Chapter – 3.

Magnetic Fields

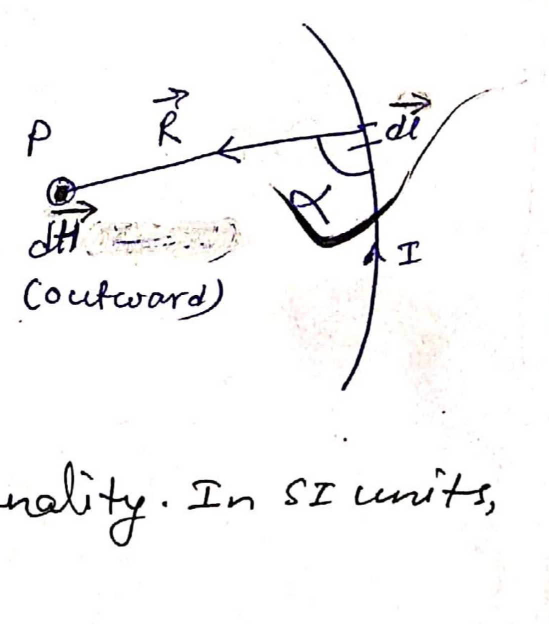
An electrostatic field is produced by static or stationary charges. If the charges are moving with constant velocity, a static magnetic (or magnetostatic) field is produced. A magnetostatic field is produced by a constant current flow (or direct current). This currents flow may be due to magnetization currents as in permanent magnets, electron-beam currents as in vaccum tubes, or conduction currents as in current-carrying wires.

Steady or Time invariant Magnetic field is the magnetic field produced by the time invariant or steady or direct current.

There are two major laws governing magnetostatic fields: ① Biot - Savart's Law and ② Ampere's Circuital law.

Biot - Savart's Law

It states that the magnetic field intensity dH produced at a point P, by the differential current element Idl is proportional to the product Idl and the sine of the angle α bet.ⁿ the element and the line joining P to the element and inversely proportional to the square of the distance R bet.ⁿ P and the element.



i.e $$dH \propto \frac{I\,dl \sin\alpha}{R^2}$$

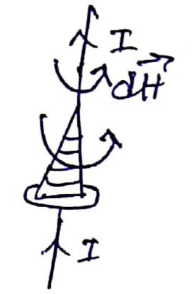
or, $$dH = \frac{K I\,dl \sin\alpha}{R^2}$$

where, K = the constant of proportionality. In SI units, $K = 1/4\pi$

$$\therefore \boxed{dH = \frac{I\,dl \sin\alpha}{4\pi R^2}}$$

From the definition of cross product

$$\boxed{\vec{dH} = \frac{I\vec{dl} \times \hat{a}_R}{4\pi R^2} = \frac{I\vec{dl} \times \vec{R}}{4\pi R^3}}$$



where, $R = |\vec{R}|$ and $\hat{a}_R = \frac{\vec{R}}{|\vec{R}|}$. The direction of $\vec{dH}$ can be determined by the right hand rule with the right hand thumb pointing the direction of the current, the right hand fingers encircling the wire in the direction of $\vec{dH}$ as in above figure. Alternatively, we can use the right handed screw rule to determine the direction of $\vec{dH}$; with the screw placed along the wire and pointed in the direction of current flow, the direction of advance of the screw is the direction of $\vec{dH}$. The unit of $\vec{dH}$ (magnetic field intensity) is Ampere per meter (A/m). Biot-Savart's law is sometimes called Ampere's law for the current element.

As, we can not isolate the differential current element the above expression of Biot-Savart's law is impossible to check experimentally. So, as the field is time invariant i.e independent of time or not a function of time we can say that the current density is not a function of time.

From Continuity Eq^n

$$\nabla \cdot \vec{J} = \frac{-\partial \rho_v}{\partial t}$$

will be $\nabla \cdot \vec{J} = 0$

Applying divergence Theorem

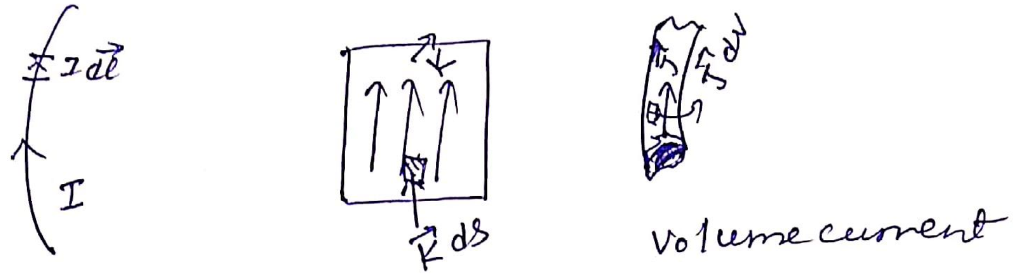
$$\oint_S \vec{J} \cdot \vec{ds} = 0$$

i.e The total current crossing any closed surface is zero, this condition may be satisfied only by assuming a current flow around a closed path.

So, the current flowing in a closed circuit must be the experimental source rather than the differential element. Now, the integral form of Biot-Savart's Law can be;

$$\boxed{\vec{H} = \oint \frac{I\,\vec{dl} \times \hat{a}_R}{4\pi R^2} = \oint \frac{I\,\vec{dl} \times \vec{R}}{4\pi R^3}}$$

As, we can have different charge configurations (Point, Line, Surface, Volume charges); we can also have different current distributions: line current, surface current and volume current:



Line current Surface current

If we define current density as $\vec{J}$, surface current density as $\vec{K}$, then the source elements are related as:

$$I\vec{dl} \equiv \vec{K}\,ds \equiv \vec{J}\,dv$$

where, $\vec{K}$ is measured in Ampere per meter (A/m)
$\vec{J}$ is measured in Ampere per meter square (A/m^2)

Thus, in terms of current distributions, the Biot-Savart's Law can be expressed as:

$$\vec{H} = \int_L \frac{I\vec{dl} \times \hat{a}_R}{4\pi R^2} \quad \text{(line current)}$$

$$\vec{H} = \int_S \frac{\vec{K}\,ds \times \hat{a}_R}{4\pi R^2} \quad \text{(surface current)}$$

$$\vec{H} = \int_{Vol.} \frac{\vec{J}\,dv \times \hat{a}_R}{4\pi R^2} \quad \text{(volume current)}$$

If the surface current density $\vec{K}$ is uniform, the total current any width b is

$\odot$ $\vec{H}$ or $I \Rightarrow$ is out $\otimes$ $\vec{H}$ or I is in

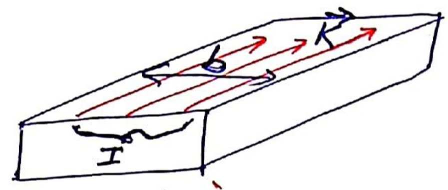
$$I = Kb$$

where, width b is measured perpendicularly to the direction in which the current is flowing.

For non uniform surface current density;

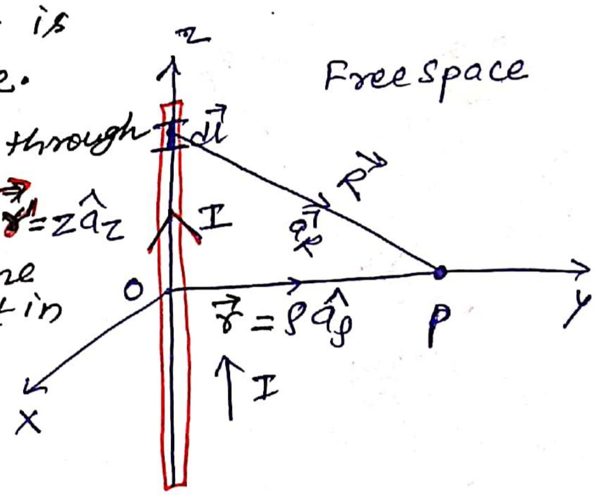
$$I = \int K \, dN$$

where dN is a differential element of the path across which the current is flowing



Magnetic Field Intensity Due to Infinite Line Current

Let us consider an infinite line current is placed along z-axis as shown in figure. Let I amount of current is flowing through the line current. Let us assume the point P at which we need to determine the magnetic field and let us choose it in $z=0$ plane for simple assumptions. Then, the position vector of point P will be $\vec{r} = \rho \hat{a}_\rho$.



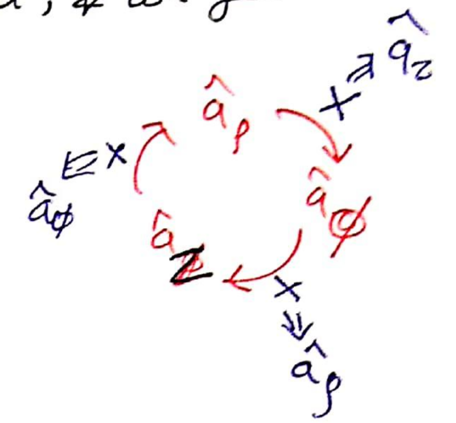
Again, let us assume the differential element $\vec{dl}$ at $(0,0,z)$ of the infinite line current then the position vector of that differential element will be $\vec{r}' = z\hat{a}_z$; so, the distance bet.^{n} point P & differential element $\vec{dl}$ will be $\vec{R} = \vec{r} - \vec{r}'$

i.e $\vec{R} = \vec{r} - \vec{r}' = \rho \hat{a}_\rho - z\hat{a}_z$

& $$\hat{a}_R = \frac{\vec{R}}{|\vec{R}|} = \frac{\rho \hat{a}_\rho - z\hat{a}_z}{\sqrt{\rho^2 + z^2}}$$

If we take $\vec{dl} = dz\,\hat{a}_z$ then $$\vec{dH} = \frac{I dz\,\hat{a}_z \times (\rho \hat{a}_\rho - z\hat{a}_z)}{4\pi (\rho^2 + z^2)^{3/2}}$$

since, the current is directed toward increasing value of z, the limits are $-\infty$ and ∞ on the integral, & we get

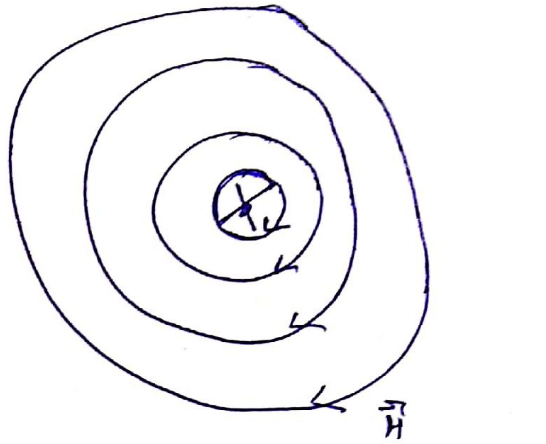


$$\vec{H}_P = \int_{-\infty}^{\infty} \frac{I\,dz\,\hat{a}_z \times (\rho\hat{a}_\rho - z\hat{a}_z)}{4\pi(\rho^2+z^2)^{3/2}}$$

$$= \frac{I}{4\pi}\int_{-\infty}^{\infty} \frac{\rho\,dz\,\hat{a}_\phi}{(\rho^2+z^2)^{3/2}}$$

$$= \frac{I\rho\hat{a}_\phi}{4\pi}\int_{-\infty}^{\infty} \frac{dz}{(\rho^2+z^2)^{3/2}}$$

$$= \frac{I\rho\hat{a}_\phi}{4\pi}\cdot \frac{z}{\rho^2\sqrt{\rho^2+z^2}}\Bigg|_{-\infty}^{\infty}$$

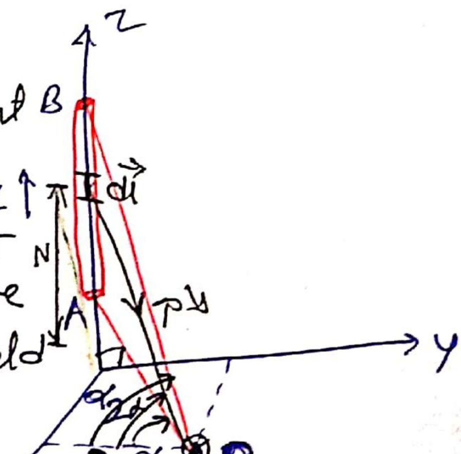


$$\therefore \boxed{\vec{H}_P = \frac{I}{2\pi\rho}\hat{a}_\phi}$$

The magnitude of the field is not a function of ϕ or z and it varies inversely as the distance from the filament or line current. The direction of magnetic field intensity vector is circumferential. Therefore, the streamlines are circles about the filament.

Magnetic field intensity due to finite length current element

Let us consider the finite length current element is placed along the z-axis as shown in figure. Let us assume a point P in z=0 plane as shown in figure where we need to determine the magnetic field intensity due to the finite current element. Let α_1 & α_2 be the angles formed by two ends of the element with y-axis as

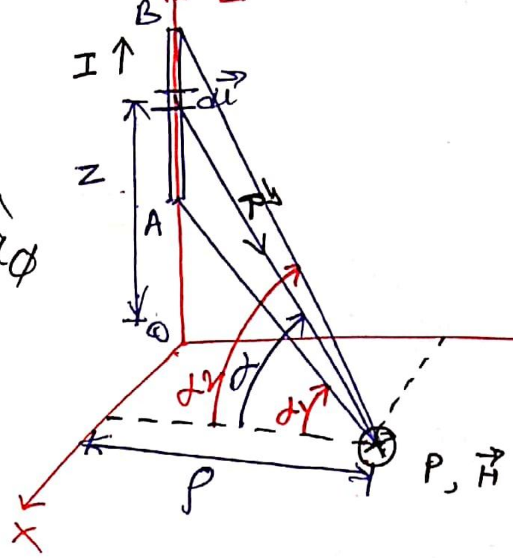


shown in figure. Then, assuming the differential element at $(0,0,z)$; we can have differential field $d\vec{H}$ at P due to an element $d\vec{l}$ at $(0,0,z)$, given as:

$$d\vec{H} = \frac{I\, d\vec{l} \times \vec{R}}{4\pi R^3}$$

But $d\vec{l} = dz\,\hat{a}_z$ and $\vec{R} = \rho\hat{a}_\rho - z\hat{a}_z$, so

$$d\vec{l} \times \vec{R} = \rho\, dz\,\hat{a}_\phi$$



Hence,
$$\vec{H}_p = \int_A^B \frac{I\rho\, dz}{4\pi\left[\rho^2+z^2\right]^{3/2}}\,\hat{a}_\phi$$

Letting $z = \rho\tan\alpha$

$dz = \rho\sec^2\alpha\, d\alpha$

Then,

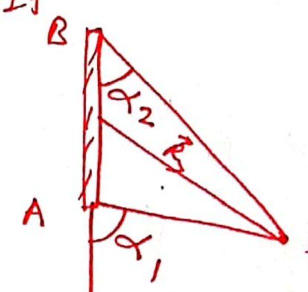
$$\vec{H}_p = \frac{I}{4\pi}\int_{\alpha_1}^{\alpha_2} \frac{\rho^2\sec^2\alpha\, d\alpha}{\rho^3\sec^3\alpha}\,\hat{a}_\phi$$

$$= \frac{I}{4\pi\rho}\,\hat{a}_\phi\int_{\alpha_1}^{\alpha_2}\cos\alpha\, d\alpha$$

$$= \frac{I}{4\pi\rho}\left[\sin\alpha\right]_{\alpha_1}^{\alpha_2}\,\hat{a}_\phi$$

$$\therefore\ \boxed{\vec{H}_p = \frac{I}{4\pi\rho}\left[\sin\alpha_2 - \sin\alpha_1\right]\hat{a}_\phi}$$

If



Then

$\vec{H} = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\hat{a}_\phi$

If one or both ends are below point P then α_1 or both α_1 & α_2 are negative. This expression can be used to find the magnetic field intensity caused by current filaments arranged as a sequence of straight line segments.

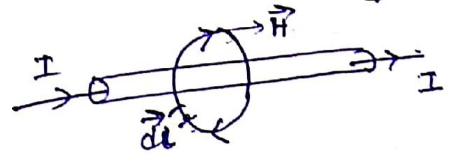
Ampere's Circuital Law or Ampere's Work law

It states that the line integral of $\vec{H}$ (tangential component) around a closed path is the same as the net current $I_{enclosed}$ enclosed by the path.

In other words, the circulation of $\vec{H}$ equals $I_{enclosed}$;

i-e

$$\boxed{\oint \vec{H} \cdot \vec{dl} = I_{enclosed}}$$

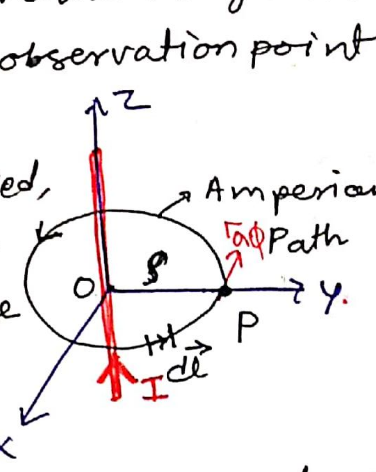


It is similar to Gauss's Law and used to determine $\vec{H}$ when the current distribution is symmetrical. Ampere's circuital law is special case of Biot-Savart's Law.

Application of Ampere's Circuital Law

① Magnetic field Intensity due to infinite long filament carrying current I. (or infinitely long current element)

Consider an infinitely long filamentary current I along the z-axis as shown in figure. To determine $\vec{H}$ at an observation point P, we choose a closed path passing through P. This path on which Ampere's law is to be applied, is known as an Amperian path (analogous to the term Gaussian Surface). Here, we choose a concentric circle around the current element as shown in figure; where $\vec{H}$ is constant provided ρ is constant. Since this path encloses the total current I; from Ampere's Law



$$I = \oint \vec{H} \cdot \vec{dl} = \int_{\phi=0}^{2\pi} H_\phi \hat{a}_\phi \cdot \rho \, d\phi \, \hat{a}_\phi$$

$$= \int_{\phi=0}^{2\pi} H_\phi \rho \, d\phi$$

$$= H_\phi \cdot 2\pi\rho$$

Conditions For Ampere's Circuital Law

① Choose a path to any section of which $\vec{H}$ is either perpendicular or tangential and along which H is constant.

② For normal component of $\vec{H}$ to the $\vec{dl}$ the dot product $\vec{H} \cdot \vec{dl}$ is zero.

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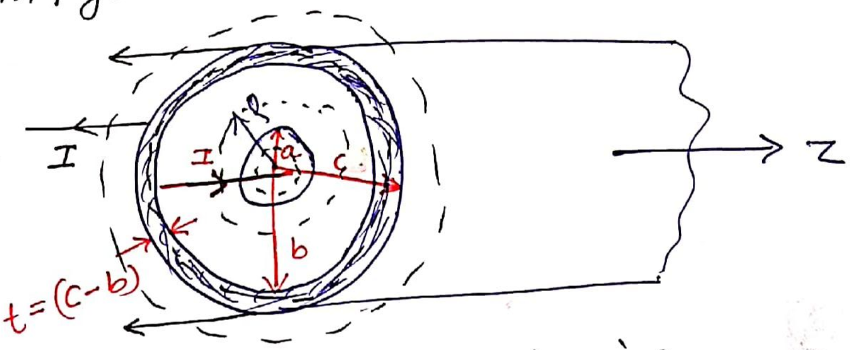
or, $H_\phi = \dfrac{I}{2\pi\rho}$

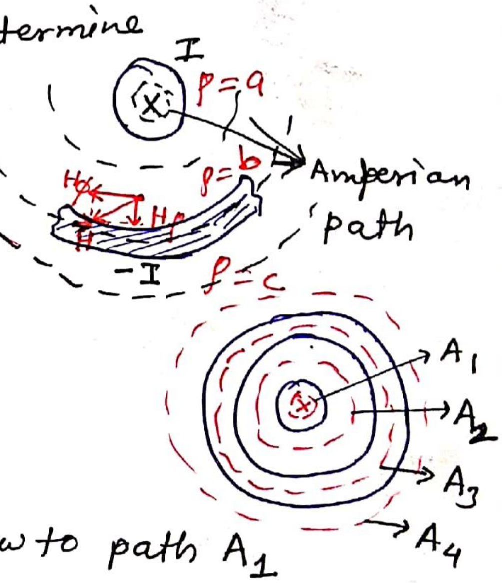
$$\therefore \boxed{\vec{H} = \frac{I}{2\pi\rho}\,\hat{a}_\phi} \rightarrow \text{tangential Component only exists.}$$

② Magnetic field intensity due to an infinitely long Co-axial Transmission line carrying a uniformly distributed current I

Consider an infinitely long transmission (Co-axial) line consisting of two Concentric cylinders having their axes along the z-axis as shown in figure. The inner Conductor has radius a and carries current I while the outer Conductor has inner radius b and outer radius c, having thickness t = c − b & carries return current −I. We want to determine $\vec{H}$ everywhere assuming that current is uniformly distributed in both conductors.



Since the current distribution is symmetrical, we apply Ampere's Law along the amperian path for each of the four possible region $0 \le \rho \le a$, $a \le \rho \le b$, $b \le \rho \le c$ & $\rho \ge c$.



For region $0 \le \rho \le a$, we apply Ampere's Law to path A_1

$$\oint_{A_1} \vec{H}\cdot\vec{dl} = I_{enclosed} \simeq \int \vec{J}\cdot\vec{ds}$$

Since, the current is uniformly distributed over the cross section,

$$\vec{J} = \frac{I}{\pi a^2}\,\hat{a}_z \,, \quad \vec{ds} = \rho\, d\phi\, d\rho\, \hat{a}_z$$

$$\therefore I_{enclosed} = \int \vec{J}\cdot\vec{ds} = \frac{I}{\pi a^2}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} \rho\, d\phi\, d\rho = \frac{\rho^2}{2}\cdot 2\pi\cdot\frac{I}{\pi a^2} = \frac{I\rho^2}{a^2}$$

So,

$$\oint \vec{H}\cdot\vec{dl} = \frac{I\rho^2}{a^2}$$

or, $H_\phi \int_{\phi=0}^{2\pi} \hat{a}_\phi \cdot \rho\, d\phi\, \hat{a}_\phi = \frac{I\rho^2}{a^2}$ [only tangential component exists]

or, $H_\phi \cdot \rho\, 2\pi = \frac{I\rho^2}{a^2}$

or, $H_\phi = \frac{I\rho}{2\pi a^2}$

$$\boxed{\therefore \vec{H} = \frac{I\rho}{2\pi a^2}\hat{a}_\phi}$$

For region $a \le \rho \le b$, we use path A_2 as the Amperian path

then,

$$\oint \vec{H}\cdot\vec{dl} = I_{enclosed} = I$$

or, $H_\phi \int_{\phi=0}^{2\pi} \rho\, d\phi = I$

or, $H_\phi \cdot 2\pi\rho = I$

or, $H_\phi = \frac{I}{2\pi\rho}$

$$\boxed{\therefore \vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi}$$

For region $b \le \rho \le c$, we use Amperian path A_3.

$$\oint \vec{H}\cdot\vec{dl} = I_{enclosed} = I + \int \vec{J}\cdot\vec{ds}$$

Since, the current is uniformly distributed in outer conductor

$$\vec{J} = \frac{-I}{\pi(c^2-b^2)}\hat{a}_z$$

So,

$$I_{enclosed} = I - \frac{I}{\pi(c^2-b^2)} \int_{\phi=0}^{2\pi} \int_{\rho=b}^{\rho} \rho \, d\rho \, d\phi$$

$$= I\left[1 - \frac{\rho^2 - b^2}{c^2 - b^2}\right]$$

$$= I \frac{\left[c^2 - \cancel{b^2} - \rho^2 + \cancel{b^2}\right]}{c^2 - b^2}$$

$$= I\left(\frac{c^2-\rho^2}{c^2-b^2}\right)$$

$$\therefore \oint \vec{H} \cdot \vec{dl} = I\left(\frac{c^2-\rho^2}{c^2-b^2}\right)$$

$$\text{or, } H_\phi \int_{\phi=0}^{2\pi} \rho \, d\phi = I\left(\frac{c^2-\rho^2}{c^2-b^2}\right)$$

$$\text{or, } H_\phi = \frac{I}{2\pi\rho}\left(\frac{c^2-\rho^2}{c^2-b^2}\right)$$

$$\boxed{\therefore \vec{H} = \frac{I}{2\pi\rho}\left(\frac{c^2-\rho^2}{c^2-b^2}\right)\hat{a}_\phi}$$

For region $\rho \geq c$, we use path A_4,

$$\oint \vec{H} \cdot \vec{dl} = I_{enclosed} = I - I = 0$$

$$\text{or, } H_\phi \int_{\phi=0}^{2\pi} \rho \, d\phi = 0$$

$$\text{or, } H_\phi = 0$$

$$\boxed{\therefore \vec{H} = 0}$$

$$\therefore \vec{H} = \begin{cases} \frac{I\rho}{2\pi a^2}\hat{a}_\phi \, ; & 0 \leq \rho \leq a \\ \frac{I}{2\pi\rho}\hat{a}_\phi \, ; & a \leq \rho \leq b \\ \frac{I}{2\pi\rho}\left(\frac{c^2-\rho^2}{c^2-b^2}\right)\hat{a}_\phi \, ; & b \leq \rho \leq c \\ 0 \, ; & \rho \geq c \end{cases}$$

From above result, if we assume 'b = 3a', 'c = 4a' for outer conductor and 'a' for inner conductor then the value of magnetic field intensity can be plotted in graph as:

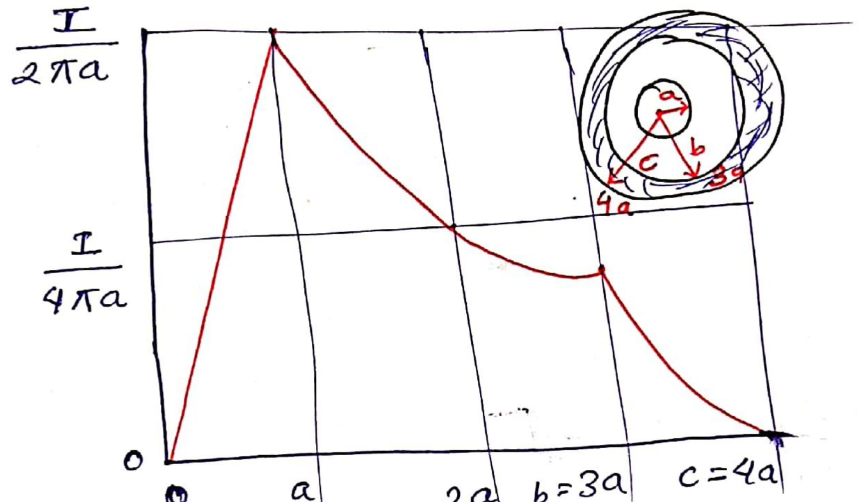
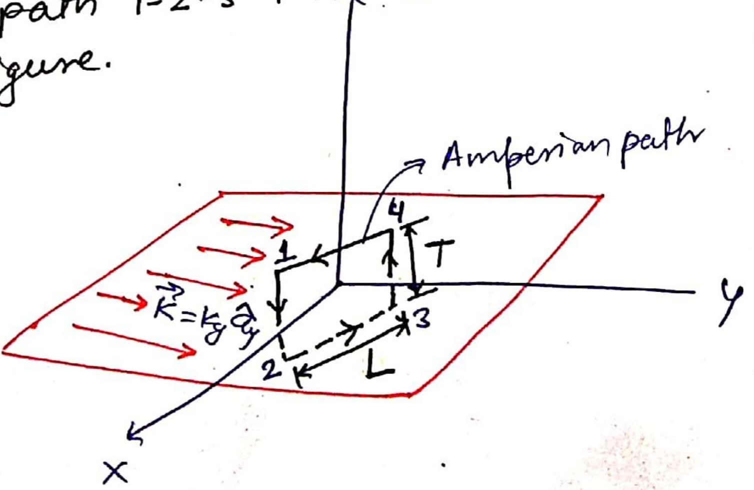
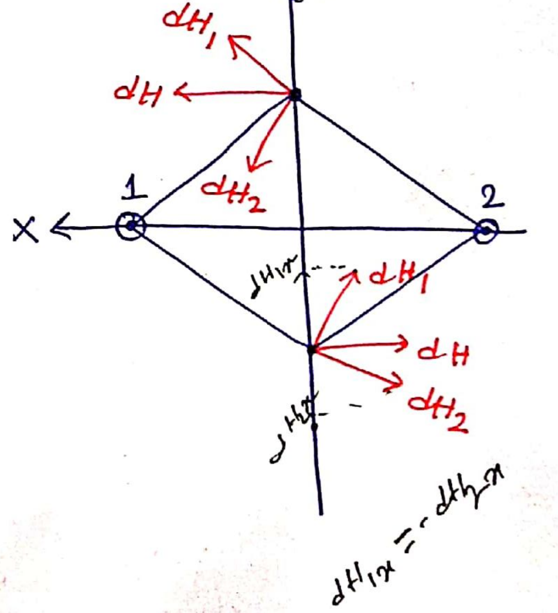


fig:- Magnetic field intensity as a function of radius of an infinitely long co-axial transmission line with dimensions shown in figure.

