{Z_0}{v} = \frac{120}{2.4\times10^{8}} = 0.50\,\mu H/m$

(b) $C = \frac{1}{Z_0 v} = [120\times2.4\times10^{8}]^{-1} = 35\,pF/m.$

(c) Let Z_L be the representation of load impedance, which is combination of $100\text{-}\Omega$ resistance in series with $0.6\ \mu H$.

So, $Z_L = R + j\omega L = 100 + j\,5\times10^8 \times 0.6\times10^{-6}$

$= 100 + j300\ \Omega$

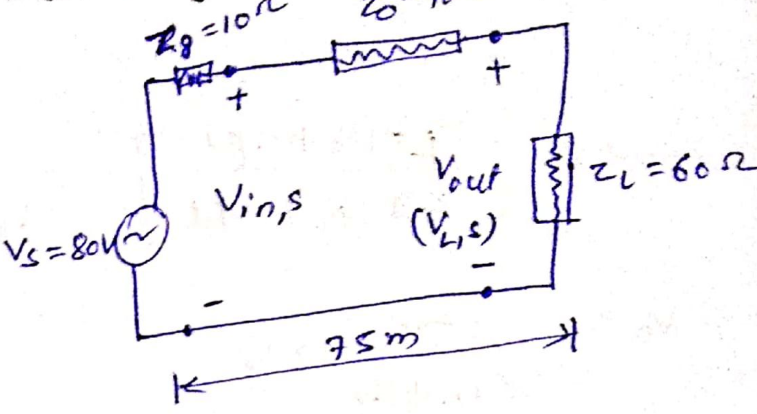
Now,

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{100 + j300 - 120}{100 + j300 + 120} = \frac{-20 + j300}{220 + j300} = 0.808\angle 40^\circ$$

$$= 0.62 + j0.52$$

& $$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.808}{1 - 0.808} = 9.4.$$

Q: Consider a two wire 40 Ω line ($z_0 = 40\,\Omega$) connecting the source of 80V, 400 KHz with series resistance 10 Ω to the load of $Z_L = 60\,\Omega$. The line is 75 m long and the velocity on the line is 2.5×10^8 m/s. Find the voltage $V_{in,s}$ at input end and $V_{L,s}$ at output end of the transmission line. [2072 Chaitra]



Sol.ⁿ

Given,
$z_0 = 40\,\Omega$
$l = 75\,m$
$f = 400\,KHz$
$V_s = 80V \angle 0^\circ$
$v = 2.5 \times 10^8\,m/s$
$Z_L = 60\,\Omega$

we have, $\beta l = \frac{\omega}{v} l = \frac{2\pi \times 400 \times 10^3}{2.5 \times 10^8} \times 75 = 43.2^\circ$

(a) The input impedance,

$$Z_{in} = \frac{z_0 (R_L + j z_0 \tan\beta l)}{z_0 + j R_L \tan\beta l}$$

$$= 40 \times \frac{60 + j40 \tan 43.2^\circ}{40 + j60 \tan 43.2^\circ}$$

$$= 37.8 - j15.73\,\Omega$$

So, effective input voltage

$$V_{in,s} = V_s \times \frac{Z_{in}}{Z_g + Z_{in}} = 80 \times \frac{37.8 - j15.73}{10 + 37.8 - j15.73} = 65.10 e^{-j4.37^\circ}\ V$$

(b) Again, Reflection Co-efficient at load is

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{60-40}{60+40} = 0.2$$

$[e^{j\theta} = \cos\theta + j\sin\theta]$

Here,

$$V_{in,s} = V_0^+ \left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right) \quad [\because z = -l]$$

$$\text{or, } V_0^+ = \frac{65.10\, e^{-j4.37^\circ}}{e^{j43.2^\circ} + 0.2\, e^{-j43.2^\circ}} = 63.078\angle -36.42^\circ \text{ or } 63.078\, e^{-j36.42^\circ}\ V$$

Thus the transmitted voltage wave, (at $z=0$)

$$V_{s,out} = V_L = V_0^+ (1 + \Gamma_L)$$

$$= 63.078\angle -36.42^\circ \times 1.2$$

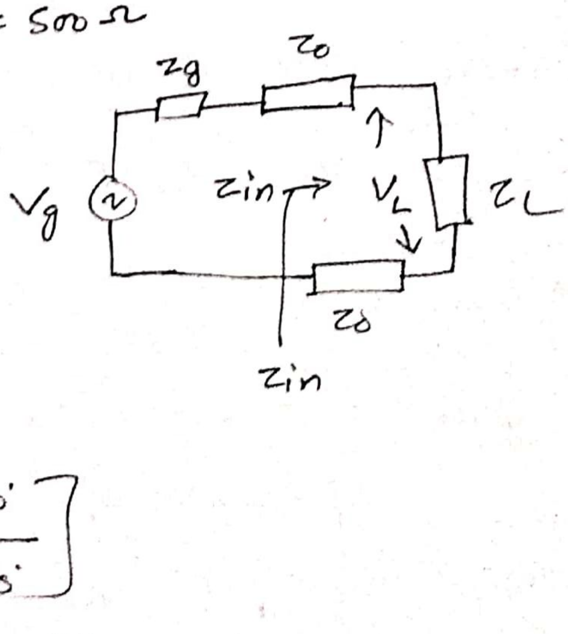
$$= 75.69\angle -36.42^\circ\ V$$

Qn: A 300 Ω transmission line is lossless, 0.25λ long, and is terminated in $Z_L = 500\ \Omega$. The line has a generator with $90\angle 0^\circ$ V in series with 100 Ω connected to the input. Find (a) the load voltage (b) Voltage at the midpoint of the line. [2072 Kartik] [2069 Jestha]

