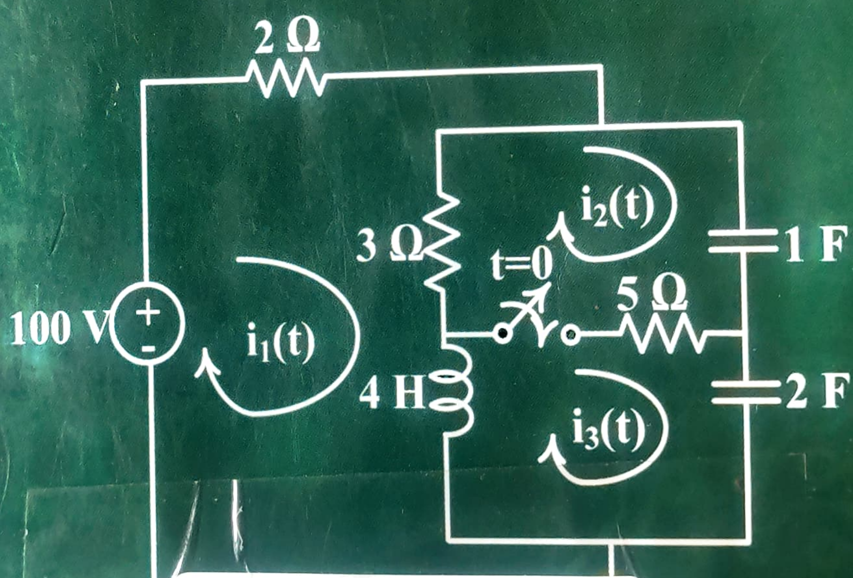
Heritage's Engineering Series

A Textbook of

Electric Circuit Theory

For Bachelor's of Engineering (B.E.)



Mukesh Gautam

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CHAPTER 1

Network Analysis

NETWORK TERMINOLOGY

Loop and Mesh

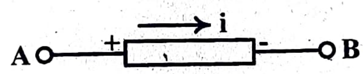
A loop is any closed path of a circuit, while mesh is a loop which does not contain any other loop within it. Therefore, all the meshes are loops and a loop is not necessarily a mesh. Loop may have other loops or meshes inside it.

Node and Junction

A point where two or more branches meet is called a node, while a junction is a point at which three or more branches are joined together. Therefore, all the junctions are nodes and a node is not necessarily a junction.

Sign Convention

An electrical load having two terminals A and B known as a two-terminal load is shown in the figure below.



If current 'i' flows in the direction shown, from A to B, then voltage 'v' drops from A to B i.e. A is at a higher potential than B. The polarity of voltage drop is as indicated in the figure.

.2 KIRCHHOFF'S LAW

a) **Kirchhoff's Current Law (KCL)**

It states that, "the sum of currents entering a node in a circuit through branches connected to that node at any instant of time is equal to the sum of currents leaving the node at that instant through the remaining branches connected to the same node".

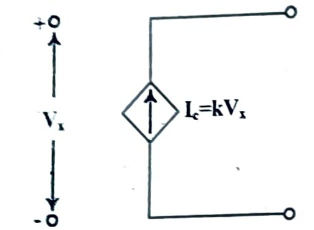
b) **Kirchhoff's Voltage Law (KVL)**

It states that, "the sum of the potential drops, at any instant of time, along the branches of a circuit when traversing a loop in a certain direction, clockwise or anti-clockwise, is equal to the sum of the potential rise in the remaining branches forming the loop".

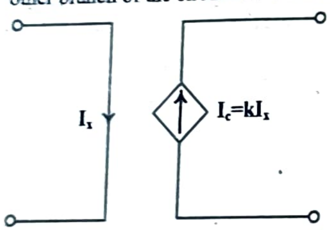
1.3 DEPENDENT OR CONTROLLED SOURCE

The source that can be controlled by (or, depends on) a voltage or current existing at some other place of the circuit is known as controlled or dependent source. Dependent or controlled source can be of four different types:

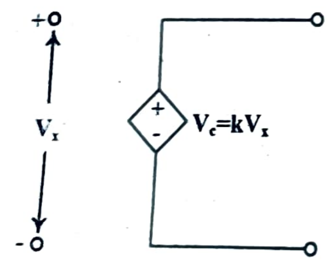
a) **Voltage Controlled Current Source (VCCS)**
VCCS is a current source whose current depends on voltage at some other place of the circuit. VCCS is shown in the figure below.



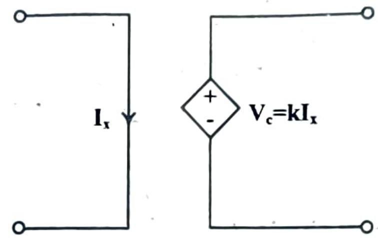
b) **Current Controlled Current Source (CCCS)**
CCCS is a current source whose current depends on current at some other branch of the circuit. CCCS is shown in the figure below.



c) **Voltage Controlled Voltage Source (VCVS)**
VCVS is a voltage source whose voltage depends on voltage at some other place of the circuit. VCVS is as shown in the figure below.



d) **Current Controlled Voltage Source (CCVS)**
CCVS is a voltage source whose voltage depends on current at some other branch of the circuit. CCVS is as shown in the figure below.



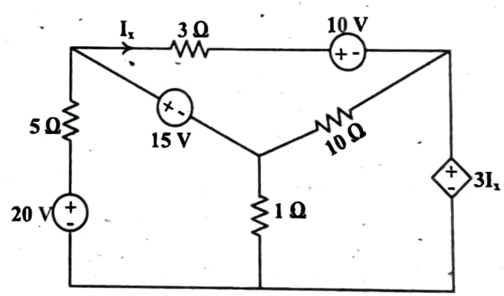
1.4 MESH ANALYSIS

Mesh analysis (or, mesh current method) is the method used to solve the planar circuit for current (and indirectly voltage) at any place in an electrical circuit. Mesh analysis makes use of KVL to arrive at a set of equations guaranteed to be solvable if the circuit has a solution.

1.5 SOLVED PROBLEMS OF NETWORK ANALYSIS USING MESH ANALYSIS

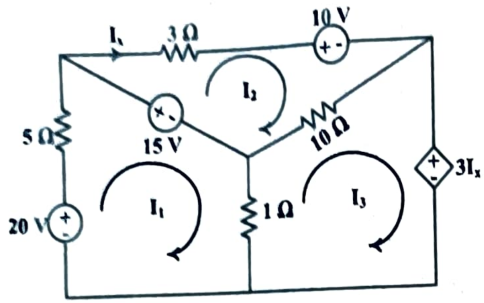
Example 1.1

Calculate the current through each resistor using mesh analysis in the circuit shown in the figure below.



Solution:

Let's consider three meshes 1, 2 and 3 with mesh currents I_1, I_2 and I_3 respectively as shown in the figure below.



Applying KVL in mesh 1,

$$20 - 15 = 5I_1 + 1 \times (I_1 - I_3)$$

or, $6I_1 - I_3 = 5$ ....(1)

Applying KVL in mesh 2,

$$15 - 10 = 3I_2 + 10 \times (I_2 - I_3)$$

or, $13I_2 - 10I_3 = 5$ ....(2)

Applying KVL in mesh 3,

$$-3I_x = 1 \times (I_3 - I_1) + 10 \times (I_3 - I_2)$$

But, $I_x = I_2$

So, $-3I_2 = 1 \times (I_3 - I_1) + 10 \times (I_3 - I_2)$

or, $I_1 + 7I_2 - 11I_3 = 0$ ....(3)

Solving equations (1), (2) and (3), we get,

$I_1 = 0.94A$,

$I_2 = 0.88$ A

and $I_3 = 0.65$ A

Now, the currents through each resistor are;

$I_{1\Omega} = I_1 - I_3 = 0.94 - 0.65 = \mathbf{0.29\ A}$

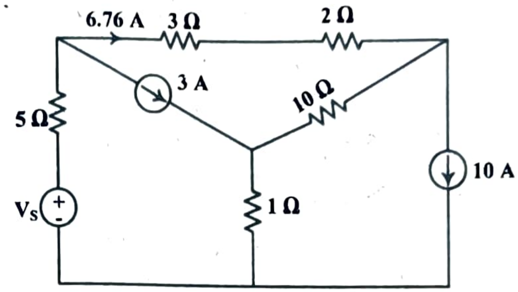
$I_{3\Omega} = I_2 = \mathbf{0.88\ A}$

$I_{5\Omega} = I_1 = \mathbf{0.94\ A}$

$I_{10\Omega} = I_2 - I_3 = \mathbf{0.23\ A}$

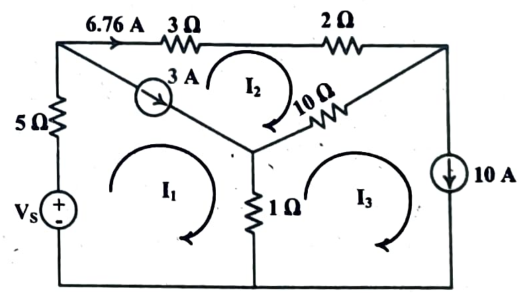
Example 1.2

Find the value of source voltage V_S for the circuit shown below using mesh analysis if the current through 3 Ω resistor is 6.76 A.



Solution:

Let's consider three meshes 1, 2 and 3 with mesh currents I_1, I_2 and I_3 respectively as shown in the figure below.



Applying KVL in super-mesh (i.e., meshes 1 and 2),

$$V_s = 5I_1 + 3I_2 + 2I_2 + 10 \times (I_2 - I_3) + 1 \times (I_1 - I_3)$$

or, $V_s = 6I_1 + 15I_2 - 11I_3$ ....(1)

In the common branch between meshes 1 and 2,

$$I_1 - I_2 = 3 \text{ ....(2)}$$

In the outer branch of mesh 3,

$$I_3 = 10 \text{ A}$$

Given that, the current through 3 Ω resistor = I_2 = 6.76 A

Then, from equation (2),

$$I_1 = 6.76 + 3 = 9.76 \text{ A}$$

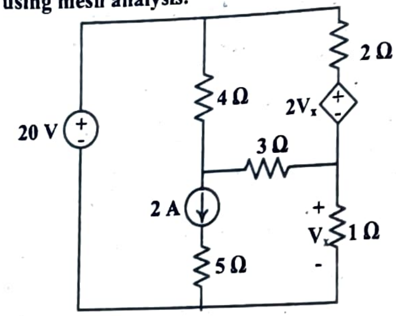
Now, from equation (1),

$$V_s = 6I_1 + 15I_2 - 11I_3$$

or, $\mathbf{V_s = 49.96\ V}$

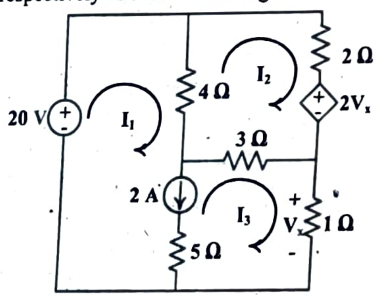
Example 1.3

In the given circuit, determine the voltage across 1 Ω resistor using mesh analysis.



Solution:

Let's consider three meshes 1, 2 and 3 with mesh currents I_1, I_2 and I_3 respectively as shown in the figure below.



Applying KVL in super-mesh (i.e., meshes 1 and 3),

$$20 = 4 \times (I_1 - I_2) + 3 \times (I_3 - I_2) + 1 \times I_3$$

or, $\quad 4I_1 - 7I_2 + 4I_3 = 20$ ....(1)

Applying KVL in mesh 2,

$$-2V_x = 2I_2 + 3 \times (I_2 - I_3) + 4 \times (I_2 - I_1)$$

But $\quad V_x = 1 \times I_3 = I_3$

Then, $-2I_3 = 2I_2 + 3 \times (I_2 - I_3) + 4 \times (I_2 - I_1)$

or, $\quad 4I_1 - 9I_2 + I_3 = 0$ ....(2)

In the common branch between meshes 1 and 3,

$$I_1 - I_3 = 2 \text{ ....(3)}$$

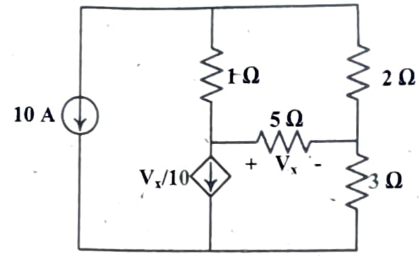
Solving equations (1), (2) and (3), we get,

$I_1 = 6.43$ A, $\qquad I_2 = 3.35$ A $\qquad$ and $\qquad I_3 = 4.43$ A

Now, the voltage across 1 Ω resistor, $V_{1\Omega} = 1 \times I_3 =$ **4.43 V**

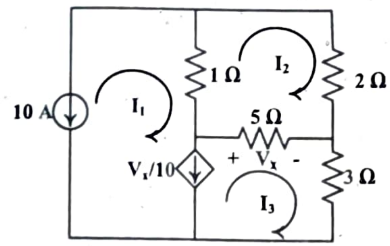
Example 1.4

Find the currents through each resistor using Mesh Analysis.



Solution:

Let's consider three meshes 1, 2 and 3 with mesh currents I_1, I_2 and I_3 respectively as shown in the figure below.



In the outer branch of mesh 1,

$$I_1 = -10 \text{ A}$$

Applying KVL in mesh 2,

$$1 \times (I_2 - I_1) + 2I_2 + 5 \times (I_2 - I_3) = 0$$

or, $\quad -I_1 + 8I_2 - 5I_3 = 0$

or, $\quad 10 + 8I_2 - 5I_3 = 0$

or, $\quad 8I_2 - 5I_3 = -10$ ....(1)

In the common branch between meshes 1 and 3,

$$I_1 - I_3 = \frac{V_x}{10}$$

But, $\quad V_x = (I_3 - I_2) \times 5$

Then, $\quad I_1 - I_3 = \dfrac{(I_3 - I_2) \times 5}{10}$

or, $\quad I_2 - 3I_3 = 20$ ....(2)

Solving equations (1) and (2), we get,

$I_2 = -6.84$ A and $I_3 = -8.95$ A

Now, the currents through each resistor are;

$I_{1\Omega} = I_2 - I_1 = -6.84 + 10 = 3.16$ **A**

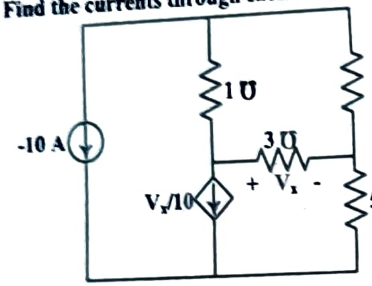
$I_{2\Omega} = -I_2 = 6.84$ **A**

$I_{3\Omega} = -I_3 = 8.95$ **A**

$I_{5\Omega} = I_2 - I_3 = 2.11$ **A**

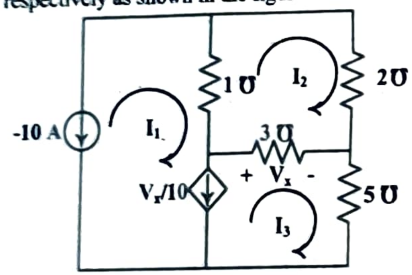
Example 1.5

Find the currents through each resistor using mesh analysis.



Solution:

Let's consider three meshes 1, 2 and 3 with mesh currents I_1, I_2 and I respectively as shown in the figure below.



In the outer loop of mesh 1,

$I_1 = 10$ A

Applying KVL in mesh 2,

$$\frac{I_2 - I_1}{1} + \frac{I_2 - I_3}{3} + \frac{I_2}{2} = 0$$

or, $$-I_1 + \frac{11}{6} I_2 - \frac{1}{3} I_3 = 0$$

or, $$-10 + \frac{11}{6} I_2 - \frac{1}{3} I_3 = 0$$

or, $$\frac{11}{6} I_2 - \frac{1}{3} I_3 = 10 \ldots\ldots\ldots\ldots(1)$$

In the common branch between meshes 1 and 3,

$$I_1 - I_3 = \frac{V_x}{10}$$

But, $$V_x = \frac{I_3 - I_2}{3}$$

Then, $$I_1 - I_3 = \frac{I_3 - I_2}{3 \times 10}$$

or, $$10 - I_3 = \frac{I_3 - I_2}{3 \times 10}$$

or, $$\frac{-1}{30} I_2 + \frac{31}{30} I_3 = 10 \ldots\ldots\ldots(2)$$

Solving equations (1) and (2), we get

$I_2 = 7.26$ A and $I_3 = 9.91$ A

Now, the currents through each resistor are;

$I_{1℧} = I_1 - I_2 = 2.74$ **A**

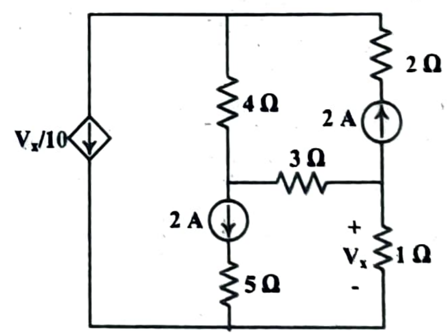
$I_{2℧} = I_2 = 7.26$ **A**

$I_{3℧} = I_3 - I_2 = 2.65$ **A**

$I_{5℧} = I_3 = 9.91$ **A**

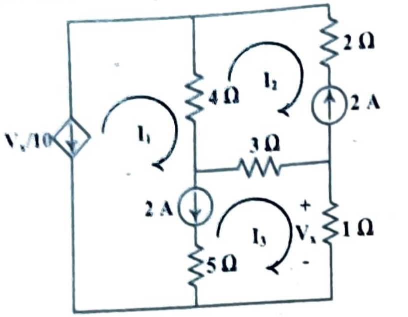
Example 1.6

Find the currents through each resistor using mesh analysis.



Solution:

Let's consider three meshes 1, 2 and 3 with mesh currents I_1, I_2 and I_3 respectively as shown in the figure below.



In the outer branch of mesh 1,

$$I_1 = \frac{-V_x}{10}$$

But, $V_x = I_3 \times 1 = I_3$

Then, $I_1 = \frac{-I_3}{10}$

or, $I_1 + \frac{1}{10} I_3 = 0$ ....(1)

In the outer branch of mesh 2,

$$I_2 = -2 \text{ A}$$

In the common branch between meshes 1 and 3,

$$I_1 - I_3 = 2 \text{ ....(2)}$$

Solving equations (1) and (2), we get,

$I_1 = 0.18$ A and $I_3 = -1.82$ A

Now, the currents through each resistor are;

$$I_{1\Omega} = -I_3 = \mathbf{1.82A}$$

$$I_{2\Omega} = -I_2 = \mathbf{2\ A}$$

$$I_{3\Omega} = I_3 - I_2 = -1.82 + 2 = \mathbf{0.18\ A}$$

$$I_{4\Omega} = I_1 - I_2 = \mathbf{2.18\ A}$$

$$I_{5\Omega} = I_1 - I_3 = \mathbf{2\ A}$$

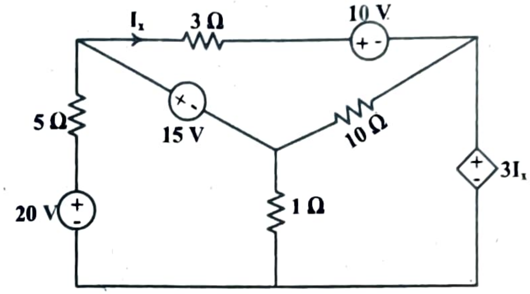
1.6 NODAL ANALYSIS

Nodal Analysis (or node voltage method) is the method of circuit analysis used to determine the values of voltages at all the principal nodes with respect to the reference node. After the determination of the node voltages, the currents flowing in each branch can also be determined. In nodal analysis, Kirchhoff's Current Law (KCL) is applied to each node and simultaneous equations are formed, whose solution gives node voltages with respect to the reference node.

1.7 SOLVED PROBLEMS OF NETWORK ANALYSIS USING NODAL ANALYSIS

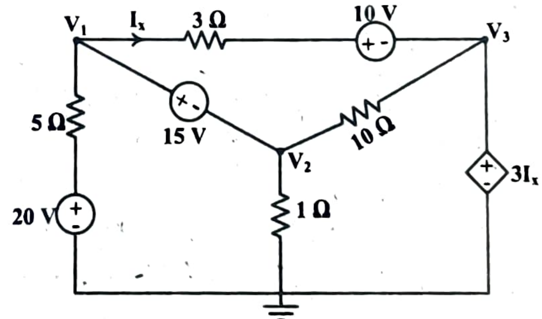
Example 1.7

Calculate the current through each resistor using nodal analysis in the circuit shown in the figure below.



Solution:

Let's consider three nodes 1, 2 and 3 with node voltages V_1, V_2 and V_3 respectively as shown in the figure below.