③ Magnetic field intensity due to infinite sheet of current

Consider an infinite current sheet in the $z=0$ plane. If the sheet has a uniform current density (surface) $\vec{K} = K_y \hat{a}_y$ A/m as shown in figure. Applying Ampere's law to the rectangular closed path 1-2-3-4-1 as in figure.





We get,

$$\oint \vec{H}\cdot d\vec{l} = I_{enclosed} = K_y L$$

Here, to evaluate the left integral part, we regard the infinite sheet as comprising of filaments; $d\vec{H}$ above or below the sheet due to a pair of filamentary currents. So, due to symmetry, the resultant $d\vec{H}$ has only an x-component and $\vec{H}$ on one side of the sheet is the negative of that on the other side. Due to infinite extent of the sheet, the sheet can be regarded as consisting of such filamentary pairs so that the characteristics of $\vec{H}$ for a pair are the same for the infinite current sheets.

i.e
$$\vec{H} = \begin{cases} H_0 \hat{a}_x; & z > 0 \\ -H_0 \hat{a}_x; & z < 0 \end{cases} \quad \text{——— (1)}$$

where, H_0 is to be determined.

From, integral part

$$\oint \vec{H}\cdot d\vec{l} = \left(\int_1^2 + \int_2^3 + \int_3^4 + \int_4^1\right) \vec{H}\cdot d\vec{l}$$

$$= (-H_z)\cdot(-T) + (-H_0)(-L) + (H_z)\cdot(T) + H_0\cdot(L)$$

$$= 0 \times T + H_0 L + 0 \times T + H_0 L \quad [\because H_z = 0]$$
[Symmetry]

$$= 2H_0 L$$

Again,

$$\oint \vec{H}\cdot d\vec{l} = I_{enclosed}$$

or, $2H_0 L = K_y L$

or, $H_0 = \dfrac{K_y}{2}$

Substituting in (1)

$$\boxed{\vec{H} = \begin{cases} \frac{1}{2} K_y \hat{a}_x ; & z > 0 \\ -\frac{1}{2} K_y \hat{a}_x ; & z < 0 \end{cases}}$$

In general form, for an infinite sheet of surface current density $\vec{K}$ A/m,

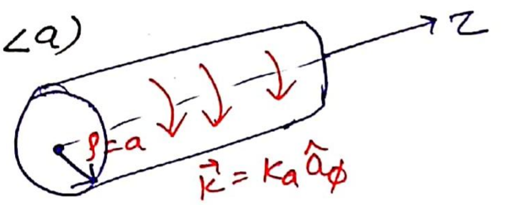
$$\boxed{\vec{H} = \frac{1}{2} \vec{K} \times \hat{a}_N}$$

where, $\hat{a}_N$ is a unit normal vector directed from the current sheet to the point of interest.

(4) Magnetic field intensity due to infinitely long solenoid of radius a and uniform current density $K_a \hat{a}_\phi$.

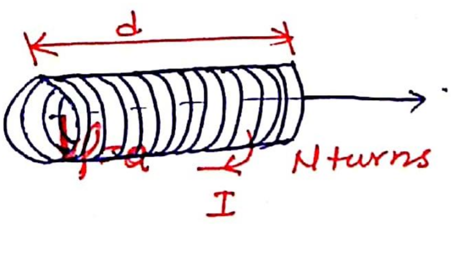
$\vec{H} = K_a \hat{a}_z \ (\rho < a)$

$\vec{H} = 0 \ (\rho > a)$



If the solenoid has a finite length d and consists of N closely wound turns of a filament that carries a current I, then field points within the solenoid is

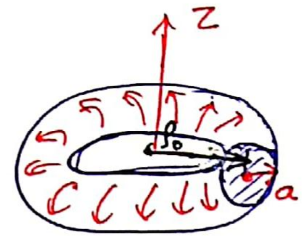
$$\vec{H} = \frac{NI}{d} \hat{a}_z \text{ (within the solenoid)}$$



For toroids, Magnetic field intensity, for ideal case is

$$\vec{H} = K_a \frac{\rho_0 - a}{\rho} \hat{a}_\phi \text{ (inside toroid)}$$

$$\vec{H} = 0 \text{ (outside Toroid)}$$

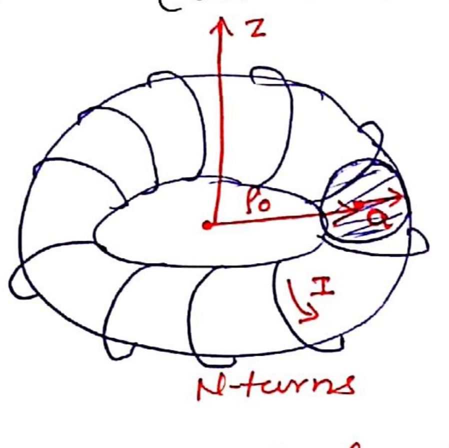


$$\vec{K} = K_a \hat{a}_z \text{ at } \rho = \rho_0 - a, z = 0$$

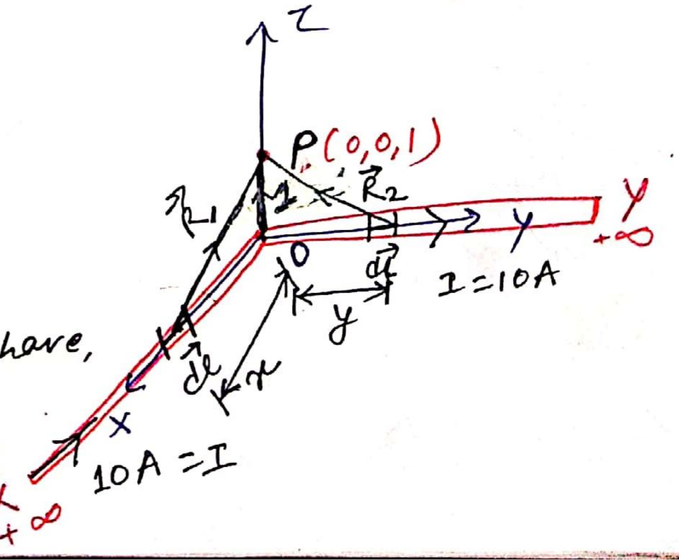
For N-turn toroid.

$$\vec{H} = \frac{NI}{2\pi\rho} \hat{a}_\phi \text{ (inside toroid)}$$

$$\vec{H} = 0 \text{ (outside toroid)}$$



Qn: A filamentary current of 10A is directed in from infinity to the origin on the positive x-axis, and then back out to infinity along the positive y-axis. Use the Biot-Savart's law to find $\vec{H}$ at P(0,0,1).



Soln

From Biot-savart's Law:

$$\vec{dH} = \frac{I \vec{dl} \times \vec{R}}{4\pi R^3}$$

Initially, considering x-axis, we have,

$$\vec{R}_1 = -x\hat{a}_x + 1\hat{a}_z$$

$$= \hat{a}_z - x\hat{a}_x$$

$R_1 = |\vec{R}_1| = \sqrt{x^2+1}$

and $I\,\vec{dl} = 10(-dx\,\hat{a}_x)$ $\quad (I = 10\,A,\ \vec{dl}_1 = dx\,\hat{a}_x)$

Thus, $\quad = -10\,dx\,\hat{a}_x$ $\quad [\therefore I\vec{dl}_1 = -10\,dx\,\hat{a}_x]$

$$\vec{H}_1 = \int_{x=\infty}^{0} \frac{-10\,dx\,\hat{a}_x \times (\hat{a}_z - x\hat{a}_x)}{4\pi\,(x^2+1)^{3/2}}$$

[direction is taken by the integration]

$$= -\frac{10}{4\pi}\int_{x=\infty}^{0} \frac{dx\,\hat{a}_y}{(x^2+1)^{3/2}}$$

$\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$

$$= -\frac{10}{4\pi}\cdot\frac{x}{(x^2+1)^{1/2}}\Big|_{\infty}^{0}\hat{a}_y$$

$$\left[\frac{10}{4\pi}\cdot\frac{\cancel{x}}{\cancel{x}(1+1/x^2)^{1/2}} = \frac{10}{4\pi}\cdot\frac{1}{(1+1/x^2)^{1/2}}\right]$$

$\therefore \vec{H}_1 = \frac{10}{4\pi}\hat{a}_y$ A/m.

Similarly, for y-axis

$\vec{R}_2 = -y\,\hat{a}_y + 1\,\hat{a}_z = \hat{a}_z - y\hat{a}_y$

$R_2 = |\vec{R}_2| = \sqrt{y^2+1}$

& $I\vec{dl} = 10\cdot dy\,\hat{a}_y$

then,

$$\vec{H}_2 = \int_{y=0}^{\infty} \frac{10\,dy\,\hat{a}_y \times (\hat{a}_z - y\hat{a}_y)}{4\pi\,(y^2+1)^{3/2}}$$

$$= \frac{10}{4\pi}\int_{y=0}^{\infty}\frac{dy\,\hat{a}_x}{(y^2+1)^{3/2}}$$

$$= \frac{10}{4\pi}\cdot\frac{y}{(y^2+1)^{1/2}}\Big|_{0}^{\infty}\hat{a}_x$$

$$= \frac{10}{4\pi}\hat{a}_x \text{ A/m}$$

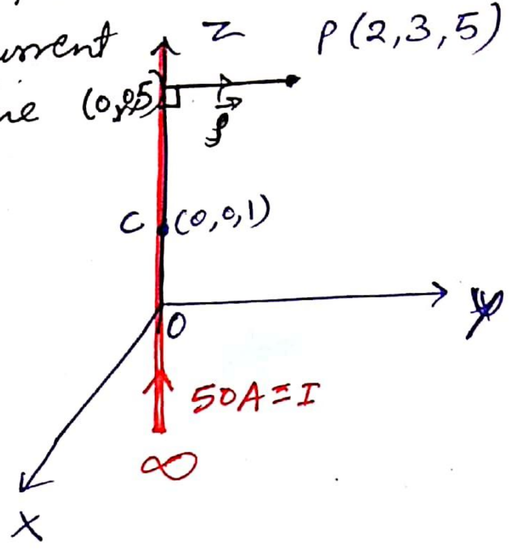
$\therefore$ Total Magnetic field at $P(0,0,1)$ is

$\vec{H} = \vec{H}_1 + \vec{H}_2$

$= \frac{10}{4\pi}\hat{a}_y + \frac{10}{4\pi}\hat{a}_x$ A/m.

Qn: Find $|\vec{H}|$ at $P(2,3,5)$ in Cartesian Co-ordinates if there is an infinitely long current filament passing through the origin and point c. The current of 50 A is directed from the origin to C, where the location of C is (a) $C(0,0,1)$; (b) $C(0,1,0)$.

Sol.: (a) Given, $I = 50\,A$ current is directed from origin to point $C(0,0,1)$



we have, if the infinitely long current filament lies along z-axis then the magnetic field intensity at any point due to this filament is

$$\vec{H} = \frac{I}{2\pi\rho}\,\hat{a}_\phi$$

where, ρ is the radial distance from conductor to the point P.

$$\therefore \vec{\rho} = (2-0)\,\hat{a}_x + (3-0)\,\hat{a}_y + (5-5)\,\hat{a}_z$$

$$= 2\hat{a}_x + 3\hat{a}_y$$

$$\rho = |\vec{\rho}| = \sqrt{2^2+3^2} = \sqrt{13}$$

$$\hat{a}_\rho = \frac{\vec{\rho}}{|\vec{\rho}|} = \frac{2\hat{a}_x + 3\hat{a}_y}{\sqrt{13}}$$

Then, magnetic field intensity at P is,

$$\vec{H} = \frac{50}{2\pi\times\sqrt{13}}\cdot\hat{a}_\phi = \frac{50}{2\pi\times\sqrt{13}}\cdot(\hat{a}_z\times\hat{a}_\rho)$$

$$= \frac{50}{2\pi\times\sqrt{13}}\left(\hat{a}_z\times\frac{\vec{\rho}}{|\vec{\rho}|}\right)$$

$$= \frac{50}{2\pi\times 13}\left[\hat{a}_z\times(2\hat{a}_x+3\hat{a}_y)\right]$$

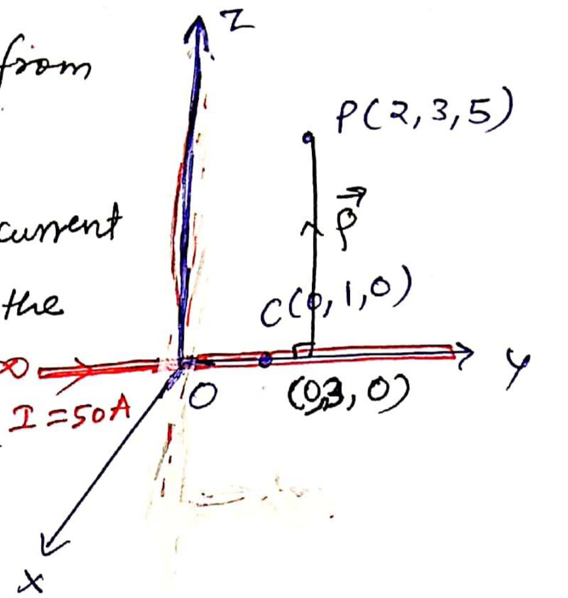
$$= \frac{25}{26\pi}\left(2\hat{a}_y + (-3)\,\hat{a}_x\right)$$

$$= -1.8373\,\hat{a}_x + 1.2248\,\hat{a}_y \ A/m.$$

(b) Again,

given, $I = 50A$ is directed from origin to point $C(0,1,0)$.

We have, if the infinitely long current filament lies along y-axis then the magnetic field intensity at any point due to this filament is



$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

where, ρ is the radial distance from conductor to the point P.

$$\therefore \vec{\rho} = (2-0)\hat{a}_x + (3-3)\hat{a}_y + (5-0)\hat{a}_z = 2\hat{a}_x + 5\hat{a}_z$$

$$\rho = |\vec{\rho}| = \sqrt{2^2+5^2} = \sqrt{29}$$

$$\hat{a}_\rho = \frac{\vec{\rho}}{|\vec{\rho}|} = \frac{2\hat{a}_x + 5\hat{a}_z}{\sqrt{29}}$$

Then, $\vec{H}$ at P is

$$\vec{H} = \frac{50}{2\pi \times \sqrt{29}}\hat{a}_\phi = \frac{50}{2\pi\times\sqrt{29}}(\hat{a}_y \times \hat{a}_\rho) = \frac{50}{2\pi\times\sqrt{29}}\left(\hat{a}_y \times \frac{\vec{\rho}}{|\vec{\rho}|}\right)$$

$$= \frac{50}{2\pi \times 29}\left[\hat{a}_y \times (2\hat{a}_x + 5\hat{a}_z)\right]$$

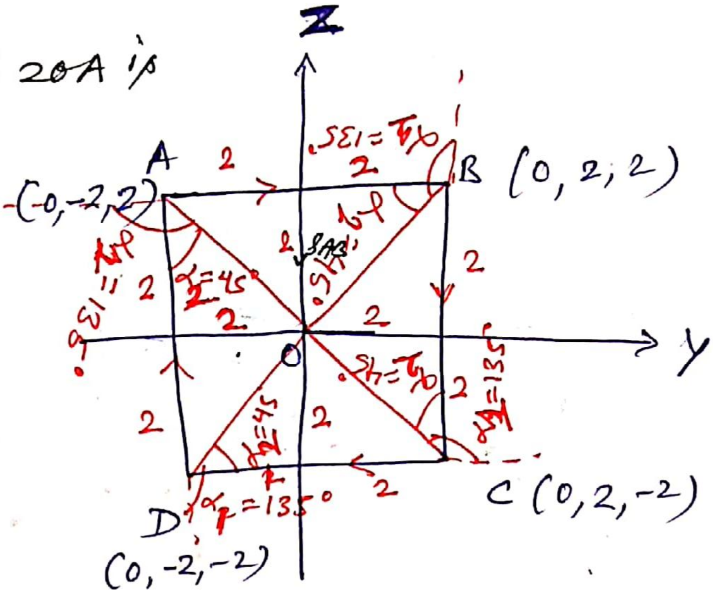
$$= \frac{25}{29\pi}(-2\hat{a}_z + 5\hat{a}_x)$$

$$= 1.372\hat{a}_x - 0.5490\hat{a}_z \text{ A/m}.$$

Qn: A current carrying square loop with vertices $A(0,-2,2)$, $B(0,2,2)$, $C(0,2,-2)$ & $D(0,-2,-2)$ is carrying a dc current of 20 A in the direction along A-B-C-D-A. Find magnetic field intensity $\vec{H}$ at center of the current carrying loop.

[2068 Chaitra]

Sol.ⁿ Given, a current of 20A is carried by the square loop A-B-C-D-A as shown in figure. From figure the center point of the square loop is origin i.e $O(0,0,0)$.



Now, taking AB - portion

$\vec{H}$ due to AB at origin is

$$\vec{H}_{AB} = \frac{I_{AB}}{4\pi \rho_{AB}} (\cos\alpha_2 - \cos\alpha_1)\, \hat{a}_\phi$$

where, $\rho_{AB} = 2$

$\cos\alpha_1 = \cos 135° = -\frac{1}{\sqrt{2}}$

$\cos\alpha_2 = \frac{1}{\sqrt{2}}$

& $\hat{a}_\phi = \hat{a}_y \times (-\hat{a}_z)$
$= -\hat{a}_x$

$$\therefore \vec{H}_{AB} = \frac{20}{4\pi \times 2}\left[+\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right](-\hat{a}_x)$$

$$= \frac{5}{2\pi}\left[+\sqrt{2}\right](-\hat{a}_x)$$

$$= -\frac{5}{\sqrt{2}\pi}\hat{a}_x \text{ A/m.}$$

Again,

taking $\vec{H}_{BC} = \frac{I_{BC}}{4\pi \rho_{BC}} [\cos\alpha_2 - \cos\alpha_1] \hat{a}_\phi$

where,

$\rho_{BC} = 2$ & $\hat{a}_\phi = (-\hat{a}_z) \times (-\hat{a}_y)$

$\cos\alpha_1 = \cos 135^\circ = -\frac{1}{\sqrt{2}}$ $= -\hat{a}_x$

$\cos\alpha_2 = \cos 45^\circ = \frac{1}{\sqrt{2}}$

$\therefore \vec{H}_{BC} = \frac{20}{4\pi \times 2} [+\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}](-\hat{a}_x)$

$= -\frac{5}{\sqrt{2}\pi} \hat{a}_x$ A/m.

Due to symmetry we can write

$\vec{H}_{CD} = \vec{H}_{AB} = -\frac{5}{\sqrt{2}\pi} \hat{a}_x$ & $\vec{H}_{DA} = \vec{H}_{BC} = \frac{-5}{\sqrt{2}\pi} \hat{a}_x$.

$\therefore$ Total magnetic field intensity at origin due to square loop is

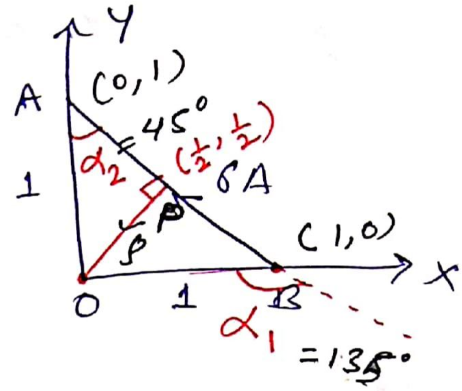
$\vec{H} = \vec{H}_{AB} + \vec{H}_{BC} + \vec{H}_{CD} + \vec{H}_{DA}$

$= -\frac{5}{\sqrt{2}\pi} \times 4 \hat{a}_x$

$= -\frac{10\sqrt{2}}{\pi} \hat{a}_x$ A/m.

Qn: Consider the figure below where AB is part of an electric circuit. Find $\vec{H}$ at the origin due to AB.

Sol: Given, AB is part of an electric circuit where, $I = 6A$ is flowing from B to A as in figure.