Sol:^{n} Given,

$Z_0 = 300\ \Omega$, $l = 0.25\lambda$, $Z_L = 500\ \Omega$

$V_g = 90\angle 0^\circ$ V, $Z_g = 100\ \Omega$



so, $$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

where, $\beta l = \frac{2\pi}{\lambda} \times 0.25\lambda = 90^\circ$.

$$\therefore Z_{in} = 300 \times \left[\frac{500 + j300\tan 90^\circ}{300 + j500\tan 90^\circ}\right]$$

$$= 300 \left[\frac{\frac{500}{\tan 90^\circ} + j300}{\frac{300}{\tan 90^\circ} + j500}\right]$$

$$= \frac{300 \times 300}{500} = 180\ \Omega$$

Again,

$$V_{in} = Z_{in} \times \frac{V_g}{Z_{in}+Z_g} = \frac{180 \times 90\angle 0^\circ}{180+100} = 57.857 \angle 0^\circ \text{ V}$$

So, with reference to load, voltage at any point on the line is

$$V(z) = V_0^+ \left(e^{-j\beta z} + \Gamma_L e^{j\beta z}\right)$$

Since, the line is lossless.

$$V_L = V_{in} \cdot e^{-\beta l} = 57.857 \angle 0^\circ - \beta l = 57.857 \angle -90^\circ \text{ V}$$

Again, $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{500-300}{500+300} = 0.25$

Again, at $z=0$ (i.e at load)

$$V(z=0) = V_L = 57.857 \angle -90^\circ \text{ V}$$

$$V(z=-0.125\lambda) = |V_{in}| \angle 0^\circ - \frac{\beta l}{2} = 57.857 \angle -45^\circ \text{ V}$$

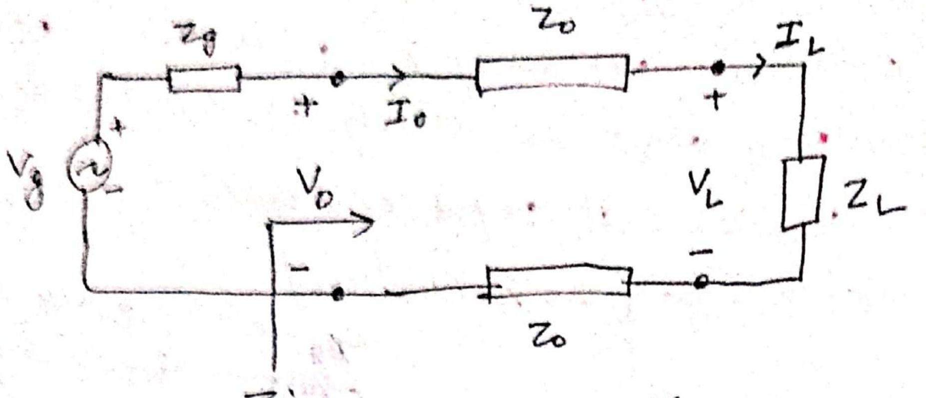
or, $\left(e^{-j\beta \times 0} + 0.25\, e^{j\beta \times 0}\right) V_0^+ = 57.857 \angle -90^\circ$

or, $V_0^+ = 46.286 \angle -90^\circ$ V

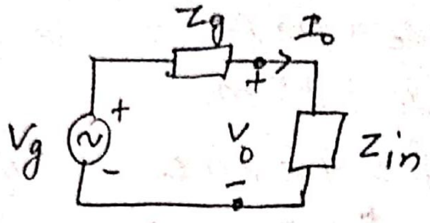
Now, at midpoint of the line i.e $z = -\frac{l}{2} = -0.125\lambda$

$$V(z=-0.125\lambda) = V_0^+\left(e^{j45^\circ} + 0.25\, e^{-j45^\circ}\right) = 47.7 \angle -59^\circ \text{ V}$$ //

Formula



$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan \beta l}{Z_0 + jZ_L \tan \beta l} \right]$$



$$V_0 = \frac{Z_{in}}{Z_{in} + Z_g} \times V_g$$

With respect to load at $z=0$. (input at $z = -l$)

$$V(z) = V_0^+ e^{-j\beta z} \left[1 + \Gamma_L e^{j2\beta z} \right]$$

$$= V_0^+ e^{-j\beta z} + \Gamma_L V_0^+ e^{j\beta z}$$

At $z=0$

$$V(0) = V_0^+ (1 + \Gamma_L)$$

At $z = l$

$$V_L = V_0^+ e^{-j\beta l} (1 + \Gamma_L e^{j2\beta l})$$

$$z_{min} = -\frac{1}{2\beta} [\phi + (2m+1)\pi], \ m = 0, 1, 2 \ldots$$

$(V_{min})\ I_{max}$

$$\Gamma = |\Gamma| e^{j\phi}$$

$$z_{max} = -\frac{1}{2\beta} [\phi + 2m\pi], \ m = 0, 1, 2, \ldots$$

$(I_{max})\ V_{max}$

Current $$I_s(z) = \frac{V_0^+}{Z_0} e^{-\gamma z} - \frac{V_0^-}{Z_0} e^{\gamma z}$$