Applying KCL at super-node (i.e., nodes 1 and 3),

$$\frac{V_1 - 20}{5} + \frac{V_1 - 10 - V_3}{3} + \frac{V_2 - V_3}{10} + \frac{V_2 - 0}{1} = 0$$

or, $\frac{8}{15} V_1 + \frac{11}{10} V_2 - \frac{13}{30} V_3 = \frac{22}{3}$ ....(1)

The voltage between nodes 1 and 2 is

$$V_1 - V_2 = 15 \text{ .... (2)}$$

The voltage between node 3 and reference node is

$$V_3 = 3I_x$$

But $\quad I_x = \dfrac{V_1 - 10 - V_3}{3}$

Then $\quad V_3 = 3\left(\dfrac{V_1 - 10 - V_3}{3}\right)$

or, $\quad V_1 - 2V_3 = 10$ ....(3)

Solving equations (1), (2) and (3), we get

$V_1 = 15.29$ V, $\quad V_2 = 0.29$ V $\quad$ and $\quad V_3 = 2.65$ V

Now, the currents through each resistor are;

$$I_{1\Omega} = \frac{V_2 - 0}{1} = 0.29 \text{ A}$$

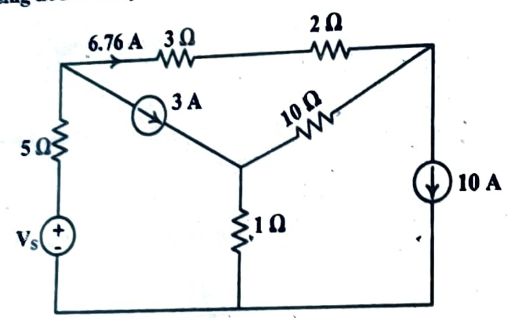
$$I_{3\Omega} = \frac{V_1 - 10 - V_3}{3} = 0.88 \text{ A}$$

$$I_{5\Omega} = \frac{20 - V_1}{5} = 0.94 \text{ A}$$

$$I_{10\Omega} = \frac{V_3 - V_2}{10} = 0.24 \text{ A}$$

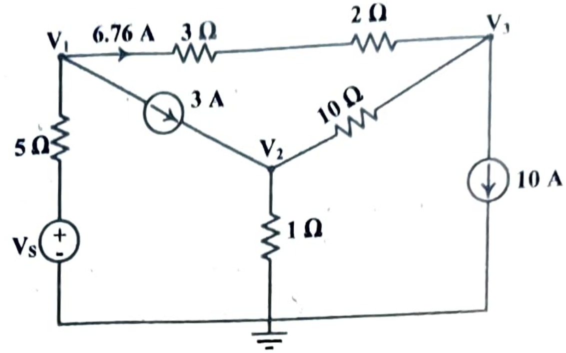
Example 1.8

Find the value of source voltage V_S for the circuit shown below using nodal analysis if the current through 3 Ω resistor is 6.76 A.



Solution:

Let's consider three nodes 1, 2 and 3 with node voltages V_1, V_2 and V_3 respectively as shown in the figure below.



Applying KCL at node 1,

$$\frac{V_1 - V_s}{5} + 3 + 6.76 = 0$$

or, $\quad \dfrac{V_1 - V_s}{5} = -9.76$

or, $\quad V_s = V_1 + 48.8$ ....(1)

Applying KCL at node 3,

$$6.76 = \frac{V_3 - V_2}{10} + 10$$

or, $\quad V_2 - V_3 = 32.4$ ....(2)

Applying KCL at node 2,

$$3 = \frac{V_2 - 0}{1} + \frac{V_2 - V_3}{10}$$

or, $\quad \dfrac{11}{10}V_2 - \dfrac{1}{10}V_3 = 3$ ....(3)

Solving equations (2) and (3), we get

$V_2 = -0.24$ V $\quad$ and $\quad V_3 = -32.64$ V

Also, $\quad \dfrac{V_1 - V_3}{5} = 6.76$

or, $\quad V_1 = 6.76 \times 5 + V_3$

or, $\quad V_1 = 6.76 \times 5 - 32.64$

or, $\quad V_1 = 1.16$ V

Now, from equation (1),

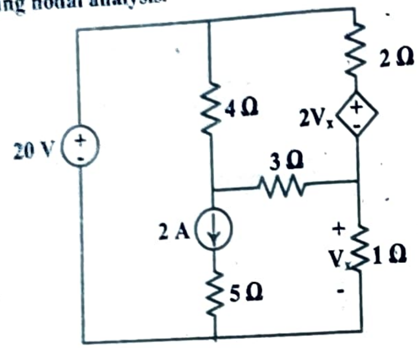
$\quad V_s = V_1 + 48.8$

or, $\quad V_s = 1.16 + 48.8$

or, $\quad V_s = $ **49.96 V**

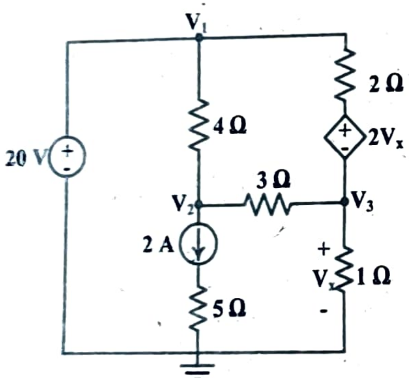
Example 1.9

In the given circuit, determine the voltage across 1 Ω resistor using nodal analysis.



Solution:

Let's consider three nodes 1, 2 and 3 with node voltages V_1, V_2 and V_3 respectively as shown in the figure below.



The voltage between node 1 and reference node is

$$V_1 = 20 \ldots\ldots\ldots\ldots\ldots(1)$$

Applying KCL at node 2,

$$-2 = \frac{V_2 - V_3}{3} + \frac{V_2 - V_1}{4}$$

or, $$\frac{-1}{4}V_1 + \frac{7}{12}V_2 - \frac{1}{3}V_3 = -2 \ldots\ldots\ldots\ldots(2)$$

Applying KCL at node 3,

$$\frac{V_3 - 0}{1} + \frac{V_3 - V_2}{3} + \frac{V_3 + 2V_x - V_1}{2} = 0$$

But, $V_x = V_3$

So, $$\frac{V_3 - 0}{1} + \frac{V_3 - V_2}{3} + \frac{V_3 + 2V_3 - V_1}{2} = 0$$

or, $$\frac{-1}{2}V_1 - \frac{1}{3}V_2 + \frac{17}{6}V_3 = 0 \ldots\ldots\ldots\ldots(3)$$

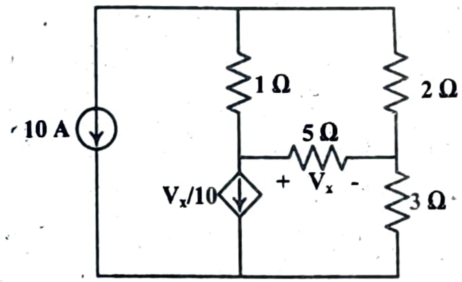
Solving equations (1), (2) and (3), we get

$V_1 = 20$ V, $V_2 = 7.67$ V and $V_3 = 4.43$ V

Now, the voltage across 1 Ω resistor, $V_{1\Omega} = V_3 = \mathbf{4.43\ V}$

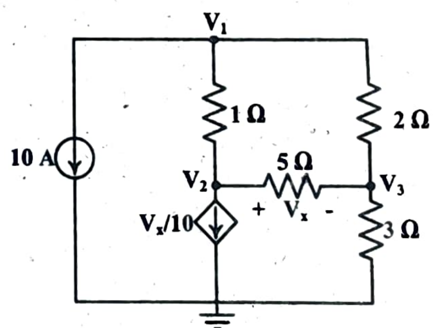
Example 1.10

Find the currents through each resistor using Nodal Analysis.



Solution:

Let's consider three nodes 1, 2 and 3 with node voltages V_1, V_2 and V_3 respectively as shown in the figure below.



Applying KCL at node 1,

$$\frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{2} = -10$$

or, $$\frac{3}{2}V_1 - V_2 - \frac{1}{2}V_3 = -10 \quad \text{...............(1)}$$

Applying KCL at node 2,

$$\frac{-V_x}{10} = \frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{5}$$

But, $V_x = V_2 - V_3$

Then, $$\frac{-(V_2 - V_1)}{10} = \frac{V_2 - V_1}{1} + \frac{V_2 - V_3}{5}$$

or, $$V_1 - \frac{13}{10}V_2 + \frac{3}{10}V_3 = 0 \quad \text{...............(2)}$$

Applying KCL at node 3,

$$\frac{V_3 - V_2}{5} + \frac{V_3 - V_1}{2} + \frac{V_3 - 0}{3} = 0$$

or, $$\frac{-1}{2}V_1 - \frac{1}{5}V_2 + \frac{31}{30}V_3 = 0 \quad \text{........................(3)}$$

Solving equations (1), (2) and (3), we get

$V_1 = -40.53$ V,

$V_2 = -37.37$ V

and $V_3 = -26.84$ V

Now, the currents through each resistor are;

$$I_{1\Omega} = \frac{V_2 - V_1}{1} = 3.16 \text{ A}$$

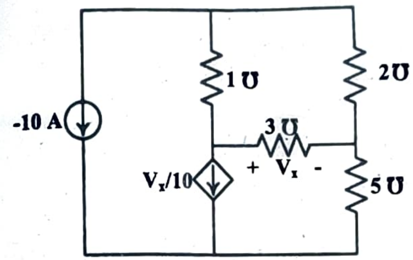
$$I_{2\Omega} = \frac{V_3 - V_1}{2} = 6.84 \text{ A}$$

$$I_{3\Omega} = \frac{0 - V_3}{3} = 8.95 \text{ A}$$

$$I_{5\Omega} = \frac{V_3 - V_2}{5} = 2.11 \text{ A}$$

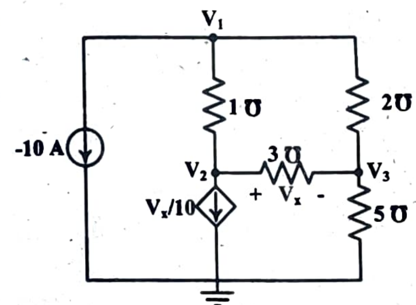
Example 1.11

Find the currents through each resistor using nodal analysis.



Solution:

Let's consider three nodes 1, 2 and 3 with node voltages V_1, V_2 and V_3 respectively as shown in the figure below.



Applying KCL at node 1,

$$(V_1 - V_2) \times 1 + (V_1 - V_3) \times 2 = 10$$

or, $$3V_1 - V_2 - 2V_3 = 10 \quad \text{...............(1)}$$

Applying KCL at node 2,

$$\frac{-V_x}{10} = (V_2 - V_1) \times 1 + (V_2 - V_3) \times 3$$

But, $V_x = V_2 - V_3$

Then, $$\frac{-(V_2 - V_1)}{10} = (V_2 - V_1) \times 1 + (V_2 - V_3) \times 3$$

or, $$V_1 - \frac{41}{10}V_2 + \frac{31}{10}V_3 = 0 \quad \text{...............(2)}$$

Applying KCL at node 3,

$$(V_3 - V_2) \times 3 + (V_3 - V_1) \times 2 + V_3 \times 5 = 0$$

or, $$-2V_1 - 3V_2 + 10V_3 = 0 \quad \text{........................(3)}$$

Solving equations (1), (2) and (3), we get

$V_1 = 5.61$ V, $V_2 = 2.87$ V and $V_3 = 1.98$ V

Now, the currents through each resistor are;

$I_{1\Omega} = (V_1 - V_2) \times 1 = 2.74$ A

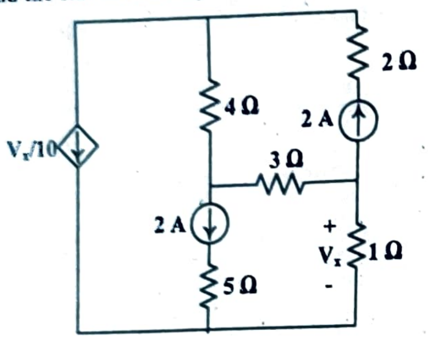
$I_{2\Omega} = (V_1 - V_3) \times 2 = 7.26$ A

$I_{3\Omega} = (V_2 - V_3) \times 3 = 2.67$ A

$I_{5\Omega} = V_3 \times 5 = 9.9$ A

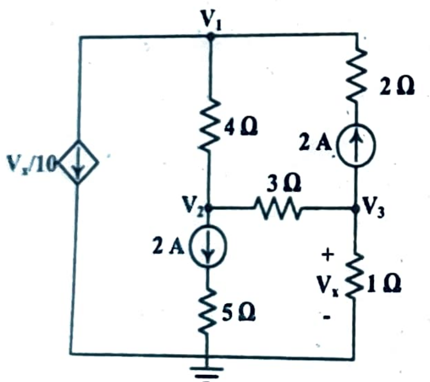
Example 1.12

Find the currents through each resistor using nodal analysis.



Solution:

Let's consider three nodes 1, 2 and 3 with node voltages V_1, V_2 and V_3 respectively as shown in the figure below.



Applying KCL at node 1,

$$\frac{V_1 - V_2}{4} + \frac{V_x}{10} = 2$$

But, $V_x = V_3$

Then, $$\frac{V_1 - V_2}{4} + \frac{V_3}{10} = 2$$

or, $$\frac{1}{4}V_1 - \frac{1}{4}V_2 + \frac{1}{10}V_3 = 2 \quad \text{..............(1)}$$

Applying KCL at node 2,

$$-2 = \frac{V_2 - V_1}{4} + \frac{V_2 - V_3}{3}$$

or, $$\frac{1}{4}V_1 - \frac{7}{12}V_2 + \frac{1}{3}V_3 = 2 \quad \text{..............(2)}$$

Applying KCL at node 3,

$$\frac{V_3 - 0}{1} + \frac{V_3 - V_2}{3} = -2$$

or, $$\frac{1}{3}V_2 - \frac{4}{3}V_3 = 2 \quad \text{.......................(3)}$$

Solving equations (1), (2) and (3), we get

$V_1 = 7.45$ V, $V_2 = -1.27$ V

and $V_3 = -1.82$ V

Now, the currents through each resistor are;

$$I_{1\Omega} = \frac{0 - V_3}{1} = \mathbf{1.82\ A}$$

$$I_{2\Omega} = \mathbf{2\ A}$$

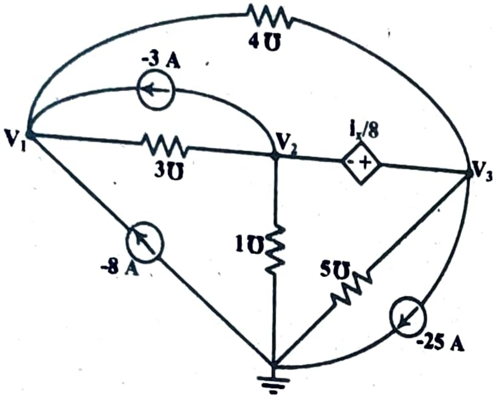
$$I_{3\Omega} = \frac{V_2 - V_3}{3} = \mathbf{0.183\ A}$$

$$I_{4\Omega} = \frac{V_1 - V_2}{4} = \mathbf{2.18\ A}$$

$$I_{5\Omega} = \mathbf{2\ A}$$

Example 1.13

Find the node voltages V_1, V_2 and V_3 for the circuit shown in the figure.



Solution:

Applying KCL at node 1,

$$-3-8=(V_1-V_2)\times 3+(V_1-V_3)\times 4$$

or, $\quad 7V_1-3V_2-4V_3=-11$ ....(1)

Applying KCL at super-node (i.e., nodes 2 and 3),

$$3+25=(V_2-V_1)\times 3+V_2\times 1+(V_3-V_1)\times 4+V_3\times 5$$

or, $\quad -7V_1+4V_2+9V_3=28$ ....(2)

The voltage between nodes 2 and 3 is

$$V_3-V_2=\frac{i_x}{8}$$

But, $\quad i_x=(V_3-V_1)\times 4$

Then, $\quad V_3-V_2=\dfrac{(V_3-V_1)\times 4}{8}$

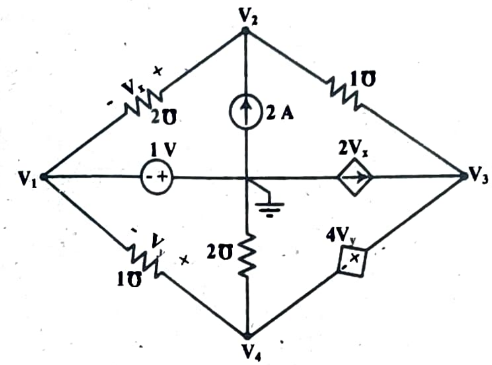
or, $\quad V_1-2V_2+V_3=0$ ....(3)

Solving equations (1), (2) and (3), we get

$V_1=1$ **V**, $\quad V_2=2$ **V** $\quad$ and $\quad V_3=3$ **V**

Example 1.14

Obtain the node voltages V_1, V_2, V_3 and V_4 for the circuit shown below.



Solution:

The voltage between node 1 and reference node is

$$V_1=-1\text{ V}$$

Applying KCL at node 2,

$$2=(V_2-V_1)\times 2+(V_2-V_3)\times 1$$

or, $\quad 2=(V_2-(-1))\times 2+(V_2-V_3)\times 1$

or, $\quad 3V_2-V_3=0$ ....(1)

Applying KCL at super–node (i.e., nodes 3 and 4),

$$2V_x=(V_3-V_2)\times 1+V_4\times 2+(V_4-V_1)\times 1$$

But, $\quad V_x=V_2-V_1$

Then, $\quad 2(V_2-V_1)=(V_3-V_2)\times 1+V_4\times 2+(V_4-V_1)\times 1$

or, $\quad V_1-3V_2+V_3+3V_4=0$

or, $\quad -1-3V_2+V_3+3V_4=0$

or, $\quad -3V_2+V_3+3V_4=1$...........(2)

The voltage between nodes 3 and 4 is

$$V_3-V_4=4V_y$$

But, $\quad V_y=V_4-V_1$

Then, $V_3 - V_4 = 4(V_4 - V_1)$

or, $4V_1 + V_3 - 5V_4 = 0$

or, $4(-1) + V_3 - 5V_4 = 0$

or, $V_3 - 5V_4 = 4$ ....(3)

Solving equations (1), (2) and (3), we get

$V_2 = 1.89$ V, $V_3 = 5.67$ V and $V_4 = 0.33$ V

1.8 ELECTRICAL RESONANCE

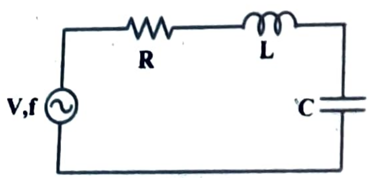
When the voltage V applied to an electrical network consisting of resistance, inductance and capacitance is in phase with the resulting current I the circuit is said to be **resonant**. The phenomenon of **resonance** is of great importance in all branches of radio, television and communication engineering, since it enables small portions of the communication frequency spectrum to be selected for amplification independently of the remainder.

At resonance, the equivalent network impedance Z is purely resistive since the supply voltage and current are in phase. The power factor of a resonant network is unity.

In the electrical works, there are two types of resonance – one associated with series circuits, when the input impedance is a minimum, and the other associated with simple parallel networks, when the input impedance is a maximum.

1.9 RESONANCE IN SERIES RLC CIRCUIT

The figure below shows a circuit comprising of resistor, inductor and capacitor connected in series and supplied by an ac source.



The above RLC series circuit has a total impedance Z given by:

$$Z = R + j(X_L - X_C)$$

or, $$Z = R + j(\omega L - 1/\omega C)$$

where, $\omega = 2\pi f$

Here, inductive reactance (X_L) and capacitive reactance (X_C) are both frequency dependent. X_L increases with the increase in frequency but X_C decreases with the increase in frequency. At a particular frequency, inductive reactance becomes equal to the capacitive reactance. At this instant, the current through the circuit attains the maximum value since equivalent impedance is the minimum.

When $X_L = X_C$, the applied voltage V and the current I are in phase. This effect is called series resonance. At resonance:

i) $V_L = V_C$

ii) $Z = R$ (i.e. the minimum circuit impedance possible in an *RLC* series circuit)

iii) $I = \frac{V}{R}$ (i.e. the maximum current possible in an *RLC* series circuit)

iv) Since $X_L = X_C$, then $2\pi f_r L = \frac{1}{2\pi f_r C}$

From which, $f_r^2 = \frac{1}{(2\pi)^2 LC}$

and $\boxed{f_r = \frac{1}{2\pi\sqrt{LC}}\ Hz}$

where f_r is the resonant frequency.

The phasor diagram of the RLC series circuit at resonance is as shown in the figure below.

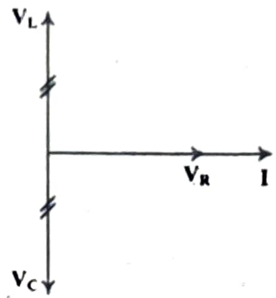


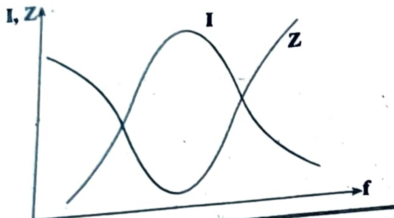
Figure: Phasor diagram of series RLC resonance circuit

At resonance, since the inductive reactance is equal to the capacitive reactance, the voltage drop across inductor becomes equal to the voltage drop across inductor in magnitude, but opposite in phase and, therefore, the circuit current I is in phase with the applied voltage i.e., the circuit behaves as a resistive circuit.

When the circuit is in resonance, the current is too large and will produce large voltage drop across inductor and capacitor, which will be equal in magnitude but opposite in phase and each may be several times greater than the applied voltage. If the resistance R would not have been present in the circuit, such a circuit, would act like a short circuit to currents of frequency to which it resonates.

Since in this resonance the voltage is maximum, it is called the voltage resonance. The series resonance circuit is also called an acceptor circuit because such a circuit accepts currents at one particular frequency but rejects currents of other frequencies. Such circuits are used in radio-receivers.

The typical graphs of current I and impedance Z against frequency f are shown in figure below.