Now, $\vec{H}$ at origin due to BA is

$$\vec{H}_{BA} = \frac{I_{BA}}{4\pi \rho_{BA}} [\cos\alpha_2 - \cos\alpha_1] \hat{a}_\phi$$

where, $\rho_{BA} = \sqrt{(0-\frac{1}{2})^2 + (0-\frac{1}{2})^2} = \sqrt{\frac{2}{4}} = \frac{1}{\sqrt{2}}$

$\cos\alpha_2 = \cos 45° = \frac{1}{\sqrt{2}}$

$\cos\alpha_1 = \cos 135° = -\frac{1}{\sqrt{2}}$

$$\hat{a}_{BA} = \frac{(0-1)\hat{a}_x + (1-0)\hat{a}_y}{\sqrt{(0-1)^2 + (1-0)^2}} = \frac{-\hat{a}_x + \hat{a}_y}{\sqrt{2}}$$

$$\hat{a}_{\rho 0} = \frac{(0-\frac{1}{2})\hat{a}_x + (0-\frac{1}{2})\hat{a}_y}{\sqrt{\frac{1}{4}+\frac{1}{4}}} = \frac{-\frac{1}{2}(\hat{a}_x + \hat{a}_y)}{\frac{\sqrt{2}}{2}} = \frac{-\hat{a}_x - \hat{a}_y}{\sqrt{2}}$$

$\therefore \hat{a}_\phi = \hat{a}_{BA} \times \hat{a}_{\rho 0}$

$$= \left(\frac{-\hat{a}_x + \hat{a}_y}{\sqrt{2}}\right) \times \left(\frac{-\hat{a}_x - \hat{a}_y}{\sqrt{2}}\right)$$

$$= \frac{\hat{a}_z}{\sqrt{2}\times\sqrt{2}} + \frac{\hat{a}_z}{\sqrt{2}\times\sqrt{2}}$$

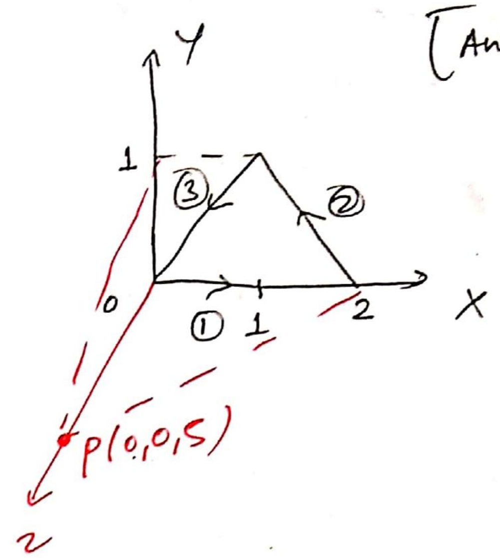
$$= \frac{2}{2}\hat{a}_z$$

$$= \hat{a}_z$$

$$\therefore \vec{H}_{BA} = \frac{6}{4\pi \times \frac{1}{\sqrt{2}}} \left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right] \hat{a}_z = \frac{3}{2\pi \times \frac{1}{\sqrt{2}}} \cdot \frac{2}{\sqrt{2}} \hat{a}_z = \frac{3}{\pi} \hat{a}_z \text{ A/m}$$

$$= 0.954 \hat{a}_z \text{ A/m.}$$

Qn: The conducting triangle loop (in figure below) carries a current of 10A. Find $\vec{H}$ at $P(0,0,5)$ due to side 1 & side 3 of the loop. [2068 Baishakh]



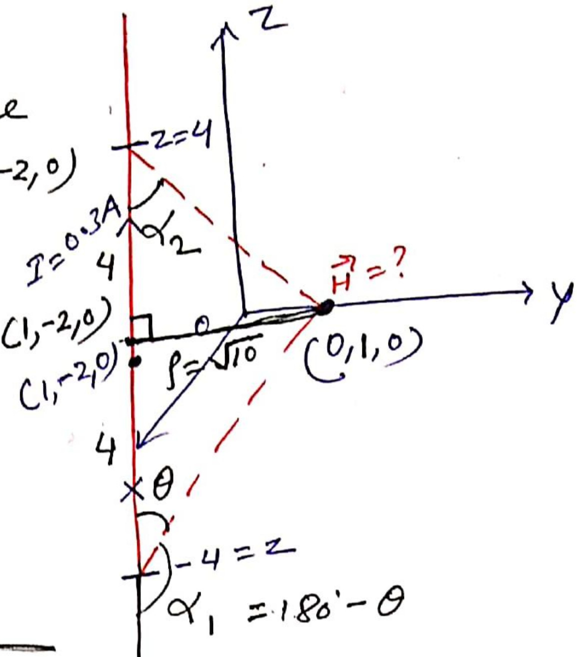
[Ans: side 1 ⇒ $-0.0592\,\hat{a}_y$ A/m.
side 3 ⇒ $-0.03\,\hat{a}_x + 0.03\,\hat{a}_y$ A/m]

3) current of 0.3A in the $\hat{a}_z$ direction in free space is in filament parallel to the z-axis and passing through the point $(1,-2,0)$. Find the magnetic field intensity $\vec{H}$ at $(0,1,0)$ if the filament lies in the interval $-4<z<4$.

[2070 Ashad]

Sol.ⁿ Given, the filament carrying $I=0.3A$ current parallel to the z-axis and passing through $(1,-2,0)$ lies in interval $-4<z<4$.

Then, $\vec{H}$ at $(0,1,0)$ due to the filament is



$$\vec{H} = \frac{I}{4\pi\rho}\left[\cos\alpha_2 - \cos\alpha_1\right]\hat{a}_\phi$$

where, $\rho = \sqrt{(0-1)^2+(1+2)^2+(0-0)^2}$

$= \sqrt{10}$

$$\cos\alpha_2 = \frac{4}{\sqrt{4^2+10}} = \frac{4}{\sqrt{26}}$$

$$\cos\alpha_1 = \cos(180°-\theta) = -\cos\theta = -\frac{4}{\sqrt{26}}$$

and $\hat{a}_\phi = \hat{a}_z \times \hat{a}_\rho$

$$= \hat{a}_z \times \frac{(-\hat{a}_x + 3\hat{a}_y)}{\sqrt{10}}$$

$$= \frac{-\hat{a}_y - 3\hat{a}_x}{\sqrt{10}}$$

$$\therefore \vec{H} = \frac{0.3}{4\pi\times\sqrt{10}}\left[\frac{4}{\sqrt{26}} + \frac{4}{\sqrt{26}}\right]\left(\frac{-\hat{a}_y - 3\hat{a}_x}{\sqrt{10}}\right)$$

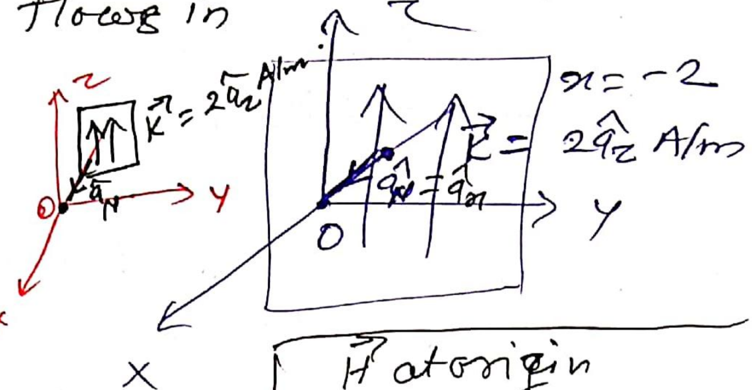
$$= \frac{0.3}{40\pi}\times\frac{8}{\sqrt{26}}\left(-\hat{a}_y - 3\hat{a}_x\right)$$

$$= 0.00375\left(-\hat{a}_y - 3\hat{a}_x\right) = -0.01124\hat{a}_x - 0.00375\hat{a}_y \text{ A/m.}$$

Qn: Find magnetic field intensity at the origin if surface current $\vec{K} = 2\hat{a}_z$ A/m flows in the plane $x = -2$.

[2068 shrawan] ⇒ Need to derive for long question

Sol.ⁿ Given, $\vec{K} = 2\hat{a}_z$ A/m flows in the plane $x = -2$.



then, $\vec{H}$ at origin is

$$\vec{H}_0 = \frac{1}{2}\vec{K} \times \hat{a}_N$$

where, $\hat{a}_N = \hat{a}_x$

$$\therefore \vec{H}_0 = \frac{1}{2} \cdot 2\hat{a}_z \times \hat{a}_x$$

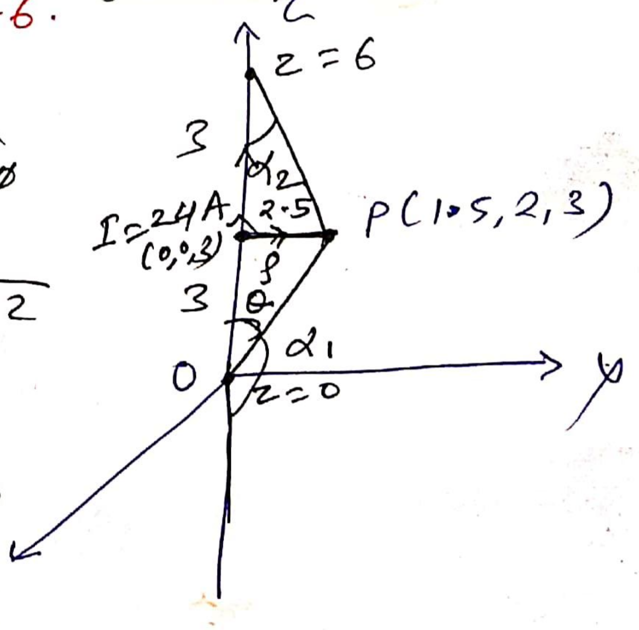
$$= \hat{a}_y$$

$\vec{H}$ at origin

$$= K_z \cdot \frac{1}{2}\hat{a}_y$$

$$= \hat{a}_y$$

Qn: Find the vector magnetic field intensity $\vec{H}$ in Cartesian co-ordinates at P(1.5, 2, 3) caused by a current filament of 24 A in the $\hat{a}_z$ direction on the z-axis and extending from $z = 0$ to $z = 6$.



Sol.ⁿ

$$\vec{H} = \frac{I}{4\pi\rho}[\cos\alpha_2 - \cos\alpha_1]\hat{a}_\phi$$

$$\rho = \sqrt{(1.5-0)^2 + (2-0)^2 + (3-3)^2}$$

$$= 2.5$$

$$\cos\alpha_2 = \frac{3}{\sqrt{3^2 + 2.5^2}} = 0.768$$

$$\cos\alpha_1 = \cos(180° - \theta)$$

$$= -\cos\theta = -\frac{3}{\sqrt{3^2 + 2.5^2}} = -0.768$$

$$\hat{a}_\phi = \hat{a}_z \times \hat{a}_\rho$$

$$= \hat{a}_z \times \frac{(1.5\hat{a}_x + 2\hat{a}_y)}{2.5}$$

$$= \frac{1.5}{2.5}\hat{a}_y - \frac{2}{2.5}\hat{a}_x$$

$$= -0.8\hat{a}_x + 0.6\hat{a}_y$$

$$\therefore \vec{H} = \frac{24}{4\pi \times 2.5}[0.768 + 0.768] \cdot [-0.8\hat{a}_x + 0.6\hat{a}_y]$$

$$= 1.173\,[-0.8\hat{a}_x + 0.6\hat{a}_y]$$

$$= -0.938\,\hat{a}_x + 0.704\,\hat{a}_y \text{ A/m}.$$

Curl of A Vector

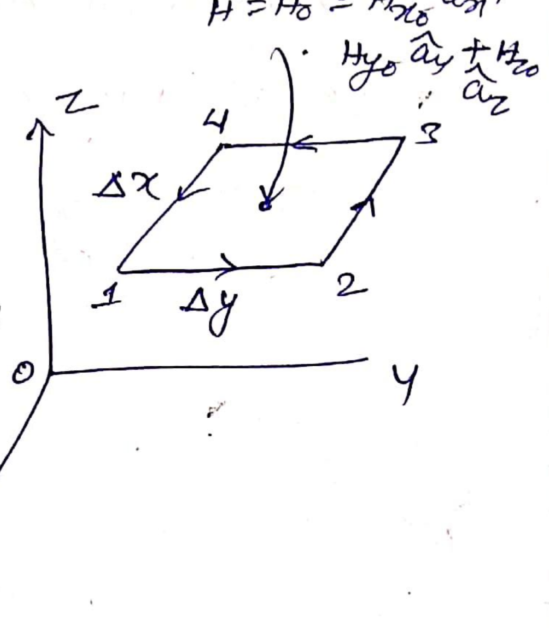
The curl of $\vec{A}$ is an axial (or rotational) vector whose magnitude is the maximum of $\vec{A}$ per unit area as the area tends to zero and whose direction is normal dir^n of the area when the area is oriented so as to make the circulation maximum.

i.e
$$\boxed{\text{curl}\,\vec{A} = \nabla \times \vec{A} = \left(\lim_{\Delta s \to 0} \frac{\oint_L \vec{A} \cdot d\vec{l}}{\Delta s} \right)_{max} \hat{a}_N}$$

where the area Δs is bounded by the curve L and $\hat{a}_N$ is the unit vector normal to the surface Δs and is determined using the right hand rule.

Point form of Ampere's Circuital Law

Let us consider an incremental closed path of sides Δx and Δy as shown in figure. We assume that some current, as yet unspecified, produces a reference value for $\vec{H}$ at the center of the small rectangle formed by closed path, as:



$$\vec{H_0} = H_{x0}\hat{a}_x + H_{y0}\hat{a}_y + H_{z0}\hat{a}_z$$

Then,
$$(\vec{H} \cdot \vec{\Delta L})_{1-2} = H_{y,1-2}\,\Delta y$$

the value of $H_{y,1-2}$ may be given in terms of the reference value H_{y0} at the center of the rectangle, the rate of change of H_y with x, and the distance $\frac{\Delta x}{2}$ from the center of the rectangle to the midpoint of side 1-2

Properties of curl

1. Curl of a vector field is another vector field
2. Curl of a scalar field V, $\nabla \times V$, makes no sense.
3. $\nabla \times (\vec{A} + \vec{B}) = \nabla \times \vec{A} + \nabla \times \vec{B}$
4. $\nabla \times (\vec{A} \times \vec{B}) = \vec{A}(\nabla \cdot \vec{B}) - \vec{B}(\nabla \cdot \vec{A}) + (\vec{B} \cdot \nabla\vec{A} - (\vec{A} \cdot \nabla)\vec{B})$
5. $\nabla \times (V\vec{A}) = V\nabla \times \vec{A} + \nabla V \times \vec{A}$
6. The divergence of the curl of a vector field vanishes.

   i.e $\nabla \cdot (\nabla \times \vec{A}) = 0$
7. The curl of the gradient of a scalar field vanishes.

   i.e $\nabla \times \nabla V = 0$.

$$H_{y,1-2} = H_{y0} + \frac{\partial H_y}{\partial x}\left(\frac{1}{2}\Delta x\right)$$

$$\therefore \left(\vec{H}\cdot\Delta\vec{L}\right)_{1-2} = \left(H_{y0} + \frac{1}{2}\frac{\partial H_y}{\partial x}\Delta x\right)\cdot \Delta y$$

Along, the path 2-3, we have

$$\left(\vec{H}\cdot\Delta\vec{L}\right)_{2-3} = H_{x,2-3}(-\Delta x) = -\left(H_{x0} + \frac{1}{2}\frac{\partial H_x}{\partial y}\Delta y\right)\Delta x$$

Continuing for the path 3-4 & 4-1 and adding the results we get

$$\oint \vec{H}\cdot\vec{dl} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_y}{\partial y}\right)\Delta x\Delta y$$

From Ampere's Circuital Law this value must be equal to the current enclosed by the path or the current crossing any surface bounded by the path. Let the general current density be $\vec{J}$ then the enclosed current is $\Delta I = J_z \Delta x \Delta y$ and

$$\oint \vec{H}\cdot\vec{dl} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\Delta x\Delta y = J_z \Delta x \Delta y$$

or, $$\frac{\oint \vec{H}\cdot\vec{dl}}{\Delta x\Delta y} = \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = J_z$$

As we cause the closed path to shrink, i.e $\Delta x \Delta y \to 0$ then the expression becomes more nearly exact and in the limit we get,

$$\lim_{\Delta x, \Delta y \to 0} \frac{\oint \vec{H}\cdot\vec{dl}}{\Delta x\Delta y} = \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} = J_z$$

If we choose the closed paths which are oriented perpendicularly to each of the remaining two co-ordinate axes, analogous processes lead to expressions for the y and z components of the current density.

$$\lim_{\Delta y, \Delta z \to 0} \frac{\oint \vec{H} \cdot \vec{dl}}{\Delta y \, \Delta z} = \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} = J_x$$

&

$$\lim_{\Delta z, \Delta x \to 0} \frac{\oint \vec{H} \cdot \vec{dl}}{\Delta z \, \Delta x} = \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} = J_y$$

Comparing the above three expressions, we see that a component of the current density is given by the limit of the quotient of the closed line integral of $\vec{H}$ about a small path in a plane normal to that component and of the area enclosed as the path shrinks to zero; called curl. The curl of any vector is a vector, and any component of curl is given by the limit of the quotient of the closed line integral of the vector about a small path in a plane normal to that component desired and the area enclosed, as the path shrinks to zero.

$$\boxed{\therefore \left(\text{Curl } \vec{H}\right)_N = \lim_{\Delta S_N \to 0} \frac{\oint \vec{H} \cdot \vec{dl}}{\Delta S_N}}$$

for Cartesian Co-ordinate

$$\text{Curl } \vec{H} = \nabla \times \vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right) \hat{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right) \hat{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) \hat{a}_z$$

or in the form of determinant

$$\nabla \times \vec{H} = \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

In Cylindrical co-ordinate

$$\nabla \times \vec{H} = \left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right)\hat{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right)\hat{a}_\phi$$

$$+ \left(\frac{1}{\rho}\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{1}{\rho}\frac{\partial H_\rho}{\partial \phi}\right)\hat{a}_z$$

In spherical Co-ordinate

$$\nabla \times \vec{H} = \frac{1}{r\sin\theta}\left(\frac{\partial(H_\phi \sin\theta)}{\partial \theta} - \frac{\partial H_\theta}{\partial \phi}\right)\hat{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial H_r}{\partial \phi} - \frac{\partial(rH_\phi)}{\partial r}\right)\hat{a}_\theta$$

$$+ \frac{1}{r}\left(\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial \theta}\right)\hat{a}_\phi$$

Physical Significance of curl

The circulation of $\vec{H}$ or $\oint \vec{H}\cdot d\vec{l}$ is obtained by multiplying the component of $\vec{H}$ parallel to the specified closed path at each point along it by the differential path length and summing the results as the differential lengths approach zero and as their number becomes infinite.

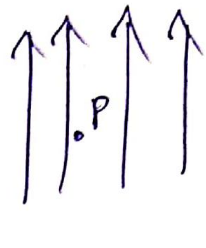
Curl is circulation per unit area. The closed path is vanishingly small and curl is defined at a point.

Physical significance of curl

The curl provides the maximum value of the circulation of the field per unit area (or circulation density) and indicates

the direction along which this maximum value occurs.
The curl of the vector field $\vec{H}$ at a point P may be regarded as a measure of the circulation or how much the field curls around P.

curl at p points out of the page



curl is zero at P

Again,

$$\text{curl } \vec{H} = \nabla \times \vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\hat{a}_x + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\hat{a}_y + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\hat{a}_z = \vec{J}$$

$$\therefore \boxed{\nabla \times \vec{H} = \vec{J}}$$

which is the point form of Ampere's Circuital Law and is Maxwell's Second Equation

from Electric field intensity of closed line

$\oint \vec{E} \cdot \vec{dl} = 0$ we get $\boxed{\nabla \times \vec{E} = 0}$

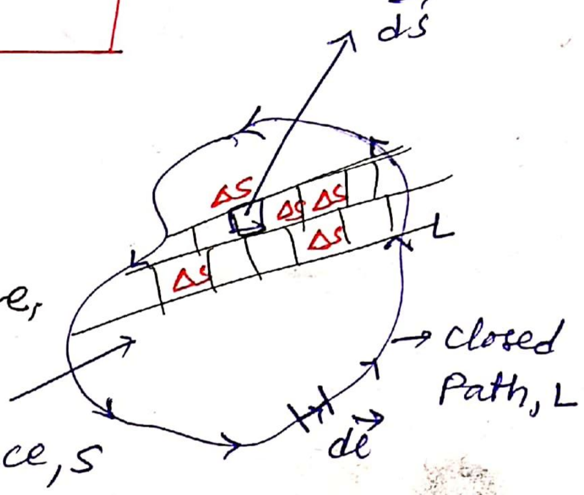
which is Maxwell's third Equation.

Stoke's Theorem

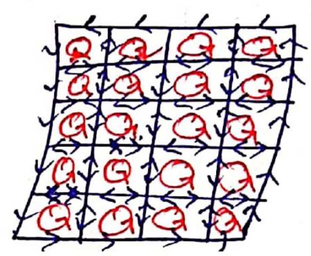
It states that the circulation of a vector field $\vec{H}$ around a (Closed) path L is equal to the surface integral of the curl of $\vec{H}$ over the open surface S bounded by L provided that $\vec{H}$ & $\nabla \times \vec{H}$ are continuous on S.

i-e
$$\boxed{\oint_L \vec{H} \cdot \vec{dl} = \int_S (\nabla \times \vec{H}) \cdot \vec{ds}}$$

Proof



Consider the surface S shown in figure, which is broken up into incremental surfaces of area ΔS. Let the kth incremental surface has area ΔS_K and bounded by path L_K, then



$$\oint_L \vec{H} \cdot \vec{dl} = \sum_K \oint_{L_K} \vec{H} \cdot \vec{dl}$$

$$= \sum_K \frac{\oint_{L_K} \vec{H} \cdot \vec{dl}}{\Delta S_K} \Delta S_K$$

From figure, there is cancellation on every interior path, so the sum of the line integrals around L_K's is the same as the line integral around the bounding curve L. Taking the limit of above expression as $\Delta S_K \to 0$ and from defination of curl.

$$\oint_L \vec{H} \cdot \vec{dl} = \int_S (\nabla \times \vec{H}) \cdot \vec{ds}$$

which is stoke's Theorem

(19)

Magnetic flux density

In free space, magnetic flux density ($\vec{B}$) is defined as:

$$\boxed{\vec{B} = \mu_0 \vec{H}} = \frac{\mu_0}{4\pi}\int_l \frac{I\,\vec{dl} \times \hat{a}_R}{R^2} = \frac{\mu_0}{4\pi}\int_l \frac{I\,\vec{dl} \times \vec{R}}{R^3}$$

where, $\vec{B}$ is measured in webers per meter square. (Wb/m^2) or in a newer unit adopted in the SI units called tesla (T). The constant μ_0 is the permeability of free space with value

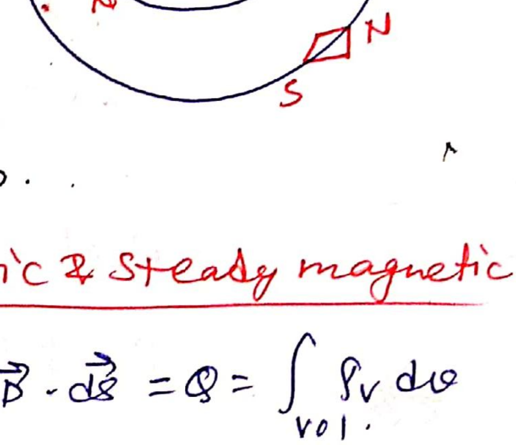
$$\boxed{\mu_0 = 4\pi \times 10^{-7} \ H/m.}$$

Magnetic flux is ~~measured~~ measured in weber and represented by ϕ then,

$$\boxed{\phi = \int_s \vec{B} \cdot \vec{ds} \ \ Wb.}$$ i-e $\vec{B} = \frac{\phi}{Area}$

Gauss's law for magnetic field. is

$$\oint_s \vec{B} \cdot \vec{ds} = 0$$



Applying divergence theorem

$$\boxed{\nabla \cdot \vec{B} = 0}$$

This is Maxwell's Fourth Equation.

Maxwell's Equations for static electric & steady magnetic fields

① $\nabla \cdot \vec{D} = \rho_v$
② $\nabla \times \vec{E} = 0$
③ $\nabla \times \vec{H} = \vec{J}$
④ $\nabla \cdot \vec{B} = 0$

① $\oint_s \vec{D} \cdot \vec{ds} = Q = \int_{vol} \rho_v \, dv$
② $\oint \vec{E} \cdot \vec{dl} = 0$.
③ $\oint_L \vec{H} \cdot \vec{dl} = \int_s \vec{J} \cdot \vec{ds}$
④ $\oint \vec{B} \cdot \vec{ds} = 0$

Gauss's Law for Magnetic field

From definition of flux, $\phi = \int_S \vec{B} \cdot \vec{ds}$ Wb

Comparing this with electric flux as: $\psi = \oint_S \vec{D} \cdot \vec{ds} = Q$

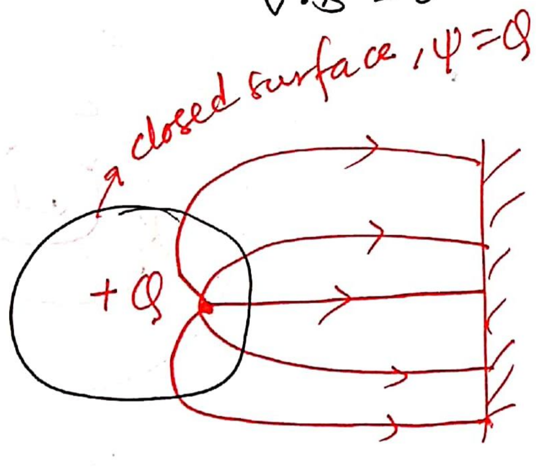
where, the charge Q is the source of the lines of electric flux and these lines begin and terminate on positive and negative charges respectively.

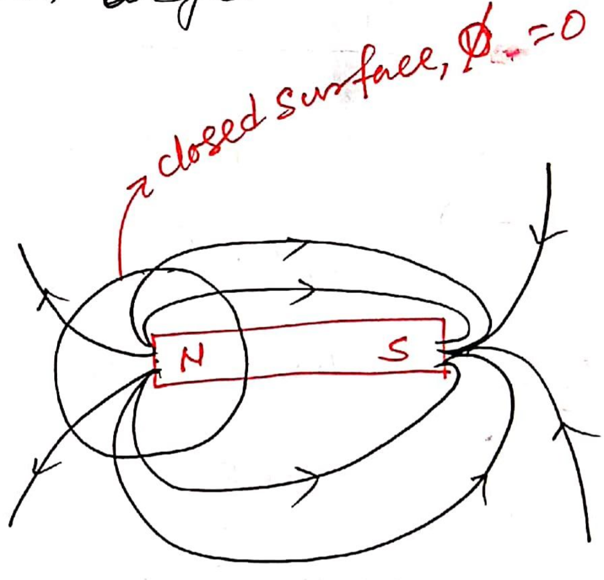
But for magnetic field there is no such source for the lines of magnetic flux. The magnetic flux lines are closed and donot terminate on a "magnetic charge". So, Gauss's law for magnetic field is (No isolated magnetic pole)

$$\boxed{\oint_S \vec{B} \cdot \vec{ds} = 0}$$ — Law of conservation of magnetic flux.