$$P_{avg} = \frac{|V_0^+|^2}{2Z_0} (1 - |\Gamma|^2)$$

Maxima & minima are separated by $\lambda/4$ distance

At Load

$$s = \frac{|V|_{max}}{|V|_{min}} = \frac{Z_L}{Z_0}$$

Qn: Two characteristics of a certain lossless transmission line are $Z_0 = 50\,\Omega$ and $\gamma = 0 + j0.2\pi\ m^{-1}$ at $f = 60\ MHz$. (a) Find L and C for the line. (b) A load $Z_L = 60 + j80\,\Omega$ is located at $z = 0$. What is the shortest distance from the load to a point at which $Z_{in} = R_{in} + j0$?

Sol.ⁿ Given, $Z_0 = 50\,\Omega$, $\gamma = 0 + j0.2\pi\ m^{-1}$ for lossless line at $f = 60\ MHz$

(a) we have, $\beta = 0.2\pi = \omega\sqrt{LC}$ & $Z_0 = 50 = \sqrt{\frac{L}{C}}$

Thus, $\frac{\beta}{Z_0} = \omega C \Rightarrow C = \frac{\beta}{\omega Z_0} = \frac{0.2\pi}{(2\pi \times 60 \times 10^6) \times 50} = 33.3\ pF/m.$

Then, $L = C Z_0^2 = (33.3 \times 10^{-12}) \times (50)^2 = 83.3\ nH/m.$

(b) A load $Z_L = 60 + j80\,\Omega$ is located at $z = 0$

then, $Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$

then, normalizing the impedances w.r.t. Z_0

$$z_{in} = \frac{Z_{in}}{Z_0} = \frac{\frac{Z_L}{Z_0} + j\tan\beta l}{1 + j\frac{Z_L}{Z_0}\tan\beta l} = \frac{z_L + j\tan\beta l}{1 + jz_L \tan\beta l}$$

where, $z_L = \frac{(60 + j80)}{50} = 1.2 + j1.6$

and let $x = \tan\beta l$ then

$$z_{in} = \left[\frac{1.2 + j(1.6 + x)}{(1 - 1.6x) + j1.2x}\right]\left[\frac{(1 - 1.6x) - j1.2x}{(1 - 1.6x) - j1.2x}\right]$$

$$z_{in} = \left[\frac{1.2(1 - 1.6x) + 1.2x(1.6 + x) - j\left[(1.2)^2 x - (1.6 + x)(1 - 1.6x)\right]}{(1 - 1.6x)^2 + (1.2)^2 x^2}\right]$$

Since, imaginary part is zero,

$$(1.2)^2 x - (1.6 + x)(1 - 1.6x) = 0 \Rightarrow 1.6x^2 + 3x - 1.6 = 0$$

$ax^2 + bx + c = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

So, $x = \tan\beta l = \frac{-3 \pm \sqrt{9 + 4\cdot(1.6)^2}}{2(1.6)} = 0.4338, -12.31$

we take +ve root and find

$$\beta l = \tan^{-1}(0.433) = 0.409$$

$$\text{or, } l = \frac{0.409}{0.2\pi} = 0.65m = 65cm.$$

Qn: A 50 Ω Lossless transmission line is 30m long and is terminated with a load $Z_L = 60 + j40\ \Omega$. The operating freq^n is 20 MHz and velocity on the line is 2.5×10^8 m/s. Find ⓐ Reflection co-efficient ⓑ SWR ⓒ Input impedance [2073 Chaitra]

Sol.^{n} ⓐ $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$ ⓑ $SWR = \frac{1+|\Gamma|}{1-|\Gamma|}$

ⓒ $Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right]$, $\beta l = \frac{\omega}{v}\times l = \frac{2\pi f \times l}{v}$

$= \frac{2\pi \times 20\times10^6 \times 30}{2.5\times10^8}$ rad

Qn: Determine the primary constants (R, L, C and G) on the transmission line when the measurement on the line at 1KHz gave the following results: $Z_0 = 710\angle{-16^\circ}$, $\alpha = 0.01$ Np/m and $\beta = 0.035$ rad/m.

[2073 Shrawan]

Sol.ⁿ Given,

$$f = 1\,KHz = 10^3\,Hz,\quad \alpha = 0.01\ Np/m$$

$$Z_0 = 710\angle{-16^\circ} \quad \& \quad \beta = 0.035\ rad/m.$$

Then, we have

$$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \quad \text{and} \quad \gamma = \alpha + j\beta = \sqrt{(R+j\omega L)\cdot(G+j\omega C)}$$

$$\text{or, } 710\angle{-16^\circ} = \sqrt{\frac{R+j\omega L}{G+j\omega C}} \qquad \text{or, } 0.01 + j0.035 = \sqrt{(R+j\omega L)\cdot(G+j\omega C)}$$