1.10 Q-FACTOR

At resonance, if R is small compared with X_L and X_C, it is possible for V_L and V_C to have voltages many times greater than the supply voltage.

$$\text{Voltage magnification at resonance} = \frac{\text{Voltage across L (or C)}}{\text{Supply Voltage V}}$$

This ratio is a measure of the quality of a circuit (as a resonator or tuning device) and is called the Q-factor.

$$\text{Hence, } Q\text{-factor} = \frac{V_L}{V} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{2\pi f_r L}{R}$$

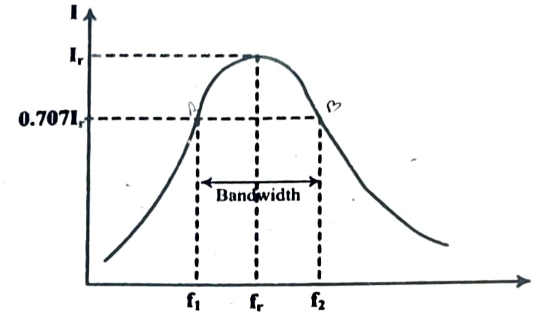
$$\text{Alternatively, } Q\text{-factor} = \frac{V_C}{V} = \frac{IX_C}{IR} = \frac{X_C}{R} = \frac{1}{2\pi f_r CR}$$

$$\text{At resonance, } f_r = \frac{1}{2\pi\sqrt{LC}}, \text{ i.e. } 2\pi f_r = \frac{1}{\sqrt{LC}}$$

$$\text{Hence, } Q\text{-factor} = \frac{2\pi f_r L}{R} = \frac{1}{\sqrt{LC}}\left(\frac{L}{R}\right) = \frac{1}{R}\sqrt{\frac{L}{C}}$$

1.11 BANDWIDTH

The figure below shows how current I varies with frequency in an *RLC* series circuit.



At resonance frequency f_r, current is a maximum value, shown as I_r. Also shown are the points A and B where the current is 0.707 of the maximum value at frequencies f_1 and f_2. The power delivered to the circuit is I^2R. At $I = 0.707I_r$, the power is half the power at frequency f_r. The corresponding frequencies f_1 and f_2 are called the half-power frequencies, f_1 being lower half-power frequency and f_2 upper half-power frequency. The difference of these frequencies is called the bandwidth.

At lower half power frequency f_1,

$$X_C - X_L = R$$

or, $$\frac{1}{2\pi f_1 C} - 2\pi f_1 L = R \quad \text{..............(a)}$$

At upper half power frequency f_2,

$$X_L - X_C = R$$

or, $$2\pi f_2 L - \frac{1}{2\pi f_2 C} = R \quad \text{..............(b)}$$

Equating equations (a) and (b),

$$\frac{1}{2\pi f_1 C} - 2\pi f_1 L = 2\pi f_2 L - \frac{1}{2\pi f_2 C}$$

or, $$\frac{1}{2\pi f_1 C} + \frac{1}{2\pi f_2 C} = 2\pi f_2 L + 2\pi f_1 L$$

or, $$\frac{1}{2\pi C}\frac{f_2 + f_1}{f_1 f_2} = 2\pi L(f_1 + f_2)$$

...diagram, at resonance,

$$I_C = I_{RL} \sin\phi_{RL}$$

or, $\dfrac{V}{X_C} = \dfrac{V}{Z_{RL}} \times \dfrac{X_L}{Z_{RL}}$

or, $Z_{RL}^2 = X_L X_C$

or, $R^2 + \omega^2 L^2 = \omega L \times \dfrac{1}{\omega C}$

or, $R^2 + \omega^2 L^2 = \dfrac{L}{C}$

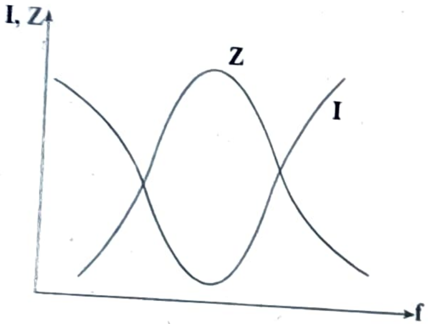
or, $\omega_r = \sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$

or, $f_r = \dfrac{1}{2\pi}\sqrt{\dfrac{1}{LC} - \dfrac{R^2}{L^2}}$

Parallel resonant circuit is sometimes called the rejector circu... because at resonant frequency the line current is minimum or it almo... rejects it.

Since in parallel resonant circuit, circulating current between the branches is many times the line current, such type of resonance is sometimes known as current resonance.

The typical graphs of current I and impedance Z against frequency f are shown in figure below.



The basic expressions for Q-factor and Bandwidth (BW) in parallel resonant circuit are same as that for series resonant circuit.

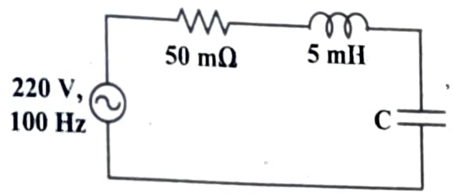
1.13 SOLVED PROBLEMS OF RESONANCE

Example 1.15

A 220 V, 100 Hz source supplies a series RLC circuit. What value of capacitor is required to produce resonance at 100 Hz, if the resistance and inductance of the circuit are 50 mΩ and 5 mH respectively? Also calculate the Q-factor and half power frequencies of the circuit. [2070 Ashad]

Solution:

The given data are shown in the circuit below:



Here; $V = 220$ V, $f_r = 100$ Hz,

$R = 50\times10^{-3}\ \Omega$, $L = 5\times10^{-3}$ H

We know, $f_r = \dfrac{1}{2\pi\sqrt{LC}}$ for a series RLC resonant circuit.

Then, $C = \dfrac{1}{(2\pi f_r)^2 L} = \mathbf{506.6\ \mu F}$

Q-factor $= \dfrac{2\pi f_r L}{R} = \mathbf{62.83}$

Lower half power frequency,

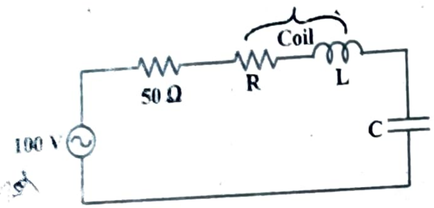
$$f_1 = f_r - \frac{R}{4\pi L} = 100 - 0.796 = \mathbf{99.204\ Hz}$$

Upper half power frequency,

$$f_2 = f_r + \frac{R}{4\pi L} = 100 + 0.796 = \mathbf{100.796\ Hz}$$

Example 1.16

A 50 Ω resistor is connected in series with a coil having resistance R and inductance L, capacitor 'C' and 100 V variable frequency supply as shown in figure below. At frequency of 200 Hz, the maximum current of 0.7 A flows through the circuit and voltage across the capacitor is 200 V. Determine the value of R, L and C.
[2068 Baisakh]



Solution:

Here, $V = 100$ V, $f_r = 200$ Hz, $I_r = 0.7$ A

Voltage across capacitor, $V_C = 200$ V

Total resistance of the circuit, $R_T = 50 + R$

We know, at resonance,

$V = I_r R_T$ or, $100 = 0.7\,(50 + R)$

or, $\mathbf{R = 92.86\ \Omega}$

Also, $V_C = I_r X_C = \dfrac{I_r}{2\pi f_r C}$

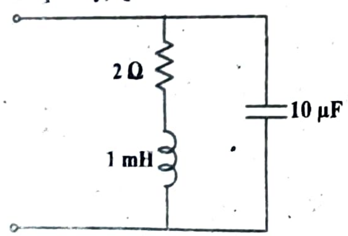
or, $C = \dfrac{I_r}{2\pi f_r V_C} = \dfrac{0.7}{2\pi\times200\times200} = \mathbf{2.78\ \mu F}$

Again, $f_r = \dfrac{1}{2\pi\sqrt{LC}}$ for a series RLC resonant circuit.

Then, $L = \dfrac{1}{(2\pi f_r)^2 C} = \mathbf{227.4\ mH}$

Example 1.17

In the parallel resonant circuit as shown in the figure below, find resonance frequency, Q factor and bandwidth. [2068 Chaitra]



Solution:

For the parallel resonant circuit shown in the figure,

The resonance frequency,

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

or, $\mathbf{f_r = 1559.4\ Hz}$

$$\text{Q-factor} = \frac{2\pi f_r L}{R} = \frac{2\pi\times1559.4\times1\times10^{-3}}{2} = 4.9$$

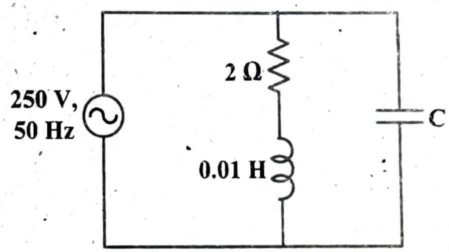
$$\text{Bandwidth, BW} = f_2 - f_1 = \frac{R}{2\pi L} = \frac{2}{2\pi\times1\times10^{-3}} = 318.3\ \text{Hz}$$

Example 1.18

An inductive circuit of resistance 2 Ω and inductance 0.01 H is connected in parallel with capacitor C and with 250 V, 50 Hz supply.

(a) Determine the value of capacitance placed in parallel to produce resonance.

(b) Determine the current taken from the source and currents through each branch.



Solution:

a) We know, for the parallel resonant circuit shown in the figure;

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \left(\frac{R}{L}\right)^2}$$

or, $(2\pi f_r)^2 + \left(\dfrac{R}{L}\right)^2 = \dfrac{1}{LC}$

or, $C = \dfrac{1}{L[(2\pi f_r)^2 + (R/L)^2]} = 721\ \mu F$

b) Here, impedance of coil branch,

$$Z_{RL} = \sqrt{R^2 + X_L^2} = 3.72\ \Omega$$

Power factor of coil branch,

$$\cos\phi_{RL} = \frac{R}{Z_{RL}} = \frac{2}{3.72} = 0.538 \text{ (lag)}$$

Current through coil branch,

$$I_{RL} = \frac{V}{Z_{RL}} = 67.2 \text{ A}$$

Current through capacitor branch,

$$I_C = \frac{V}{X_C} = V \times (2\pi f_r C) = 250 \times 2\pi \times 50 \times 721 \times 10^{-6}$$

or, $I_C = 56.63$ **A**

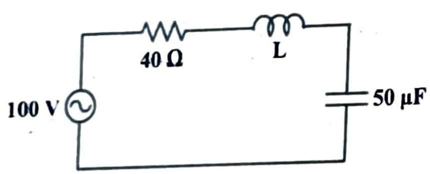
Current taken from the source,

$$I_0 = I_{RL}\cos\phi_{RL} = 67.2 \times 0.538 = \mathbf{36.15\ A}$$

Example 1.19

A 50μF Capacitor, when connected in series with a coil having 40Ω resistor,resonates at 1000Hz. Find the inductance of the coil. Also obtain the circuit current if theapplied voltage is 100V. Also calculate the voltage across the capacitor and the coil atresonance.

Solution:



Here, $f_r = \dfrac{1}{2\pi\sqrt{LC}}$ for a series RLC resonant circuit.

Then, $L = \dfrac{1}{(2\pi f_r)^2 C} = \dfrac{1}{(2\pi\times 1000)^2 \times 50\times 10^{-6}} = \mathbf{0.51\ mH}$

Circuit current at resonance, $I_r = \dfrac{V}{R} = \dfrac{100}{40} = \mathbf{2.5\ A}$

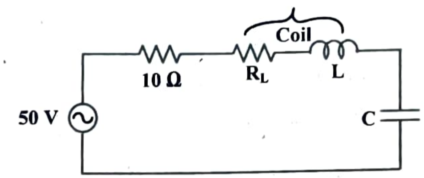
Voltage across the capacitor, $V_C = I_r X_C = \dfrac{I_r}{2\pi f_r C} = \mathbf{7.96\ V}$

Voltage across the coil, $V_{coil} = I_r Z_{coil} = I_r\sqrt{R^2 + X_L^2} = \mathbf{100.32\ V}$

Example 1.20

A 10 Ω resistor is placed in series with a coil of self resistance R_L and inductance L and a pure capacitor 'C' across a 50 V variable frequency supply. The current is maximum and has value of 1 A when the frequency is 500 Hz. At this frequency, voltage across the capacitor is 300 V. Calculate (i) capacitance of the capacitor (ii) resistance and inductance of the coil (iii) Power consumed in the circuit (iv) Voltage across the resistor and the coil.

Solution:



Here, $V = 50$ V, $I_r = 1$ A, $f_r = 500$ Hz

Voltage across the capacitor, $V_C = 300$ V

Total resistance of the circuit, $R_T = 10 + R_L$

We know,

$$V_C = I_r X_C = \frac{I_r}{2\pi f_r C}$$

or, $C = \dfrac{I_r}{2\pi f_r V_C} = \mathbf{1.061\ \mu F}$

$$I_r = \frac{V}{R_T} = \frac{50}{10 + R_L}$$

or, $1 = \dfrac{50}{10 + R_L}$

or, $\mathbf{R_L = 40\ \Omega}$

Inductance of the coil at resonance is;

$$L = \frac{1}{(2\pi f_r)^2 C} = \frac{1}{(2\pi\times 500)^2\times 1.061\times 10^{-6}} = \mathbf{95.5\ mH}$$

Power consumed in the circuit, $P = I_r^2 R_T = \mathbf{50\ W}$

Voltage across the resistor, $V_R = I_r R = 1 \times 10 = \mathbf{10\ V}$

Voltage across the coil, $V_{coil} = = I_r Z_{coil} = I_r\sqrt{R_L^2 + X_L^2} = \mathbf{302.7\ V}$

Multiple Choice Questions

1. In a series RLC circuit, the maximum voltage across the capacitor occurs at a frequency
   a) double the resonant frequency
   b) equal to resonant frequency
   c) $\sqrt{2}$ times the resonant frequency
   d) below the resonant frequency
2. In a series RLC circuit, the voltage across inductor will be maximum
   a) at resonant frequency
   b) just after resonant frequency
   c) just before resonant frequency
   d) just before and after resonant frequency
3. For a series RLC circuit, the power factor at the lower half power frequency is
   a) 0.5 lagging b) 0.5 leading
   c) unity d) 0.707 leading
4. Q-factor of a series RLC circuit possessing resonant frequency of 10 Hz and bandwidth of 5 Hz is
   a) 0.5 b) 2
   c) 2.5 d) 50
5. When Q-factor of a circuit is high, then
   a) power factor of the circuit is high
   b) impedance of the circuit is high
   c) bandwidth is large
   d) none of these
6. A high Q coil has
   a) large bandwidth b) high losses
   c) low losses d) flat response
7. In a series resonant circuit, with the increase in inductance (L),
   a) resonant frequency will decrease
   b) bandwidth will decrease
   c) Q will increase
   d) all of the above
8. Which of the following will not be affected due to change in R?
   a) Bandwidth b) Q
   c) Resonant frequency d) None of these
9. Higher the Q of a series RLC circuit, narrower its
   a) pass-band b) resonant curve
   c) bandwidth d) all of these

ANSWERS

1.(d), 2.(b), 3.(d), 4.(b), 5.(c), 6.(b), 7.(d), 8.(c), 9.(d)

□□□

CHAPTER 2

Transient Analysis

2.1 INTRODUCTION

Whenever a circuit is switched from one condition to another, either by a change in the applied source or a change in the circuit elements, there is a transition period during which the branch currents and element voltages change from their former values to new ones. This period is called the transient. After the transient has passed, the circuit is said to be in the steady state.

During the switching in various circuits, the change in branch currents and element voltages depend on the type of circuit element and their inherent properties. Thus, for the determination of circuit variables after the instant of switching, we should first know the properties of various circuit elements such as resistor, inductor and capacitor.

a) Resistor is the circuit element which opposes the flow of current in the circuit. It does not store energy as it is only power dissipating circuit element. Hence, there is not any change in behavior of resistor during switching.

b) Inductor is the circuit element which opposes the rate of flow of current. If the current is increasing, the inductor tries to decrease it and vice-versa. In this manner, inductor stores energy in the form of electromagnetic fields. Hence, transients occur in the circuit having inductor because of the changes in stored energy in it.

   For an inductor, if $v_L(t)$ be the voltage across inductor and $i_L(t)$ be the current through it, then,

$$v_L(t) = L\frac{di_L(t)}{dt}$$

or, $$i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt$$

c) Capacitor is the circuit element which stores the energy in the form of electrostatic fields. Hence, transient occurs in the circuit having capacitor because of the changes in stored energy in it.

For a capacitor, if $v_c(t)$ be the voltage across the capacitor and $i_c(t)$ be the current through it, then, $i_c(t) = C\dfrac{dv_c(t)}{dt}$

$$\text{or, } v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)\, dt$$

2.2 INITIAL CONDITIONS

During the transient analysis of the circuit, we come across two types of variables, dependent variable and independent variable. Time (t) is an independent variable and voltage v(t) & current i(t) are dependent variables as they are function of time.

While solving transient analysis problem, our main objective is to determine the expressions for the dependent variable as a function of time. But because of the presence of energy storing elements such as inductor and capacitor in the circuit, we encounter differential or integro-differential equations. The complete solution of those differential or integro-differential equations is not obtained without initial conditions. Only general solution is obtained without initial conditions.

Hence, the values of the dependent variables and their higher order derivatives just after the instant of switching are referred to as initial conditions.

Let t = 0 be the instant of switching in a circuit, then $i(0^+)$, $v(0^+)$, $\frac{di}{dt}(0^+)$, $\frac{dv}{dt}(0^+)$ etc. are initial conditions.

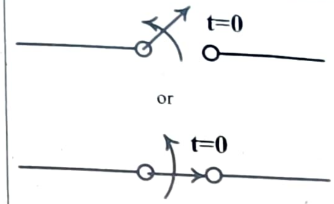
The circuit changes are assumed to occur at time t = 0, and represented by a switch. For convenience, it is defined that:

$t = 0^-$, the instant prior to $t = 0$ and

$t = 0^+$, the instant immediately after switching.

The following table explains the meaning of two states of switch during switching.

Table 2.1: Two States of switch during switching

	
At the instant prior to t = 0 i.e. at $t = 0^-$, the switch is open/off.	At the instant prior to t = 0 i.e. at $t = 0^-$, the switch is close/on.
At the instant immediately after switching i.e. at $t = 0^+$, the switch is close/on.	At the instant immediately after switching i.e. at $t = 0^+$, the switch is open/off.

The voltage-current relationships of the three circuit elements R, L and C are shown in the table below.

Table 2.2: Relationship for the parameters

Parameter	Voltage-current Relationships
Resistance (R)	$v_R(t) = R\, i_R(t)$
Inductance (L)	$v_L(t) = L\dfrac{di_L(t)}{dt}$ or, $i_L(t) = \dfrac{1}{L}\displaystyle\int_{-\infty}^{t} v_L(t)\, dt$ or, $i_L(t) = i_L(0^+) + \dfrac{1}{L}\displaystyle\int_{0}^{t} v_L(t)\, dt$
Capacitance (C)	$i_c(t) = C\dfrac{dv_c(t)}{dt}$ or, $v_c(t) = \dfrac{1}{C}\displaystyle\int_{-\infty}^{t} i_c(t)\, dt$ or, $v_c(t) = v_c(0^+) + \dfrac{1}{C}\displaystyle\int_{0}^{t} i_c(t)\, dt$

2.3 PROCEDURE FOR SOLVING INITIAL CONDITIONS PROBLEM

1. Draw the equivalent circuit at $t = 0^-$ by short-circuiting the inductor and open-circuiting the capacitor. From the equivalent circuit, find the current through the inductor $i_L(0^-)$ and the voltage across the capacitor $v_c(0^-)$. The main objective of the equivalent circuit at $t = 0^-$ is to determine voltages across capacitors and currents through inductors at this instant.
2. Draw the equivalent circuit at $t = 0^+$.
   a) For an inductor, find the current as $i_L(0^+) = i_L(0^-)$. Then, if the current is zero, replace the inductor by an open-circuit. Otherwise, if the current is non-zero, replace it by the constant current source of values same as obtained current value.
   b) For a capacitor, find the voltage as $v_c(0^+) = v_c(0^-)$. Then, if the voltage is zero, replace the capacitor by a short circuit. Otherwise, if the voltage is non-zero, replace it by the constant voltage source of values same as obtained voltage value.
   c) Resistors are left in the network without change.
3. Compute the currents and voltages as desired using circuit analysis techniques.

The equivalent circuit for the three parameters (R, L and C) at $t = 0^-$ and $t = 0^+$ are shown in the table below.

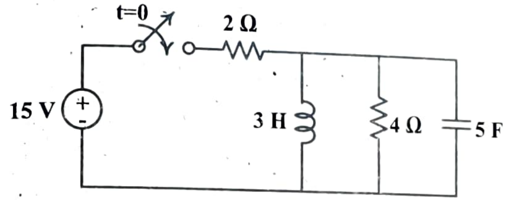
Table 2.3: Equivalent circuit of the parameters

Element with initial conditions	Equivalent circuit at $t = 0^-$	Equivalent circuit at $t = 0^+$
	S.C.	O.C.
I_0		I_0
	O.C.	S.C.
+ - V_0		+ - V_0

2.4 SOLVED PROBLEMS OF INITIAL CONDITIONS

Example 2.1

For the circuit shown in the figure below, find the current through and voltage across each circuit elements at $t = 0^+$. Also find $\frac{dv_c}{dt}$ and $\frac{di_L}{dt}$ at $t=0^+$.



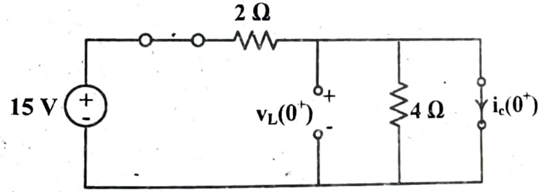
Solution:

At $t = 0^-$, the circuit is not energized since switch is open.