Applying divergence theorem we get

$$\nabla \cdot \vec{B} = 0.$$





electric charge

$$\psi = \oint_S \vec{D} \cdot \vec{ds} = Q$$

magnetic charge

(No isolated pole)

$$\phi = \oint_S \vec{B} \cdot \vec{ds} = 0$$

Magnetic Scalar and Vector Potentials

Relating to electrostatic potential V to the field $\vec{E}$ ($\vec{E} = -\nabla V$); we can assume that there must be the magnetic potential too for magnestatic field $\vec{H}$. So, we define the scalar potential V_m or vector magnetic potential $\vec{A}$ associated with the magnetic field $\vec{H}$. As from previous electrostatic potential we can write,

$$\vec{H} = -\nabla V_m$$

Again, we have

$$\nabla \times \vec{H} = \vec{J}$$

or, $\nabla \times (-\nabla V_m) = \vec{J}$

from property of curl $\nabla \times \nabla V = 0$

$\therefore \vec{J} = 0$ $\Rightarrow \nabla^2 V_m = 0$ Laplace's eqn.

i-e if $\vec{H}$ is to be defined as the gradient of a scalar magnetic potential, then current density must be zero throughout the region in which the scalar magnetic potential is so defined.

$$\boxed{\therefore \vec{H} = -\nabla V_m \quad (\vec{J} = 0)}$$

It is applicable in case of permanent magnets. The dimention of V_m is Ampere.

This scalar potential also satisfies Laplace's Equation.

In free space,

$$\nabla \cdot \vec{B} = \mu_0 \nabla \cdot \vec{H} = 0$$

and hence

$$\mu_0 \nabla \cdot (-\nabla V_m) = 0$$

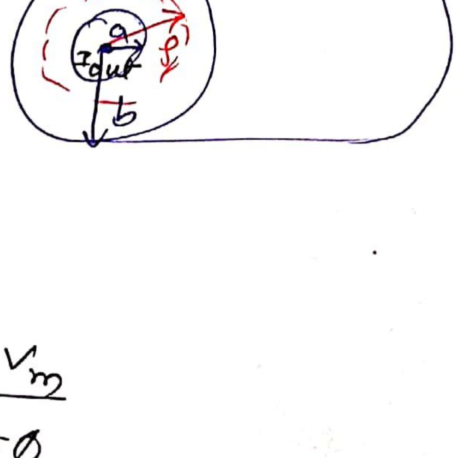
$$\boxed{\text{or, } \nabla^2 V_m = 0 \quad (\vec{J} = 0)}$$

This equation is also valid for homogeneous magnetic materials.

The value of electric potential V is a single valued function of position but V_m is not a single valued function of position.

eg: The co-axial line has field intensity in the region $a < \rho < b$, $\vec{J} = 0$ is

$$\vec{H} = \frac{I}{2\pi\rho} \hat{a}_\phi$$



Then, $\vec{H} = -\nabla V_m$.

$$\text{or, } \frac{I}{2\pi\rho} = -\nabla V_m \Big|_\phi = -\frac{1}{\rho} \frac{\partial V_m}{\partial \phi}$$

$$\text{or, } \frac{\partial V_m}{\partial \phi} = \frac{-I}{2\pi}$$

$$\text{Thus, } V_m = \frac{-I}{2\pi} \phi + C$$

where, $C = 0$ is assumed then $V_m = -\frac{I}{2\pi} \phi$

If we let V_m be zero at $\phi = 0$ and proceed counter clockwise around the circle, the magnetic potential goes negative linearly. When we made a complete cycle the potential is $-I$, but that was the point at which we said the potential was zero. So, value of V_m get multiple values at a point.

$$\therefore \boxed{V_{m(P)} = \frac{I}{2\pi}\left(2n - \frac{1}{4}\right)\pi} \quad (n = 0, \pm 1, \pm 2, \cdots)$$

or, $$V_{m(P)} = I\left(n - \frac{1}{8}\right) \quad (n = 0, \pm 1, \pm 2, \cdots)$$

If $\vec{J} = 0$ then $\nabla \times \vec{H} = 0$ but if we take the closed path enclosing the current the result of $\oint \vec{H} \cdot \vec{dl} = I$ increases by I each time. If no current is enclosed by a path then a single valued potential function may be defined as:

$$V_{m,ab} = -\int_b^a \vec{H} \cdot \vec{dl} \quad \text{(specified path)}$$

Vector Magnetic potential.

This vector field is one which is extremly useful in studying radiation from antennas, from apertures, and radiation leakage from transmission lines, waveguides, and microwave ovens. The vector magnetic potential may be used in regions where the current density is zero or nonzero.

From Maxwell's equation,

$$\nabla \cdot \vec{B} = 0$$

3) then we define a vector (vector magnetic potential), $\vec{A}$ such that

$$\nabla \cdot \vec{B} = \nabla \cdot (\nabla \times \vec{A}) = 0$$

i.e $\boxed{\vec{B} = \nabla \times \vec{A}}$.

where, $\vec{A}$ = Vector magnetic potential, units is weber/m. (wb/m)

From,
$$\vec{B} = \mu_0 \vec{H}$$

or,
$$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{\nabla \times \vec{A}}{\mu_0}$$

and
$$\boxed{\nabla \times \vec{H} = \frac{1}{\mu_0} \nabla \times \nabla \times \vec{A} = \vec{J}}$$

The curl of a curl of a vector is not zero and is given by a fairly complicated expression, which we need to know in general form.

So, from Biot Savart's Law for differential current elements, the definition of $\vec{A}$ may be determined as:

$$\boxed{\vec{A} = \oint_L \frac{\mu_0 I \vec{dl}}{4\pi R}} \qquad \boxed{\vec{A} = \int_S \frac{\mu_0 \vec{K}\, ds}{4\pi R}} \qquad \boxed{\vec{A} = \int_{vol.} \frac{\mu_0 \vec{J}\, dv}{4\pi R}}$$

Again, this may be written in differential form as:

$$\boxed{d\vec{A} = \frac{\mu_0 I \vec{dl}}{4\pi R}}$$

For current sheet, $\vec{K}$
$$I\vec{dl} = \vec{K}\, ds$$

For volume with a density, $\vec{J}$
$$I\vec{dl} = \vec{J}\, dv$$

Poisson's Equation for Magnetic Field
$$\nabla^2 \vec{A} = -\mu_0 \vec{J}$$

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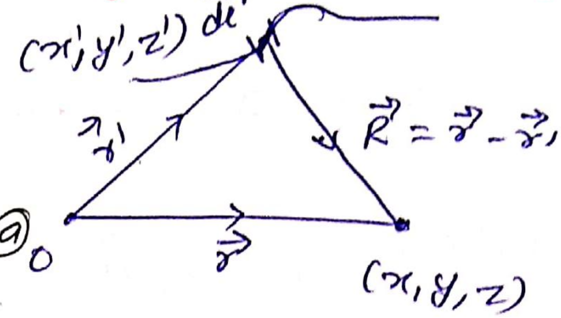
Qn: If magnetic flux density, $\vec{B} = \frac{\mu_0}{4\pi} \int_L \frac{I \, \vec{dl}' \times \vec{R}}{|\vec{R}|^3}$, then derive the expression for vector magnetic potential.

Sol.ⁿ We have,

$$\vec{B} = \frac{\mu_0}{4\pi} \int_L \frac{I \, \vec{dl} \times \vec{R}}{|\vec{R}|^3} \quad \text{— (a)}$$



where, $\vec{R}$ is the distance vector from the line element $\vec{dl}'$ at source point (x', y', z') to the field point (x, y, z) as shown in figure and $|\vec{R}| = R$, i.e

$$R = |\vec{R}| = |\vec{r} - \vec{r}'| = [(x-x')^2 + (y-y')^2 + (z-z')^2]^{\frac{1}{2}}$$

Hence,
$$\nabla\left(\frac{1}{R}\right) = -\frac{(x-x')\hat{a}_x + (y-y')\hat{a}_y + (z-z')\hat{a}_z}{[(x-x')^2 + (y-y')^2 + (z-z')^2]^{3/2}}$$

$$= -\frac{\vec{R}}{R^3}$$

or,
$$\frac{\vec{R}}{R^3} = -\nabla\left(\frac{1}{R}\right) = \frac{\hat{a}_R}{R^2}$$

where, the differentiation is with respect to x, y & z.

Substituting this into (a), we obtain

$$\vec{B} = -\frac{\mu_0}{4\pi} \int_L I \, \vec{dl}' \times \nabla\left(\frac{1}{R}\right)$$

From vector identity

$$\nabla \times (f\vec{F}) = f \, \nabla \times \vec{F} + (\nabla f) \times \vec{F}$$

contd....

Contd...

where f is a scalar field and $\vec{F}$ is a vector field.

Taking $f = \frac{1}{R}$ and $\vec{F} = \vec{dl}'$ we have

$$\vec{dl}' \times \nabla\left(\frac{1}{R}\right) = \frac{1}{R} \nabla \times \vec{dl}' - \nabla \times \left(\frac{\vec{dl}'}{R}\right)$$

Since ∇ operates with respect to (x, y, z) while $\vec{dl}'$ is a function of (x', y', z'), $\nabla \times \vec{dl}' = 0$

Hence

$$\vec{dl}' \times \nabla\left(\frac{1}{R}\right) = -\nabla \times \frac{\vec{dl}'}{R}$$

then, above equation reduces to

$$\vec{B} = \nabla \times \int_L \frac{\mu_0 I \vec{dl}'}{4\pi R}$$

Comparing this equation with $\vec{B} = \nabla \times \vec{A}$

we get,

$$\vec{A} = \int_L \frac{\mu_0 I \vec{dl}'}{4\pi R}$$

Verifying this we can apply stoke's theorem

$$\phi = \int_S \vec{B} \cdot \vec{ds} = \int_S (\nabla \times \vec{A}) \cdot \vec{ds} = \oint_L \vec{A} \cdot \vec{dl}$$

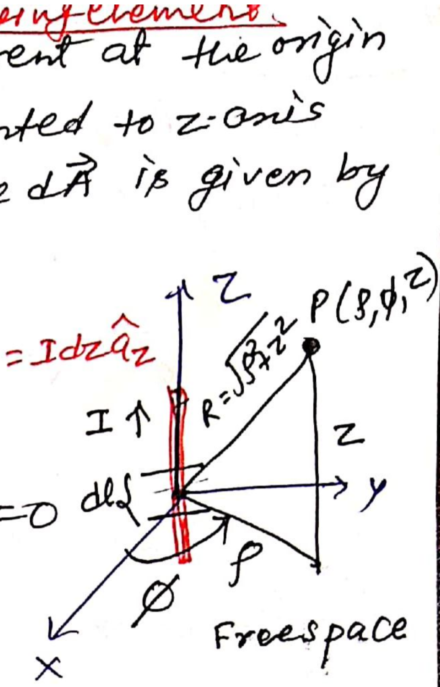
or, $$\boxed{\phi = \oint_L \vec{A} \cdot \vec{dl}}$$

Using Vector Magnetic Potential

1) Magnetic field intensity due to Current carrying element.

Let us Consider the filament carrying current at the origin in freespace as shown in figure. Let it is extended to z-axis so that $\vec{dl} = dz\,\hat{a}_z$ then at $P(\rho,\phi,z)$ the $d\vec{A}$ is given by

$$d\vec{A} = \frac{\mu_0 I dz\,\hat{a}_z}{4\pi\sqrt{\rho^2+z^2}}$$



or, $$dA_z = \frac{\mu_0 I dz}{4\pi\sqrt{\rho^2+z^2}}, \quad dA_\phi = 0 \text{ \& } dA_\rho = 0$$

where, dirn of dA_z is same as dirn of $\vec{dl}$.

then, magnetic field intensity due to the differential element dl is

$$d\vec{H} = \frac{1}{\mu_0}\nabla \times d\vec{A} = \frac{1}{\mu_0}\left(-\frac{\partial dA_z}{\partial \rho}\right)\hat{a}_\phi$$

or, $$d\vec{H} = \frac{Idz}{4\pi}\cdot\frac{-\rho}{(\rho^2+z^2)^{3/2}}\hat{a}_\phi$$

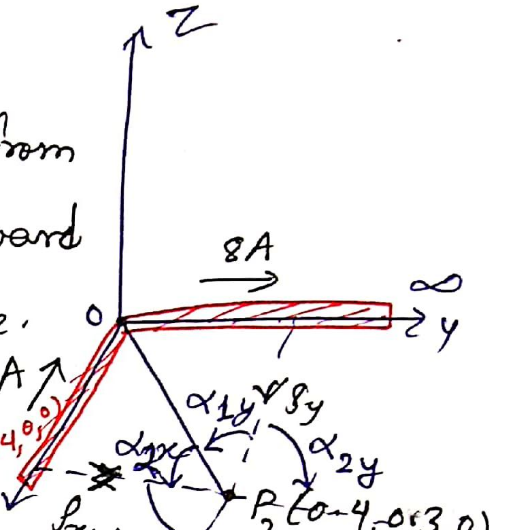
which is same as given by Biot & Savart's Law.

Again, Integrating the above we can get the complete value of $\vec{H}$ as:

$$\vec{H} = \int_{-\infty}^{\infty}\frac{Idz}{4\pi}\frac{\rho}{(\rho^2+z^2)^{3/2}}\hat{a}_\phi$$

$$\therefore \boxed{\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi}$$

Qn: Determine $\vec{H}$ at P_2 (0.4, 0.3, 0) in the field of an 8A filamentary current directed inward from infinity to the origin on the positive x-axis and then outward to ∞ along the y-axis.



Sol.ⁿ Given, the 8A = I current is flowing from ∞ to origin in x-axis and flowing outward from origin to y-axis as shown in figure. The angles α_{1x} & α_{2x}, α_{1y} & α_{2y} are formed at point P_2 by the current carrying elements.

So, the field due to x-axis filament is

$$\vec{H}_x = \frac{I}{4\pi\rho_x}\left[\sin\alpha_{2x} - \sin\alpha_{1x}\right]\hat{a}_\phi$$

where,

$$\rho_x = 0.3$$

$$\alpha_{2x} = \tan^{-1}(0.4/0.3) = 53.1^\circ$$

$$\alpha_{1x} = -90^\circ$$

$$\hat{a}_\phi = -\hat{a}_z$$

$$\therefore \vec{H}_x = \frac{8}{4\pi(0.3)}(\sin 53.1^\circ + 1)\,\hat{a}_\phi$$

$$= \frac{12}{\pi}\,\hat{a}_\phi \text{ A/m.}$$

$$\text{or, } \vec{H}_x = -\frac{12}{\pi}\,\hat{a}_z \text{ A/m.}$$

(b) Again, the field due to y-axis filament is

$$\vec{H}_y = \frac{I}{4\pi\rho_y}(\sin\alpha_{2y} - \sin\alpha_{1y})$$

where,

$$\rho_y = 0.4$$

$$\alpha_{2y} = 90^\circ$$

$$\alpha_{1y} = -\tan^{-1}(0.3/0.4) = -36.9^\circ$$

$$\hat{a}_\phi = -\hat{a}_z$$

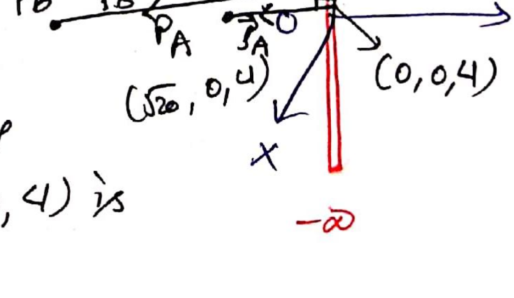
$$\therefore \vec{H}_y = \frac{8}{4\pi(0.4)}[\sin 90^\circ - \sin(-36.9^\circ)]$$

$$= -\frac{8}{\pi}\hat{a}_z \text{ A/m}.$$

$$\therefore \vec{H} = \vec{H}_x + \vec{H}_y = \frac{-20}{\pi}\hat{a}_z = -6.37\hat{a}_z \text{ A/m}.$$

Qn: A current filament carrying 15A in the $\hat{a}_z$ direction lies along the entire z-axis. find $\vec{H}$ in cartesian co-ordinates at: (a) $P_A(\sqrt{20}, 0, 4)$ (b) $P_B(2, -4, 4)$

Sol.ⁿ Given, a 15A current is flowing in the current element placed at z-axis from $-\infty$ to ∞.



Then, the magnetic field intensity due to the filament to $P_A(\sqrt{20}, 0, 4)$ is

(a) $$\vec{H}_A = \frac{I}{2\pi\rho_A}\hat{a}_{\phi A}$$

where, $\vec{\rho}_A = (\sqrt{20} - 0)\hat{a}_x + (0-0)\hat{a}_y + (4-4)\hat{a}_z = \sqrt{20}\,\hat{a}_x$

$$\hat{a}_{\rho_A} = \hat{a}_x \text{ \& } \rho_A = \sqrt{20}$$

and $\hat{a}_{\phi_A} = \hat{a}_z \times \hat{a}_{\rho_A}$

$$= \hat{a}_z \times (\hat{a}_x)$$

$$= \hat{a}_y$$

$$\therefore \vec{H}_A = \frac{15}{2\pi \times \sqrt{20}} \hat{a}_y = 0.5338 \hat{a}_y \text{ A/m}$$

Again,

(b) At point $P_B(2,-4,4)$

$$\vec{H}_B = \frac{I}{2\pi \rho_B} \hat{a}_\phi$$

where,

$$\vec{\rho}_B = (2-0)\hat{a}_x + (-4-0)\hat{a}_y + (4-4)\hat{a}_z$$

$$= 2\hat{a}_x - 4\hat{a}_y$$

$$\rho_B = \sqrt{2^2 + (-4)^2} = \sqrt{20}$$

$$\hat{a}_{\rho_B} = \frac{2\hat{a}_x - 4\hat{a}_y}{\sqrt{20}}$$

$$\& \ \hat{a}_{\phi_B} = \hat{a}_z \times \hat{a}_{\rho_B} = \hat{a}_z \times \frac{(2\hat{a}_x - 4\hat{a}_y)}{\sqrt{20}}$$

$$= \frac{2}{\sqrt{20}} \hat{a}_y + \frac{4}{\sqrt{20}} \hat{a}_x$$

$$\therefore \vec{H}_B = \frac{15}{2\pi \times \sqrt{20}} \left(\frac{2\hat{a}_y + 4\hat{a}_x}{\sqrt{20}} \right)$$

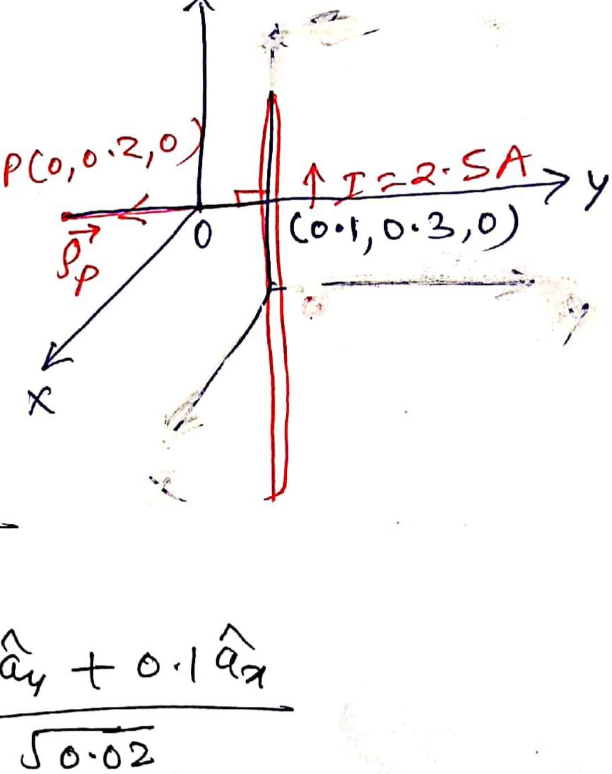
$$= \frac{15}{40\pi} (2\hat{a}_y + 4\hat{a}_x)$$

$$= 0.477\hat{a}_x + 0.2387\hat{a}_y \text{ A/m}.$$

18) Qn: Express the value of $\vec{H}$ in cartesian components at $P(0, 0.2, 0)$ in the field of: (a) a current filament, 2.5 A in the $\hat{a}_z$ direction at $x = 0.1$, $y = 0.3$ (b) a Coaxial centered on the z axis, with $a = 0.3$, $b = 0.5$, $c = 0.6$, $I = 2.5$ A in $\hat{a}_z$ direction in center of conductor. (c) three current sheets $2.7\hat{a}_x$ A/m at $y = 0.1$, $-1.4\hat{a}_x$ A/m at $y = 0.15$; and $-1.3\hat{a}_x$ A/m at $y = 0.25$.

Sol:
(a) Given, $2.5A = I$ current is flowing in a filament passing through $x = 0.1, y = 0.3$ in $\hat{a}_z$ direction. then the field $\vec{H}$ at $P(0, 0.2, 0)$ is

$$\vec{H}_p = \frac{I}{2\pi \rho_p} \hat{a}_{\phi p}$$



where, $\vec{\rho}_p = -0.1\hat{a}_x - 0.1\hat{a}_y$

$$\rho_p = \sqrt{0.02}$$

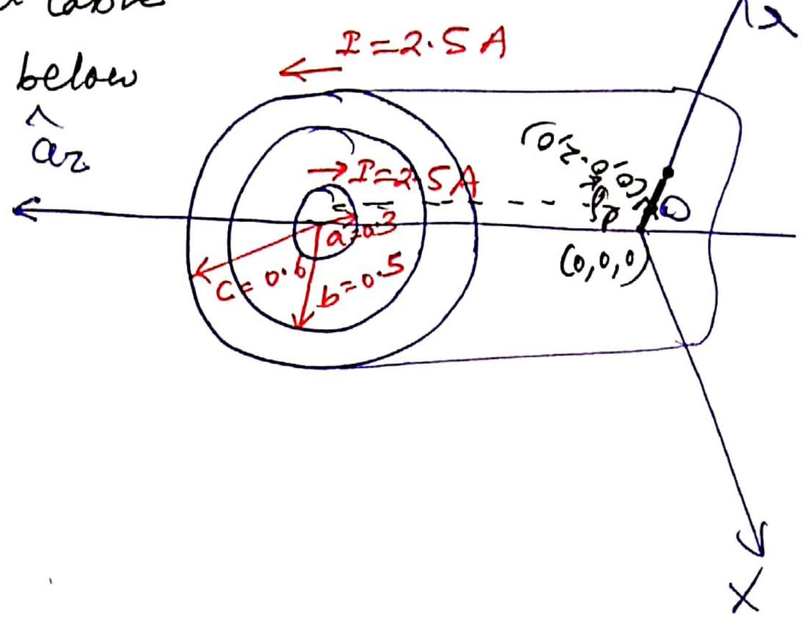
$$\hat{a}_{\rho p} = \frac{-0.1\hat{a}_x - 0.1\hat{a}_y}{\sqrt{0.02}}$$

$$\& \ \hat{a}_{\phi p} = \hat{a}_z \times \hat{a}_{\rho p} = \frac{-0.1\hat{a}_y + 0.1\hat{a}_x}{\sqrt{0.02}}$$

$$\therefore \vec{H}_p = \frac{2.5}{2\pi \times \sqrt{0.02}} \cdot \frac{(0.1\hat{a}_x - 0.1\hat{a}_y)}{\sqrt{0.02}}$$

$$= 1.989\,\hat{a}_x - 1.989\,\hat{a}_y \text{ A/m.}$$

(19) (B) Again, for co-axial cable the point $P(0, 0.2, 0)$ lies below the inner conductor i.e in $0 < \rho < a$. where, the field is given by



$$\vec{H}_P = \frac{I \rho_P}{2\pi a^2} \hat{a}_{\phi_P}$$

where, $\vec{\rho}_P = (0-0)\hat{a}_x + (0.2-0)\hat{a}_y + (0-0)\hat{a}_z = 0.2\hat{a}_y$

$\rho_P = 0.2$

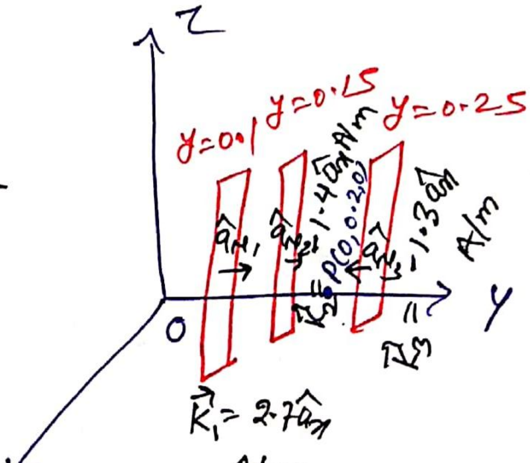
$\hat{a}_{\rho_P} = \hat{a}_y$

& $\hat{a}_{\phi_P} = \hat{a}_z \times \hat{a}_{\rho_P} = \hat{a}_z \times \hat{a}_y = -\hat{a}_x$

$$\therefore \vec{H}_P = \frac{2.5 \times 0.2}{2\pi \times (0.3)^2} (-\hat{a}_x)$$

$$= -0.884 \hat{a}_x \text{ A/m}.$$

(C) Now, for sheets at $y = 0.1$, $y = 0.15$, & $y = 0.25$ the field is given by



$$\vec{H}_P = \frac{1}{2}\vec{K}_1 \times \hat{a}_{N_1} + \frac{1}{2}\vec{K}_2 \times \hat{a}_{N_2} + \frac{1}{2}\vec{K}_3 \times \hat{a}_{N_3}$$

$$= \frac{1}{2} \times 2.7\hat{a}_x \times \hat{a}_y + \frac{1}{2} \times (-1.4)\hat{a}_x \times \hat{a}_y + \frac{1}{2} \times (-1.3)\hat{a}_x \times (-\hat{a}_y)$$

$$\vec{H}_P = \frac{1}{2}\times 2.7\,\hat{a}_z - \frac{1.4}{2}\hat{a}_z + \frac{1.3}{2}\hat{a}_z$$
$$= 1.3\,\hat{a}_z \text{ A/m}.$$

Qn: Calculate the value of vector current density: (a) in cartesian co-ordinates at $P_A(2,3,4)$ if $\vec{H} = x^2 z\,\hat{a}_y - y^2 x\,\hat{a}_z$ (b) in cylindrical co-ordinates at $P_B(1.5, 90°, 0.5)$ if $\vec{H} = \frac{2}{\rho}(\cos 0.2\phi)\,\hat{a}_\rho$ (c) in spherical co-ordinates at $P_C(2, 30°, 20°)$ if $\vec{H} = \frac{1}{\sin\theta}\hat{a}_\theta$.