Let $R+j\omega L = a$ & $G+j\omega C = b$ then

$$682.49 - j195.7 = \sqrt{\frac{a}{b}} \quad -(1) \quad \& \quad 0.01 + j0.035 = \sqrt{ab} \quad -(2)$$

Dividing (1) by (2)

$$\frac{682.49 - j195.7}{0.01 + j0.035} = \frac{\sqrt{a/b}}{\sqrt{ab}}$$

$$\text{or, } -18.59 - j19505.2 = \frac{1}{b}$$

$$\text{or, } b = -4.886\times10^{-8} + j5.13\times10^{-5}$$

$$\text{So, } a = \frac{(0.01 + j0.035)^2}{-4.886\times10^{-8} + j5.13\times10^{-5}} = 13.66 + j21.92$$

Again,

$\omega = 2\pi \times 10^3 = 6283.185$ rad/s.

and $a = R + j\omega L = 13.66 + j21.92$

$\therefore R = 13.66\ \Omega/m$.

& $L = \dfrac{21.92}{6283.185} = 3.4886$ mH/m

Again,

$b = G + j\omega C = -48.86 \times 10^{-9} + j51.3 \times 10^{-6}$

$\therefore G = -48.86$ nS/m.

& $C = \dfrac{51.3 \times 10^{-6}}{6283.185} = 8.165$ nF/m

Qn: A lossless 75 Ω transmission line having length of 0.6λ is connected to a $100 + j50\ \Omega$ load. Find (a) Reflection co-efficient (b) SWR (c) Load admittance (d) Z_{in} at 0.4λ from the load (e) The location of V_{max} & V_{min} with respect to load (f) Z_{in} at the generator.

Qn:- A lossless transmission line with $Z_0 = 50\Omega$ has a length of 0.4λ. The operating freq^n is 300MHz and it is terminated with a load $Z_L = 40 + j30\Omega$. Find.

[2071 chaitra]

ⓐ Reflection Co-efficient (Γ)
ⓑ Standing wave ratio on the line (SWR)
ⓒ Input Impedance (Z_{in})

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Sol^n Given, $Z_0 = 50\Omega$, $l = 0.4\lambda$, $f = 300$ MHz, $Z_L = 40 + j30\Omega$

ⓐ Reflection co-efficient, $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{40 + j30 - 50}{40 + j30 + 50}$

$= 0.33 \angle 90^\circ$

ⓑ SWR, $S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = \dfrac{1 + 0.33}{1 - 0.33} = 1.985$

ⓒ $Z_{in} = Z_0 \left[\dfrac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$

where, $\beta l = \dfrac{2\pi}{\lambda} \times 0.4\lambda = 0.8\pi = 144^\circ$

$\therefore Z_{in} = 50 \left[\dfrac{(40 + j30) + j\,50 \tan 144^\circ}{50 + j(40 + j30) \tan 144^\circ} \right]$

$= 25.5 + j5.9\,\Omega$

Qn: If a transmission line having a characteristic impedance, $Z_0 = 200\Omega$ is operating at freq^n 15MHz, with propagation constant $\gamma = j0.5\,m^{-1}$ then determine ⓐ velocity of propagation ⓑ wavelength ⓒ Inductance ⓓ Capacitance. [2068 Shrawan]

Sol^n Given, $Z_0 = 200\Omega$, $f = 15$MHz, $\gamma = j0.5\,m^{-1}$

Since, $\gamma = \alpha + j\beta = j0.5\,m^{-1}$

$\alpha = 0$ & $\beta = 0.5$ i.e line is lossless.

then, (a) velocity, $v = \frac{\omega}{\beta} = \frac{2\pi \times 15 \times 10^6}{0.5} = 1.88 \times 10^8$ m/s.

(b) wavelength, $\lambda = \frac{v}{f} = \frac{2\pi}{\beta} = 12.566$ m.

(c) Inductance, L can be calculated as:

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

For lossless line, $R = 0 = G$.

$$\therefore \gamma = j0.5 = j\omega\sqrt{LC}$$

$$\text{or, } 0.5 = 2\pi \times 15 \times 10^6 \sqrt{LC}$$

$$\text{or, } LC = 2.8144 \times 10^{-17} \quad \text{—— (a)}$$

Again, for lossless line

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$\text{or, } 200 = \sqrt{\frac{L}{C}} \quad \text{——}$$

$$\therefore L = 40000\,C \quad \text{—— (b)}$$

From (a) & (b)

$$40000\,C^2 = 2.8144 \times 10^{-17}$$

$$\text{or, } C = 26.525F$$

$$\& \; L = 1.061\,\mu H$$

(d) $C = 26.525F$.