So, $i_L(0^-) = 0$

and $v_c(0^-) = 0$

Then, at $t = 0^+$, $i_L(0^+) = 0$, i.e. the inductor behaves as open circuit (O.C.) and $v_c(0^+) = 0$, i.e. the capacitor behaves as short circuit (S.C.). The equivalent circuit at $t=0^+$ is shown below;



Here, the current does not flow through 4 Ω resistor as it is short circuited.

$$i_c(0^+) = \frac{15}{2} = 7.5\text{ A}$$

Since there is short circuit in parallel, $v_L(0^+) = 0$ V

Now, current and voltage of each circuit elements are;

For 2 Ω resistor, $\mathbf{i_{2\Omega}(0^+) = 7.5}$ **A** and $\mathbf{v_{2\Omega}(0^+) = 7.5\times2 = 15}$ **V**

For 4Ω resistor, $\mathbf{i_{4\Omega}(0^+) = 0}$ and $\mathbf{v_{4\Omega}(0^+) = 0}$

For 3 H inductor, $\mathbf{i_L(0^+) = 0}$ and $\mathbf{v_L(0^+) = 0}$ **V**

For 5 F capacitor, $\mathbf{i_c(0^+) = 7.5}$ **A** and $\mathbf{v_c(0^+) = 0}$

Also, $v_L(0^+) = L\frac{di_L}{dt}(0^+)$

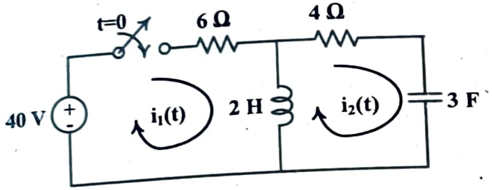
or, $\frac{di_L}{dt}(0^+) = 0$

And, $i_c(0^+) = C\frac{dv_c}{dt}(0^+)$

or, $\frac{dv_c}{dt}(0^+) = \frac{7.5}{5} = 1.5$ V/s

Example 2.2

For the circuit shown in the figure below, find the current through and voltage across each circuit elements at t=0⁺. Also find $\frac{dv_c}{dt}$ and $\frac{di_L}{dt}$ at t=0⁺.



Solution:

At $t = 0^-$, the circuit is not energized since switch is open.

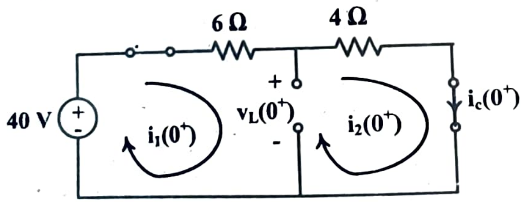
So, $i_L(0^-) = 0$

and $v_c(0^-) = 0$

Then, at $t = 0^+$, $i_L(0^+) = 0$, i.e. the inductor behaves as open circuit (O.C.)

and $v_c(0^+) = 0$, i.e. the capacitor behaves as short circuit (S.C.)

The equivalent circuit at $t=0^+$ is shown below;



Here, the current does not flow through 4 Ω resistor as it is short circuited.

$$i_1(0^+) = i_2(0^+) = i_c(0^+) = \frac{40}{6+4} = 4 \text{ A}$$

Now, current and voltage of each circuit elements are;

For 4Ω resistor, $i_{4\Omega}(0^+) = 4$ A and $v_{4\Omega}(0^+) = 4\times4 = 16$ V

For 6Ω resistor, $i_{6\Omega}(0^+) = 4$ A and $v_{6\Omega}(0^+) = 4\times6 = 24$ V

For 2 H inductor, $i_L(0^+) = 0$ and $v_L(0^+) = v_{4\Omega}(0^+) = 16$ V

For 3 F capacitor, $i_c(0^+) = 4$ A and $v_c(0^+) = 0$

Also, $v_L(0^+) = L\frac{di_L}{dt}(0^+)$

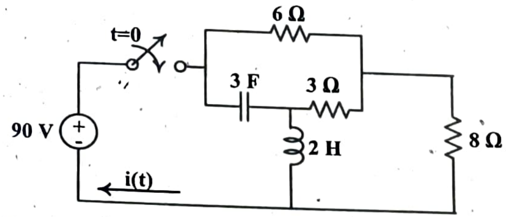
or, $\frac{di_L}{dt}(0^+) = \frac{16}{2} = 8$ A/s

And, $i_c(0^+) = C\frac{dv_c}{dt}(0^+)$

or, $\frac{dv_c}{dt}(0^+) = \frac{4}{3} = 1.33$ V/s

Example 2.3

For the circuit shown in the figure below, find i, $i_{6\Omega}$, $i_{3\Omega}$, $i_{8\Omega}$, i_c, i_L, v_c, v_L, q_c, $v_{3\Omega}$, $v_{6\Omega}$, $v_{8\Omega}$, $\frac{dv_c}{dt}$ and $\frac{di_L}{dt}$ at t=0⁺.



Solution:

At $t=0^-$, the circuit is not energized since switch is open.

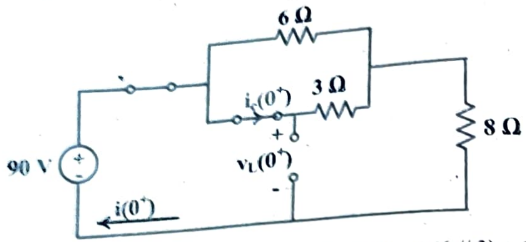
So, $i_L(0^-) = 0$

and $v_c(0^-) = 0$

Then, at $t = 0^+$, $i_L(0^+) = 0$, i.e. the inductor behaves as open circuit (O.C.)

and $v_c(0^+) = 0$, i.e. the capacitor behaves as short circuit (S.C.)

The equivalent circuit at $t=0^+$ is shown below;



Here, the equivalent resistance of the circuit, $R_{eq} = (6 \,//\, 3) + 8 = 10$

Then, $i(0^+) = i_{8\Omega}(0^+) = \frac{90}{10} = 9$ A

Using current divider rule,

$$i_{3\Omega}(0^+) = i_c(0^+) = \frac{6}{6+3} \times 9 = 6 \text{ A}$$

Then, $i_{6\Omega}(0^+) = 9 - 6 = 3$ A

Now, current and voltage of each circuit elements are;

For 3Ω resistor, $i_{3\Omega}(0^+) = 6$ A and $v_{3\Omega}(0^+) = 6\times3 = 18$ V

For 6 Ω resistor, $i_{6\Omega}(0^+) = 3$ A and $v_{6\Omega}(0^+) = 3\times6 = 18$ V

For 8 Ω resistor, $i_{8\Omega}(0^+) = 9$ A and $v_{8\Omega}(0^+) = 9\times8 = 72$ V

For 2 H inductor, $i_L(0^+) = 0$ and $v_L(0^+) = 90$ V.

For 3 F capacitor, $i_c(0^+) = 6$ A and $v_c(0^+) = 0$

Also, $v_L(0^+) = L\frac{di_L}{dt}(0^+)$

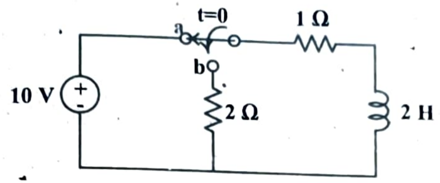
or, $\frac{di_L}{dt}(0^+) = \frac{90}{2} = 45$ A/s

And, $i_c(0^+) = C\frac{dv_c}{dt}(0^+)$

or, $\frac{dv_c}{dt}(0^+) = \frac{6}{3} = 2$ V/s

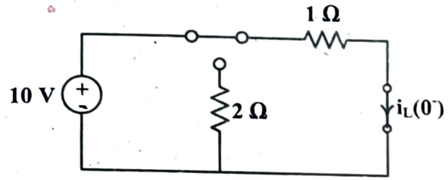
Example 2.4

The switch has been at position 'a' for a long time. Then it is quickly thrown to position 'b' at t=0. Find $i_{1\Omega}$, $i_{2\Omega}$, i_L, v_L, $v_{1\Omega}$, $v_{2\Omega}$, v_{ab} and $\frac{di_L}{dt}$ at $t=0^+$.



Solution:

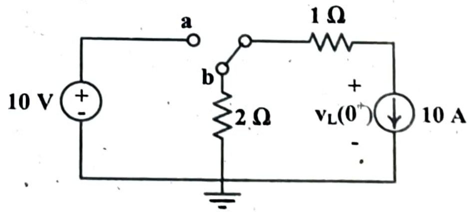
The equivalent circuit at $t=0^-$ is shown below;



Here, $i_L(0^-) = \frac{10}{1}$ A $= 10$ A

Then, at $t=0^+$, $i_L(0^+) = 10$ A

The equivalent circuit at $t=0^+$ is shown below;



Applying KVL in the circuit,

$$0 = 10\times2 + 10\times1 + v_L(0^+)$$

or, $v_L(0^+) = -30$ V

Now, current and voltage of each circuit elements are;

For 2 Ω resistor, $i_{2\Omega}(0^+) = 10$ A and $v_{2\Omega}(0^+) = 10\times2 = 20$ V

For 1 Ω resistor, $i_{1\Omega}(0^+) = 10$ A and $v_{1\Omega}(0^+) = 10\times1 = 10$ V

For 2 H inductor, $i_L(0^+) = 10$ A and $v_L(0^+) = -30$ V

Now, $V_{ab}(0^+) = V_a(0^+) - V_b(0^+) = 10 - (-20) = 30$ V

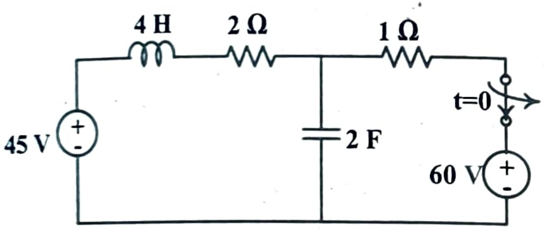
Also, $v_L(0^+) = L\dfrac{di_L}{dt}(0^+)$

or, $\dfrac{di_L}{dt}(0^+) = -\dfrac{30}{2} = -15$ A/s

Imp

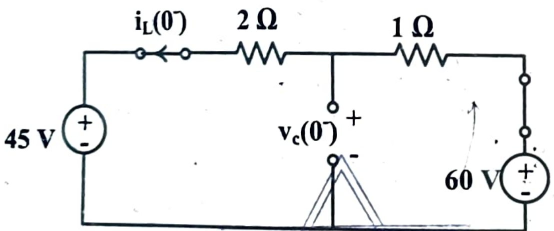
Example 2.5

For the circuit shown in the figure below, find $i_{1\Omega}$, $i_{2\Omega}$, i_c, i_L, v_c, v_L, $v_{1\Omega}$, $v_{2\Omega}$, $\dfrac{dv_c}{dt}$ and $\dfrac{di_L}{dt}$ at $t=0^+$.



Solution:

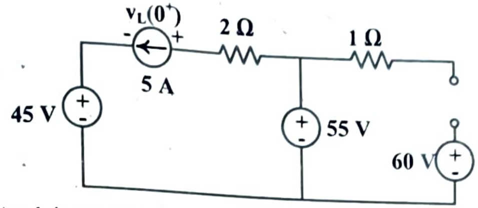
The equivalent circuit at $t=0^-$ is shown below;



Here, $i_L(0^-) = \dfrac{60-45}{2+1} = 5$ A and $v_c(0^-) = 60 - 5\times1 = 55$ V

Then, at $t = 0^+$, $i_L(0^+) = 10$ A and $v_c(0^+) = 55$ V

The equivalent circuit at $t=0^+$ is shown below;



Applying KVL in the circuit,

$$55 - 45 = 5\times2 + v_L(0^+)$$

or, $v_L(0^+) = 0$

Now, current and voltage of each circuit elements are;

For 2 Ω resistor, $i_{2\Omega}(0^+) = 5$ A and $v_{2\Omega}(0^+) = 5\times2 = 10$ V

For 1 Ω resistor, $i_{1\Omega}(0^+) = 0$ and $v_{1\Omega}(0^+) = 0$

For 4 H inductor, $i_L(0^+) = 5$ A and $v_L(0^+) = 0$

For 2 F capacitor, $i_c(0^+) = 5$ A and $v_c(0^+) = 55$ V

Also, $v_L(0^+) = L\dfrac{di_L}{dt}(0^+)$

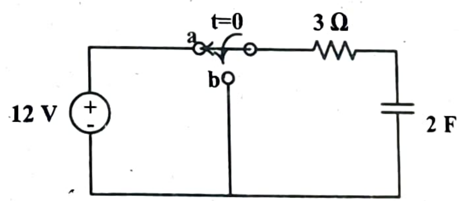
or, $\dfrac{di_L}{dt}(0^+) = 0$

And, $i_c(0^+) = C\dfrac{dv_c}{dt}(0^+)$

or, $\dfrac{dv_c}{dt}(0^+) = \dfrac{5}{2} = 2.5$ V/s

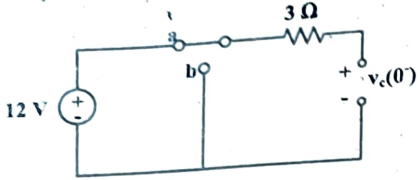
Example 2.6

The switch has been at position 'a' for a long time. Then it is quickly thrown to position 'b' at t=0. Find $i_{3\Omega}$; i_c, v_c, $v_{3\Omega}$ and $\dfrac{dv_c}{dt}$ at $t=0^+$.



Solution:

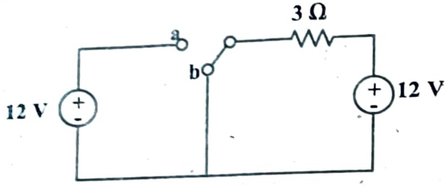
The equivalent circuit at $t=0^-$ is shown below;



Here, $v_c(0^-) = 12$ V

Then, at $t=0^+$, $v_c(0^+) = 12$ V

The equivalent circuit at $t=0^+$ is shown below;



Here, current flowing through the circuit $= \frac{12}{3} = 4$ A

Now, current and voltage of each circuit elements are;

For 3Ω resistor, $i_{3\Omega}(0^+) = 4$ A and $v_{3\Omega}(0^+) = 4\times3 = 12$ V

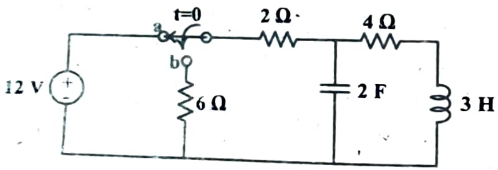
For 2 F capacitor, $i_c(0^+) = 4$ A and $v_c(0^+) = 12$ V

Also, $i_c(0^+) = C\frac{dv_c}{dt}(0^+)$

or, $\frac{dv_c}{dt}(0^+) = \frac{4}{2} = 2$ V/s

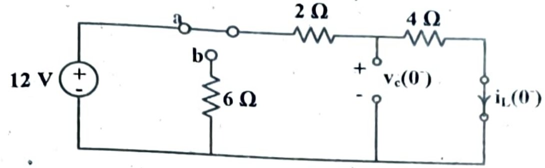
Example 2.7

For the circuit shown in the figure below, find $i_{2\Omega}$, $i_{4\Omega}$, $i_{6\Omega}$, i_c, i_L, v_c, v_L, q_c, $v_{2\Omega}$, $v_{4\Omega}$, $v_{6\Omega}$, $\frac{dv_c}{dt}$ and $\frac{di_L}{dt}$ at $t=0^+$.



Solution:

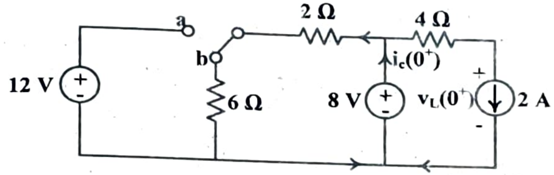
The equivalent circuit at $t=0^-$ is shown below;



Here, $i_L(0^-) = \frac{12}{2+4} = 2$ A and $v_c(0^-) = 2\times4 = 8$ V

Then, at $t=0^+$, $i_L(0^+) = 2$ A and $v_c(0^+) = 8$ V

The equivalent circuit at $t=0^+$ is shown below;



Here, $i_{2\Omega}(0^+) = i_{6\Omega}(0^+) = \frac{8}{2+6} = 1$ A

Applying KVL in right mesh,

$$8 = 4\times2 + v_L(0^+)$$

or, $\quad v_L(0^+) = 0$

Now, current and voltage of each circuit elements are;

For 2 Ω resistor, $i_{2\Omega}(0^+) = 1$ A and $v_{2\Omega}(0^+) = 1\times2 = 2$ V

For 6Ω resistor, $i_{6\Omega}(0^+) = 1$ A and $v_{6\Omega}(0^+) = 1\times6 = 6$ V

For 3 H inductor, $i_L(0^+) = 2$ A and $v_L(0^+) = 0$

For 2 F capacitor, $i_c(0^+) = 1 + 2 = 3$ A and $v_c(0^+) = 8$ V

Also, $v_L(0^+) = L\frac{di_L}{dt}(0^+)$

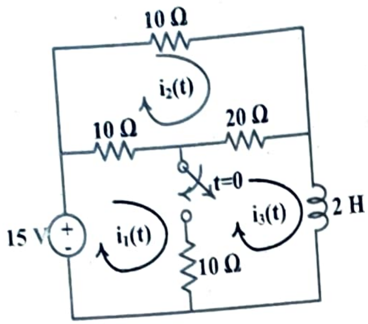
or, $\frac{di_L}{dt}(0^+) = 0$

And, $i_c(0^+) = C\frac{dv_c}{dt}(0^+)$

or, $\frac{dv_c}{dt}(0^+) = \frac{3}{2} = 1.5$ V/s

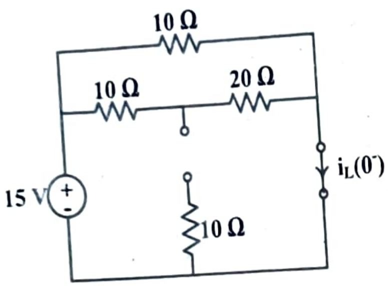
Example 2.8

For the circuit shown in the figure below, find the mesh currents at t=0$^+$.



Solution:

At t=0$^-$, the equivalent circuit is;

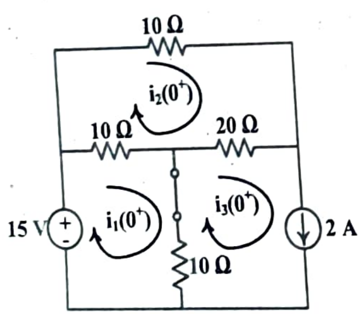


Here, $R_{eq} = 10//(10+20) = 7.5\ \Omega$

$$i_L(0^-) = \frac{15}{7.5} = 2\text{ A}$$

Then, at $t = 0^+$, $i_L(0^+) = 2$ A

The equivalent circuit at $t = 0^+$ is;



Let $i_1(0^+), i_2(0^+)$ and $i_3(0^+)$ be the mesh currents at $t = 0^+$, then

Applying KVL in mesh 1,

$$15 = 10\,[i_1(0^+) - i_2(0^+)] + 10\,[i_1(0^+) - i_3(0^+)]$$

or, $\quad 15 = 20i_1(0^+) - 10i_2(0^+) - 10i_3(0^+)$ ....(1)

Applying KVL in mesh 2,

$$0 = 10\,[i_2(0^+) - i_1(0^+)] + 20\,[i_2(0^+) - i_3(0^+)] + 10i_2(0^+)$$

or, $\quad -10i_1(0^+) + 40i_2(0^+) - 20i_3(0^+) = 0$ ....(2)

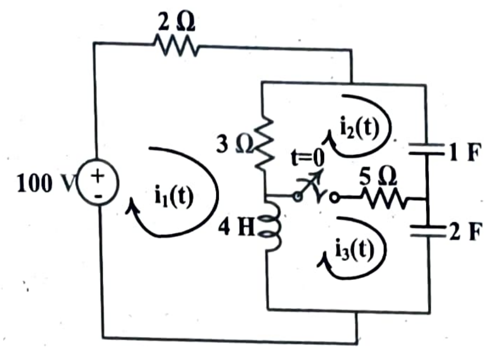
In the outer branch of mesh 3, $i_3(0^+) = 2$ A ....(3)

Solving equations (1), (2) and (3), we get,

$$\mathbf{i_1(0^+) = 2.57\ A,\ i_2(0^+) = 1.64\ A,\ i_3(0^+) = 2\ A}$$

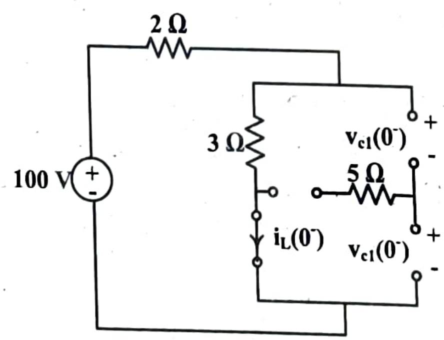
Example 2.9

In the circuit shown in the figure below, find the mesh currents i_1, i_2 and i_3 at t=0$^+$.



Solution:

The equivalent circuit at t=0$^-$ is shown below;



Here, $i_L(0^-) = \frac{100}{2+3} = 20$ A

and $v_{c1}(0^-) + v_{c2}(0^-) = 20 \times 3 = 60$ V

Using voltage divider rule for capacitance,

$$v_{c1}(0^-) = \frac{C_2}{C_1 + C_2} \times [v_{c1}(0^-) + v_{c2}(0^-)] = \frac{2}{1+2} \times 60 = 40 \text{ V}$$

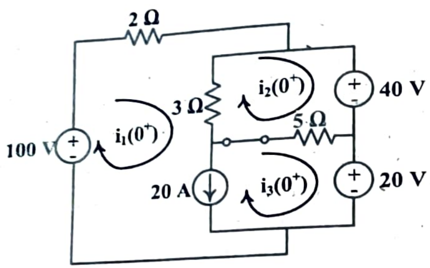
$$v_{c2}(0^-) = 60 - 40 = 20 \text{ V}$$

Then, at $t=0^+$, $i_L(0^+) = 20$ A

$$v_{c1}(0^+) = 40 \text{ V}$$

$$v_{c2}(0^+) = 20 \text{ V}$$

The equivalent circuit at $t=0^+$ is shown below;



Applying KVL in mesh 2,

$$-40 = 3\,[i_2(0^+) - i_1(0^+)] + 5\,[i_2(0^+) - i_3(0^+)]$$

or, $-40 = -3i_1(0^+) + 8i_2(0^+) - 5i_3(0^+)$ ....(1)

Applying KVL in mesh 1 and 3,

$$100 - 20 = 3\,[i_1(0^+) - i_2(0^+)] + 5\,[i_3(0^+) - i_2(0^+)] + 2i_1(0^+)$$

or, $5i_1(0^+) - 8i_2(0^+) + 5i_3(0^+) = 80$ ....(2)

In the common branch between mesh 1 and 2,

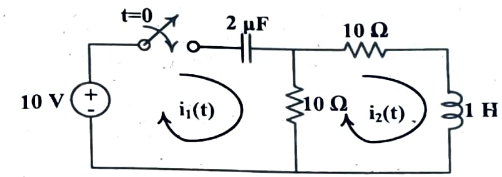
$i_1(0^+) - i_3(0^+) = 20$ ....(3)

Solving equations (1), (2) and (3), we get,

$$\mathbf{i_1(0^+) = 20\text{ A},\ i_2(0^+) = 2.5\text{ A},\ i_3(0^+) = 0}$$

Example 2.10

For the circuit shown in the figure below, find i_1, i_2, $\frac{di_1}{dt}$, $\frac{di_2}{dt}$, $\frac{d^2i_1}{dt^2}$ and $\frac{d^2i_2}{dt^2}$ at $t=0^+$.