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Soln (a) Given, $\vec{H} = x^2 z\,\hat{a}_y - y^2 x\,\hat{a}_z$

the vector current density $\vec{J}$ can be evaluate by using point form of Ampere's circuital law as:

$$\vec{J}_A = \nabla\times\vec{H}.$$
$$= \nabla\times(x^2 z\,\hat{a}_y - y^2 x\,\hat{a}_z)$$
$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & x^2 z & -y^2 x \end{vmatrix}$$

$$= \left[\frac{\partial(-y^2x)}{\partial y} - \frac{\partial(x^2z)}{\partial z}\right]\hat{a}_x + \left[\frac{\partial 0}{\partial z} - \frac{\partial(-y^2x)}{\partial x}\right]\hat{a}_y + \left[\frac{\partial(x^2z)}{\partial x} - \frac{\partial 0}{\partial y}\right]\hat{a}_z$$

$$= [-2yx - x^2]\,\hat{a}_x + y^2\hat{a}_y + 2xz\,\hat{a}_z$$

At $P_A(2,3,4)$,
$$\vec{J}_A = [-2\times 3\times 2 - 2^2]\,\hat{a}_x + 3^2\hat{a}_y + 2\times 2\times 4\,\hat{a}_z$$

$\vec{J}_A = -16\hat{a}_x + 9\hat{a}_y + 16\hat{a}_z \ A/m^2$

b) Again,

Given, $\vec{H} = \frac{2}{\rho}(\cos 0.2\phi)\hat{a}_\rho$

$\therefore \vec{J}_B = \nabla \times \vec{H}$

$$= \begin{vmatrix} \hat{a}_\rho & \rho\hat{a}_\phi & \hat{a}_z \\ \frac{\partial}{\partial \rho} & \frac{1}{\rho}\frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ \frac{2}{\rho}(\cos 0.2\phi) & \rho \cdot 0 & 0 \end{vmatrix}$$

or Simplify using formula

$$\nabla \times \vec{H} = \hat{a}_\rho\left(\frac{1}{\rho}\frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z}\right) + \hat{a}_\phi\left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho}\right) + \hat{a}_z\left(\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi}\right)\frac{1}{\rho}$$

$$= (0-0)\hat{a}_\rho + \left[\frac{\partial\left[\frac{2}{\rho}\cos 0.2\phi\right]}{\partial z} - 0\right]\rho\hat{a}_\phi + \left\{0 - \frac{1}{\rho}\frac{\partial\left[\frac{2}{\rho}\cos 0.2\phi\right]}{\partial \phi}\right\}\hat{a}_z$$

$$= -\frac{2}{\rho^2}\frac{\partial(\cos 0.2\phi)}{\partial \phi}\hat{a}_z$$

$$= \frac{2}{\rho^2} \times 0.2 \sin 0.2\phi \ \hat{a}_z$$

$$= \frac{0.4}{\rho^2}\sin 0.2\phi \ \hat{a}_z$$

At $P_B(1.5, 90^\circ, 0.5)$

$$\vec{J}_B = \frac{0.4}{(1.5)^2}\sin 0.2 \times 90^\circ \ \hat{a}_z = 0.0549\,\hat{a}_z \ A/m^2$$

(e) Now, Given, $\vec{H} = \frac{1}{\sin\theta}\hat{a}_\theta$

$\therefore \vec{J}_c = \nabla \times \vec{H}$

$$= \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\hat{a}_\phi \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial\theta} & \frac{\partial}{\partial\phi} \\ 0 & r\cdot\frac{1}{\sin\theta} & r\sin\theta\cdot 0 \end{vmatrix}$$

$$= \frac{1}{r^2\sin\theta}\begin{vmatrix} \hat{a}_r & r\hat{a}_\theta & r\sin\theta\hat{a}_\phi \\ \partial/\partial r & \partial/\partial\theta & \partial/\partial\phi \\ 0 & \frac{r}{\sin\theta} & 0 \end{vmatrix}$$

$$= \frac{1}{r^2\cancel{\sin\theta}}\left[\frac{\partial}{\partial r}\left(\frac{r}{\sin\theta}\right) - 0\right]\hat{a}_\phi \cdot \cancel{(r\sin\theta)}$$

$$= \frac{1}{r}\cdot\frac{1}{\sin\theta}\hat{a}_\phi$$

$$= \frac{1}{r\sin\theta}\hat{a}_\phi$$

At $P_c(2, 30^\circ, 20^\circ)$

$$\vec{J}_c = \frac{1}{2\times\sin 30^\circ}\hat{a}_\phi = 1\,\hat{a}_\phi = \hat{a}_\phi \text{ A/m}^2$$

$$\nabla\times\vec{H} = \frac{1}{r\sin\theta}\left(\frac{\partial(H_\phi\sin\theta)}{\partial\theta} - \frac{\partial H_\theta}{\partial\phi}\right)\hat{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial(rH_\phi)}{\partial r}\right)\hat{a}_\theta + \frac{1}{r}\left(\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial\theta}\right)\hat{a}_\phi$$

or

simplify form formula

Qn: The magnetic field intensity is given in a certain region of space as $\vec{H} = \frac{x+2y}{z^2}\hat{a}_y + \frac{2}{z}\hat{a}_z$ A/m. Find the total current passing through the surface $z=4$, $1<x<2$, $3<y<5$, in the $\hat{a}_z$ direction.

[2069 chaitra]

Sol.ⁿ Given, $\vec{H} = \frac{x+2y}{z^2}\hat{a}_y + \frac{2}{z}\hat{a}_z$ A/m.

and the surface is $z=4$, $1<x<2$, $3<y<5$ in $\hat{a}_z$ dirⁿ

then,

$$\vec{J} = \nabla \times \vec{H}$$

$$= \begin{vmatrix} \hat{a}_x & \hat{a}_y & \hat{a}_z \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 0 & \frac{x+2y}{z^2} & \frac{2}{z} \end{vmatrix}$$

$$= \left[\frac{\partial}{\partial y}(2/z) - \frac{\partial\left(\frac{x+2y}{z^2}\right)}{\partial z}\right]\hat{a}_x + \left[-\frac{\partial}{\partial x}(2/z) + \frac{\partial}{\partial z}0\right]\hat{a}_y$$

$$+ \left[\frac{\partial}{\partial x}\left(\frac{x+2y}{z^2}\right) - \frac{\partial}{\partial y}0\right]\hat{a}_z$$

$$= -(x+2y)\frac{\partial(1/z^2)}{\partial z}\hat{a}_x + \frac{1}{z^2}\frac{\partial x}{\partial x}\hat{a}_z$$

$$= -(x+2y)(-2)z^{-3}\hat{a}_x + \frac{1}{z^2}\hat{a}_z$$

$$= \frac{+2(x+2y)}{z^3}\hat{a}_x + \frac{1}{z^2}\hat{a}_z \text{ A/m}^2$$

$$= \frac{2x+4y}{z^3}\hat{a}_x + \frac{1}{z^2}\hat{a}_z \text{ A/m}^2$$

Again,

$$I = \int_S \vec{J} \cdot \vec{ds}$$

$$= \int_{S,\ z=4} \left[\frac{2x+4y}{z^3} \hat{a}_x + \frac{1}{z^2} \hat{a}_z \right] \cdot \left[dx\, dy\, \hat{a}_z \right]$$

$$= \int_{S,\ z=4} \frac{1}{z^2}\, dx\, dy$$

$$= \int_{x=1}^{2} \int_{y=3}^{5} \frac{1}{(4^2)}\, dx\, dy$$

$$= \frac{1}{16} \cdot [2-1] \cdot [5-3]$$

$$= \frac{2}{16}$$

$$= \frac{1}{8} \text{ A}$$

Qn: The magnetic field intensity in a certain region of space is given as $\vec{H} = (2\rho + z)\hat{a}_\rho + \frac{2}{z}\hat{a}_z$ A/m. Find the total current passing through the surface $\rho = 2, \frac{\pi}{4} < \phi < \frac{\pi}{2}, 3 < z < 5$, in the $\hat{a}_\rho$ direction. [2070 Chaitra]

Sol.^{n} Given,

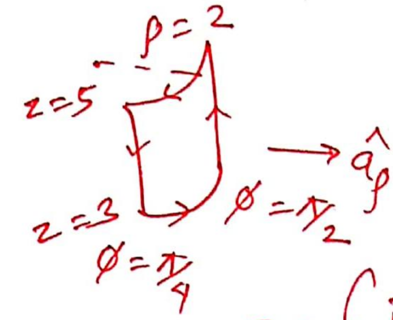
$$\vec{H} = (2\rho + z)\hat{a}_\rho + \frac{2}{z}\hat{a}_z \text{ A/m}.$$

then,

$$\vec{J} = \nabla \times \vec{H} = \left(\frac{1}{\rho} \frac{\partial H_z}{\partial \phi} - \frac{\partial H_\phi}{\partial z} \right) \hat{a}_\rho + \left(\frac{\partial H_\rho}{\partial z} - \frac{\partial H_z}{\partial \rho} \right) \hat{a}_\phi$$

$$+\frac{1}{\rho}\left(\frac{\partial(\rho H_\phi)}{\partial \rho}-\frac{\partial H_\rho}{\partial \phi}\right)\hat{a}_z$$

$$=0+\left[\frac{\partial(2\rho+z)}{\partial z}-0\right]\hat{a}_\phi+0$$



$\therefore \vec{J} = \therefore \hat{a}_\phi \; A/m^2$

$\Leftarrow I = \oint_L \vec{H}\cdot\vec{dl}$.

Again, $I=\int_s \vec{J}\cdot\vec{ds}$

$$=\int \hat{a}_\phi \cdot \rho\, d\phi\, dz\, \hat{a}_\rho$$

$\therefore \rho = 2$

$$=2\int 0$$

$$=0$$

Qn: Given, $\vec{H}=10\sin\theta\,\hat{a}_r$ in free space. Find the current in $\hat{a}_r$ direction having $r=3$, $0\le\theta\le 90°$, $0\le\phi\le 90°$. [2072 Kartik]

Sol.^{n} Given, $\vec{H}=10\sin\theta\,\hat{a}_r$

we have, $\vec{J}=\nabla\times\vec{H}=\frac{1}{r\sin\theta}\left[\cancel{\frac{\partial(0\times\sin\theta)}{\partial\theta}}-\frac{\partial\cdot 0}{\partial\phi}\right]\hat{a}_r+\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial(10\sin\theta)}{\partial\phi}\right.$

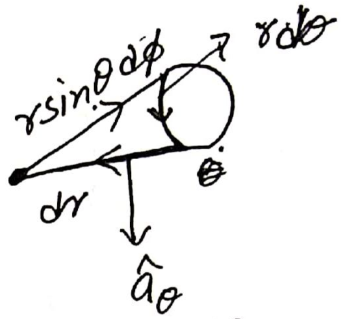
$$-\left.\cancel{\frac{\partial(r\times 0)}{\partial r}}\right]\hat{a}_\theta+\frac{1}{r}\left[\frac{\partial(r\times 0)}{\partial r}-\frac{\partial(10\sin\theta)}{\partial\theta}\right]\hat{a}_\phi$$

$$=\frac{1}{r}\left[-10\cos\theta\right]\hat{a}_\phi$$

$$=-\frac{10\cos\theta}{r}\hat{a}_\phi$$

Qn: Verify Stoke's theorem for the field $\vec{H} = \left(\frac{3r^2}{\sin\theta}\right)\hat{a}_\theta + 54r\cos\theta\,\hat{a}_\phi$ A/m in freespace for the conical surface defined by $\theta = 20°$, $0 \le \phi \le 2\pi$, $0 \le r \le 5$. Let the +ve direction of $\vec{ds}$ be $\hat{a}_\theta$. [2074 Ashwin]

Sol^n $\oint_L \vec{H}\cdot\vec{dl} = \int_S (\nabla\times\vec{H})\cdot\vec{ds}$



L.H.S $= \oint_L \vec{H}\cdot\vec{dl} = \oint \vec{H}\cdot\vec{dl}$

$\theta = 20°$, $r = 5$

$= \oint [H_r dr + H_\theta r d\theta + H_\phi r\sin\theta\, d\phi]$

$= \int_{2\pi}^{0} \left[\left(\frac{3r^2}{\sin\theta}\right)\hat{a}_\theta + 54r\cos\theta\,\hat{a}_\phi\right]\cdot r\sin\theta\, d\phi\,\hat{a}_\phi$

$\begin{pmatrix}\theta = 20°\\ r = 5\end{pmatrix}$ $= -\int_0^{2\pi} 54r^2\sin\theta\cdot\cos\theta\, d\phi$

$\begin{pmatrix}r = 5\\ \theta = 20°\end{pmatrix}$

$= -\frac{54\times 25}{2}\int_0^{2\pi}\sin 2\theta\, d\phi$

$= -675\times\sin 2\times 20°\times 2\pi$

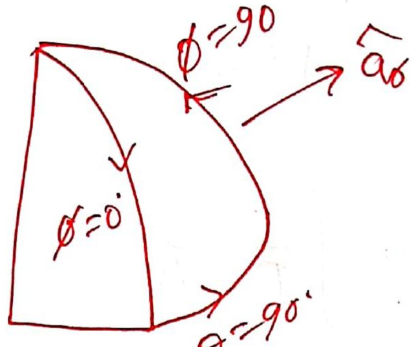
$= -2726.165$ A

(*)

Again, $I = \int_s \vec{J} \cdot d\vec{s}$

$$= \int_{\substack{s \\ (r=3)}} \frac{-10\cos\theta}{r}\, \hat{a}_\phi \cdot r^2 \sin\theta \cdot d\theta \cdot d\phi\, \hat{a}_r$$

$$= 0$$



(*) R.H.S. $= \int_s (\nabla \times \vec{H}) \cdot d\vec{s}$

where,

$$\nabla \times \vec{H} = \frac{1}{r\sin\theta}\left(\frac{\partial(H_\phi \sin\theta)}{\partial\theta} - \frac{\partial H_\theta}{\partial\phi}\right)\hat{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial H_r}{\partial\phi} - \frac{\partial(rH_\phi)}{\partial r}\right)\hat{a}_\theta$$
$$+ \frac{1}{r}\left(\frac{\partial(rH_\theta)}{\partial r} - \frac{\partial H_r}{\partial\theta}\right)\hat{a}_\phi$$

Here, $H_\theta = \frac{3r^2}{\sin\theta}$ and $H_\phi = 54r\cos\theta$

$$\nabla \times \vec{H} = \frac{1}{r\sin\theta}\left(\frac{\partial(54r\cos\theta \cdot \sin\theta)}{\partial\theta}\right)\hat{a}_r - \frac{1}{r}\frac{\partial(r \cdot 54r\cos\theta)}{\partial r}\hat{a}_\theta$$
$$+ \frac{1}{r}\left(\frac{\partial(r \cdot 3r^2/\sin\theta)}{\partial r}\right)\hat{a}_\phi$$

$$= \frac{1}{r\sin\theta} \cdot 27\cancel{r} \times 2\cos 2\theta\, \hat{a}_r - 54 \times 2\cos\theta\, \hat{a}_\theta + \frac{1}{r} \cdot \frac{3 \times 3r^2}{\sin\theta}\hat{a}_\phi$$

$$= \frac{54\cos 2\theta}{\sin\theta}\hat{a}_r - 108\cos\theta\, \hat{a}_\theta + \frac{9r}{\sin\theta}\hat{a}_\phi \quad (*)(*)$$

Magnetic Force

रविन कासुला
इंजिनियर

The electric field causes a force to be exerted on a charge which may be either stationary or in motion. The steady magnetic field is capable of exerting a force only on a moving charge. So, a magnetic field may be produced by moving charges and may exert forces on moving charges; a magnetic field cannot arise from stationary charges and cannot exert any forces on a stationary charge.

Force on a Moving charge

A charged particle in motion in a magnetic field of flux density $\vec{B}$ is found experimentally to experience a force whose magnitude is proportional to the product of magnitudes of the charge Q, its velocity $\vec{v}$, and the flux density $\vec{B}$, and to the sine of angle between $\vec{v}$ & $\vec{B}$. The direction of force is perpendicular to both $\vec{v}$ & $\vec{B}$ and is given by a unit vector in the direction of $\vec{v} \times \vec{B}$.

$$\boxed{\therefore \quad \vec{F} = Q\vec{v} \times \vec{B}}$$

The force on a moving particle due to combined electric & magnetic fields is obtained as:

$$\boxed{\vec{F} = Q(\vec{E} + \vec{v} \times \vec{B})}$$

This equation is known as Lorentz force equation.

Force on a differential current element

The force on a charged particle moving through a steady magnetic field ~~may~~ be written ~~as~~ the differential force exerted on a differential ~~element~~ of charge,

$$d\vec{F} = dQ\,\vec{v} \times \vec{B}$$

$$\therefore \int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \int_{r=0}^{5} \int_{\phi=0}^{2\pi} \left(-108 \cos\theta \, \hat{a}_\theta \right) \cdot r\sin\theta \, d\phi \cdot dr \, \hat{a}_\theta \quad (\theta = 20^\circ)$$

$$= -108 \left[\cos\theta \cdot \sin\theta \right]_{20^\circ} \int_{r=0}^{5} \int_{\phi=0}^{2\pi} r \, d\phi \, dr$$

$$= -108 \cos 20^\circ \times \sin 20^\circ \cdot \frac{(25-0)}{2} (2\pi - 0)$$

$$= -2726.165 \text{ A}$$

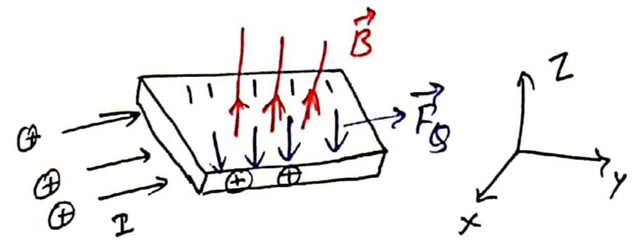
$$\therefore \oint_L \vec{H} \cdot \vec{dl} = \int_S (\nabla \times \vec{H}) \cdot \vec{ds} = -2726.165 \text{ A}.$$

Physically, the differential element of charge consists of a large number of very small discrete charges occupying a volume, which is larger (much more) than the average separation betn the charges. The differential force is thus merely the sum of the forces on the individual charges. This sum, or resultant force, is not a force applied to a single object.

If the considered charges are electrons in motion in a conductor, we can show that the force is transferred to the conductor and that the sum of this extremely large number of small forces is of practical importance. Within the conductor, the electrons are in motion throughout a region of immobile positive ions which form a crystalline array giving the conductor its solid properties. A magnetic field which exerts forces on the electrons tends to cause them to shift position slightly and produces a small displacement between the centers of "gravity" of the positive & negative charges. The Coulomb forces betn electrons & positive ions however, tend to resist such a displacement. Any attempt to move the electrons, therefore results in an attractive force betn electrons and the positive ions of the crystalline lattice. The magnetic force is thus transferred to the crystalline lattice or to the conductor itself. The coulomb forces are so much greater than the magnetic forces in good conductors that the actual displacement of the electrons is almost immeasurable. The charge separation that does result, however, is disclosed by the presence of a slight potential difference across the conductor sample in a dirn perpendicular to both the magnetic field and the velocity of the charges. The voltage is known as Hall voltage and the effect is called the Hall effect.

Again, we have $\vec{J} = \rho_v \vec{v}$

The differential element of charges may also be expressed in terms of volume charge density,



$$dQ = \rho_v \, dv$$

Thus, $d\vec{F} = \rho_v \, dv \, \vec{v} \times \vec{B}$

or, $\boxed{d\vec{F} = \vec{J} \times \vec{B} \, dv}$

Also, $\vec{J} \, dv = \vec{K} \, ds = I \, d\vec{l}$

Therefore the Lorentz force equation may be

$d\vec{F} = \vec{K} \times \vec{B} \, ds$ – for surface current density

and $d\vec{F} = I \, d\vec{l} \times \vec{B}$ – for differential current filament.

Then,

$$\boxed{\vec{F} = \int_{vol.} \vec{J} \times \vec{B} \, dv}$$

$$\boxed{\vec{F} = \int_s \vec{K} \times \vec{B} \, ds}$$

& $\boxed{\vec{F} = \oint I \, d\vec{l} \times \vec{B} = -I \oint \vec{B} \times d\vec{l}}$

Applying this to a straight conductor in a uniform magnetic field, we can have

$$\boxed{\vec{F} = I \vec{L} \times \vec{B}}$$

& the magnitude of the force is given by

$$F = BIL \sin\theta$$

where; θ is the angle betn the vectors representing the dirn of current flow and the dirn of the magnetic flux density.

Force Between Differential current elements

The magnetic field at point 2 due to a current element at point 1 was found to be

$$d\vec{H}_2 = \frac{I_1 d\vec{L}_1 \times \hat{a}_{R_{12}}}{4\pi R_{12}^2}$$

& the differential force on differential current element is

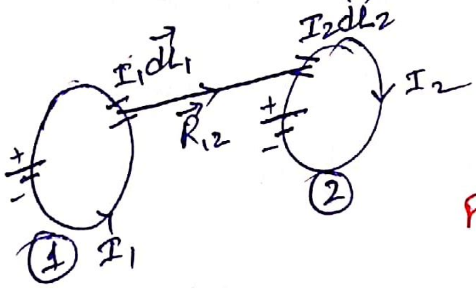
$$d\vec{F} = I d\vec{L} \times \vec{B}$$

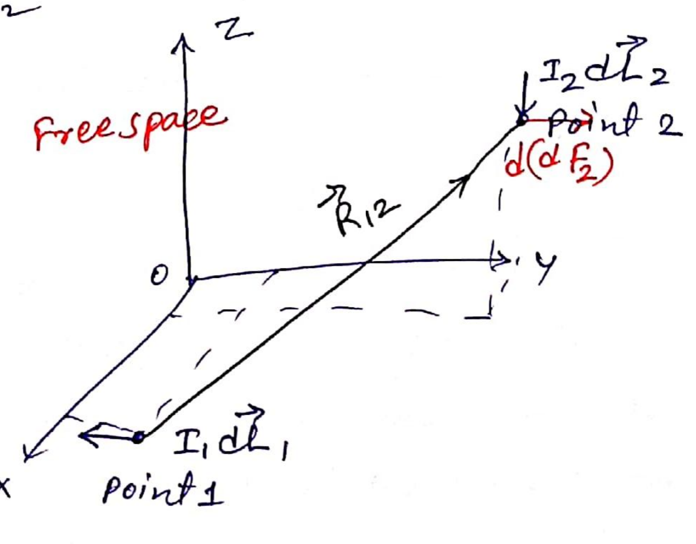
and applying this to the case of two differential current element by letting $\vec{B}$ be $d\vec{B}_2$ (the differential flux density at point 2 caused by current element 1), by identifying $I d\vec{L}$ as $I_2 d\vec{L}_2$ and by symbolizing the differential amount of differential force on element 2 as $d(d\vec{F}_2)$;

$$d(d\vec{F}_2) = I_2 d\vec{L}_2 \times d\vec{B}_2$$

Since, $d\vec{B}_2 = \mu_0 d\vec{H}_2$, we obtain the force betn two differential current elements as:

$$d(d\vec{F}_2) = \mu_0 \frac{I_1 I_2}{4\pi R_{12}^2} d\vec{L}_2 \times (d\vec{L}_1 \times \hat{a}_{R_{12}})$$





As, we have seen that force exerted by one charge to another point charge is negative of that on the second. That is the total force on the system was zero. This is not the case with the differential current elements. The reason for this different behaviour lies with the nonphysical nature of the current element.