Qn. A load impedance of $(40+j70)\Omega$ terminates a 100Ω transmission line that is 0.3λ long. Find the reflection co-efficient at the load and the voltage at the input of the line: [2069 Ashad]

Sol^n Given, $Z_L = (40+j70)\,\Omega$; $Z_0 = 100\,\Omega$, $l = 0.3\lambda$

then,
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{40+j70-100}{40+j70+100} = -0.142 + j571$$

Again,
$$\beta l = \frac{2\pi}{\lambda} \times 0.3\lambda = 0.6\pi \times \frac{180}{\pi} = 108^\circ$$

and $\tan\beta l = \tan 108^\circ = -3.0777$

$$\therefore Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

$$= 100 \times \left[\frac{(40+j70) + j100(-3.0777)}{100 + j(40+j70)(-3.0777)}\right]$$

$$= 36.53 - j61.11\,\Omega$$

Let V_g be the generator voltage then

$$V_{in} = V_g \times \frac{Z_{in}}{Z_{in} + Z_g}$$ [if source or generator given]

At $z = -l$

$$V_{in} = V_0^+ \left(e^{+j\beta l} + \Gamma_L e^{-j\beta l}\right)$$

$$= V_0^+ \left[e^{j108^\circ} + (-0.142 + j571) \cdot e^{-j108^\circ}\right]$$

$$= V_0^+ [542.87 - j175.11] \text{ V}$$

Qn: A lossless line having an air dielectric has a characteristic impedance of 400 Ω. The line is operating at 200 MHz and $Z_{in} = 200 - j200\ \Omega$. Find (a) SWR (b) Z_L, if the line is 1m long. (c) the distance from load to the nearest voltage maximum. [2074 Ashwin]

Sol.ⁿ $Z_0 = 400\ \Omega$, $Z_{in} = 200 - j200\ \Omega$, $f = 200$ MHz

$\mu = \mu_0$ & $\varepsilon = \varepsilon_0$, $l = 1m$

(a) SWR = ?

Here,

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

where,

$$\beta = \omega\sqrt{\mu_0\varepsilon_0} = 2\pi \times 200 \times 10^6 \times \frac{1}{c} = \frac{4\pi \times 10^8}{3\times 10^8} = 4.19$$

$$\& \ \beta l = 4.19 \text{ rad} = \left(\frac{4.19 \times 180}{\pi}\right)^\circ = 240^\circ$$

So,

$$200 - j200 = 400\left[\frac{Z_L + j\,400 \tan 240^\circ}{400 + jZ_L \tan 240^\circ}\right]$$

$$\text{or, } 0.5 - j0.5 = \frac{Z_L + j692.82}{400 + j1.73205 Z_L}$$

$$\text{or, } 200 - j200 + j0.866 Z_L + 0.866 Z_L = Z_L + j692.82$$

$$\text{or, } (200 + 0.866 Z_L) + j0.866 Z_L = Z_L + j692.82$$

Let $Z_L = a + jb$ then

$$(200 + 0.866a + j0.866b) + j(0.866a) - 0.866b = a + j(692.82 + b)$$

$$\text{or, } (200 + 0.866a - 0.866b) + j(0.866a + 0.866b) = a + j(692.82 + b)$$

$200 + 0.866a - 0.866b = a$

or, $0.134a + 0.866b = 200$ —— (1)

And $0.866a + 0.866b = 692.82 + b$

or, $0.866a - 0.134b = 692.32$ —— (2)

Solving (1) & (2)

$$0.134 \times 0.134a + 0.866 \times \cancel{0.134b} = 200 \times 0.134$$

$$+\ 0.866 \times 0.866a - \cancel{0.134 \times 0.866b} = 692.32 \times 0.866$$

$$0.767912\, a = 626.34912$$

or, $a = 815.65$

and $b = 104.74$

(c) $Z_L = a + jb = 815.65 + j104.74\ \Omega$

& $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{815.65 + j104.74 - 400}{815.65 + j104.74 + 400}$

$= 0.3467 + j0.0563$

$= 0.351 \angle 9.219^\circ = |\Gamma| \angle \phi$

$SWR = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$

$= \dfrac{1 + 0.351}{1 - 0.351}$

$= 2.08166$

(c) For nearest voltage maxima from Load, we have voltage will be maximum at

$Z_{max} = \dfrac{-1}{2\beta}(\phi + 2m\pi)$

For nearest (first) maxima $m = 0$

$Z_{max} = \dfrac{-1}{2\beta} \times \phi = -\dfrac{9.219^\circ \times \frac{\pi}{180}}{2 \times 4.19} = -0.0192\text{m}$ from Load

Qn: A lossless transmission line with $Z_0 = 50\Omega$ with length 1.5m connects a voltage $V_g = 60V$ source to a terminal load of $Z_L = 50 + j50\ \Omega$. If the operating freq^n $f = 100MHz$, generator impedance, $Z_g = 50\ \Omega$ and speed of wave equal to the speed of the light. Find the distance of the first voltage maximum from the load. What is the power delivered to load.? [2070 Chaitra]

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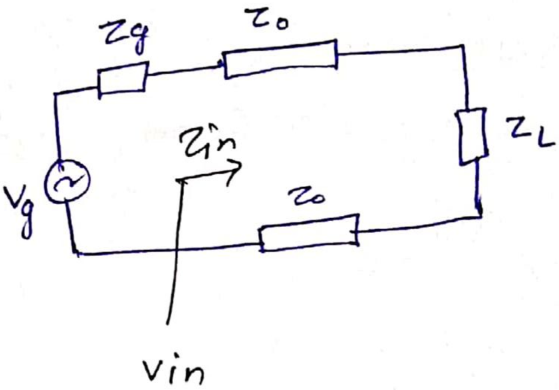
Sol.^{n} Given, $Z_0 = 50\Omega,\ l = 1.5m,\ V_g = 60V,\ Z_L = 50 + j50\Omega$