Solution:

Applying KVL in mesh 1 for $t>0$,

$$10 = \frac{1}{2\times10^{-6}} \int_{-\infty}^{t} i_1\,dt + 10\,(i_1 - i_2)$$

or, $$10 = 5 \times 10^5 \int_{-\infty}^{t} i_1\,dt + 10\,i_1 - 10\,i_2$$

Differentiating the above equation,

$$0 = 5\times10^5\, i_1 + 10\frac{di_1}{dt} - 10\frac{di_2}{dt} \text{ ....(1)}$$

Applying KVL in mesh 2 for $t>0$,

$$10(i_2 - i_1) + 10\,i_2 + 1\frac{di_2}{dt} = 0$$

or, $$20\,i_2 - 10\,i_1 + \frac{di_2}{dt} = 0 \text{ ....(2)}$$

Differentiating equation (1) w.r.t. time,

$$0 = 5\times10^5\frac{di_1}{dt} + 10\frac{d^2i_1}{dt^2} - 10\frac{d^2i_2}{dt^2} \text{ ....(3)}$$

ifferentiating equation (2) w.r.t. time,

$$20\frac{di_2}{dt} - 10\frac{di_1}{dt} + \frac{d^2i_2}{dt^2} = 0 \text{ ....(4)}$$

At $t=0^-$, the circuit is not energized since switch is open.

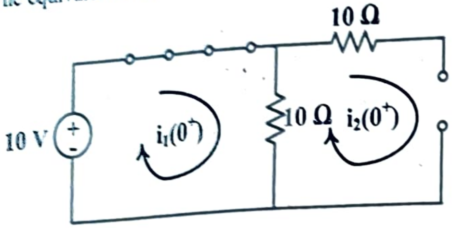
So, $i_L(0^-) = 0$

and $v_c(0^-) = 0$

Then, at $t=0^+$, $i_L(0^+) = 0$, i.e. the inductor behaves as open circuit (O.C.)

and $v_c(0^+) = 0$, i.e. the capacitor behaves as short circuit (S.C.)

The equivalent circuit at $t=0^+$ is shown below;



Here, $i_1(0^+) = \frac{10}{10} = 1$ A

$i_2(0^+) = 0$

Writing equation (2) at $t = 0^+$, we get

$$20\, i_2(0^+) - 10\, i_1(0^+) + \frac{di_2}{dt}(0^+) = 0$$

or, $$20\times0 - 10\times1 + \frac{di_2}{dt}(0^+) = 0$$

or, $$\frac{di_2}{dt}(0^+) = 10 \text{ A/s}$$

Writing equation (1) at $t = 0^+$, we get

$$0 = 5\times10^5\, i_1(0^+) + 10\frac{di_1}{dt}(0^+) - 10\frac{di_2}{dt}(0^+)$$

or, $$0 = 5\times10^5\times1 + 10\frac{di_1}{dt}(0^+) - 10\times10$$

or, $$\frac{di_1}{dt}(0^+) = -49{,}990 \text{ A/s}$$

Writing equation (4) at $t = 0^+$, we get

$$20\frac{di_2}{dt}(0^+) - 10\frac{di_1}{dt}(0^+) + \frac{d^2i_2}{dt^2}(0^+) = 0$$

or, $$20\times10 - 10\times(-49990) + \frac{d^2i_2}{dt^2}(0^+) = 0$$

or, $$\frac{d^2i_2}{dt^2}(0^+) = -500{,}100 \text{ A/s}^2$$

Writing equation (3) at $t = 0^+$, we get

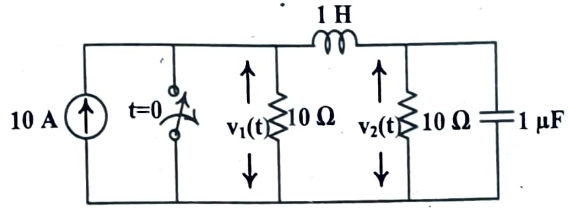
$$0 = 5\times10^5\frac{di_1}{dt}(0^+) + 10\frac{d^2i_1}{dt^2}(0^+) - 10\frac{d^2i_2}{dt^2}(0^+)$$

or, $$0 = 5\times10^5\times(-49990) + 10\frac{d^2i_1}{dt^2}(0^+) - 10\times(-500100)$$

or, $$\frac{d^2i_1}{dt^2}(0^+) = 2.499\times10^9 \text{ A/s}^2$$

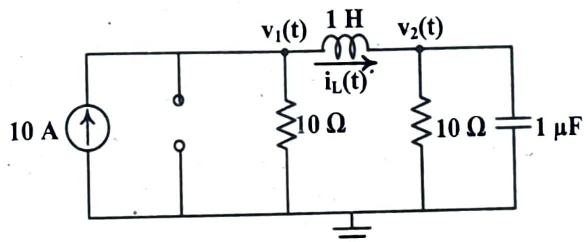
Example 2.11

In the circuit shown in the figure below, the switch is opened at $t = 0$. Find v_1, v_2, $\frac{dv_1}{dt}$, $\frac{dv_2}{dt}$, $\frac{d^2v_1}{dt^2}$ and $\frac{d^2v_2}{dt^2}$ at $t=0^+$.



Solution:

Defining voltage nodes and reference node so as to apply nodal analysis for $t > 0$;



Applying KCL at node 1 for $t>0$,

$$10 = \frac{1}{10}v_1(t) + i_L(t)$$

or, $$10 = \frac{1}{10}v_1(t) + \frac{1}{1}\int_{-\infty}^{t}(v_1-v_2)dt$$

Differentiating above equation w.r.t. time,

$$0 = \frac{1}{10}\frac{dv_1}{dt} + v_1 - v_2 \text{ ....(1)}$$

Applying KCL at node 2 for t>0,

$$i_L(t) = \frac{1}{10}v_2(t) + 1\times10^{-6}\frac{dv_2}{dt} \quad \text{..........(2)}$$

or, $$\frac{1}{1}\int_{-\infty}^{t}(v_1-v_2)dt = \frac{1}{10}v_2(t) + 1\times10^{-6}\frac{dv_2}{dt}$$

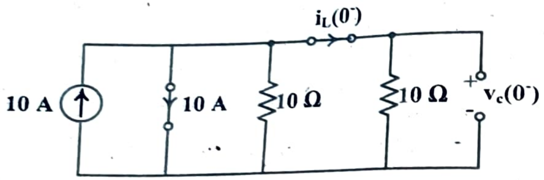
Differentiating above equation w.r.t. time,

$$v_1 - v_2 = \frac{1}{10}\frac{dv_2}{dt} + 1\times10^{-6}\frac{d^2v_2}{dt^2} \quad \text{..........(3)}$$

Differentiating equation (1) w.r.t. time,

$$0 = \frac{1}{10}\frac{d^2v_1}{dt^2} + \frac{dv_1}{dt} - \frac{dv_2}{dt} \quad \text{..........(4)}$$

The equivalent circuit at $t=0^-$ is shown below;



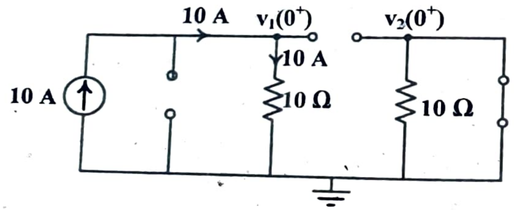
Here, whole the current from the current source passes through short circuit path.

So, $i_L(0^-) = 0$ and $v_c(0^-) = 0$

Then, at $t=0^+$, $i_L(0^+) = 0$, i.e. the inductor behaves as open circuit (O.C.)

and $v_c(0^+) = 0$, i.e. the capacitor behaves as short circuit (S.C.)

The equivalent circuit at $t=0^+$ is shown below;



Here, $v_1(0^+) = 10\times10 = 100$ V and $v_2(0^+) = 0$

Writing equation (1) at $t = 0^+$, we get

$$0 = \frac{1}{10}\frac{dv_1}{dt}(0^+) + v_1(0^+) - v_2(0^+)$$

or, $$0 = \frac{1}{10}\frac{dv_1}{dt}(0^+) + 100 - 0$$

or, $$\frac{dv_1}{dt}(0^+) = -1000 \text{ V/s}$$

Writing equation (2) at $t = 0^+$, we get

$$i_L(0^+) = \frac{1}{10}v_2(0^+) + 1\times10^{-6}\frac{dv_2}{dt}(0^+)$$

or, $$0 = \frac{1}{10}\times0 + 1\times10^{-6}\frac{dv_2}{dt}(0^+)$$

or, $$\frac{dv_2}{dt}(0^+) = 0$$

Writing equation (4) at $t = 0^+$, we get

$$0 = \frac{1}{10}\frac{d^2v_1}{dt^2}(0^+) + \frac{dv_1}{dt}(0^+) - \frac{dv_2}{dt}(0^+)$$

or, $$0 = \frac{1}{10}\frac{d^2v_1}{dt^2}(0^+) + (-1000) - 0$$

or, $$\frac{d^2v_1}{dt^2}(0^+) = 10^4 \text{V/s}^2$$

Writing equation (3) at $t = 0^+$, we get

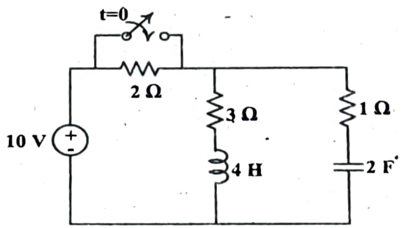
$$v_1(0^+) - v_2(0^+) = \frac{1}{10}\frac{dv_2}{dt}(0^+) + 1\times10^{-6}\frac{d^2v_2}{dt^2}(0^+)$$

or, $$100 - 0 = \frac{1}{10}\times0 + 1\times10^{-6}\frac{d^2v_2}{dt^2}(0^+)$$

or, $$\frac{d^2v_2}{dt^2}(0^+) = 10^8 \text{ V/s}^2$$

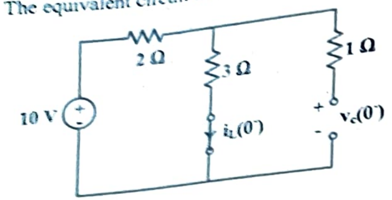
Example 2.12

For the circuit shown in the figure below, find $i_{2\Omega}$, $i_{3\Omega}$, $i_{1\Omega}$, i_c, i_L, v_c, v_L, q_c, $v_{2\Omega}$, $v_{3\Omega}$ and $v_{1\Omega}$ at $t=0^+$. [2068 Shrawan]



Solution:

The equivalent circuit at $t=0^-$ is shown below;

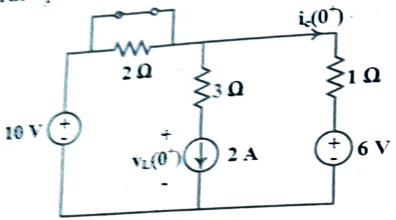


Here, $i_L(0^-) = \frac{10}{2+3}\ A = 2\ A$

and $v_c(0^-) = v_{3\Omega}(0^-) = 2\times3 = 6\ V$

Hence, at $t = 0^+$, $i_L(0^+) = 2\ A$ and $v_c(0^+) = 6\ V$

The equivalent circuit at $t=0^+$ is shown below;



Applying KVL in left mesh,

$$10 = 2\times3 + v_L(0^+)$$

or, $v_L(0^+) = 4\ V$

Applying KVL in outer loop,

$$10 = i_c(0^+)\times1 + 6$$

or, $i_c(0^+) = 4\ A$

Now, current and voltage of each circuit elements are;

For 2 Ω resistor, $i_{2\Omega}(0^+) = 0$ A and $v_{2\Omega}(0^+) = 0$ V

For 3 Ω resistor, $i_{3\Omega}(0^+) = 2$ A and $v_{3\Omega}(0^+) = 2\times3 = 6$ V

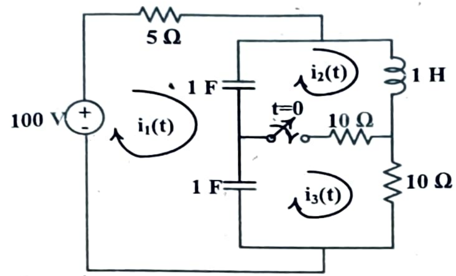
For 1 Ω resistor, $i_{1\Omega}(0^+) = 4$ A and $v_{1\Omega}(0^+) = 4\times1 = 4$ V

For 4 H inductor, $i_L(0^+) = 2$ A and $v_L(0^+) = 4$ V

For 2F capacitor, $i_c(0^+) = 4$ A , $v_c(0^+) = 6$ V and $q_c(0^+) = 2\times6 = 12$ C

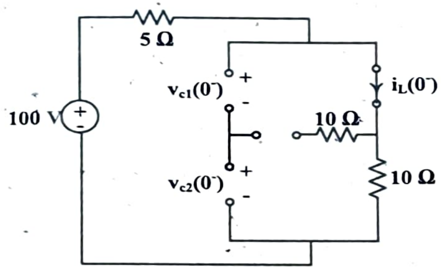
Example 2.13

In the circuit shown in the figure below, find the mesh currents i_1, i_2 and i_3 at $t = 0^+$. **[2071 Chaitra]**



Solution:

The equivalent circuit at $t = 0^-$ is shown below;



Here, $i_L(0^-) = \frac{100}{5+10} = \frac{20}{3}$ A

and $v_{c1}(0^-) + v_{c2}(0^-) = \frac{20}{3}\times10 = \frac{200}{3}$ V

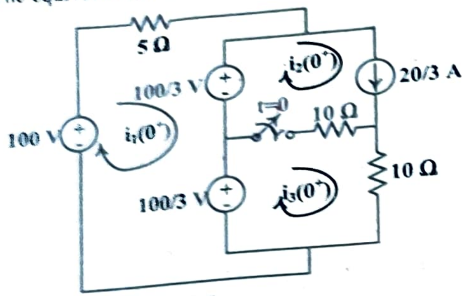
Since both the capacitors have equal capacitances, the voltage divides equally.

So, $v_{c1}(0^-) = v_{c2}(0^-) = \frac{100}{3}$ V

Hence, at $t=0^+$, $i_L(0^+) = \frac{20}{3}$ A

$v_{c1}(0^+) = v_{c2}(0^+) = \frac{100}{3}$ V

The equivalent circuit at $t=0^+$ is shown below;



Applying KVL in mesh 1,

$$100 = 5 \times i_1(0^+) + \frac{100}{3} + \frac{100}{3}$$

or, $\quad i_1(0^+) = \frac{20}{3}$ A

In mesh 2,

$$i_2(0^+) = \frac{20}{3}\ \text{A}$$

Applying KVL in mesh 3,

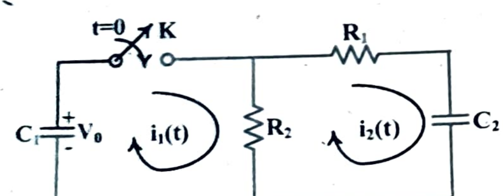
$$\frac{100}{3} = 10\,(\,i_3(0^+) - i_2(0^+)\,) + 10\,i_3(0^+)$$

or, $\quad \frac{100}{3} = 10\,(\,i_3(0^+) - \frac{20}{3}\,) + 10\,i_3(0^+)$

or, $\quad i_3(0^+) = 5$ A

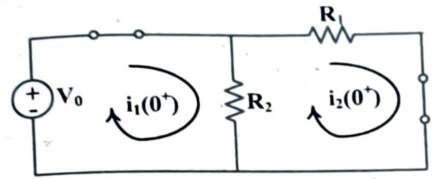
Example 2.14

In the given network, the capacitor C_1 is charged to voltag V_0 and switch K is closed at t = 0. When $R_1 = 2$ MΩ, $V_0 = 1000$ $R_2 = 1$ MΩ, $C_1 = 10$ µF and $C_2 = 20$ µF, solve for i_1, i_2, $\frac{di_2}{dt}$ and $\frac{d}{dt}$ at $t = 0^+$. [2075 Ashwi



Solution:

The equivalent circuit at $t=0^+$ is shown below;



Here, $\quad i_2(0^+) = \frac{V_0}{R_1} = \frac{1000}{2 \times 10^6} = 0.5\text{mA}$

Applying KVL in left mesh for $t = 0^+$,

$$1000 = 1 \times 10^6\,[i_1(0^+) - i_2(0^+)]$$

or, $\quad i_1(0^+) = 10^{-3} + i_2(0^+)$

or, $\quad \mathbf{i_1(0^+) = 1.5\ mA}$

Applying KVL in left mesh for $t > 0$,

$$0 = \frac{1}{C_1}\int_{-\infty}^{t} i_1\,dt + 10^6\,(i_1 - i_2)$$

or, $\quad 0 = \frac{1}{10\times10^{-6}}\int_{-\infty}^{t} i_1\,dt + 10^6\,(i_1 - i_2)$

or, $\quad 0 = 10^5\int_{-\infty}^{t} i_1\,dt + 10^6\,i_1 - 10^6\,i_2$

Differentiating the above equation,

$$0 = 10^5\,i_1 + 10^6\frac{di_1}{dt} - 10^6\frac{di_2}{dt} \quad \text{..........(1)}$$

Applying KVL in right mesh for $t > 0$,

$$10^6\,(i_2 - i_1) + 2 \times 10^6\,i_2 + \frac{1}{C_2}\int_{-\infty}^{t} i_2\,dt = 0$$

or, $\quad 10^6\,(i_2 - i_1) + 2 \times 10^6\,i_2 + \frac{1}{20 \times 10^{-6}}\int_{-\infty}^{t} i_2\,dt = 0$

Differentiating the above equation,

or, $3 \times 10^6 \frac{di_2}{dt} - 10^6 \frac{di_1}{dt} + 5 \times 10^4 i_2 = 0$ ....(2)

Differentiating equation (1) w.r.t. time,

$$0 = 10^5 \frac{di_1}{dt} + 10^6 \frac{d^2 i_1}{dt^2} - 10^6 \frac{d^2 i_2}{dt^2} \text{ ....(3)}$$

Differentiating equation (2) w.r.t. time,

$$3 \times 10^6 \frac{d^2 i_2}{dt^2} - 10^6 \frac{d^2 i_1}{dt^2} + 5 \times 10^4 \frac{di_2}{dt} = 0 \text{ ....(4)}$$

Writing equations (1) and (2) at $t = 0^+$, we get

$$0 = 10^5 i_1(0^+) + 10^6 \frac{di_1}{dt}(0^+) - 10^6 \frac{di_2}{dt}(0^+)$$

or, $10^6 \frac{di_1}{dt}(0^+) - 10^6 \frac{di_2}{dt}(0^+) = -150$ ....(5)

and $3 \times 10^6 \frac{di_2}{dt}(0^+) - 10^6 \frac{di_1}{dt}(0^+) + 5 \times 10^4 i_2(0^+) = 0$

or, $-10^6 \frac{di_1}{dt}(0^+) + 3 \times 10^6 \frac{di_2}{dt}(0^+) = -25$ ....(6)

Solving equations (5) and (6), we get,

$$\frac{di_1}{dt}(0^+) = -2.375 \times 10^{-4} \text{ A/s}$$

and $\frac{di_2}{dt}(0^+) = -8.75 \times 10^{-5}$ A/s

Writing equations (3) and (4) at $t = 0^+$, we get

$$0 = 10^5 \frac{di_1}{dt}(0^+) + 10^6 \frac{d^2 i_1}{dt^2}(0^+) - 10^6 \frac{d^2 i_2}{dt^2}(0^+)$$

or, $10^6 \frac{d^2 i_1}{dt^2}(0^+) - 10^6 \frac{d^2 i_2}{dt^2}(0^+) = 23.75$ ....(7)

and $3 \times 10^6 \frac{d^2 i_2}{dt^2}(0^+) - 10^6 \frac{d^2 i_1}{dt^2}(0^+) + 5 \times 10^4 \frac{di_2}{dt}(0^+) = 0$

or, $-10^6 \frac{d^2 i_1}{dt^2}(0^+) + 3 \times 10^6 \frac{d^2 i_2}{dt^2}(0^+) = 4.375$ ....(8)

Solving equations (5) and (6), we get,

$$\frac{d^2 i_1}{dt^2}(0^+) = 3.78 \times 10^{-5} \text{ A/s}^2$$

and $\frac{d^2 i_2}{dt^2}(0^+) = 1.41 \times 10^{-5}$ A/s²

2.5 DIFFERENTIAL EQUATIONS

During the transient analysis of the circuits, the differential equations are formed because of the presence of energy storing elements such as capacitors and inductors in the circuits. The order of the differential equation depends on the number of energy storing elements in the circuit. Hence, in order to obtain the solution of the differential equation, we should apply either classical method or Laplace Transform.