Now, for the total force betn two filamentary circuits is obtained by integrating twice:

$$\vec{F}_2 = \mu_0 \frac{I_1 I_2}{4\pi} \oint_{L_2} \left[\vec{dL}_2 \times \oint_{L_1} \frac{\vec{dL}_1 \times \hat{a}_{R12}}{R_{12}^2} \right]$$

$$= \mu_0 \frac{I_1 I_2}{4\pi} \oint_{L_2} \left[\oint_{L_1} \frac{\hat{a}_{R12} \times \vec{dL}_1}{R_{12}^2} \right] \times \vec{dL}_2$$

As from above derivations,
the force on a filamentary closed circuit is

$$\vec{F} = - I \oint \vec{B} \times \vec{dL}$$

and if the magnetic flux density is uniform, then

$$\vec{F} = - I \vec{B} \times \oint \vec{dL}$$

For closed line integrals in an electrostatic potential field was found to be zero as: $\oint \vec{dL} = 0$, therefore the force on a closed filamentary circuit in a uniform magnetic field is zero.

If the field is not uniform, the total force need not be zero.

Magnetic Torque & Moment

If the current loop or filament is placed parallel to a magnetic field, it experiences a force that tends to rotate it.

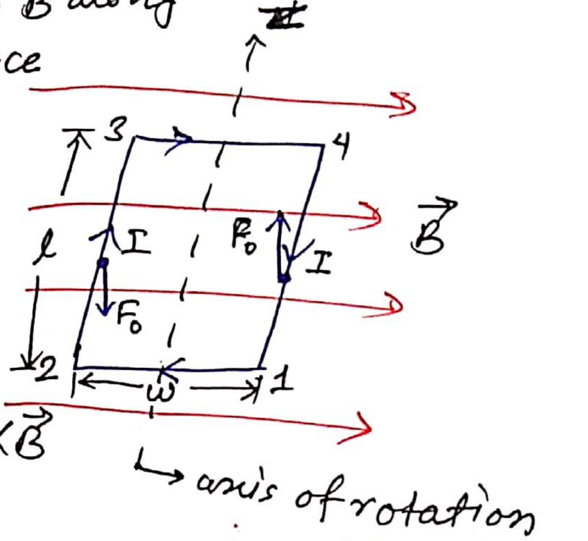
The torque, $\vec{T}$ (or mechanical moment of force) on the current loop is the vector product of the force $\vec{F}$ and the moment arm $\vec{r}$.

That is,

$$\boxed{\vec{T} = \vec{r} \times \vec{F}}$$

and its units are Newton-meters (N-m).

Let us apply this to a rectangular loop of length l and width w placed in a uniform magnetic field $\vec{B}$ as shown in figure. Here, we notice that $d\vec{l}$ is parallel to $\vec{B}$ along sides 12 and 34 of the loop and no force is exerted on those sides. Thus,

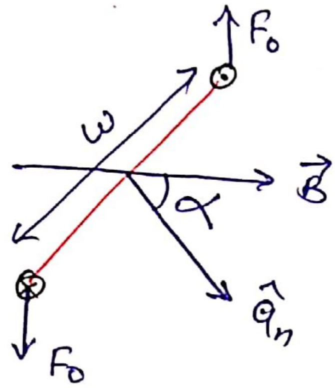


$$\vec{F} = I\int_2^3 d\vec{l} \times \vec{B} + I\int_4^1 d\vec{l} \times \vec{B}$$

$$= I\int_0^l dz\,\hat{a}_z \times \vec{B} + I\int_l^0 dz\,\hat{a}_z \times \vec{B}$$

$$= \vec{F}_0 - \vec{F}_0$$

$$= 0$$



where, $|\vec{F}_0| = IBl$ because $\vec{B}$ is uniform. However, $\vec{F}_0$ & $-\vec{F}_0$ acts at different points on the loop, thereby creating a couple. If the normal to the plane of the loop makes an angle α with $\vec{B}$ the torque on the loop is

$$|\vec{T}| = |\vec{F}_0|\, w \sin\alpha$$

or, $T = BIlw \sin\alpha$

But $wl = S$, area of the loop

$$\therefore \boxed{T = BIS \sin\alpha}$$

Now, we define a quantity, $\boxed{\vec{m} = IS\,\hat{a}_n}$ as the magnetic dipole moment (in A/m²) of the loop. where, $\hat{a}_n$ is the normal vector to the plane of the loop and its direction is determined by the right hand rule: fingers in the dir? of current & thumb along $\hat{a}_n$.

The magnetic dipole moment is the product of current and area of the loop; its direction is normal to the loop.

so, $\boxed{\vec{T} = \vec{m} \times \vec{B}}$

Magnetic Dipole

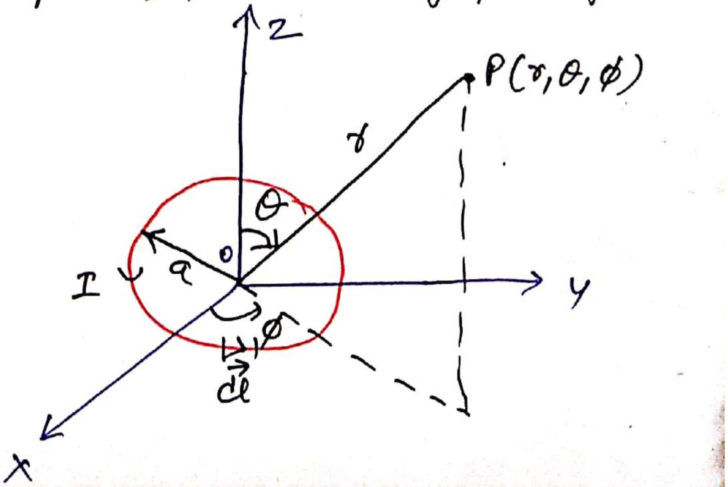
A bar magnet or a small filamentary current loop is usually referred to as a magnetic dipole. Let us consider a small current loop as shown in figure. Let us determine the magnetic field $\vec{B}$ at an observation point $P(r,\theta,\phi)$ due to a circular loop carrying current I as in figure. The magnetic vector potential at P is

$$\vec{A} = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l}}{r}$$

It can be shown that at far field ($r \gg a$, so that the loop appears small at the observation point), $\vec{A}$ has only ϕ component and it is given by

$$\vec{A} = \frac{\mu_0 I \pi a^2 \sin\theta\, \hat{a}_\phi}{4\pi r^2}$$

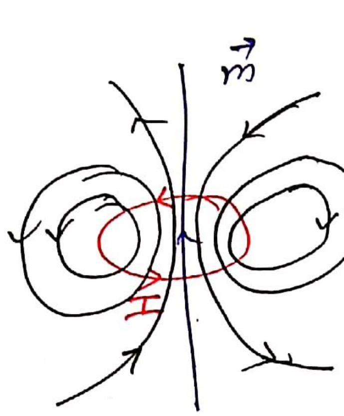
or, $$\vec{A} = \frac{\mu_0\, \vec{m} \times \hat{a}_r}{4\pi r^2}$$



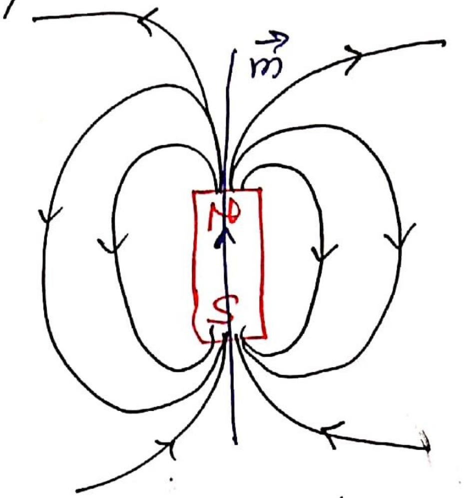
where, $\vec{m} = I\pi a^2 \hat{a}_z$, the magnetic moment of the loop and $\hat{a}_z \times \hat{a}_r = \sin\theta\, \hat{a}_\phi$. We determine the magnetic flux density $\vec{B}$ from $\vec{B} = \nabla \times \vec{A}$ as:

$$\boxed{\vec{B} = \frac{\mu_0 m}{4\pi r^3}\left(2\cos\theta\, \hat{a}_r + \sin\theta\, \hat{a}_\theta\right)}$$

This is the magnetic flux density due to the magnetic dipole or a small current loop.



Small Current Loop



Bar magnet

For Bar magnet, the torque is given by

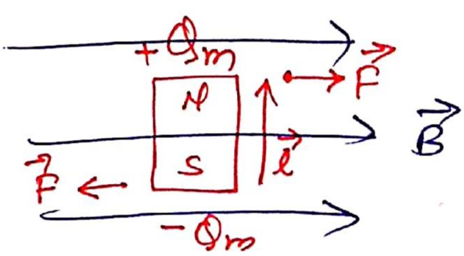
$$\vec{T} = \vec{m} \times \vec{B} = Q_m \vec{l} \times \vec{B}$$

where, Q_m is isolated magnetic charge (pole strength)
$\vec{l}$ is length of the magnet, pointing in the dir.^{n} south to north.
$\vec{m} = Q_m \vec{l}$

Force on magnetic charge

$\vec{F} = Q_m \vec{B}$ & Torque is

$T = Q_m l B = ISB$



Hence, $IS = Q_m l$ (Bar magnet & small current loop)

Magnetization

The magnetization ($\vec{M}$) (in amperes/meter) is the magnetic dipole per unit volume.

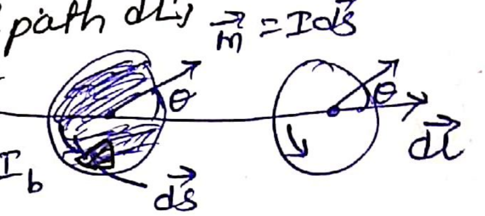
If there are N atoms in a given volume ΔV and the kth atom has a magnetic moment $\vec{m}_k$, then

$$\boxed{\vec{M} = \lim_{\Delta V \to 0} \frac{\sum_{k=1}^{N} \vec{m}_k}{\Delta V}}$$

The magnetization is due to the movement of bound charges (orbital electrons, electron spin and nuclear spin) and the current produced is bound current or Amperian current.

A medium for which $\vec{M}$ is zero everywhere is said to be magnetized.

Let us consider the effect of some allignment of the magnetic dipoles as the result of the application of magnetic field as shown in figure. The figure shows several magnetic moments $\vec{m}$ that makes an angle θ with the element of path $d\vec{L}$; each moment consists of a bound current I_b circulating about an area $d\vec{S}$.



Let us consider a small volume, $d\vec{S} \cdot d\vec{L} = dS\cos\theta\, dL$, within which there are $n\, d\vec{S} \cdot d\vec{L}$ magnetic dipoles are present. In changing from a random orientation to this partial alignment, the bound current crossing the surface enclosed by the path has increased by I_b for each of the $n\, d\vec{S} \cdot d\vec{L}$ dipoles, thus

$$dI_b = n I_b\, d\vec{S} \cdot d\vec{L} = \vec{M} \cdot d\vec{L}$$

and within an entire closed contour,

$$I_b = \oint \vec{M} \cdot d\vec{L}$$

Now, we have from Ampere's circuital law for total current, I_T

$$\oint \frac{\vec{B}}{\mu_0} \cdot d\vec{L} = I_T$$

5) i.e $I_T = I_b + I$ where, I_b = bound current
I = free (charge) current

Now, we have,

$$I = I_T - I_b = \oint \frac{\vec{B}\cdot\vec{dl}}{\mu_0} - \vec{M}\cdot\vec{dl} = \oint \left(\frac{\vec{B}}{\mu_0} - \vec{M}\right)\cdot\vec{dl}$$

We may now define the new $\vec{H}$ in terms of $\vec{B}$ & $\vec{M}$ as

$$\boxed{\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}}$$

and $\boxed{\vec{B} = \mu_0\vec{H} + \mu_0\vec{M} = \mu_0(\vec{H} + \vec{M})}$

Also, $I_b = \oint_s \vec{J}_b \cdot \vec{ds}$, $I = \oint \vec{H}\cdot\vec{dl}$

$I_T = \oint_s \vec{J}_T \cdot \vec{ds}$ $\quad I = \oint \vec{J}\cdot\vec{ds}$

From Stoke's theorem

$$\nabla \times \vec{M} = \vec{J}_b$$

$$\nabla \times \frac{\vec{B}}{\mu_0} = \vec{J}_T$$

& $\nabla \times \vec{H} = \vec{J}$

For a Linear isotropic Media where magnetic susceptibility χ_m can be defined, we can write

$$\vec{M} = \chi_m \vec{H}$$

thus, $\vec{B} = \mu_0(\vec{H} + \vec{M}) = \mu_0(\vec{H} + \chi_m\vec{H}) = \mu_0(1+\chi_m)\vec{H}$
$= \mu_0\mu_r\vec{H}$

or, $\boxed{\vec{B} = \mu_0\mu_r\vec{H}} = \boxed{\mu\vec{H}}$

where, $\boxed{\mu_r = 1 + \chi_m}$ is relative permeability.

& permeability of material, $\boxed{\mu = \mu_0\mu_r}$

(27.6) Qn: Evaluate the closed line integral of $\vec{H}$ about the rectangular path $P_1(2,3,4)$ to $P_2(4,3,4)$ to $P_3(4,3,1)$ to $P_4(2,3,1)$ to P_1, given $\vec{H} = 3z\hat{a}_x - 2x^3\hat{a}_z$ A/m.

Sol.ⁿ

Given, $\vec{H} = 3z\hat{a}_x - 2x^3\hat{a}_z$ A/m.

and given closed path is $P_1(2,3,4)$ to $P_2(4,3,4)$ to $P_3(4,3,1)$ to $P_4(2,3,1)$ and to P_1.

$\therefore$ The closed line integral is

$$\oint_L \vec{H}\cdot\vec{dl} = \int_{P_1}^{P_2} \vec{H}\cdot\vec{dl} + \int_{P_2}^{P_3} \vec{H}\cdot\vec{dl} + \int_{P_3}^{P_4} \vec{H}\cdot\vec{dl} + \int_{P_4}^{P_1} \vec{H}\cdot\vec{dl}.$$

$$= \int_{\substack{P_1 \\ (y=3,\,z=4)}}^{P_2} (3z\hat{a}_x - 2x^3\hat{a}_z)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z) + \int_{\substack{P_2 \\ (x=4,\,y=3)}}^{P_3} (3z\hat{a}_x - 2x^3\hat{a}_z)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$+ \int_{\substack{P_3 \\ (y=3,\,z=1)}}^{P_4} (3z\hat{a}_x - 2x^3\hat{a}_z)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$+ \int_{\substack{P_4 \\ (x=2,\,y=3)}}^{P_1} (3z\hat{a}_x - 2x^3\hat{a}_z)\cdot(dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z)$$

$$= \int_{\substack{x=2 \\ (y=3,\,z=4)}}^{4} (3\times 4\,dx - \cancel{2x^3 dz}^{\,0}) + \int_{\substack{z=4 \\ (x=4,\,y=3)}}^{1} -2\times 4^3\,dz + \int_{\substack{x=4 \\ (y=3,\,z=1)}}^{2} 3\times 1\,dx$$

$$+ \int_{\substack{z=1, \\ (y=3,\,x=2)}}^{4} -2\times(2)^3\,dz$$

$= 12 \times [4-2] + 3 \times [2-4] - 128 \times [1-4] - 16 \times [4-1]$

$= 24 - 6 + 384 - 48$

$= 354\ A$

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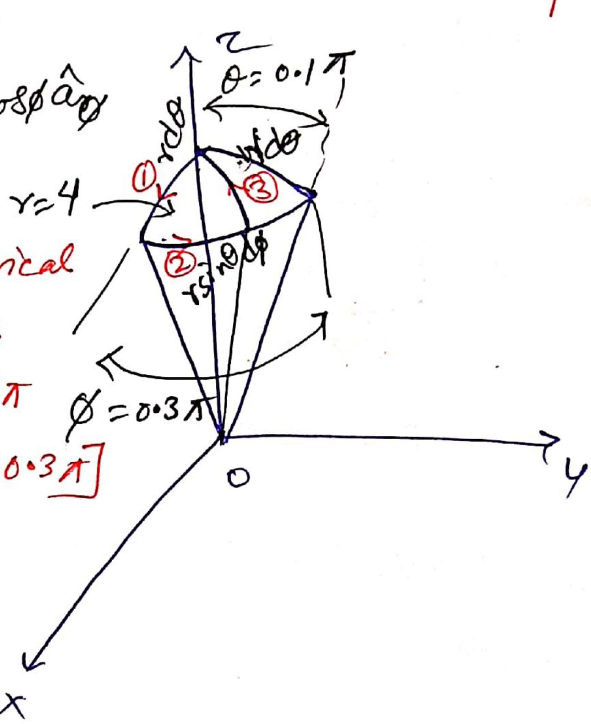
Qn: Evaluate Both sides of stoke's theorem for the surface specified by $r = 4,\ 0 \le \theta \le 0.1\pi,\ 0 \le \phi \le 0.3\pi$, and ~~arcs~~ ~~Given~~. $\vec{H} = 6r\sin\phi\,\hat{a}_r + 18r\sin\theta\cos\phi\,\hat{a}_\phi$. $\hat{a}_r$ is dir.^{n} of surface.

Sol.^{n} We have, $\vec{H} = 6r\sin\phi\,\hat{a}_r + 18r\sin\theta\cos\phi\,\hat{a}_\phi$

and $\vec{dl} = dr\,\hat{a}_r + r\,d\theta\,\hat{a}_\theta + r\sin\theta\,d\phi\,\hat{a}_\phi$



[The first path segment is described in spherical coordinates by $r = 4,\ 0 \le \theta \le 0.1\pi,\ \phi = 0$; the second one by $r = 4,\ \theta = 0.1\pi,\ 0 \le \phi \le 0.3\pi$ and the third by $r = 4,\ 0 \le \theta \le 0.1\pi,\ \phi = 0.3\pi$]

then, from stoke's theorem, we have

$$\oint_L \vec{H}\cdot\vec{dl} = \int_S (\nabla\times\vec{H})\cdot\vec{ds}$$

Taking Left only we have

$$\oint_L \vec{H}\cdot\vec{dl} = \int_1 \vec{H}\cdot\vec{dl} + \int_2 \vec{H}\cdot\vec{dl} + \int_3 \vec{H}\cdot\vec{dl}$$

$$= \int_1 H_\theta\, r\,d\theta + \int_2 H_\phi\, r\sin\theta\,d\phi + \int_3 H_\theta\, r\,d\theta$$

[Since, $r = 4$ and $dr = 0$]

Also, $H_\theta = 0$, so

$$\oint_L \vec{H}\cdot\vec{dl} = \int_{\phi=0}^{0.3\pi} [18(4)\sin 0.1\pi\cos\phi]\,4\sin 0.1\pi\,d\phi$$

$$= 288\sin^2 0.1\pi\,\sin 0.3\pi = 22.2\,A$$

Taking right part

$\nabla\times\vec{H} = \frac{1}{r\sin\theta}(36r\sin\theta\cos\theta\cos\phi)\hat{a}_r + \frac{1}{r}\left(\frac{1}{\sin\theta}\cancel{6}r\cos\phi - 36r\right.$

$\left.\sin\theta\cos\phi\right)\hat{a}_\theta$

since, $\vec{ds} = r^2\sin\theta\, d\theta\, d\phi\, \hat{a}_r$.

$$\int_S(\nabla\times\vec{H})\cdot\vec{ds} = \int_{\phi=0}^{0.3\pi}\int_{\theta=0}^{0.1\pi}(36\cos\theta\cdot\cos\phi)\,16\sin\theta\, d\theta\, d\phi$$

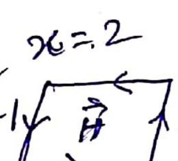
$$= \int_0^{0.3\pi} 576\left(\frac{1}{2}\sin^2\theta\right)\Big|_0^{0.1\pi}\cos\phi\, d\phi$$

$$= 288\sin^2 0.1\pi\sin 0.3\pi$$

$$= 22.2\,A$$

Qn: Evaluate both sides of stoke's theorem for the field $\vec{H} = 6xy\hat{a}_x - 3y^2\hat{a}_y$ A/m and the rectangular path around the region, $2\le x\le 5$, $-1\le y\le 1$, $z=0$. Let the positive direction of $\vec{ds}$ be $\hat{a}_z$. [2071 Chaitra]

Sol^n Magnetic field in the rectangular region $2\le x\le 5$, $-1\le y\le 1$, $z=0$ is $\vec{H} = 6xy\hat{a}_x - 3y^2\hat{a}_y$ A/m.



z=0 plane

From Stoke's Theorem.

$$\oint_L\vec{H}\cdot\vec{dl} = \int_S(\nabla\times\vec{H})\cdot\vec{ds}$$

Using $\vec{dl} = dx\,\hat{a}_x + dy\,\hat{a}_y + dz\,\hat{a}_z$ for Cartesian Co-ordinates, we can evaluate left part of stoke's Theorem as:

$$\oint_L\vec{H}\cdot\vec{dl} = \int_{x=2}^{5}6x\times(-1)dx + \int_{y=-1}^{1}(-3y^2)dy + \int_{x=5}^{2}6x\times 1\,dx + \int_{y=1}^{-1}(-3y^2)dy$$

$$= -6\times\frac{x^2}{2}\Big|_2^5 - \frac{3y^3}{3}\Big|_{-1}^{1} + 6\frac{x^2}{2}\Big|_5^2 - 3\frac{y^3}{3}\Big|_1^{-1}$$

$$= -3\times 21 - 2 + 3\times(-21) + 2 = -126\,A$$

Taking. $\nabla\times\vec{H} = \left[\frac{\partial(0)}{\partial y} - \frac{\partial(-3y^2)}{\partial z}\right]\hat{a}_x + \left(\frac{\partial(-3y^2)}{\partial x} - \frac{\partial(6xy)}{\partial y}\right]\hat{a}_z$

$+ \left[\frac{\partial(6xy)}{\partial z} - \frac{\partial(0)}{\partial x}\right]\hat{a}_y$

$= 0\hat{a}_x + \left[-6x\right]\hat{a}_z + 0\,\hat{a}_y$

$= -6x\,\hat{a}_z$

and $\int_S (\nabla\times\vec{H})\cdot\vec{dS} = \int_S -6x\,\hat{a}_z \cdot dx\,dy\,\hat{a}_z$

$= -6\int_{x=2}^{5}\int_{y=-1}^{1} x\,dx\,dy$

$= -6\left.\frac{x^2}{2}\right|_2^5 [1+1]$

$= -6\times 21$

$= -126\ A$

$\therefore \oint_L \vec{H}\cdot\vec{dl} = \int_S (\nabla\times\vec{H})\cdot\vec{dS}$

Qn: Evaluate the closed line integral of $\vec{H}$ from $P_1(5,4,1)$ to $P_2(5,6,1)$ to $P_3(0,6,1)$ to $P_4(0,4,1)$ to P_1 using straight line segments, if $\vec{H} = 0.1y^3\hat{a}_x + 0.4x\hat{a}_z$ A/m. [2072 Chaitra]

Sol.^{n} Given, $\vec{H} = 0.1y^3\hat{a}_x + 0.4x\hat{a}_z$ A/m. and a closed line is from $P_1(5,4,1)$ to $P_2(5,6,1)$ to $P_3(0,6,1)$ to $P_4(0,4,1)$ to P_1.

$\therefore$ The closed line integral is

$$\oint \vec{H}\cdot\vec{dl} = \int_{P_1}^{P_2}\vec{H}\cdot\vec{dl} + \int_{P_2}^{P_3}\vec{H}\cdot\vec{dl} + \int_{P_3}^{P_4}\vec{H}\cdot\vec{dl} + \int_{P_4}^{P_1}\vec{H}\cdot\vec{dl}$$

$$= \int_{\substack{y=4 \\ (x=5,\ z=1)}}^{6} (0.1y^3\hat{a}_x + 0.4x\hat{a}_z)\cdot(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z) + \int_{\substack{x=5 \\ (y=6,\ z=1)}}^{0} (0.1y^3\hat{a}_x + 0.4x\hat{a}_z)\cdot(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z)$$

$$+ \int_{\substack{y=6 \\ (x=0,\ z=1)}}^{4} (0.1y^3\hat{a}_x + 0.4x\hat{a}_z)\cdot(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z)$$

$$+ \int_{\substack{x=0 \\ (y=4,\ z=1)}}^{5} (0.1y^3\hat{a}_x + 0.4x\hat{a}_z)\cdot(dx\hat{a}_x + dy\hat{a}_y + dz\hat{a}_z)$$

$$= 0 + \int_{x=5}^{0} 0.1\times(6)^3\,dx + 0 + \int_{x=0}^{5} 0.1\times(4)^3\,dx$$

$$= 0.1\times 216\,[0-5] + 0.1\times 64\,[5-0]$$

$$= -108 + 32$$

$$= -76\,A$$

Qn: Evaluate both sides of stokes theorem for the field $\vec{A} = 6xy\hat{a}_x - 3y^2\hat{a}_y$ A/m and the rectangular path around the region; $2 \le x \le 5, -1 \le y \le 1, z=0$. Let the positive direction of $\vec{ds}$ be $\hat{a}_z$.

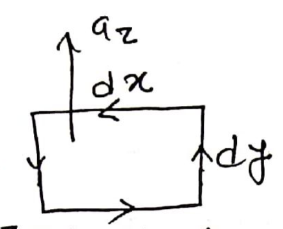
[2071 Chaitra]

Ans: $-126A$, $-126A$

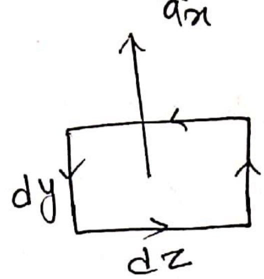
[solution in page - 218.]