$f = 100MHz,\ Z_g = 50\Omega\ \&\ c = 3\times10^8 m/s.$

Here, $Z_{in} = Z_0\left[\dfrac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right]$



where, $\beta l = \dfrac{2\pi f}{c}\times l$

$= \dfrac{2\pi\times100\times10^6}{3\times10^8}\times1.5$

$= \pi$

$= 180°$

$\therefore\ Z_{in} = 50\times\left[\dfrac{50 + j50 + j50\tan180°}{50 + j(50+j50)\tan180°}\right]$

$= \dfrac{50\times(50+j50)}{50}$

$= 50 + j50\ \Omega$

Then, $V_{in} = \dfrac{V_g\times Z_{in}}{Z_{in} + Z_g} = \dfrac{60\times(50+j50)}{50+j50+50} = \dfrac{60\times50(1+j)}{50(2+j)}$

$= 60\times\left(\dfrac{1+j}{2+j}\right) = 37.947\angle18.43°\ V$

Again, $\Gamma_L = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{50 + j50 - 50}{50 + j50 + 50} = \dfrac{j50}{100 + j50} = \dfrac{j}{2 + j}$

$= 0.447 \angle 63.43$

$= |\Gamma| \angle \phi$

Since the line is lossless line

Voltage at Load is $V_L = 37.947 \angle 18.43^\circ - \beta l$

$= 37.947 \angle -161.57^\circ$ V

From Load, Voltage will be maximum at Zmax, which is given by

$$Z_{max} = -\frac{1}{2\beta}(\phi + 2m\pi) \quad \text{where } m = 0, 1, 2, \ldots$$

First maxima is at $m = 0$.

i-e $Z_{max} = -\dfrac{1}{2\beta}(\phi + 0) = -\dfrac{\phi}{2\beta} \quad [\because |\Gamma| \angle \phi]$

$\Rightarrow \dfrac{\phi \times \pi \times \lambda}{2 \times 2\pi \times 180^\circ} = \dfrac{\phi \times \pi \times c}{2 \times 2\pi \times f \times 180^\circ}$

$= \dfrac{-63.43^\circ \times 3 \times 10^8 \times \cancel{\pi}}{2 \times 2\cancel{\pi} \times 10^8 \times 180^\circ}$

$= -0.2643$ m from load.

Now, Current flow in Load is given by

$$I_L = \frac{V_L}{Z_L} = \frac{37.947 \angle -161.57^\circ}{50 + j50} = \frac{39.947 \angle -161.57^\circ}{70.7107 \angle 45^\circ}$$

$= 0.5649 \angle 153.43^\circ$ A

So, Average power delivered to load is

$$P_L = \frac{1}{2} |I_L|^2 \cdot Re\{Z_L\}$$

$= \frac{1}{2} \times (0.5649)^2 * 50$

$= 7.98$ Watts.

Qn: 50Ω lossless line has a length of 0.4λ. The operating freq^n is 300 MHz. A load $Z_L = 40+j30\Omega$ is connected at $z=0$ and the thevenin equivalent source at $z=-l$ is $12\angle 0^\circ$ in series with $Z_{Th} = 50+j0\Omega$. Find (a) The reflection co-efficient (Γ) (b) the voltage standing wave ratio (VSWR) and (c) The input impedance (Z_{in}). [2071 Shrawan]

Sol.^{n} Given, $Z_0 = 50\Omega$, $l = 0.4\lambda$, $f = 300$ MHz

$Z_L = 40+j30\,\Omega$, $V_g = 12\angle 0^\circ$ V, $Z_g = Z_{Th} = 50+j0\,\Omega = 50\Omega$

Then,

(a) Reflection co-efficient, $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0}$

$$= \frac{40+j30-50}{40+j30+50}$$

$$= 0.33\angle 90^\circ$$

(b) $$VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.33}{1-0.33} = 1.98$$

(c) Input impedance, $Z_{in} = Z_0 \left[\dfrac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right]$

where, $\beta l = \dfrac{2\pi}{\lambda} \times 0.4\lambda = 144^\circ$

$$\therefore Z_{in} = 50 \times \left[\frac{40+j30+j50\tan 144^\circ}{50+j(40+j30)\tan 144^\circ}\right]$$

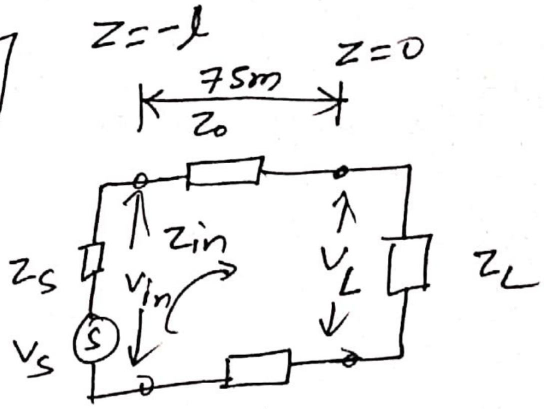
$$= 25.5 + j5.9\,\Omega$$

Qn: Consider a two wire 40Ω line ($Z_0 = 40\Omega$) connecting the source of 80V, 400 KHz with series resistance 10Ω to the load of $Z_L = 60\Omega$. The line is 75m long and the velocity on the line is 2.5×10^8 m/s. Find the voltage $V_{in,s}$ at input end and $V_{L,s}$ at output end of the transmission line.
[2072 chaitra]