2.6 SOLVING FIRST ORDER LINEAR DIFFERENTIAL EQUATION USING CLASSICAL METHOD

The standard form of first order linear differential equation is;

$$\frac{dy(t)}{dt} + Py(t) = Q$$

where, $y(t)$ is dependent variable (voltage, current etc.)

P is constant

Q may be either constant or variable depending on source

The solution of above differential can be expressed as

$$y(t) = y_N(t) + y_F(t)$$

where, $y_N(t)$ is called natural response

$y_F(t)$ is called forced response

The natural response is the response which is independent of the source or forcing function. It only depends on the circuit parameters. It is given by;

$y_N(t) = K e^{-Pt}$; where the value of K can be determined using initial condition.

The forced response depends on source or forcing function. It varies with the nature of source.

a) For constant or DC source, the forced response is given by;

$$y_F(t) = \frac{Q}{P}$$

b) For exponential source:

If the exponential source is given by $V(t) = a e^{-bt}$; where a and b are constants.

Then, the trial solution for the forced response is assumed as;

$$y_F(t) = Ae^{-bt};\ \text{if } b \neq P$$

$$y_F(t) = A\,te^{-bt};\ \text{if } b = P$$

where, A is constant whose value can be determined after replacement of y(t) in original differential equation by $y_F(t)$.

c) For sinusoidal source:

If the sinusoidal source is given by $V(t) = V_m \sin(\omega t + \phi)$ or $V_m \cos(\omega t + \phi)$.

Then, the trial solution for the forced response is assumed as;

$$y_F(t) = A\cos(\omega t + \phi) + B\sin(\omega t + \phi)$$

where, A and B are constants whose values can be determined after replacement of y(t) in original differential equation by $y_F(t)$.

2.7 SOLVING SECOND ORDER LINEAR DIFFERENTIAL EQUATION USING CLASSICAL METHOD

The standard form of second order linear differential equation is;

$$\frac{d^2y(t)}{dt^2} + R\frac{dy(t)}{dt} + Py(t) = Q$$

where, y(t) is dependent variable (voltage, current etc.)

P and R are constants

Q may be either constant or variable depending on source

The solution of above differential can be expressed as

$$y(t) = y_N(t) + y_F(t)$$

where, $y_N(t)$ is called natural response

$y_F(t)$ is called forced response

For the determination of natural response, the auxiliary equation is formed as;

$$s^2 + Rs + P = 0$$

and the natural response depends on the nature of roots of the auxiliary equation.

Case I: The roots are real and unequal.

Let the roots be $s = s_1, s_2$

Then, $y_N(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$

Case II: The roots are real and equal.

Let the roots be $s = s_1, s_2 (=s_1)$

Then, $y_N(t) = (K_1 + K_2 t)\, e^{s_1 t}$

Case III: The roots are complex conjugates.

Let the roots be $s = \alpha \pm j\beta$

Then, $y_N(t) = e^{\alpha t}(K_1\cos\beta t + K_2\sin\beta t)$

Here, in all three cases, the values of K_1 and K_2 can be determined using initial conditions.

The forced response depends on source or forcing function. It varies with the nature of source.

a) For constant or DC source, the forced response is given by;

$$y_F(t) = \frac{Q}{P}$$

b) For exponential source:

If the exponential source is given by $V(t) = a\,e^{-bt}$; where a and b are constants.

Then, the trial solution for the forced response is assumed as;

$$y_F(t) = Ae^{-bt};\ \text{if } b \neq -s_1 \neq -s_2$$

$$y_F(t) = A\,te^{-bt};\ \text{if } b = -s_1 \text{ or } -s_2 \text{ (equal to any one)}$$

$$y_F(t) = A\,t^2e^{-bt};\ \text{if } b = -s_1 = -s_2 \text{ (equal to both)}$$

where, A is constant whose value can be determined after replacement of y(t) in original differential equation by $y_F(t)$.

c) For sinusoidal source:

If the sinusoidal source is given by $V(t) = V_m \sin(\omega t + \phi)$ or $V_m \cos(\omega t + \phi)$.

Then, the trial solution for the forced response is assumed as;

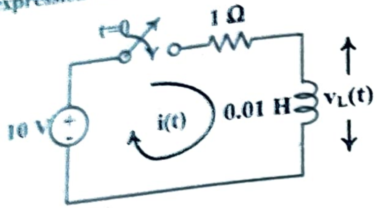
$$y_F(t) = A\cos(\omega t + \phi) + B\sin(\omega t + \phi)$$

where, A and B are constants whose values can be determined after replacement of y(t) in original differential equation by $y_F(t)$.

2.8 SOLVED PROBLEMS OF TRANSIENT ANALYSIS BY CLASSICAL METHOD

Example 2.15

A DC source of 10 V is suddenly applied at time t = 0 to a series R-L circuit comprising R = 1 Ω and L = 0.01 H. Obtain the expression for i(t) and $v_L(t)$ in the circuit.



Solution:

Applying KVL in the circuit for t>0,

$$10 = 1\times i(t) + 0.01\frac{di(t)}{dt}$$

or, $$\frac{di(t)}{dt} + 100\, i(t) = 1000 \;\ldots\ldots\ldots\ldots(1)$$

The above equation is first order linear differential equation whose solution is given by,

$$i(t) = Ke^{-100t} + \frac{1000}{100}$$

or, $$i(t) = Ke^{-100t} + 10 \ldots\ldots\ldots\ldots(2)$$

The above equation is the general solution of differential equation and the value of K can be determined using initial conditions.

At $t=0^-$, $i_L(0^-) = 0$ [By circuit inspection]

Then, $i_L(0^-) = 0 = i(0^+)$

Writing equation (2) at $t=0^+$ and substituting $i(0^+)$, we get

$$0 = K + 10$$

or, $$K = -10$$

Substituting the value of K in general solution, we get

$$i(t) = -10e^{-100t} + 10$$

or, $$i(t) = 10(1-e^{-100t})\ \text{A}$$

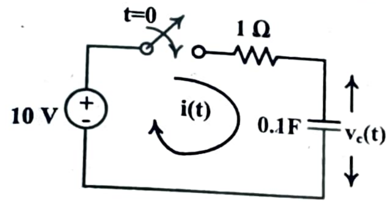
The current i(t) is same for both resistor and inductor in the given circuit.

So, $v_L(t) = L\frac{di(t)}{dt} = 0.01\frac{d[10(1-e^{-100t})]}{dt} = -0.01\times 10\times(-100)\,e^{-100t}$

Hence, $v_L(t) = 10e^{-100t}$ V

Example 2.16

A DC source of 10 V is suddenly applied at time t = 0 to a series R–C circuit comprising R = 1 Ω and C = 0.1 F. Obtain the expression for i(t) and $v_c(t)$ in the circuit.



Solution:

Applying KVL in the circuit for t>0,

$$10 = 1\times i(t) + \frac{1}{0.1}\int_{-\infty}^{t} i(t)\,dt$$

Differentiating w.r.t time,

or, $$\frac{di(t)}{dt} + 10\, i(t) = 0 \;\ldots\ldots\ldots\ldots(1)$$

The above equation is first order linear differential equation whose solution is given by,

$$i(t) = Ke^{-10t} + \frac{0}{10}$$

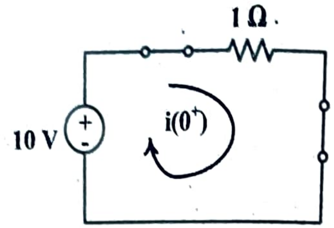
or, $$i(t) = Ke^{-10t} \;\ldots\ldots\ldots\ldots(2)$$

The above equation is the general solution of differential equation (1) and the value of K can be determined using initial conditions.

At $t=0^-$, $v_c(0^-) = 0$ [By circuit inspection]

Then, $v_c(0^+) = 0$ i.e. the capacitor behaves as short circuit.

At $t = 0^+$, the equivalent circuit is;



Here, $i(0^+) = 10$ A

Writing equation (2) at $t = 0^+$ and substituting $i(0^+)$, we get

$$10 = K$$

or, $K = 10$

Substituting the value of K in general solution, we get

$$\mathbf{i(t) = 10e^{-10t}\ A}$$

The current i(t) is same for both resistor and capacitor in the given circuit.

So, $v_c(t) = \frac{1}{C}\int_{-\infty}^{t} i_c(t)\, dt$

or, $v_c(t) = v_c(0^+) + \frac{1}{C}\int_{0}^{t} i_c(t)\, dt$

or, $v_c(t) = 0 + \frac{1}{0.1}\int_{0}^{t} 10\, e^{-10t}\, dt$

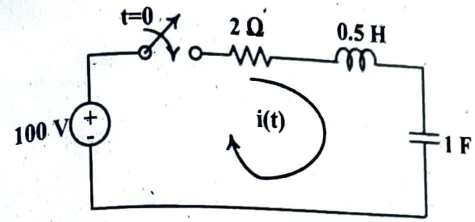
or, $v_c(t) = 10 \times 10\left[\frac{e^{-10t}}{-10}\right]_0^t$

or, $v_c(t) = -10\,(e^{-10t} - 1)$

or, $\mathbf{v_c(t) = 10(1 - e^{-10t})\ V}$

Example 2.17

A DC source of100 V is suddenly applied at time t = 0 to a series RLC circuit comprising R = 2 Ω, L=0.5 H and C =1F. Obtain the expression for current in the circuit.



Solution:

Let i(t) be the current flowing through the circuit at any time t.

Then, applying KVL in the circuit for t>0, we get,

$$100 = 2\, i(t) + 0.5\frac{di(t)}{dt} + v_c(t) \ ....(1)$$

$$100 = 2\, i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{1}\int_{-\infty}^{t} i(t)\, dt$$

Differentiating this equation w.r.t. time, we get

$$0 = 2\frac{di(t)}{dt} + 0.5\frac{d^2i(t)}{dt^2} + i(t)$$

or, $\frac{d^2i(t)}{dt^2} + 4\frac{di(t)}{dt} + 2\, i(t) = 0 \ ....(2)$

The above equation is second order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

where, $i_F(t) = 0$

For $i_N(t)$:

The auxiliary equation is;

$$s^2 + 4s + 2 = 0$$

or, $s = -0.586, -3.414$ (The roots are real and unequal)

So, $i_N(t) = K_1\, e^{-0.586t} + K_2\, e^{-3.414t}$

Then, $i(t) = i_N(t) + i_F(t)$

or, $i(t) = K_1\, e^{-0.586t} + K_2\, e^{-3.414t} \ ....(3)$

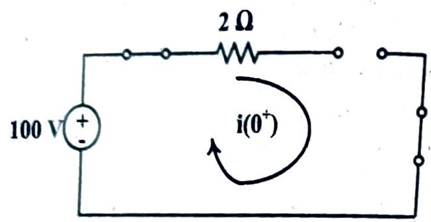
The above equation is the general solution of the differential equation (2) and the values of K_1 and K_2 can be determined using initial conditions.

$$\frac{di(t)}{dt} = -0.586\, K_1\, e^{-0.586t} - 3.414\, K_2\, e^{-3.414t} \quad \text{..........(4)}$$

At $t=0^-$, $i_L(0^-) = 0$ and $v_c(0^-) = 0$

Then, at $t=0^+$, $i_L(0^+) = 0$ and $v_c(0^+) = 0$

The equivalent circuit at $t=0^+$ is shown below;



Here, $i(0^+)= 0$

Writing equation (1) at $t=0^+$, we get,

$$100 = 2\, i(0^+) + 0.5\frac{di}{dt}(0^+) + v_c(0^+)$$

or, $\frac{di}{dt}(0^+) = 200$ A/s

Writing equations (3) and (4) at $t=0^+$ and substituting the values of $i(0^+)$ and $\frac{di}{dt}(0^+)$, we get,

$$0 = K_1 + K_2 \quad \text{..........(a)}$$

and $200 = -0.586\, K_1 - 3.414\, K_2$ ....(b)

Solving equations (a) and (b), we get,

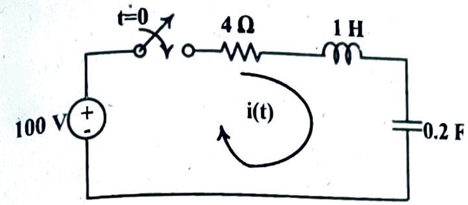
$K_1 = 70.72$ and $K_2 = -70.72$

Substituting the values of K_1 and K_2 in general solution gives the complete solution as;

$$\mathbf{i(t) = 70.72\, e^{-0.586t} - 70.72\, e^{-3.414t}\ A}$$

Example 2.18

A DC source of100 V is suddenly applied at time t = 0 to a series RLC circuit comprising R = 4 Ω, L=1 H and C =0.2F. Obtain the expression for current in the circuit.



Solution:

Let i(t) be the current flowing through the circuit at any time t.

Then, applying KVL in the circuit for t>0, we get,

$$100 = 4\, i(t) + 1\frac{di(t)}{dt} + v_c(t) \quad \text{..........(1)}$$

$$100 = 4\, i(t) + 1\frac{di(t)}{dt} + \frac{1}{0.2}\int_{-\infty}^{t} i(t)\, dt$$

Differentiating this equation w.r.t. time, we get

$$0 = 4\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} + 5\, i(t)$$

or, $$\frac{d^2i(t)}{dt^2} + 4\frac{di(t)}{dt} + 5\, i(t) = 0 \quad \text{..........(2)}$$

The above equation is second order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

where, $i_F(t) = 0$

For $i_N(t)$:

The auxiliary equation is;

$$s^2 + 4s + 5 = 0$$

or, $s = -2 \pm j$ (The roots are complex conjugates)

So, $i_N(t) = e^{-2t}(K_1 \cos t + K_2 \sin t)$

Then, $i(t) = i_N(t) + i_F(t)$

or, $i(t) = e^{-2t}(K_1 \cos t + K_2 \sin t)$..........(3)

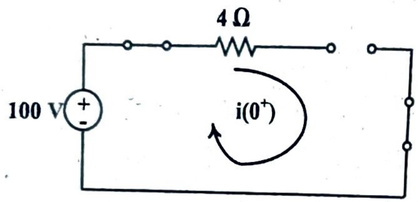
The above equation is the general solution of the differential equation (2) and the values of K_1 and K_2 can be determined using initial conditions.

$$\frac{di(t)}{dt} = -2e^{-2t}(K_1 \cos t + K_2 \sin t) + e^{-2t}(-K_1 \sin t + K_2 \cos t) \quad \text{.......(4)}$$

At $t=0^-$, $i_L(0^-) = 0$ and $v_c(0^-) = 0$

Then, at $t=0^+$, $i_L(0^+) = 0$ and $v_c(0^+) = 0$

The equivalent circuit at $t=0^+$ is shown below;



Here, $i(0^+) = 0$

Writing equation (1) at $t=0^+$, we get,

$$100 = 4\,i(0^+) + 1\frac{di}{dt}(0^+) + v_c(0^+)$$

or, $$\frac{di}{dt}(0^+) = 100 \text{ A/s}$$

Writing equations (3) and (4) at $t=0^+$ and substituting the values of $i(0^+)$ and $\frac{di}{dt}(0^+)$, we get,

$$0 = K_1$$

or, $$K_1 = 0$$

and $$100 = -2K_1 + K_2$$

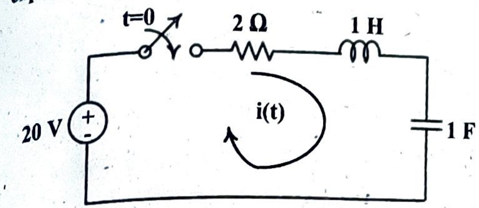
or, $$K_2 = 100$$

Substituting the values of K_1 and K_2 in general solution gives the complete solution as;

$$i(t) = 100\, e^{-2t} \sin t \text{ A}$$

Example 2.19

A DC source of 20 V is suddenly applied at time t = 0 to a series RLC circuit comprising R = 2 Ω, L = 1 H and C = 1 F. Obtain the expression for current in the circuit.



Solution:

Let i(t) be the current flowing through the circuit at any time t.

Then, applying KVL in the circuit for t>0, we get,

$$20 = 2\,i(t) + 1\frac{di(t)}{dt} + v_c(t) \quad \text{............(1)}$$

$$20 = 2\,i(t) + 1\frac{di(t)}{dt} + \frac{1}{1}\int_{-\infty}^{t} i(t)\,dt$$

Differentiating this equation w.r.t. time, we get

$$0 = 2\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} + i(t)$$

or, $$\frac{d^2i(t)}{dt^2} + 2\frac{di(t)}{dt} + i(t) = 0 \quad \text{...........(2)}$$

The above equation is second order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

where, $i_F(t) = 0$

For $i_N(t)$:

The auxiliary equation is;

$$s^2 + 2s + 1 = 0$$

or, $s = -1, -1$ (The roots are real and equal).

So, $i_N(t) = (K_1 + K_2 t)\, e^{-t}$

Then, $i(t) = i_N(t) + i_F(t)$

or, $i(t) = (K_1 + K_2 t)\, e^{-t}$ ....(3)

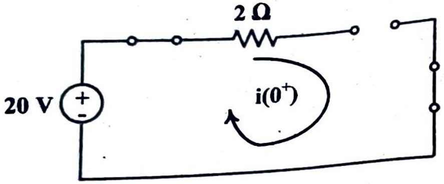
The above equation is the general solution of the differential equation (2) and the values of K_1 and K_2 can be determined using initial conditions.

$$\frac{di(t)}{dt} = -e^{-t}(K_1 + K_2 t) + e^{-t} K_2 \quad \text{.............(4)}$$

At $t = 0^-, i_L(0^-) = 0$ and $v_c(0^-) = 0$

Then, at $t=0^+$, $i_L(0^+) = 0$ and $v_c(0^+) = 0$

The equivalent circuit at $t=0^+$ is shown below;



Here, $i(0^+) = 0$

Writing equation (1) at $t=0^+$, we get,

$$20 = 2\, i(0^+) + 1 \frac{di}{dt}(0^+) + v_c(0^+)$$

or, $\frac{di}{dt}(0^+) = 20$ A/s

Writing equations (3) and (4) at $t=0^+$ and substituting the values of $i(0^+)$ and $\frac{di}{dt}(0^+)$, we get,

$$0 = K_1$$

or, $K_1 = 0$

and $20 = -K_1 + K_2$

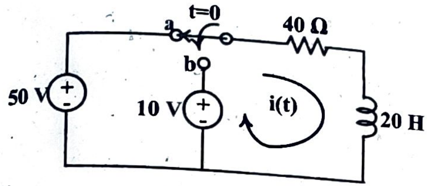
or, $K_2 = 20$

Substituting the values of K_1 and K_2 in general solution gives the complete solution as;

$$i(t) = 20\, t\, e^{-t} \text{ A}$$

Example 2.20

The switch in the figure below has been in position 'a' for a long time. Then, it is moved to 'b' at t=0. Obtain the expression for current i(t) for t>0.



Solution:

Applying KVL in the circuit for $t>0$,

$$10 = 40 \times i(t) + 20 \frac{di(t)}{dt}$$

or, $\frac{di(t)}{dt} + 2\, i(t) = 0.5$ ....(1)

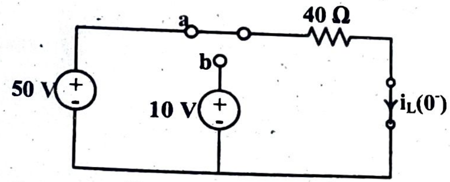
The above equation is first order linear differential equation whose solution is given by;

$$i(t) = Ke^{-2t} + \frac{0.5}{2}$$

or, $i(t) = Ke^{-2t} + 0.25$...........(2)

The above equation is the general solution of differential equation (1) and the value of K can be determined using initial conditions.

At $t = 0^-$, the equivalent circuit is as follows;



Here, $i_L(0^-) = \frac{50}{40} = 1.25$ A

Then, $i_L(0^+) = 1.25 \text{ A} = i(0^+)$

Writing equation (2) at $t=0^+$ and substituting $i(0^+)$, we get

$$1.25 = K + 0.25$$

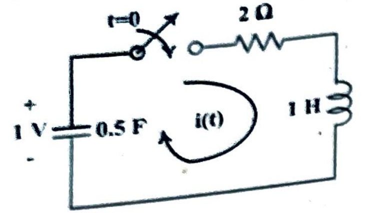
or, $K = 1$

Substituting the value of K in general solution, we get the complete solution of current;

$$i(t) = e^{-2t} + 0.25 \text{ A}$$

Example 2.21

The capacitor shown in the circuit below is charged initially to a voltage of 1 V with polarity as indicated in the figure. Find the expression for i(t).