Closed Loop Consideration (Stoke's Theorem)

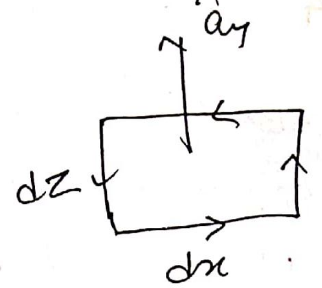
① Cartesian Co-ordinates.



$z = \text{constant}$



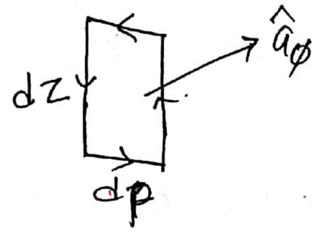
$x = \text{constant}$



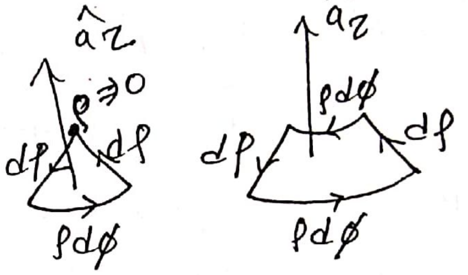
$y = \text{constant}$

② Cylindrical Co-ordinates.

$\rho = \text{constant}$



$\phi = \text{constant}$

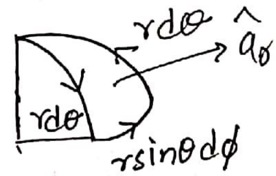


$z = \text{constant}$ (if $\rho = 0$ to a) $z = \text{constant}$ (if $\rho = a$ to b)

③ Spherical Co-ordinates

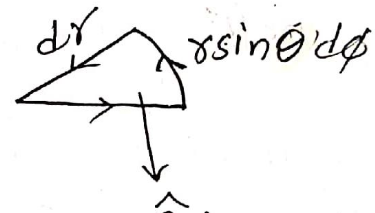
$r = \text{constant}$

(if $\alpha_1 \le \theta \le \alpha_2$
$\beta_1 \le \phi \le \beta_2$).



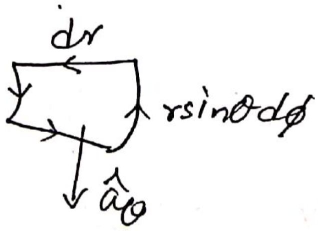
$r = \text{constant}$

(if $0 \le \theta \le \alpha_1$, $0 \le \phi \le \beta_1$)



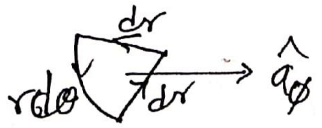
$\theta = \text{constant}$

(if $0 \le r \le a$)



$\theta = \text{constant}$

(if $a \le r \le b$)



$\phi = \text{constant}$

(if $0 \le r \le a$)

$\phi = \text{constant}$

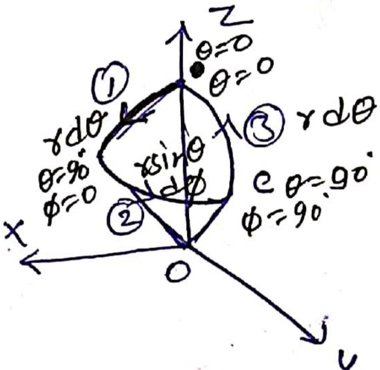
(if $a \le r \le b$)

n: Evaluate Both sides of stoke's theorem for the field $\vec{G} = 10\sin\theta\, \hat{a}_\phi$ and the surface $r=3, 0 \le \theta \le 90°, 0 \le \phi \le 90°$. Let the surface have the $\hat{a}_r$ direction.

Sol.^{n}

Given, $\vec{G} = 10\sin\theta\, \hat{a}_\phi$ and the surface $r=3, 0 \le \theta \le 90°$, $0 \le \phi \le 90°$.

Considering a contour, c for given surface; that consists of the three joined arcs of radius 3 that sweep out 90° in xy, xz and zy planes. Their centers at the origin.



From Stoke's theorem

$$\oint_C \vec{G} \cdot \vec{dl} = \int_S (\nabla \times \vec{G}) \cdot \vec{ds}$$

Taking left only.

$$\oint_C \vec{G} \cdot \vec{dl} = \int_1 \cancel{10\sin\theta\, \hat{a}_\phi \cdot r d\theta\, \hat{a}_\theta}^{\,0} + \int_2 10\sin\theta \cdot \hat{a}_\phi \cdot r\sin\theta\, d\phi\, \hat{a}_\phi$$

$$+ \int_3 \cancel{10\sin\theta \cdot \hat{a}_\phi \cdot r d\theta\, \hat{a}_\theta}^{\,0}$$

$$= \int_{2\,(\theta=90°\ \&\ r=3)} 10\sin^2\theta \cdot r\, d\phi$$

$$= \int_{\phi=0}^{90°} 10 \sin^2 90° \times 3\, d\phi$$

$$= 10 \times 1 \times 3 \int_{\phi=0}^{90° = \frac{\pi}{2}} d\phi$$

$$= 30 \times \frac{\pi}{2} = 15\pi \text{ A}$$

Again, taking right part only.

$$\int_S (\nabla \times \vec{V}) \cdot \vec{dS} \quad \text{where, } \nabla \times \vec{V} = \frac{1}{r\sin\theta}\left(\frac{\partial (V_\phi \sin\theta)}{\partial \theta} - \frac{\partial V_\theta}{\partial \phi}\right)\hat{a}_r$$

$$+ \frac{1}{r}\left(\frac{1}{\sin\theta}\cdot\frac{\partial V_r}{\partial \phi} - \frac{\partial (r V_\phi)}{\partial r}\right)\hat{a}_\theta + \frac{1}{r}\left(\frac{\partial (r V_\theta)}{\partial r} - \frac{\partial V_r}{\partial \theta}\right)\hat{a}_\phi$$

$$= \frac{1}{r\sin\theta}\cdot\frac{\partial}{\partial \theta}(10\sin\theta \times \sin\theta)\,\hat{a}_r - \frac{1}{r}\,10\sin\theta\,\hat{a}_\theta$$

$$= \frac{10}{r\sin\theta} \times \frac{\partial \sin^2\theta}{\partial \theta}\,\hat{a}_r - \frac{1}{r}\,10\sin\theta\,\hat{a}_\theta$$

$$= \frac{20}{r\sin\theta} \cdot \sin\theta \cdot \cos\theta\,\hat{a}_r - \frac{1}{r}\,10\sin\theta\,\hat{a}_\theta$$

$$= \frac{20\cos\theta}{r}\,\hat{a}_r - \frac{1}{r}\,10\sin\theta\,\hat{a}_\theta$$

$$\therefore \int_S (\nabla \times \vec{V}) \cdot \vec{dS} = \int_S \frac{20\cos\theta}{r}\hat{a}_r \cdot r^2\sin\theta\, d\phi\, d\theta\, \hat{a}_r$$

$$= \int_{\phi=0}^{90^\circ}\int_{\theta=0}^{90^\circ} 20 r \sin\theta \cdot \cos\theta \, d\theta\, d\phi$$

$(r=3)$

$$= \frac{60}{2}\int_{\phi=0}^{90^\circ = \frac{\pi}{2}}\int_{\theta=0}^{90^\circ} \sin 2\theta\, d\theta\, d\phi$$

$$= 30 \times \left(\frac{-\cos 2\theta}{2}\right)\Big|_0^{90^\circ} \cdot \left[\frac{\pi}{2} - 0\right]$$

$$= 30 \times \frac{\pi}{2} \times \left[-\cos 180^\circ + \cos 0^\circ\right] = 15\pi A$$

$$\therefore \oint_C \vec{V}\cdot\vec{dl} = \int_S (\nabla \times \vec{V}) \cdot \vec{dS} = 15\pi A$$

Qn: Given, the field $\vec{H} = 20\rho^2 \hat{a}_\phi$ A/m: (a) Determine the current density $\vec{J}$; (b) integrate $\vec{J}$ over the circular surface $\rho = 1$, $0 < \phi < 2\pi$, $z = 0$, to determine the total current passing through that surface in the $\hat{a}_z$ direction. (c) find the total current once more, this time by a line integral around the circular path $\rho = 1$, $0 < \phi < 2\pi$, $z = 0$.

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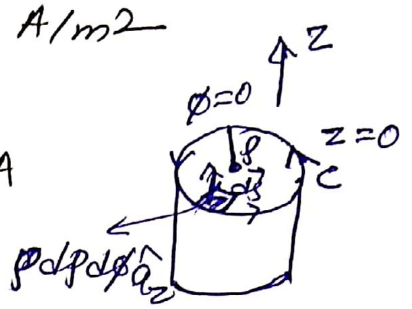
Sol.ⁿ (a) $\vec{J} = \nabla \times \vec{H} = \frac{1}{\rho} \frac{d(\rho H_\phi)}{d\rho} \hat{a}_z = 60\rho \hat{a}_z$ A/m2

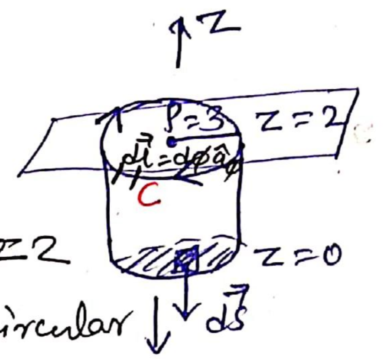
(b) $I = \int_S \vec{J} \cdot \vec{ds} = \int_{\phi=0}^{2\pi} \int_{\rho=0}^{1} \rho \, d\rho \cdot d\phi \, \hat{a}_z = 40\pi$ A



(c) $I = \oint_C \vec{H} \cdot \vec{dl} = \int_{\phi=0}^{2\pi} 20\rho^2 \hat{a}_\phi \Big|_{\rho=1} = 40\pi$ A.

Qn: Given, the field $\vec{H} = \frac{1}{2}\cos\frac{\phi}{2} \hat{a}_\rho - \sin\frac{\phi}{2} \hat{a}_\phi$ A/m, evaluate both sides of Stoke's theorem for the path formed by the intersection of the cylinder $\rho = 3$ and the plane $z = 2$ and for the surface defined by $\rho = 3$, $0 \le z \le 2$ and $z = 0$, $0 \le \rho \le 3$.

Sol.ⁿ Given, $\vec{H} = \frac{1}{2}\cos\frac{\phi}{2} \hat{a}_\rho - \sin\frac{\phi}{2} \hat{a}_\phi$ A/m



Here, the cylinder is cut by $z = 2$ plane so, it will be the cylinder from $z = 0$ to $z = 2$ and $\rho = 3$. the path (closed) will be the circular path at $z = 2$.

From Stoke's theorem.

$$\oint_C \vec{H} \cdot \vec{dl} = \int_S (\nabla \times \vec{H}) \cdot \vec{ds}.$$

Taking left only

$$\oint_C \vec{H} \cdot \vec{dl} = \int_{\phi=0}^{2\pi} \left(\frac{1}{2}\cos\frac{\phi}{2} \hat{a}_\rho - \sin\frac{\phi}{2} \hat{a}_\phi\right) \cdot (d\phi \, \hat{a}_\phi)$$

$$= \int_{\phi=0}^{2\pi} \sin\frac{\phi}{2}\, d\phi = 12A$$

Again, taking right part only; as we choose clockwise contour current will flow in negative z-axis.

So,

$\int_S (\nabla \times \vec{H}) \cdot \vec{ds}$ will be calculated

$$\nabla \times \vec{H} = \frac{1}{\rho}\left(\frac{\partial(\rho H_\phi)}{\partial \rho} - \frac{\partial H_\rho}{\partial \phi}\right)\hat{a}_z$$

$$= \frac{1}{\rho}\left(-\sin\frac{\phi}{2} + \frac{1}{4}\sin\frac{\phi}{2}\right)\hat{a}_z$$

$$= \frac{-3}{4\rho}\sin\frac{\phi}{2}\,\hat{a}_z \text{ A/m}.$$

We use the surface towards $-\hat{a}_z$ direction as given $z=0$.

$$\therefore \int_S (\nabla \times \vec{H}) \cdot \vec{ds} = \frac{-3}{4}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{3} \frac{1}{\rho}\sin\frac{\phi}{2}\,\hat{a}_z \cdot (-\hat{a}_z)\,\rho\, d\rho\, d\phi = 9A$$

Note that if the radial component of $\vec{H}$ were not included in the computation of $\nabla \times \vec{H}$, then the factor of 3/4 in front of the above integral would change to a factor of 1, and the result would have been 12A. [i.e $\frac{\partial H_\rho}{\partial \phi} = 0$ taken]

What would appear to be a violation of Stoke's theorem is likely the result of a missing term in the ϕ component of $\vec{H}$, having zero curl, which would have enabled the original line integral to have a value of 9A.

Qn: Given the vector magnetic potential $\vec{A} = -(\rho^2/4)\,\hat{a}_z$ Wb/m. Calculate the total magnetic flux crossing the surface $\phi = \pi/2$, $1 \le \rho \le 2$ m and $0 \le z \le 5$ m. [2072 Chaitra] [2069 Ashad]

Sol.^{n} Given, the vector magnetic potential is

$$\vec{A} = -\frac{\rho^2}{4}\hat{a}_z \text{ wb/m.}$$

We have,

$$\vec{B} = \nabla \times \vec{A} = \frac{1}{\rho}\left[\cancel{\frac{\partial(\rho \times 0)}{\partial \rho}}^{\,0} - \frac{\partial \times 0}{\partial \phi}\right]\hat{a}_z + \cancel{\left(\frac{1}{\rho}\frac{\partial(-\rho^2/4)}{\partial \phi} - \frac{\partial \times 0}{\partial z}\right)\hat{a}_\rho}^{\,0} + \left(\frac{\partial \times 0}{\partial z} - \frac{\partial(-\rho^2/4)}{\partial \rho}\right)\hat{a}_\phi$$

$$= -\frac{\partial(-\rho^2/4)}{\partial \rho}\hat{a}_\phi$$

$$= \frac{2\rho}{4}\hat{a}_\phi$$

$$= \frac{\rho}{2}\hat{a}_\phi$$

Again, the magnetic flux, $\phi_m = \int_S \vec{B} \cdot \vec{ds}$ through the surface $\phi = \frac{\pi}{2}$, $1 \le \rho \le 2$ m and $0 \le z \le 5$ m

or, $\phi_m = \int_S \frac{\rho}{2}\hat{a}_\phi \cdot d\rho . dz\, \hat{a}_\phi$

$$= \int_{\rho=1}^{2}\int_{z=0}^{5} \frac{\rho}{2}\, d\rho\, dz$$

$$= \frac{1}{2} \cdot \frac{\rho^2}{2}\Big|_1^2 [5-0]$$

$$= \frac{[4-1]}{4} \times 5$$

$$= \frac{15}{4} \text{ wb.} = 3.75 \text{ wb.}$$

Qn: For magnetic vectorpotential given in cylindrical co-ordinate system as $\vec{A} = 5r^3 \hat{a}_z$ wb/m in freespace find the magnetic field intensity, $\vec{H}$. [2069 Ashad]

$\left[-\frac{15r^2}{\mu_0} \text{ A/m}\right]$

Sol.^{n} $\vec{B} = \nabla \times \vec{A}$

& $\vec{H} = \frac{\vec{B}}{\mu_0}$

Qn: Given, the vector magnetic potential $\vec{A} = \frac{\rho^2}{8}\hat{a}_z$ wb/m. Calculate the total magnetic flux crossing the surface $\phi = \frac{\pi}{4}$, $1 \le \rho \le 3m$, $0 \le z \le 5m$. [2071 Chaitra]

Sol.ⁿ Given,

the vector magnetic potential is

$$\vec{A} = \frac{\rho^2}{8}\hat{a}_z \text{ wb/m}.$$

then, we have,

$$\vec{B} = \nabla \times \vec{A} = \left[\frac{\partial \times 0}{\partial z} - \frac{\partial\left(\frac{\rho^2}{8}\right)}{\partial \rho}\right]\hat{a}_\phi$$

$$= -\frac{2\rho}{8}\hat{a}_\phi$$

$$= -\frac{\rho}{4}\hat{a}_\phi$$

Again, the magnetic flux, ϕ_m crossing the surface $\phi = \pi/4$, $1 \le \rho \le 3m$, $0 \le z \le 5m$ is

$$\phi_m = \int_S \vec{B} \cdot \vec{ds}$$

$$= \int_S -\frac{\rho}{4}\hat{a}_\phi \cdot d\rho\, dz\, \hat{a}_\phi$$

$$= \int_{\rho=1}^{3}\int_{z=0}^{5} -\frac{\rho}{4}\, d\rho\, dz$$

$$= -\frac{1}{4} \times \frac{\rho^2}{2}\Big|_1^3 [5-0]$$

$$= -\frac{[9-1]}{8} \times 5$$

$$= -5 \text{ Wb}.$$

n: A current distribution gives rise to the vector magnetic potential $\vec{A} = x^2y\hat{a}_x + y^2x\hat{a}_y - 4xyz\hat{a}_z$ wb/m. Calculate (a) $\vec{B}$ at $(-1,2,5)$ (b) the flux through the surface defined by $z=1$, $0 \le x \le 1$, $-1 \le y \le 4$. [Ans (a) $20\hat{a}_x + 40\hat{a}_y + 3\hat{a}_z$ wb/m^2 (b) 20 wb]

Sol.^{n} Given, $\vec{A} = x^2y\hat{a}_x + y^2x\hat{a}_y - 4xyz\hat{a}_z$ wb/m.

we have, (a) $\vec{B} = \nabla \times \vec{A} = \left(\frac{\partial(-4xyz)}{\partial y} - \frac{\partial(y^2x)}{\partial z}\right)\hat{a}_x + \left(\frac{\partial(x^2y)}{\partial z} - \frac{\partial(-4xyz)}{\partial x}\right)\hat{a}_y + \left(\frac{\partial(y^2x)}{\partial x} - \frac{\partial(x^2y)}{\partial y}\right)\hat{a}_z$

$= -4xz\,\hat{a}_x + 4yz\,\hat{a}_y + (y^2 - x^2)\,\hat{a}_z$

$\vec{B}$ at $(-1,2,5)$ is $= -4\times(-1)\times 5\hat{a}_x + 4\times 2\times 5\,\hat{a}_y + (2^2-(-1)^2)\hat{a}_z$

$= 20\hat{a}_x + 40\hat{a}_y + 3\hat{a}_z$ wb/m^2.

Again, (b) the flux through the surface defined by $z=1$, $0 \le x \le 1$, $-1 \le y \le 4$ is

$$\phi = \int_s \vec{B}\cdot d\vec{s} = \int_{x=0}^{1}\int_{y=-1}^{4} [-4xz\,\hat{a}_x + 4yz\,\hat{a}_y + (y^2-x^2)\hat{a}_z]\cdot dx\,dy\,\hat{a}_z$$

$$= \int_{x=0}^{1}\int_{y=-1}^{4} (y^2-x^2)\,dx\,dy$$

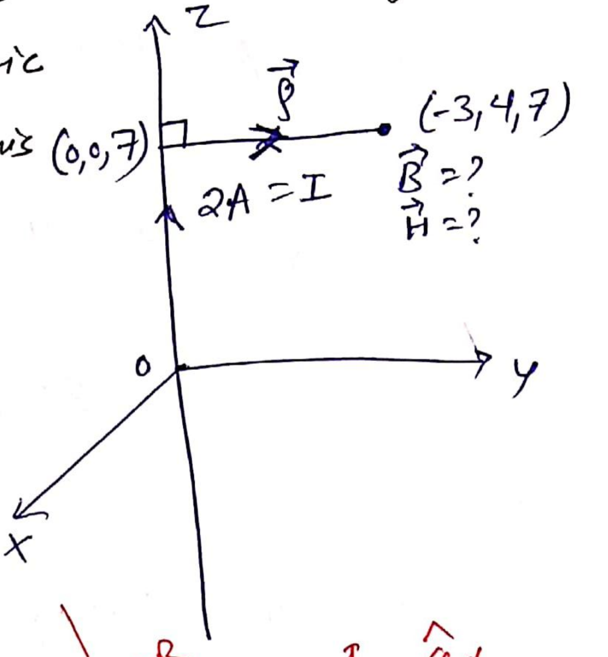
$$= -\frac{x^3}{3}\Big|_0^1 \cdot [4-(-1)] + \frac{y^3}{3}\Big|_{-1}^{4} \cdot [1-0]$$

$$= \frac{5\times}{3}[-1+0] + \frac{[4^3-(-1)^3]}{3}\times 1$$

$$= -\frac{5}{3} + \frac{64+1}{3} = \frac{60}{3} = 20 \text{ wb}.$$

Qn: An infinitely long filamentary wire carries a current of 2A in the +z-dir.^{n} Calculate (a) $\vec{B}$ at (-3, 4, 7) (b) The flux through the square loop described by $2 \le \rho \le 6$, $0 \le z \le 4$, $\phi = 90°$ [Ans: (a) $80\,\hat{a}_\phi$ nwb/m^2 (b) 1.756 μwb.]

Sol.^{n} Given, I = 2A current flows in $\hat{a}_z$ dir.^{n} in infinitely long filamentary wire. then, the magnetic field intensity at (-3, 4, 7) due to this filamentary wire is



$$\vec{H} = \frac{I}{2\pi\rho}\hat{a}_\phi$$

where, $\vec{\rho} = (-3-0)\hat{a}_x + (4-0)\hat{a}_y$

$= -3\hat{a}_x + 4\hat{a}_y$

$\rho = \sqrt{(-3)^2 + (4^2)} = 5$

$\hat{a}_\rho = \dfrac{-3\hat{a}_x + 4\hat{a}_y}{5}$

and $\hat{a}_\phi = \hat{a}_z \times \hat{a}_\rho = \hat{a}_z \times \left(\dfrac{-3\hat{a}_x + 4\hat{a}_y}{5}\right)$

$= \dfrac{-3\hat{a}_y - 4\hat{a}_x}{5}$

$\therefore\ \vec{H} = \dfrac{2}{2\pi \times 5} \times \left(\dfrac{-3\hat{a}_y - 4\hat{a}_x}{5}\right) = \dfrac{-3\hat{a}_y - 4\hat{a}_x}{25\pi}$ A/m.

Again, $\vec{B} = \mu_0\vec{H} = 4\cancel{\pi} \times 10^{-7} \times \left(\dfrac{-3\hat{a}_y - 4\hat{a}_x}{25\cancel{\pi}}\right)$

$= -\dfrac{12\hat{a}_y - 16\hat{a}_x}{25} \times 10^{-7}$ wb/m^2

$= \left[-\dfrac{1200}{25}\hat{a}_y - \dfrac{1600}{25}\right]$ nwb/m^2

$= -64\hat{a}_x - 48\hat{a}_y$ nwb/m^2

OR

$\vec{B} = \dfrac{\mu_0 I}{2\pi\rho}\hat{a}_\phi$

$= \dfrac{4\pi \times 10^{-7} \times 2}{2\pi \times 5}\hat{a}_\phi$

$= 80\,\hat{a}_\phi$ nwb/m^2

Again, changing $\vec{B}$ to cylindrical co-ordinates at (-3, ...

$$\vec{B}_{cyl.} = \left[\left(-64\hat{a}_x - 48\hat{a}_y\right)\cdot\hat{a}_\rho\right]\hat{a}_\rho + \left[\left(-64\hat{a}_x - 48\hat{a}_y\right)\cdot\hat{a}_\phi\right]\hat{a}_\phi$$

$$= \left[-64\cos\phi - 48\sin\phi\right]\hat{a}_\rho + \left[-64(-\sin\phi) - 48\cos\phi\right]\hat{a}_\phi$$

where, $\phi = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(-\frac{4}{3}\right) = 126.87^\circ$

$$\therefore \vec{B}_{cyl.} = \cancel{(+38.4 - 38.4)}^{\,0}\,\hat{a}_\rho + (51.2 + 28.8)\,\hat{a}_\phi$$

$$= 80\hat{a}_\phi \text{ nwb/m}^2$$

Again,

the flux through the square loop described by

$2 \le \rho \le 6,\ 0 \le z \le 4,\ \phi = 90^\circ$ is

$$\phi_m = \int_s \vec{B}\cdot d\vec{s} = \int_{\rho=2}^{6}\int_{z=0}^{4} \frac{4\pi\times10^{-7}\times2}{2\pi\rho}\,\hat{a}_\phi \cdot d\rho\, dz\, \hat{a}_\phi$$

$$= 16\times10^{-7}\ln 3 = 1.756\,\mu\text{wb}.$$

$\vec{B} = \frac{\mu_0 I}{2\pi\rho}\hat{a}_\phi$ & $\phi_m = \int_s \vec{B}\cdot d\vec{s} = \int_s \frac{\mu_0 I}{2\pi\rho}\hat{a}_\phi \cdot d\rho\, dz\, \hat{a}_\phi$

$$= \frac{\mu_0 I}{2\pi}\int_s \frac{d\rho}{\rho}\, dz$$

Q: Find $\vec{J}$ to $\vec{A} = \frac{10}{\rho^2}\hat{a}_z$ Wb/m in free space.