Sol.^{n}

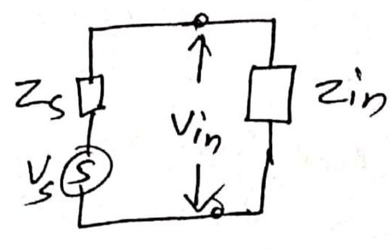
$$\beta = \frac{\omega}{v} = \frac{2\pi f}{v} = \frac{2\pi \times 4\times10^5}{2.5\times10^8}, \quad l = 75m.$$

$$Z_{in} = Z_0\left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$



$$V_{in} = \frac{Z_{in}\cdot V_S}{Z_{in} + Z_S}$$

|||



Again,
$z = -l = -75m$ at input

$$V_{in} = V_0^+ e^{j\beta l}(1 + \Gamma_L e^{-2\beta l})$$

or, $$V_0^+ = \frac{V_{in}}{e^{\beta l}(1 + \Gamma_L e^{-2\beta l})}$$

$z = 0$ at Load.

& $$V_L = V_0^+ (1 + \Gamma_L)$$

A lossless transmission line with $Z_0 = 50\,\Omega$ is 200 m long. It terminated with a load $Z_L = 30+j60\,\Omega$, and operated at a freq^n of 0.5 MHz. Let the velocity, $v = 0.6\times c$ on the line where, c = velocity of light $= 3\times10^8$ m/s. Find.

(a) The reflection Co-efficient [2070 Ashad]

(b) The VSWR on the line.

(c) The input impedance (Z_{in})

Sol.^{n} Given, $Z_0 = 50\,\Omega$, $l = 200\,m$, $Z_L = 30+j60\,\Omega$, $f = 0.5\times10^6\,Hz$

$v = 0.6\times3\times10^8$ m/s

(a) $$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30+j60-50}{30+j60+50} = 0.632 \angle 71.565^\circ$$

(b) $$\text{VSWR}, s = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.632}{1-0.632} = 4.435$$

(c) $$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right]$$

where, $$\beta l = \frac{\omega}{v}\times l = \frac{2\pi\times0.5\times10^6}{0.6\times3\times10^8}\times200 = \frac{2\pi}{1.8} = 200^\circ.$$

$\therefore \tan\beta l = 0.364.$

$$\therefore Z_{in} = 50\times\left[\frac{30+j60+j50\times0.364}{50+j(30+j60)\times0.364}\right]$$

$$= 50\times\left[\frac{30+j78.2}{28.16+j10.92}\right]$$

$$= 93.11 + j102.74\,\Omega$$

$$= 138.66\angle47.82^\circ\,\Omega$$

Qn: A $300\,\Omega$ transmission line is lossless, 0.25λ long, and is terminated with $Z_L = 500\,\Omega$. The line has a generator with $90\angle 0°$ V in series with $100\,\Omega$ connected to the input. (a) Find the Load voltage (b) Find the Voltage at the midpoint of the line. [2068 Jestha]

Sol.^{n}

$Z_0 = 300\,\Omega$

$l = 0.25\lambda$ $\qquad \beta l = \frac{2\pi}{\lambda} \times 0.25\lambda = 0.5\pi$ rad.

$Z_L = 500\,\Omega$

$Z_s = 100\,\Omega$

$V_s = 90\angle 0°$

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \right]$$

then,
$$V_{in} = \frac{Z_{in} \cdot V_s}{Z_{in} + Z_s}$$

Since, line is lossless

(a)
$$V_L = V_{in} \cdot e^{-j\beta l}$$

(b) voltage at midpoint
$$V(0.125\lambda) = V_{in} \cdot e^{-j\beta \frac{l}{2}}$$

Q: A lossless 75Ω transmission line having length 0.6λ is connected to a 100 + j50 Ω load. Find (a) Reflection co-efficient (b) Standing wave ratio (c) Load admitance, Y_L (d) Z_{in} at 0.4λ from the load (e) Location of V_{max} & V_{min} with respect to load. (f) Z_{in} at the generator.

Sol.ⁿ Given, $Z_0 = 75\Omega$ and lossless line of length, $l = 0.6\lambda$.

$Z_L = 100 + j50\Omega$.

(a) $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{100 + j50 - 75}{100 + j50 + 75} = 0.307 \angle 47.49^\circ$

(b) Standing wave ratio, $s = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = \dfrac{1 + 0.307}{1 - 0.307} = 1.89$

(c) $Y_L = \dfrac{1}{Z_L} = \dfrac{1}{100 + j50} = \dfrac{100 - j50}{(100)^2 + (50)^2} = 8 - j4$ m mho.

(d) Z_{in} at 0.4λ is

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

where, $\tan\beta l = \tan\left(\frac{2\pi}{\lambda} \times 0.4\lambda\right) = \tan(144^\circ) = -0.7265$

$$\therefore Z_{in} = 75 \times \left[\frac{100 + j50 + j75 \times (-0.7265)}{75 + j(100 + j50)(-0.7265)}\right]$$

$$= 75 \times \left[\frac{100 - j4.4875}{111.325 - j72.65}\right]$$

$$= 48.63 + j28.71\ \Omega$$

(e) Location of V_{max} is given by

$$Z_{max} = -\frac{\phi}{2\beta} = -\frac{47.49^\circ \times \pi \times \lambda}{180^\circ \times 2 \times 2\pi} = -0.065\lambda$$

And $z_{min} = -\left(0.065\lambda + \frac{\lambda}{4}\right) = -0.315\lambda$ for V_{min}.