Solution:

Applying KVL in the circuit for t>0, we get,

$$0 = 2\,i(t) + 1\frac{di(t)}{dt} + v_c(t) \;\ldots\ldots\ldots\ldots(1)$$

$$0 = 2\,i(t) + 1\frac{di(t)}{dt} + \frac{1}{0.5}\int_{-\infty}^{t} i(t)\,dt$$

Differentiating this equation w.r.t. time, we get

$$0 = 2\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} + 2\,i(t)$$

or, $$\frac{d^2i(t)}{dt^2} + 2\frac{di(t)}{dt} + 2\,i(t) = 0 \;\ldots\ldots\ldots\ldots(2)$$

The above equation is second order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

where, $i_F(t) = 0$

For $i_N(t)$:

The auxiliary equation is;

$$s^2 + 2s + 2 = 0$$

or, $s = -1 \pm j$ (The roots are complex conjugates)

So, $i_N(t) = e^{-t}(K_1 \cos t + K_2 \sin t)$

Then, $i(t) = i_N(t) + i_F(t)$

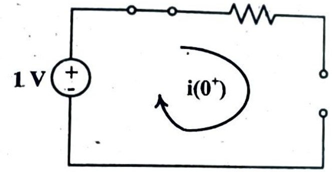
$$i(t) = e^{-t}(K_1 \cos t + K_2 \sin t) \ldots\ldots\ldots\ldots(3)$$

The above equation is the general solution of the differential equation (2) and the values of K_1 and K_2 can be determined using initial conditions.

$$\frac{di(t)}{dt} = -e^{-t}(K_1 \cos t + K_2 \sin t) + e^{-t}(-K_1 \sin t + K_2 \cos t) \ldots\ldots\ldots(4)$$

At $t=0^+$, $v_c(0^+) = -1$ V [Negative sign is introduced because polarity of the initial voltage across capacitor does not conform with the given direction of current]

The equivalent circuit at $t=0^+$ is shown below;



Here, $i(0^+) = 0$

Writing equation (1) at $t=0^+$, we get,

$$0 = 4\,i(0^+) + 1\frac{di}{dt}(0^+) + v_c(0^+)$$

or, $$\frac{di}{dt}(0^+) = 1 \text{ A/s}$$

Writing equations (3) and (4) at $t=0^+$ and substituting the values of $i(0^+)$ and $\frac{di}{dt}(0^+)$, we get,

$$0 = K_1$$

or, $K_1 = 0$

and $1 = -K_1 + K_2$

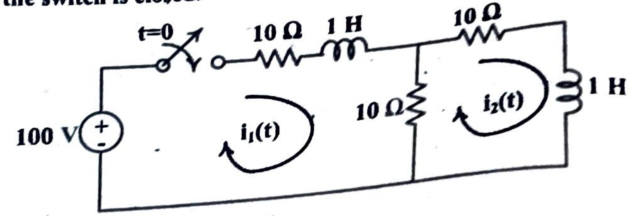
or, $K_2 = 1$

Substituting the values of K_1 and K_2 in general solution gives the complete solution as;

$$i(t) = e^{-t}\sin t \text{ A}$$

Example 2.22

In the network shown in the figure below, the switch is closed at $t = 0$. With the network parameter values shown, find the expression for $i_1(t)$ and $i_2(t)$ if the network is un-energized before the switch is closed.



Solution:

Applying KVL in mesh 1 for t>0,

$$100 = 10\, i_1 + 10\, (i_1 - i_2) + 1\frac{di_1}{dt}$$

or, $$100 = 20\, i_1 - 10\, i_2 + \frac{di_1}{dt} \quad \text{.....................(1)}$$

Applying KVL in mesh 2 for t>0,

$$-10\, (i_1 - i_2) + 10\, i_2 + 1\frac{di_2}{dt} = 0$$

or, $$10\, i_1 = 20\, i_2 + \frac{di_2}{dt}$$

or, $$i_1 = 2\, i_2 + \frac{1}{10}\frac{di_2}{dt} \quad \text{...............(2)}$$

From equations (1) and (2),

$$100 = 20\left(2\, i_2 + \frac{1}{10}\frac{di_2}{dt}\right) - 10\, i_2 + \frac{d}{dt}\left(2\, i_2 + \frac{1}{10}\frac{di_2}{dt}\right)$$

or, $$100 = 40\, i_2 + 2\frac{di_2}{dt} - 10\, i_2 + 2\frac{di_2}{dt} + \frac{1}{10}\frac{d^2 i_2}{dt^2}$$

or, $$100 = \frac{1}{10}\frac{d^2 i_2}{dt^2} + 4\frac{di_2}{dt} + 30\, i_2$$

or, $$\frac{d^2 i_2}{dt^2} + 40\frac{di_2}{dt} + 300\, i_2 = 1000 \quad \text{...........(3)}$$

The equation (3) is second order linear differential equation whose solution can be expressed as;

$$i_2(t) = i_{2N}(t) + i_{2F}(t)$$

where, $$i_{2F}(t) = \frac{1000}{300} = \frac{10}{3}$$

For $i_{2N}(t)$, the auxiliary equation is;

$$s^2 + 40s + 300 = 0$$

or, $s = -10, -30$ (The roots are real and unequal)

So, $$i_{2N}(t) = K_1\, e^{-10t} + K_2\, e^{-30t}$$

Then, $$i_2(t) = K_1\, e^{-10t} + K_2\, e^{-30t} + \frac{10}{3} \quad \text{...........(4)}$$

The equation (4) is the general solution of differential equation (3). For complete solution, the values of K_1 and K_2 are required which can be determined using initial conditions.

$$\frac{di_2(t)}{dt} = -10\, K_1\, e^{-10t} - 30\, K_2\, e^{-30t} \quad \text{...........(5)}$$

Since the circuit is un-energized before switch is closed at $t = 0$,

$$i_1(0^+) = i_1(0^+) = 0$$

Writing equation (2) at $t = 0^+$,

$$i_1(0^+) = 2\, i_2(0^+) + \frac{1}{10}\frac{di_2}{dt}(0^+)$$

or, $$\frac{di_2}{dt}(0^+) = 0$$

Writing equations (4) and (5) at $t=0^+$ and substituting the values of $i_2(0^+)$ and $\frac{di_2}{dt}(0^+)$, we get,

$$0 = K_1 + K_2 + \frac{10}{3}$$

or, $$K_1 + K_2 = -\frac{10}{3} \quad \text{...........(6)}$$

and $$0 = -10\, K_1 - 30\, K_2 \quad \text{...........(7)}$$

Solving equations (6) and (7), we get

$$K_1 = -5 \text{ and } K_2 = \frac{5}{3}$$

Substituting the values of K_1 and K_2 in general solution, we get the complete solution as;

$$i_2(t) = -5\, e^{-10t} + \frac{5}{3}\, e^{-30t} + \frac{10}{3}\ \text{A}$$

Again, from equation (2),

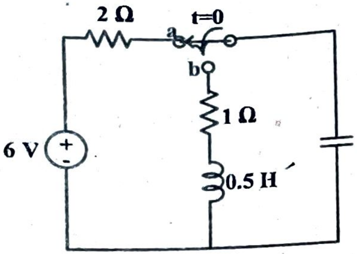
$$i_1(t) = 2\, i_2(t) + \frac{1}{10}\frac{di_2}{dt}$$

or, $$i_1(t) = 2\left(-5\, e^{-10t} + \frac{5}{3}e^{-30t} + \frac{10}{3}\right) + \frac{1}{10}\frac{d}{dt}\left(-5\, e^{-10t} + \frac{5}{3}e^{-30t} + \frac{10}{3}\right)$$

or, $$i_1(t) = -5\, e^{-10t} - \frac{5}{3}\, e^{-30t} + \frac{20}{3}\ \text{A}$$

Example 2.23

The switch in the figure below has been in position 'a' for a long time. Then, it is moved to 'b' at t=0. Obtain the expression for voltage across capacitor for t>0.



Solution:

Let i(t) be the current flowing through the circuit for t>0.

Then, applying KVL in the circuit for t>0, we get,

$$0 = 0.5\frac{di}{dt} + i + v_c$$

Since, $i_c = C\frac{dv_c}{dt} = i$

$$\text{Hence, } 0 = 0.5\left(1\frac{d^2v_c}{dt^2}\right) + 1\frac{dv_c}{dt} + v_c$$

or, $$\frac{d^2v_c}{dt^2} + 2\frac{dv_c}{dt} + 2\,v_c = 0 \;\ldots\ldots\ldots\ldots(1)$$

The above equation is second order linear differential equation whose solution can be expressed as;

$$v_c(t) = v_{cN}(t) + v_{cF}(t)$$

where, $v_{cF}(t) = 0$

For $v_{cN}(t)$:

The auxiliary equation is;

$$s^2 + 2s + 2 = 0$$

or, $s = -1 \pm j$ (The roots are complex conjugates)

So, $v_{cN}(t) = e^{-t}\,(K_1 \cos t + K_2 \sin t)$

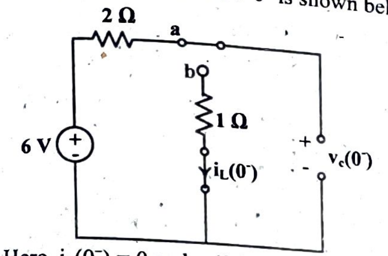
Then, $v_c(t) = v_{cN}(t) + v_{cF}(t)$

or, $v_c(t) = e^{-t}\,(K_1 \cos t + K_2 \sin t) \ldots\ldots\ldots\ldots(2)$

The above equation is the general solution of the differential equation (1) and the values of K_1 and K_2 can be determined using initial conditions.

$$\therefore \frac{dv_c(t)}{dt} = -e^{-t}\,(K_1 \cos t + K_2 \sin t) + e^{-t}\,(-K_1 \sin t + K_2 \cos t) \;\ldots\ldots\ldots(3)$$

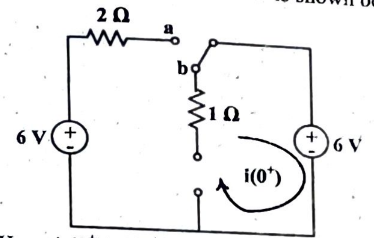
The equivalent circuit at $t=0^-$ is shown below;



Here, $i_L(0^-) = 0$ and $v_c(0^-) = 6$ V;

Hence, at $t=0^+$, $i_L(0^+) = 0$ and $v_c(0^+) = 6$ V

The equivalent circuit at $t=0^+$ is shown below;



Here, $i_c(0^+) = i(0^+) = 0$

or, $1\frac{dv_c}{dt}(0^+) = 0$

or, $\frac{dv_c}{dt}(0^+) = 0$

Writing equations (2) and (3) at $t=0^+$ and substituting the values of $v_c(0^+)$ and $\frac{dv_c}{dt}(0^+)$, we get,

$6 = K_1$

or, $K_1 = 6$

and $0 = -K_1 + K_2$

or, $K_2 = K_1 = 6$

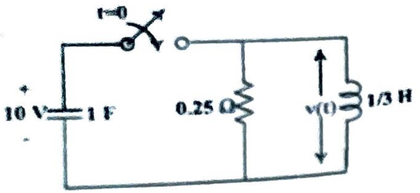
Substituting the values of K_1 and K_2 in general solution gives the complete solution as;

$$v_c(t) = e^{-t}\,(6\cos t + 6 \sin t)$$

or, $$\mathbf{v_c(t) = 6\,e^{-t}\,(\cos t + \sin t)\ V}$$

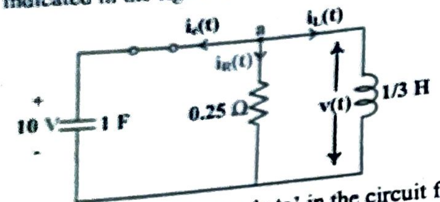
Example 2.24

Figure below shows a parallel circuit where capacitor has an initial voltage of 10 V with polarity indicated in the figure. The switch is closed at t=0. Find v(t) for t>0.



Solution:

Let $i_c(t)$, $i_L(t)$ and $i_R(t)$ be the current flowing through capacitor, inductor and resistor respectivelyfor time t>0 with directions as indicated in the figure below.



Then, applying KCL at node 'a' in the circuit for t>0, we get,

$$i_c(t) + i_R(t) + i_L(t) = 0$$

or, $$C\frac{dv_c(t)}{dt} + \frac{v_R(t)}{R} + \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt = 0$$

But voltages across all the components are equal as they are in parallel,

So, $$1\frac{dv(t)}{dt} + \frac{v(t)}{0.25} + \frac{1}{1/3}\int_{-\infty}^{t} v(t)\,dt = 0$$

Differentiating this equation w.r.t. time, we get

or, $$\frac{d^2v(t)}{dt^2} + 4\frac{dv(t)}{dt} + 3\,v(t) = 0 \;\ldots\ldots\ldots\ldots(1)$$

The above equation is second order linear differential equation whose solution can be expressed as;

$$v(t) = v_N(t) + v_F(t)$$

where, $v_F(t) = 0$

For $v_N(t)$:

The auxiliary equation is;

$$s^2 + 4s + 3 = 0$$

or, $s = -1, -3$ (The roots are real and unequal)

So, $v_N(t) = K_1 e^{-t} + K_2 e^{-3t}$

Then, $v(t) = v_N(t) + v_F(t)$

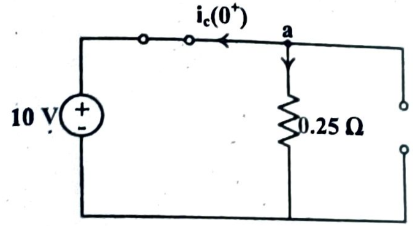
or, $$v(t) = K_1 e^{-t} + K_2 e^{-3t} \;\ldots\ldots\ldots\ldots(2)$$

The above equation is the general solution of the differential equation (1) and the values of K_1 and K_2 can be determined using initial conditions.

$$\frac{dv(t)}{dt} = -K_1 e^{-t} - 3K_2 e^{-3t} \;\ldots\ldots\ldots\ldots(3)$$

At $t=0^+$, $i_L(0^+) = 0$ and $v_c(0^+) = v(0^+) = 10$ V

The equivalent circuit at $t=0^+$ is shown below;



Here, current flowing through the circuit $= \frac{10}{0.25} = 40$ A

and $i_c(0^+) = -40$ A

But $i_c(0^+) = C\frac{dv}{dt}(0^+)$

or, $\frac{dv}{dt}(0^+) = -40$ V/s

Writing equations (2) and (3) at $t=0^+$ and substituting the values of $v(0^+)$ and $\frac{dv}{dt}(0^+)$, we get,

$$10 = K_1 + K_2 \;\ldots\ldots\ldots\ldots(a)$$

and $$-40 = -K_1 - 3K_2 \;\ldots\ldots\ldots\ldots(b)$$

Solving equations (a) and (b), we get,

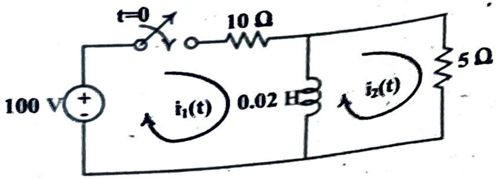
$$K_1 = -5 \text{ and } K_2 = 15$$

Substituting the values of K_1 and K_2 in general solution gives the complete solution as;

$$v(t) = -5\,e^{-t} + 15\,e^{-3t}\,\text{V}$$

Example 2.25

In the two mesh network shown in the figure below, find the currents which result when the switched is closed.



Solution:

Applying KVL in mesh 1 for $t > 0$,

$$100 = 10\, i_1 + 0.02 \frac{d}{dt}(i_1 - i_2)$$

or, $$100 = 10\, i_1 + 0.02 \frac{di_1}{dt} - 0.02 \frac{di_2}{dt} \quad \text{..................(1)}$$

Applying KVL in outer loop for $t > 0$,

$$100 = 10\, i_1 + 5\, i_2$$

or, $$5\, i_2 = 100 - 10\, i_1$$

or, $$i_2 = 20 - 2\, i_1 \quad \text{...........(2)}$$

From equations (1) and (2),

$$100 = 10\, i_1 + 0.02 \frac{di_1}{dt} - 0.02 \frac{d}{dt}(20 - 2\, i_1)$$

or, $$100 = 10\, i_1 + 0.06 \frac{di_1}{dt}$$

or, $$\frac{di_1}{dt} + 166.67\, i_1 = 1666.67 \quad \text{..............(3)}$$

The equation (3) is first order linear differential equation whose solution is given by;

$$i_1(t) = K\, e^{-166.67t} + \frac{1666.67}{166.67}$$

or, $$i_1(t) = K\, e^{-166.67t} + 10 \quad \text{...........(4)}$$

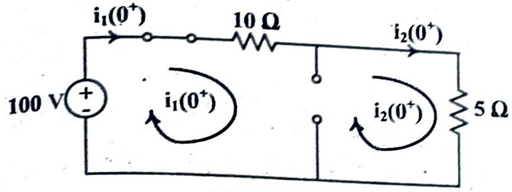
The equation (4) is the general solution of differential equation (3) where the value of K can be determined using initial condition.

Since the circuit is un-energized before the switch is closed,

$$i_L(0^-) = 0$$

Then, $i_L(0^+) = 0$

The equivalent circuit at $t = 0^+$ is as follows;



Here, $i_1(0^+) = i_2(0^+) = \dfrac{100}{10+5} = 6.67$ A

Writing equation (4) at $t = 0^+$ and substituting the value of $i_1(0^+)$, we get

$$6.67 = K + 10$$

or, $$K = -3.33$$

Substituting the value of K in equation (4), we get complete solution as;

$$\mathbf{i_1(t) = -3.33\, e^{-166.67t} + 10 \text{ A}}$$

From equation (2),

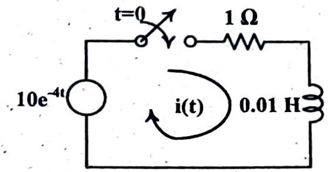
$$i_2(t) = 20 - 2\, i_1(t)$$

or, $$i_2(t) = 20 - 2\,(-3.33\, e^{-166.67t} + 10)$$

or, $$\mathbf{i_2(t) = 6.66\, e^{-1666.67}\, t \text{ A}}$$

Example 2.26

An exponential source $V(t) = 10e^{-4t}$ is suddenly applied at time $t = 0$ to a series R–L circuit comprising $R = 1\ \Omega$ and $L = 0.01$ H. Obtain the expression for the current i(t) in the circuit.



Solution:

Applying KVL in the circuit for $t>0$,

$$10e^{-4t} = 1 \times i(t) + 0.01 \frac{di(t)}{dt}$$

or, $$\frac{di(t)}{dt} + 100\, i(t) = 1000\, e^{-4t} \quad \text{...........(1)}$$

The above equation is first order linear differential equation wh
solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

where, $i_N(t) = Ke^{-100t}$

For $i_F(t)$:

Let the trial solution be $i_F(t) = A\,e^{-4t}$.

Then, replacing i(t) in the differential equation (1) by $i_F(t)$, we get

$$\frac{d}{dt}(A\,e^{-4t}) + 100\,A\,e^{-4t}) = 1000\,e^{-4t}$$

or, $-4\,A\,e^{-4t} + 100A\,e^{-4t} = 1000\,e^{-4t}$

or, $A = 10.41$

So, $i_F(t) = 10.41\,e^{-4t}$

Then, $i(t) = Ke^{-100t} + 10.41\,e^{-4t}$ ....(2)

The above equation is the general solution of differential equation (1)
and the value of K can be determined using initial conditions.

At $t=0^-$, $i_L(0^-) = 0$ [By circuit inspection]

Then, $i_L(0^+) = 0 = i(0^+)$

Writing equation (2) at $t=0^+$ and substituting $i(0^+)$, we get

$$0 = K + 10.41$$

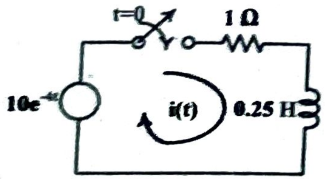
or, $K = -10.41$

Substituting the value of K in general solution, we get

$$\mathbf{i(t) = -10.41e^{-100t} + 10.41\,e^{-4t}\ A}$$

Example 2.27

An exponential source $V(t) = 10e^{-4t}$ is suddenly applied at time $t = 0$ to a series R–L circuit comprising $R = 1\ \Omega$ and $L = 0.25$ H. Obtain the expression for the current i(t) in the circuit.



Solution:

Applying KVL in the circuit for t>0,

$$10\,e^{-4t} = 1 \times i(t) + 0.25\frac{di(t)}{dt}$$

or, $$\frac{di(t)}{dt} + 4\,i(t) = 40\,e^{-4t} \quad ....(1)$$

The above equation is first order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

where, $i_N(t) = Ke^{-4t}$

For $i_F(t)$:

Let the trial solution be $i_F(t) = A\,t\,e^{-4t}$

Then, replacing i(t) in the differential equation (1) by $i_F(t)$, we get

$$\frac{d}{dt}(A\,t\,e^{-4t}) + 4(A\,t\,e^{-4t}) = 40\,e^{-4t}$$

or, $-4\,A\,t\,e^{-4t} + A\,e^{-4t} + 4A\,t\,e^{-4t} = 40\,e^{-4t}$

or, $A = 40$

So, $i_F(t) = 40\,te^{-4t}$

Then, $i(t) = K\,e^{-4t} + 40t\,e^{-4t}$ ....(2)

The above equation is the general solution of differential equation (1) and the value of K can be determined using initial conditions.

At $t=0^-$, $i_L(0^-) = 0$ [By circuit inspection]

Then, $i_L(0^+) = 0 = i(0^+)$

Writing equation (2) at $t=0^+$ and substituting $i(0^+)$, we get

$$0 = K + 0$$

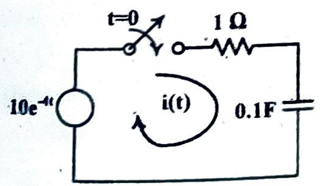
or, $K = 0$

Substituting the value of K in general solution, we get

$$\mathbf{i(t) = 40\,te^{-4t}A}$$

Example 2.28

An exponential source $V(t) = 10e^{-4t}$ is suddenly applied at time $t = 0$ to a series R–C circuit comprising $R = 1\ \Omega$ and $C = 0.1$F. Obtain the expression for the current i(t) in the circuit.



Solution:

Applying KVL in the circuit for t>0,

$$10\,e^{-4t} = 1\times i(t) + \frac{1}{0.1}\int_{-\infty}^{t} i(t)\,dt$$

Differentiating above equation w.r.t. time,

$$-40\,e^{-4t} = 1\times\frac{di(t)}{dt} + 10\times i(t)$$

or, $$\frac{di(t)}{dt} + 10\,i(t) = -40\,e^{-4t} \quad\text{.............(1)}$$

The above equation is first order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

where, $i_N(t) = Ke^{-10t}$

For $i_F(t)$:

Let the trial solution be $i_F(t) = A\,e^{-4t}$.