[Ans:- $-\frac{40}{\mu_0 \rho^4}\hat{a}_z$ A/m²]

Soln: we have, $\vec{B} = \nabla\times\vec{A} = \frac{20}{\rho^3}\hat{a}_\phi \left[\frac{\partial(10/\rho^2)}{\partial\rho}\hat{a}_\phi\right]$

Again, $\vec{H} = \frac{20}{\rho^3}\hat{a}_\phi \cdot \frac{1}{\mu_0}$.

& $\vec{J} = \nabla\times\vec{H} = \frac{1}{\mu_0\rho}\frac{\partial}{\partial}\left(\rho\cdot\frac{20}{\rho^3}\right)\hat{a}_z = -\frac{40}{\mu_0\rho^4}\hat{a}_z$ A/m²

Qn: The magnetic vector potential of a current distribution in free space is given by $\vec{A} = 15e^{-\rho}\sin\phi\,\hat{a}_z$ Wb/m. Find $\vec{H}$ at $(3, \pi/4, -10)$. Calculate the flux through $\rho = 5$, $0 \le \phi \le \pi/2$, $0 \le z \le 10$. [Ans:- $\vec{H} = (14\hat{a}_\rho + 42\hat{a}_\phi)\times 10^4$ A/m, $\Psi = -1.011$ Wb.]

Soln

$$\vec{B} = \nabla\times\vec{A} = \frac{1}{\rho}\frac{\partial A_z}{\partial\phi}\hat{a}_\rho - \frac{\partial A_z}{\partial\rho}\hat{a}_\phi$$

$$= \frac{15}{\rho}e^{-\rho}\cos\phi\,\hat{a}_\rho + 15e^{-\rho}\sin\phi\,\hat{a}_\phi$$

$$\vec{B} \text{ at } (3, \pi/4, -10) = 5e^{-3}\frac{1}{\sqrt{2}}\hat{a}_\rho + 15e^{-3}\frac{1}{\sqrt{2}}\hat{a}_\phi$$

$$\vec{H} = \frac{\vec{B}}{\mu_0} = (14\hat{a}_\rho + 42\hat{a}_\phi)\times 10^4 \text{ A/m.}$$

$$\phi_m = \int_S \vec{B}\cdot d\vec{s} = \int_{\phi=0}^{\pi/2}\int_{z=0}^{10} \frac{15}{\rho}e^{-\rho}\cos\phi\,\rho\,d\phi\,dz$$

$$= -150e^{-5} = -1.011 \text{ wb.}$$

Magnetic properties of Material

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ईन्जिनियर

According to magnetic properties, materials can be classified as:

(a) Non magnetic - where Magnetization is zero eg:- Vaccum or free Space.

(b) Diamagnetic - net magnetic moment of material is zero, however shows net magnetic moment opposing the field, when the field is applied. eg:- Bismuth.

(c) Paramagnetic - Magnetic moment aligned in the direction of field and attracted to magnet. eg: Aluminium

(d) Ferromagnetic - Magnetic moment strongly aligned in the direction of magnetic field and attracted to magnet. eg:- Iron.

(e) Antiferromagnetic - Magnetic moment aligned in antiparallel fashion and so non-magnetic even in the presence of magnetic field. eg:- Manganese oxide.

(f) FerriMagnetic - less magnetic than Ferromagnetic material. Magnetic moments aligned in antiparallel fashion with net magnetic moment. eg:- Iron ferrite.

(g) Superparamagnetic - It consists of ferromagnetic materials suspended in dielectric binders. eg: audio tape.

Magnetic Boundary Condition

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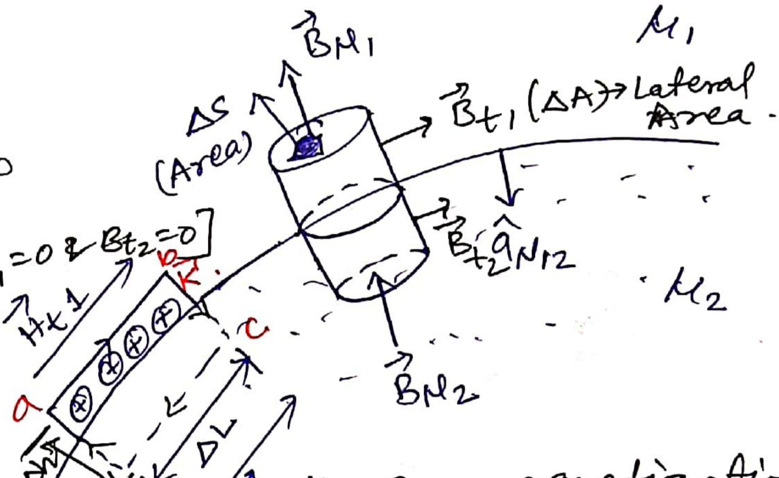
Let us consider two magnetic materials with relative permeabilities, μ_1 & μ_2 placed together forming the magnetic boundary betn two materials as shown in figure. Let the materials be isotropic homogeneous linear materials.

Then, the boundary condition on the normal components is determined by allowing the surface to cut a small cylindrical gaussian surface. Applying Gauss's Law for the magnetic field, we have

$$\oint_S \vec{B} \cdot d\vec{S} = 0$$



or, $B_{N1} \cdot \Delta S + \cancel{B_{t1}}\overset{0}{\Delta A} + \cancel{B_{t2}\Delta A}^{0} - B_{N2} \cdot \Delta S = 0$ [$\because B_{t1} = 0$ & $B_{t2} = 0$]

or, $B_{N1}\Delta S - B_{N2}\Delta S = 0$

or, $\boxed{B_{N2} = B_{N1}}$ i.e $\vec{B}_{N2} = \vec{B}_{N1}$

For magnetization

$$M_{N2} = \chi_{m2} \frac{\mu_1}{\mu_2} H_{N1}$$

$$= \frac{\chi_{m2}\mu_1}{\chi_{m1}\mu_2} M_{N1}$$

Again, $\mu_2 H_{N2} = \mu_1 H_{N1}$

or, $\boxed{H_{N2} = \frac{\mu_1}{\mu_2} H_{N1}}$ i.e $\vec{H}_{N2} = \frac{\mu_1}{\mu_2} \vec{H}_{N1}$

Also, from Ampere's Circuital Law.

$$\oint \vec{H} \cdot d\vec{l} = I$$

we can find the boundary condition for tangential components. So, taking a small closed path in a plane normal to the boundary surface as shown in figure and taking clockwise direction around the path we have,

$$H_{t_1}\Delta L + H_{N_1}\cancel{\frac{\Delta h}{2}} - H_{N_2}\cancel{\frac{\Delta h}{2}} - H_{t_2}\Delta L + H_{N_1}\cancel{\frac{\Delta h}{2}} + H_{N_2}\cancel{\frac{\Delta h}{2}}$$
$$= K\Delta L$$

or, $H_{t_1}\Delta L - H_{t_2}\Delta L = K\Delta L$ [$\because \Delta h \to 0$]

where, we assume that the boundary may carry a surface current $\vec{K}$ whose component to the plane of the closed path is K. Thus,

$$\boxed{H_{t_1} - H_{t_2} = K}$$

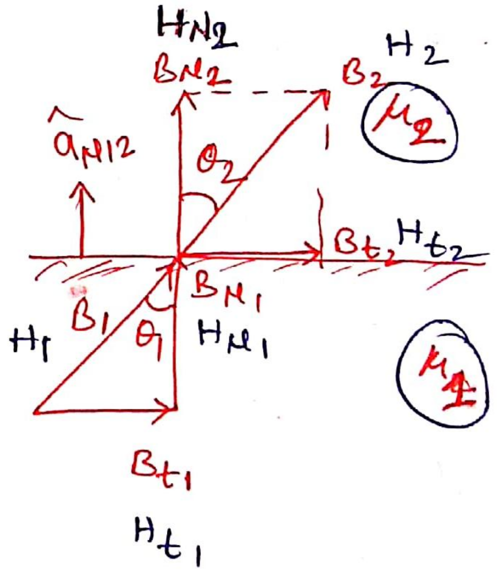
For the exact directions we use cross product as!

$$\boxed{(\vec{H_1} - \vec{H_2}) \times \hat{a}_{N12} = \vec{K}}$$

where, $\hat{a}_{N12}$ is the unit vector normal to the boundary directed from region 1 to region 2. Also, for vector form of tangential components, we have

$$\boxed{\vec{H}_{t_1} - \vec{H}_{t_2} = \hat{a}_{N12} \times \vec{K}}$$

And, for tangential $\vec{B}$, we have



$$\boxed{\frac{B_{t_1}}{\mu_1} - \frac{B_{t_2}}{\mu_2} = K}$$

For Magnetization.

$$M_{t_2} = \frac{\chi_{m2}}{\chi_{m1}} M_{t_1} - \chi_{m2} K$$

If the surface current density is zero, then, the relation will be simple to calculate and it will be zero (i.e K=0) if the free current density is zero that means no conductor is present. (Media are not Conductors)

$$B_1 \cos\theta_1 = B_{N1} = B_{N2} = B_2 \cos\theta_2$$

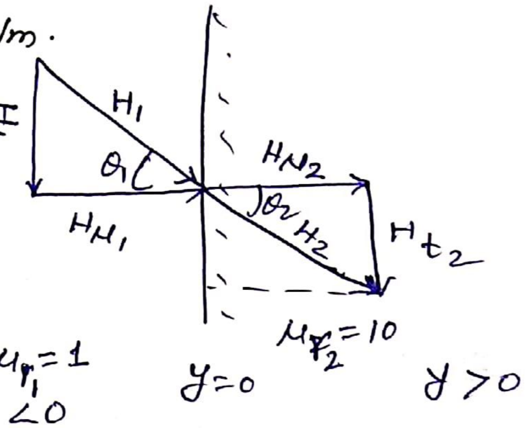
& $$\frac{B_1}{\mu_1} \cos\theta_1 = H_{t_1} = H_{t_2} = \frac{B_2}{\mu_2} \sin\theta_2$$

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}}$$

If $K = 0$, $\vec{H}_{t_1} = \vec{H}_{t_2}$ or $\frac{\vec{B}_{t_1}}{\mu_1} = \frac{\vec{B}_{t_2}}{\mu_2}$

n: The region $y<0$ (Region 1) is air and $y>0$ (Region 2) has $\mu_r = 10$. There is a uniform magnetic field $\vec{H} = 5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z$ A/m in region 1, find $\vec{B}$ and $\vec{H}$ in region 2. [2073 Shrawan]

Sol.ⁿ Given, Region 1 ($y<0$) has air i.e $\mu_{r_1} = 1$ and Region 2 ($y>0$) has $\mu_{r_2} = 10$. Also, $\vec{H}_1 = 5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z$ A/m.



Since, the boundary is $y=0$ plane, the normal unit vector $\hat{a}_{n_{12}} = \hat{a}_y$.

So, $\vec{H}_{N_1} = (\vec{H} \cdot \hat{a}_{n_{12}})\hat{a}_{n_{12}}$

$= 6\hat{a}_y$ A/m.

And, $\vec{H}_{t_1} = \vec{H}_1 - \vec{H}_{N_1} = 5\hat{a}_x + 7\hat{a}_z$ A/m.

Again, $\vec{B}_1 = \mu_0 \mu_{r_1} \vec{H}_1 = \mu_0 \times 1 (5\hat{a}_x + 6\hat{a}_y + 7\hat{a}_z)$ wb/m²

& $\vec{B}_{N_1} = \mu_0 \mu_{r_1} \vec{H}_{N_1} = \mu_0 \times 1 (6\hat{a}_y) = 6\mu_0 \hat{a}_y$ wb/m²

Now, using Boundary Conditions for magnetic materials.

$\vec{B}_{N_2} = \vec{B}_{N_1} = 6\mu_0 \hat{a}_y = 75.4 \times 10^{-7} \hat{a}_y$ wb/m²

$= 7.54 \hat{a}_y$ μwb/m²

and $\vec{H}_{t_2} = \vec{H}_{t_1}$ [for non conducting materials]

$= 5\hat{a}_x + 7\hat{a}_z$ A/m.

& $\vec{B}_{t_2} = \mu_0 \mu_{r_2} \vec{H}_{t_2} = \mu_0 \times 10 \times (5\hat{a}_x + 7\hat{a}_z)$

$= 628.32 \times 10^{-7} \hat{a}_x + 879.65 \hat{a}_z \times 10^{-7}$ wb/m²

$= 62.83 \hat{a}_x + 87.96 \hat{a}_z$ μwb/m²

$\therefore \vec{B}_2 = \vec{B}_{N_2} + \vec{B}_{t_2} = 7.54\hat{a}_y + 62.83\hat{a}_x + 87.96\hat{a}_z$ μwb/m²

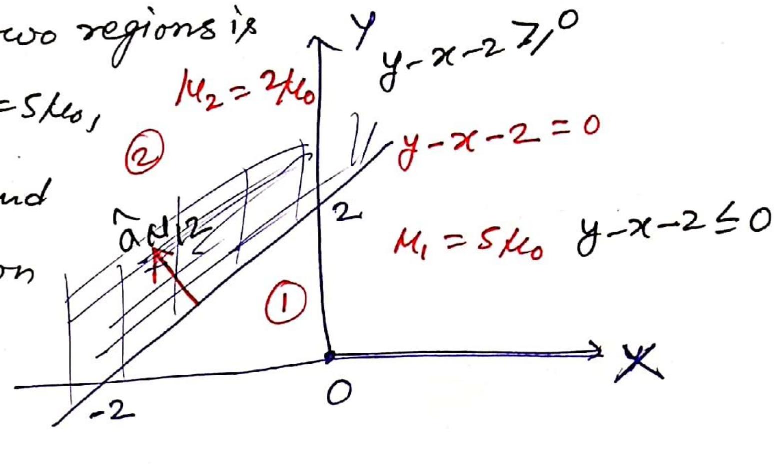
and

$$\vec{H}_2 = \frac{\vec{B}_2}{\mu_0 \mu_{r_2}} = \frac{(62.83\hat{a}_x + 7.54\hat{a}_y + 87.96\hat{a}_z)}{4\pi \times 10^{-7} \times 10} \times 10^{-6} \text{ A/m}$$

$= 5\hat{a}_x + 0.6\hat{a}_y + 7\hat{a}_z$ A/m.

Qn: Given, that $\vec{H_1} = -2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z$ A/m in region $y-x-2 \le 0$ where $\mu_1 = 5\mu_0$; calculate (a) $\vec{M_1}$ and $\vec{B_1}$ (b) $\vec{H_2}$ & $\vec{B_2}$ in region $y-x-2 \geq 0$ where $\mu_2 = 2\mu_0$.

Sol.ⁿ Given, the boundary betⁿ two regions is $y-x-2=0$ and; region 1 has $\mu_1 = 5\mu_0$, $\vec{H_1} = -2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z$ A/m and region 2 has $\mu_2 = 2\mu_0$ as shown in figure



(a) we have,

$$\vec{B_1} = \mu_1 \vec{H_1} = 5 \times \mu_0 (-2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z)$$
$$= -12.57\hat{a}_x + 37.7\hat{a}_y + 25.13\hat{a}_z \ \mu Wb/m^2$$

and, the normal unit vector $\hat{a}_{n12}$ is given by

$$\hat{a}_{n12} = \frac{\nabla f}{|\nabla f|} \quad \text{where, } f(x,y) = y-x-2$$
$$\& \ \nabla f = \hat{a}_y - \hat{a}_x$$

$$\therefore \hat{a}_{n12} = \frac{-\hat{a}_x + \hat{a}_y}{\sqrt{2}}$$

So, $\vec{M_1} = (\mu_{r1} - 1)\vec{H_1} = (5-1)\cdot(-2\hat{a}_x + 6\hat{a}_y + 4\hat{a}_z)$
$$= -8\hat{a}_x + 24\hat{a}_y + 16\hat{a}_z \text{ A/m}.$$

(b) Again,

we have,

$$\vec{H}_{n1} = (\vec{H_1} \cdot \hat{a}_{n12})\hat{a}_{n12} = -4\hat{a}_x + 4\hat{a}_y \text{ A/m}.$$

$$\& \ \vec{H}_{t1} = \vec{H}_1 - \vec{H}_{n1} = 2\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z \text{ A/m}.$$

Using boundary conditions, we have

$$\vec{H}_{t2} = \vec{H}_{t1} = 2\hat{a}_x + 2\hat{a}_y + 4\hat{a}_z \text{ A/m}.$$

$$\vec{B}_{n2} = \vec{B}_{n1} = (\vec{B_1} \cdot \hat{a}_{n12})\hat{a}_{n12} \Rightarrow \frac{\mu_1}{\mu_2}\vec{H}_{n1} = \vec{H}_{n2}$$

$\therefore \vec{H}_{\mu_2} = -10\hat{a}_x + 10\hat{a}_y$ A/m.

Again,

$\therefore \vec{H}_2 = \vec{H}_{\mu_2} + \vec{H}_{t_2} = -8\hat{a}_x + 12\hat{a}_y + 4\hat{a}_z$ A/m

and $\vec{B}_2 = \mu_2 \vec{H}_2 = \mu_0 \mu_{r_2} \vec{H}_2 = -20.11\hat{a}_x + 30.16\hat{a}_y + 10.05\hat{a}_z \, \mu wb/m^2$

If θ_1 & θ_2 are to be calculated then

$\cos\theta_1 = \dfrac{\vec{H}_1}{|\vec{H}_1|} \cdot \hat{a}_N$ $\left[\because \hat{a}_{\vec{H}_1} \cdot \hat{a}_N = |\hat{a}_{\vec{H}_1}| \cdot |\hat{a}_N| \cos\theta_1 \right]$

or, $\theta_1 = \cos^{-1}\left(\dfrac{\vec{H}_1}{|\vec{H}_1|} \cdot \hat{a}_N \right) = 40.89^\circ$

$\cos\theta_2 = \dfrac{\vec{H}_2}{|\vec{H}_2|} \cdot \hat{a}_N$

or, $\theta_2 = \cos^{-1}\left(\dfrac{\vec{H}_2}{|\vec{H}_2|} \cdot \hat{a}_N \right) = 19.10^\circ$

∴ Region 2 ... stric...

1: Region 1, described by $3x+4y \geq 10$, is free space whereas region 2, described by $3x+4y \leq 10$, is a magnetic material for which $\mu = 10\mu_0$. Assuming that the boundary betn the material and free space is current free, find $\vec{B}_2$ if $\vec{B}_1 = 0.1\hat{a}_x + 0.4\hat{a}_y + 0.2\hat{a}_z$ wb/m². [Ans: $-1.052\hat{a}_x + 1.264\hat{a}_y + 2\hat{a}_z$ wb/m²]

Qn: A unit normal vector from region 2 ($\mu = 2\mu_0$) to region 1 ($\mu = \mu_0$) is $\hat{a}_{N21} = (6\hat{a}_x + 2\hat{a}_y - 3\hat{a}_z)/7$. If $\vec{H}_1 = 10\hat{a}_x + \hat{a}_y + 12\hat{a}_z$ A/m. and $\vec{H}_2 = H_{2x}\hat{a}_x - 5\hat{a}_y + 4\hat{a}_z$ A/m, determine

(a) H_{2x} (b) The surface current density $\vec{K}$ on the interface

(c) The angles $\vec{B}_1$ & $\vec{B}_2$ make with the normal to the interface

[Ans: (a) 5.833, (b) $4.86\hat{a}_x - 8.64\hat{a}_y + 3.95\hat{a}_z$ A/m (c) 76.27°, 77.62°]

Qn: Point (2,3,1) lies on the planar boundary separating region 1 from region 2. The unit vector $\hat{a}_{N12} = 0.6\hat{a}_x + 0.48\hat{a}_y + 0.64\hat{a}_z$ is directed from region 1 to region 2. Let $\mu_{r1} = 2$, $\mu_{r2} = 8$, and $\vec{H}_1 = 100\hat{a}_x - 300\hat{a}_y + 200\hat{a}_z$ A/m. Find $\vec{H}_2$.

[Ans: $80.2\hat{a}_x - 315.8\hat{a}_y + 178.9\hat{a}_z$ A/m]

Qn: Let $\mu_{r1} = 2$ in region 1, defined by $2x+3y-4z > 1$, while $\mu_{r2} = 5$ in region 2 where $2x+3y-4z < 1$. In region 1, $\vec{H}_1 = 50\hat{a}_x - 30\hat{a}_y + 20\hat{a}_z$ A/m. Find: (a) $\vec{H}_{N1}$, (b) $\vec{H}_{t1}$, (c) $\vec{H}_{t2}$ (d) $\vec{H}_{N2}$ (e) θ_1 (f) θ_2.

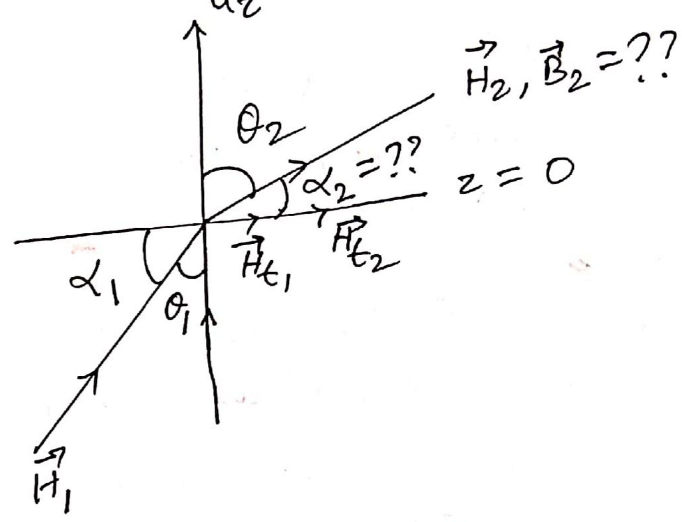
$\cos\theta_1 = \frac{\vec{H}_1}{|\vec{H}_1|} \cdot \hat{a}_{N21}$

[Sol: $\hat{a}_{N21} = 0.37\hat{a}_x + 0.56\hat{a}_y - 0.74\hat{a}_z \Rightarrow \hat{a}_{N21} = \frac{\nabla(2x+3y-4z)}{|\nabla(2x+3y-4z)|}$ increasing value of the function is the direction]

Ans (a) $\vec{H}_{N1} = -4.83\hat{a}_x - 7.24\hat{a}_y + 9.66\hat{a}_z$ A/m (b) $\vec{H}_{t1}$ (c) $\vec{H}_{t2} = 54.83\hat{a}_x - 22.76\hat{a}_y + 10.34\hat{a}_z$ A/m

(d) $\vec{H}_{N2} = -1.93\hat{a}_x - 2.90\hat{a}_y + 3.86\hat{a}_z$ A/m, (e) $102° = \theta_1$ (f) $\theta_2 = 95°$

Qn: Flux density at medium with $\mu_1 = 15$ is $\vec{B_1} = 1.2\hat{a}_x + 8\hat{a}_y + 4\hat{a}_z$ T. Find $\vec{B}$, $\vec{H}$ and the angles betn the field vectors and tangent to the interface at second medium, if second medium has $\mu_2 = 1$ and interface plane is $z=0$. [2073 chaitra]



Q. Let us assume that $\mu = \mu_1 = 4\,\mu H/m$ in region 1 where $z > 0$, while $\mu_2 = 7\,\mu H/m$ wherever $z < 0$. Moreover, let $\vec{K} = 80\hat{a}_x$ A/m on the surface $z = 0$ and a field, $\vec{B}_1 = 2\hat{a}_x - 3\hat{a}_y + \hat{a}_z$ mT, in region 1, then find $\vec{B}_2$. (2074 Ashwin)

Sol.^{n} Here, the normal from region 1 to region 2 is $\hat{a}_{N12} = -\hat{a}_z$

So, normal component of $\vec{B}_1$ is

$$\vec{B}_{N1} = (\vec{B}_1 \cdot \hat{a}_{N12})\,\hat{a}_{N12} = [(2\hat{a}_x - 3\hat{a}_y + \hat{a}_z)\cdot(-\hat{a}_z)](-\hat{a}_z)$$
$$= \hat{a}_z \text{ mT}$$

thus, $\vec{B}_{N2} = \vec{B}_{N1} = \hat{a}_z$ mT

Now, the tangential component is

$$\vec{B}_{t1} = \vec{B}_1 - \vec{B}_{N1} = 2\hat{a}_x - 3\hat{a}_y \text{ mT}$$

and $\vec{H}_{t1} = \dfrac{\vec{B}_{t1}}{\mu_1} = 500\hat{a}_x - 750\hat{a}_y$ A/m

thus, $\vec{H}_{t2} = \vec{H}_{t1} - \hat{a}_{N12} \times \vec{K} = 500\hat{a}_x - 750\hat{a}_y - (-\hat{a}_z)\times 80\hat{a}_x$

[From boundary Cond.^{n}]

$$= 500\hat{a}_x - 750\hat{a}_y + 80\hat{a}_y = 500\hat{a}_x - 670\hat{a}_y \text{ A/m}$$

$[\vec{H}_{t1} - \vec{H}_{t2} = \hat{a}_{N12} \times \vec{K}]$

and $\vec{B}_{t2} = \mu_2 \vec{H}_{t2} = 3.5\hat{a}_x - 4.69\hat{a}_y$ mT

$\therefore \vec{B}_2 = \vec{B}_{N2} + \vec{B}_{t2} = 3.5\hat{a}_x - 4.69\hat{a}_y + \hat{a}_z$ mT