Since V_{max} & V_{min} are in $\lambda/4$ difference.

Now,

(f) Z_{in} at generator is

$$Z_{in} = Z_0 \left[\frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l}\right]$$

where, $\beta l = \frac{2\pi}{\lambda} \times 0.6\lambda = 1.2\pi = 216^\circ$

$\tan\beta l = 0.7265$

$$\therefore Z_{in} = 75 \times \left[\frac{100 + j50 + j75 \times 0.7265}{75 + j(100 + j50) \times 0.7265}\right]$$

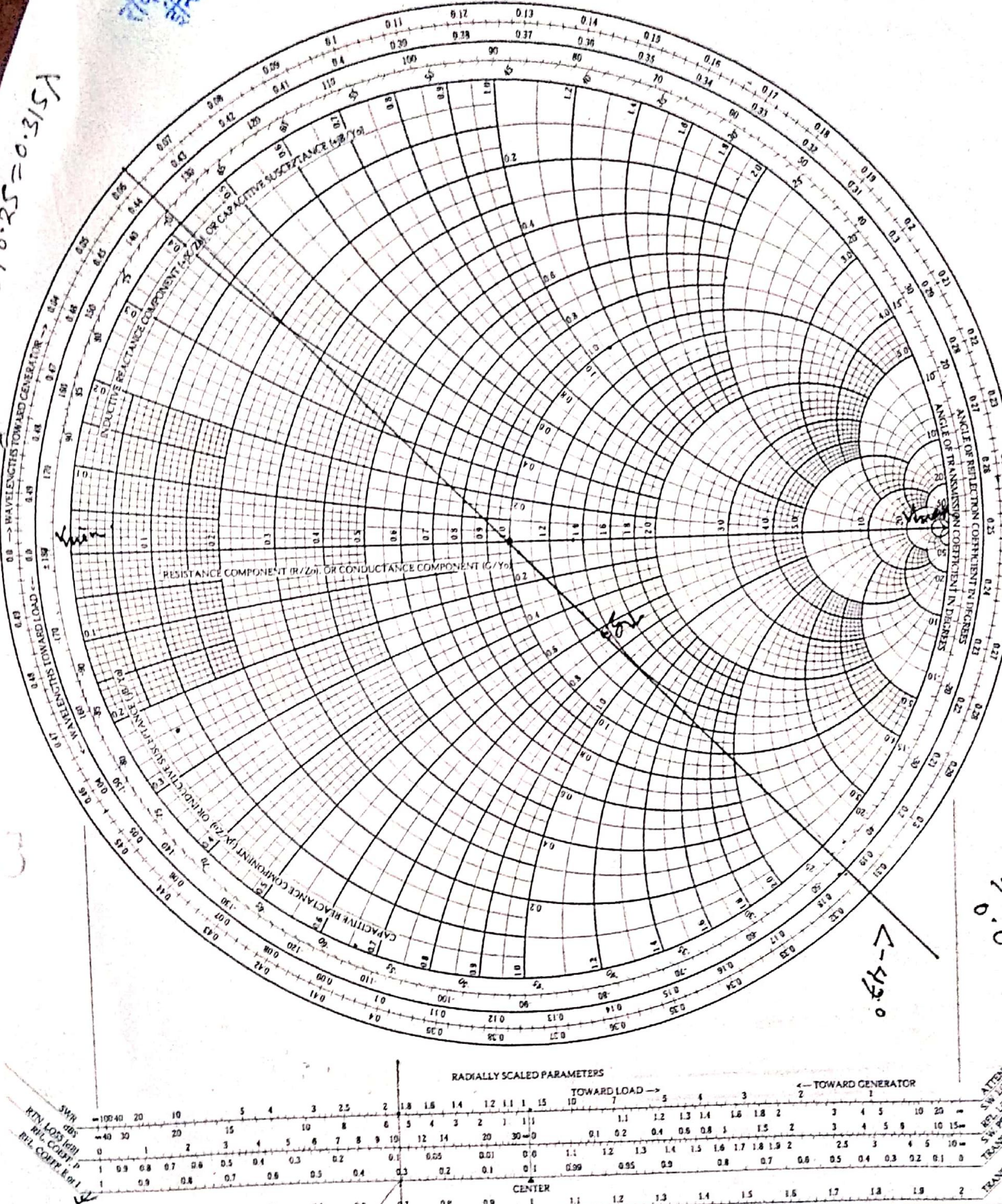
$$= 75 \times \left[\frac{100 + j104.4875}{38.675 + j72.65}\right]$$

$$= 126.87 - j35.696\ \Omega$$

The Complete Smith Chart

Black Magic Design

$0.065 + 0.25 = 0.315\lambda$



$\angle -47.0$

$$\begin{array}{r} 0.314 \\ -0.25 \\ \hline 0.064\lambda \end{array} \Rightarrow V_{max}$$

$|\Gamma| = 0.307 \angle -47°$