Then, replacing i(t) in the differential equation (1) by $i_F(t)$, we get

$$\frac{d}{dt}(A\,e^{-4t}) + 10\,(A\,e^{-4t}) = -40\,e^{-4t}$$

or, $-4\,A\,e^{-4t} + 10A\,e^{-4t} = -40\,e^{-4t}$

or, $A = -6.67$

So, $i_F(t) = -6.67\,e^{-4t}$

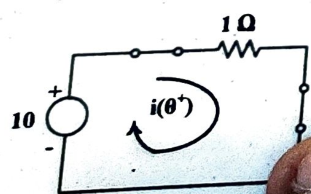
Then, $i(t) = K\,e^{-10t} - 6.67\,e^{-4t}$ ....(2)

The above equation is the general solution of differential equation (1) and the value of K can be determined using initial conditions.

At $t=0^-$, $v_c(0^-) = 0$ [By circuit inspection]

Then, $v_c(0^+) = 0$

The equivalent circuit at $t = 0^+$ is as follows;



Here, $i(0^+) = 10$ A

Writing equation (2) at $t=0^+$ and substituting $i(0^+)$, we get

$$10 = K - 6.67$$

or, $$K = 16.67$$

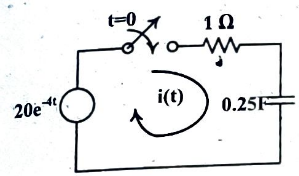
Substituting the value of K in general solution, we get

$$\mathbf{i(t) = 16.67\,e^{-10t} - 6.67\,e^{-4t}\ A}$$

Example 2.29

An exponential voltage $V(t) = 20e^{-4t}$ is suddenly applied at time $t = 0$ to a series R–C circuit comprising $R = 1\ \Omega$ and $C = 0.25$F. Obtain the particular solution for the current i(t) in the circuit.

[2068 Shrawan]



Solution:

Applying KVL in the circuit for t>0,

$$20\,e^{-4t} = 1\times i(t) + \frac{1}{0.25}\int_{-\infty}^{t} i(t)\,dt$$

Differentiating above equation w.r.t. time,

$$-80\,e^{-4t} = 1\times\frac{di(t)}{dt} + 4\times i(t)$$

or, $$\frac{di(t)}{dt} + 4\,i(t) = -80\,e^{-4t} \quad\text{............(1)}$$

The above equation is first order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

where, $i_N(t) = Ke^{-4t}$

For $i_F(t)$:

Let the trial solution be $i_F(t) = A\,t\,e^{-4t}$.

Then, replacing i(t) in the differential equation (1) by $i_F(t)$, we get

$$\frac{d}{dt}\left(A\,t\,e^{-4t}\right) + 4\left(A\,t\,e^{-4t}\right) = -80\,e^{-4t}$$

or, $\quad -4\,A\,t\,e^{-4t} + A\,e^{-4t} + 4A\,t\,e^{-4t} = -80\,e^{-4t}$

or, $\quad A = -80$

So, $\quad i_F(t) = -80\,t\,e^{-4t}$

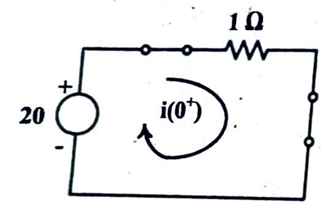
Then, $i(t) = K\,e^{-4t} - 80\,t\,e^{-4t}$ ....(2)

The above equation is the general solution of differential equation (1) and the value of K can be determined using initial conditions.

At $t=0^-$, $v_c(0^-) = 0$ [By circuit inspection]

Then, $v_c(0^+) = 0$

The equivalent circuit at $t = 0^+$ is as follows;



Here, $i(0^+) = 20$ A

Writing equation (2) at $t=0^+$ and substituting $i(0^+)$, we get

$$20 = K$$

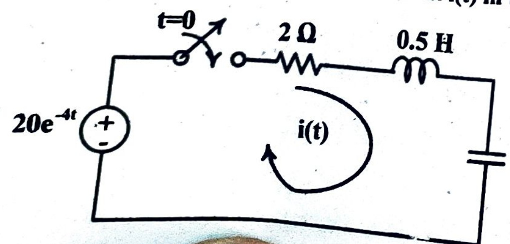
or, $\quad K = 20$

Substituting the value of K in general solution, we get

$$i(t) = 20\,e^{-4t} - 80\,t\,e^{-4t}\ \text{A}$$

Example 2.30

An exponential source $V(t) = 20e^{-4t}$ is suddenly applied at time $t = 0$ to a series RLC circuit comprising $R = 2\ \Omega$, $L=0.5$ H and $C=1$F. Obtain the expression for the current i(t) in the circuit.



Solution:

Applying KVL in the circuit for $t > 0$, we get,

$$20\,e^{-4t} = 2\,i(t) + 0.5\frac{di(t)}{dt} + v_c(t) \quad ....(1)$$

$$20\,e^{-4t} = 2\,i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{1}\int_{-\infty}^{t} i(t)\,dt$$

Differentiating this equation w.r.t. time, we get

$$-80\,e^{-4t} = 2\frac{di(t)}{dt} + 0.5\frac{d^2i(t)}{dt^2} + i(t)$$

or, $$\frac{d^2i(t)}{dt^2} + 4\frac{di(t)}{dt} + 2\,i(t) = -160\,e^{-4t} \quad ....(2)$$

The above equation is second order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

For $i_N(t)$:

The auxiliary equation is;

$$s^2 + 4s + 2 = 0$$

or, $\quad s = -0.586, -3.414$ (The roots are real and unequal)

So, $\quad i_N(t) = K_1\,e^{-0.586t} + K_2\,e^{-3.414t}$

For $i_F(t)$:

Let the trial solution be $i_F(t) = A\,e^{-4t}$

Then, replacing i(t) in the differential equation (2) by $i_F(t)$, we get

$$\frac{d^2}{dt^2}(A\,e^{-4t}) + 4\frac{d}{dt}(A\,e^{-4t}) + 2(A\,e^{-4t}) = -160\,e^{-4t}$$

or, $\quad (-4)^2\,A\,e^{-4t} + 4\,(-4)\,A\,e^{-4t} + 2\,A\,e^{-4t} = -160\,e^{-4t}$

or, $\quad A = -80$

So, $\quad i_F(t) = -80\,e^{-4t}$

Then, $i(t) = i_N(t) + i_F(t)$ ....(3)

or, $i(t) = K_1\,e^{-0.586t} + K_2\,e^{-3.414t} - 80\,e^{-4t}$

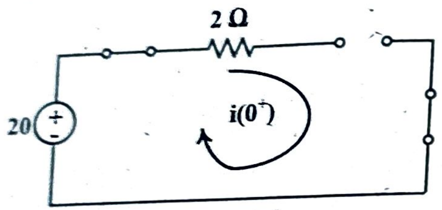
The above equation is the general solution of the differential equ... (2) and the values of K_1 and K_2 can be determined using ... conditions.

$$\frac{di(t)}{dt} = -0.586\, K_1\, e^{-0.586t} - 3.414\, K_2\, e^{-3.414t} + 320\, e^{-4t}$$

At $t=0^-$, $i_L(0^-) = 0$ and $v_c(0^-) = 0$

Then, at $t=0^+$, $i_L(0^+) = 0$ and $v_c(0^+) = 0$

The equivalent circuit at $t=0^+$ is shown below;



Here, $i(0^+) = 0$

Writing equation (1) at $t=0^+$, we get,

$$20 = 2\, i(0^+) + 0.5\frac{di}{dt}(0^+) + v_c(0^+)$$

or, $\frac{di}{dt}(0^+) = 40$ A/s

Writing equations (3) and (4) at $t=0^+$ and substituting the values ... $i(0^+)$ and $\frac{di}{dt}(0^+)$, we get,

$$0 = K_1 + K_2 - 80$$

or, $K_1 + K_2 = 80$ ....(a)

and $40 = -0.586\, K_1 - 3.414\, K_2 + 320$

or, $0.586\, K_1 + 3.414\, K_2 = 280$ ....(b)

Solving equations (a) and (b), we get,

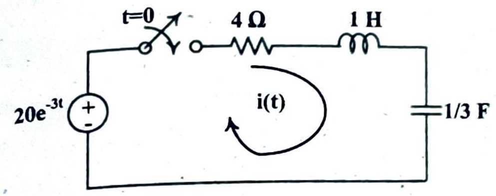
$K_1 = -2.43$ and $K_2 = 82.43$

Substituting the values of K_1 and K_2 in general solution gives ... complete solution as;

$$i(t) = -2.43\, e^{-0.586t} + 82.43\, e^{-3.414t} - 80\, e^{-4t} \text{ A}$$

Example 2.31

An exponential source $V(t) = 20e^{-3t}$ is suddenly applied at time $t = 0$ to a series RLC circuit comprising $R = 4\,\Omega$, $L = 1$ H and $C = 1/3$ F. Obtain the expression for the current i(t) in the circuit.



Solution:

Applying KVL in the circuit for $t>0$, we get,

$$20\, e^{-3t} = 4\, i(t) + 1\frac{di(t)}{dt} + v_c(t) \text{ ....(1)}$$

$$20\, e^{-3t} = 4\, i(t) + 1\frac{di(t)}{dt} + \frac{1}{1/3}\int_{-\infty}^{t} i(t)\, dt$$

Differentiating this equation w.r.t. time, we get

$$-60\, e^{-3t} = 4\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} + 3\, i(t)$$

or, $$\frac{d^2i(t)}{dt^2} + 4\frac{di(t)}{dt} + 3\, i(t) = -60\, e^{-3t} \text{ ....(2)}$$

The above equation is second order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

For $i_N(t)$:

The auxiliary equation is;

$$s^2 + 4s + 3 = 0$$

or, $s = -1, -3$ (The roots are real and unequal)

So, $i_N(t) = K_1\, e^{-t} + K_2\, e^{-3t}$

For $i_F(t)$:

Let the trial solution be $i_F(t) = A\,t\,e^{-3t}$

Then, replacing i(t) in the differential equation (2) by $i_F(t)$, we get

$$\frac{d^2}{dt^2}(A\,t\,e^{-3t}) + 4\frac{d}{dt}(A\,t\,e^{-3t}) + 3(A\,t\,e^{-3t}) = -60\,e^{-3t}$$

or, $$\frac{d}{dt}(A\,e^{-3t} - 3\,A\,t\,e^{-3t}) + 4\,A\,e^{-3t} - 12\,A\,t\,e^{-3t} + 3\,A\,t\,e^{-3t} = -60\,e^{-3t}$$

or, $$-3Ae^{-3t} - 3Ae^{-3t} + 9A\,te^{-3t} + 4Ae^{-3t} - 12Ate^{-3t} + 3Ate^{-3t} = -60\,e^{-3t}$$

or, $A = 30$

So, $i_F(t) = 30\,t\,e^{-3t}$

Then, $i(t) = i_N(t) + i_F(t)$

or, $$i(t) = K_1\,e^{-t} + K_2\,e^{-3t} + 30\,t\,e^{-3t} \quad \text{...........(3)}$$

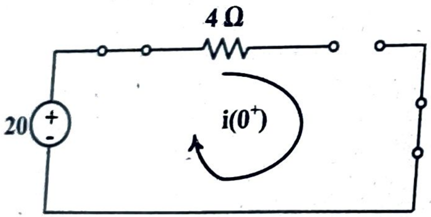
The above equation is the general solution of the differential equation (2) and the values of K_1 and K_2 can be determined using initial conditions.

$$\frac{di(t)}{dt} = -K_1\,e^{-t} - 3\,K_2\,e^{-3t} + 30\,e^{-3t} - 90\,t\,e^{-3t} \quad \text{...........(4)}$$

At $t=0^-$, $i_L(0^-) = 0$ and $v_c(0^-) = 0$

Then, at $t=0^+$, $i_L(0^+) = 0$ and $v_c(0^+) = 0$

The equivalent circuit at $t=0^+$ is shown below;



Here, $i(0^-)= 0$

Writing equation (1) at $t=0^+$, we get,

$$20 = 4\,i(0^+) + 1\,\frac{di}{dt}(0^+) + v_c(0^+)$$

or, $$\frac{di}{dt}(0^+) = 20 \text{ A/s}$$

Writing equations (3) and (4) at $t=0^+$ and substituting the values of $i(0^+)$ and $\frac{di}{dt}(0^+)$, we get,

$$0 = K_1 + K_2 \quad \text{...........(a)}$$

and $$20 = -K_1 - 3\,K_2 + 30$$

or, $$K_1 + 3\,K_2 = 10 \quad \text{...........(b)}$$

Solving equations (a) and (b), we get,

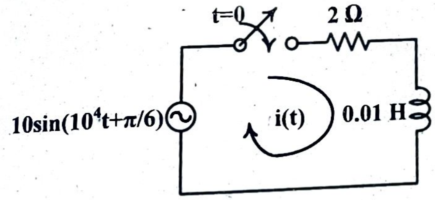
$$K_1 = -5 \text{ and } K_2 = 5$$

Substituting the values of K_1 and K_2 in general solution gives the complete solution as;

$$i(t) = -5\,e^{-t} + 5\,e^{-3t} + 30\,t\,e^{-3t} \text{ A}$$

Example 2.32

A sinusoidal source $V(t) = 10 \sin\left(10^4 t + \frac{\pi}{6}\right)$ is suddenly applied at time t = 0 to a series RL circuit comprising R = 2 Ω and L= 0.01 H. Obtain the expression for the current i(t) in the circuit.



Solution:

Applying KVL in the circuit for t>0,

$$10 \sin\left(10^4 t + \frac{\pi}{6}\right) = 2\times i(t) + 0.01\frac{di(t)}{dt}$$

or, $$\frac{di(t)}{dt} + 200\,i(t) = 1000 \sin\left(10^4 t + \frac{\pi}{6}\right) \quad \text{...........(1)}$$

The above equation is first order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

where, $i_N(t) = Ke^{-200t}$

For $i_F(t)$:

Let the trial solution be $i_F(t) = A\cos\left(10^4 t + \frac{\pi}{6}\right) + B\sin\left(10^4 t + \frac{\pi}{6}\right)$

Then, replacing $i(t)$ in the differential equation (1) by $i_F(t)$, we get

$$\frac{d}{dt}\left(A\cos\left(10^4 t+\frac{\pi}{6}\right)+B\sin\left(10^4 t+\frac{\pi}{6}\right)\right)$$

$$+ 200\left(A\cos\left(10^4 t+\frac{\pi}{6}\right)+B\sin\left(10^4 t+\frac{\pi}{6}\right)\right) = 1000\sin\left(10^4 t+\frac{\pi}{6}\right)$$

or, $$-10^4 A\sin\left(10^4 t+\frac{\pi}{6}\right) + 10^4 B\cos\left(10^4 t+\frac{\pi}{6}\right) + 200A\cos\left(10^4 t+\frac{\pi}{6}\right)$$

$$+ 200B\sin\left(10^4 t+\frac{\pi}{6}\right) = 1000\sin\left(10^4 t+\frac{\pi}{6}\right)$$

Equating the coefficients of sin terms,

$$-10^4 A + 200\,B = 1000 \quad \text{.....................(a)}$$

Equating the coefficients of cos terms,

$$200\,A + 10^4 B = 0 \quad \text{.......................(b)}$$

Solving equations (a) and (b),

$A = -0.1$ and $B = 0.002$

So, $i_F(t) = -0.1\cos\left(10^4 t + \frac{\pi}{6}\right) + 0.002\sin\left(10^4 t + \frac{\pi}{6}\right)$

Then,

$$i(t) = K e^{-200t} - 0.1\cos\left(10^4 t + \frac{\pi}{6}\right) + 0.002\sin\left(10^4 t + \frac{\pi}{6}\right) \quad \text{.........(2)}$$

The above equation is the general solution of differential equation (1) and the value of K can be determined using initial conditions.

At $t=0^-$, $i_L(0^-) = 0$ [By circuit inspection]

Then, $i_L(0^+) = 0 = i(0^+)$

Writing equation (2) at $t=0^+$ and substituting $i(0^+)$, we get

$$0 = K - 0.1\cos\left(\frac{\pi}{6}\right) + 0.002\sin\left(\frac{\pi}{6}\right)$$

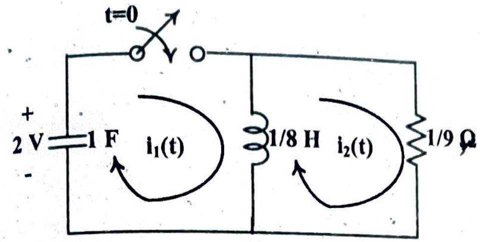
or, $K = 0.0856$

Substituting the value of K in general solution, we get

$$\mathbf{i(t) = 0.0856\, e^{-200t} - 0.1\cos\left(10^4 t + \frac{\pi}{6}\right) + 0.002\sin\left(10^4 t + \frac{\pi}{6}\right) A}$$

Example 2.33

For the circuit shown in the figure below, find the mesh currents i_1 and i_2 for $t > 0$.



Solution:

Applying KVL in outer loop for $t > 0$,

$$\frac{1}{1}\int_{-\infty}^{t} i_1(t)\,dt + \frac{1}{9} i_2(t) = 0$$

Differentiating above equation w.r.t. time,

$$i_1(t) + \frac{1}{9}\frac{di_2(t)}{dt} = 0$$

or, $$i_1(t) = -\frac{1}{9}\frac{di_2(t)}{dt} \quad \text{...........(1)}$$

Applying KVL in right mesh for $t > 0$,

$$\frac{1}{8}\frac{d}{dt}(i_2(t) - i_1(t)) + \frac{1}{9} i_2(t) = 0$$

or, $$\frac{1}{8}\frac{di_2(t)}{dt} - \frac{1}{8}\frac{di_1(t)}{dt} + \frac{1}{9} i_2(t) = 0 \quad \text{..........(2)}$$

From equations (1) and (2),

$$\frac{1}{8}\frac{di_2(t)}{dt} - \frac{1}{8}\frac{d}{dt}\left(\frac{-1}{9}\frac{di_2(t)}{dt}\right) + \frac{1}{9} i_2(t) = 0$$

or, $$\frac{1}{72}\frac{d^2 i_2}{dt^2} + \frac{1}{8}\frac{di_2}{dt} + \frac{1}{9} i_2 = 0$$

or, $$\frac{d^2 i_2}{dt^2} + 9\frac{di_2}{dt} + 8\, i_2 = 0 \quad \text{..............(3)}$$

The above equation (3) is second order linear differential equation whose solution can be expressed as;

$$i_2(t) = i_{2N}(t) + i_{2F}(t)$$

where, $i_{2F}(t) = 0$

For $i_{2N}(t)$:

The auxiliary equation is;

$$s^2 + 9s + 8 = 0$$

or, $s = -1, -8$ (Roots are real and unequal)

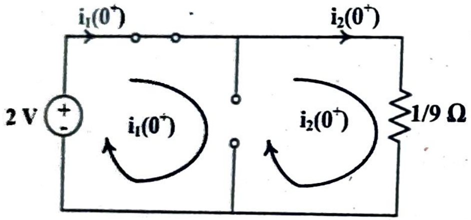
So, $i_{2N}(t) = K_1 e^{-t} + K_2 e^{-8t}$

Then, $i_2(t) = K_1 e^{-t} + K_2 e^{-8t}$ ....(4)

The equation (4) is the general solution of the differential equation (3) where values of K_1 and K_2 can be determined using initial conditions.

$$\frac{di_2(t)}{dt} = -K_1 e^{-t} - 8K_2 e^{-8t} \quad ....(5)$$

The equivalent circuit at $t = 0^+$ is;



Here, $v_c(0^+) = -2$ V

and $i_1(0^+) = i_2(0^+) = \frac{2}{1/9} = 18$ A

Writing equation (1) at $t = 0^+$,

$$i_1(0^+) = -\frac{1}{9}\frac{di_2}{dt}(0^+)$$

or, $\frac{di_2}{dt}(0^+) = -162$ A/s

Writing equations (4) and (5) at $t = 0^+$ and substituting values of $i_2(0^+)$ and $\frac{di_2}{dt}(0^+)$, we get,

$$18 = K_1 + K_2 \quad ....(a)$$

$$-162 = -K_1 - 8K_2 \quad ....(b)$$

Solving equations (a) and (b),

$$K_1 = -2.57 \text{ and } K_2 = 20.57$$

Substituting values of K_1 and K_2 in equation (4), we get complete solution as;

$$\mathbf{i_2(t) = -2.57\ e^{-t} + 20.57\ e^{-8t}\ A}$$

Again, from equation (1),

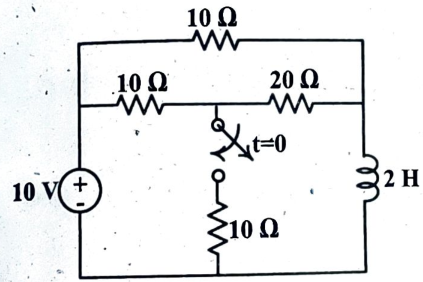
$$i_1(t) = -\frac{1}{9}\frac{di_2(t)}{dt}$$

or, $i_1(t) = -\frac{1}{9}\frac{d}{dt}\left(-2.57\ e^{-t} + 20.57\ e^{-8t}\right)$

or, $\mathbf{i_1(t) = -0.28\ e^{-t} + 18.28\ e^{-8t}\ A}$

Example 2.32

For the circuit shown in the figure below, find the current through the inductor for t>0. **[2068 Chaitra]**



Solution:

Applying KVL in mesh 1 for t>0,

$$10 = 10(i_1 - i_2) + 10(i_1 - i_3)$$

or, $10 = 20i_1 - 10i_2 - 10i_3$ ....(1)

Applying KVL in mesh 2 for t>0,

$$10(i_2 - i_1) + 20(i_2 - i_3) + 10i_2 = 0$$

or, $-10i_1 + 40i_2 - 20i_3 = 0$ ....(2)

Eliminating i_1 from equations (1) and (2), we get,

$$i_2 = \frac{10 + 50i_3}{70} \quad ....(3)$$

Eliminating i_2 from equations (1) and (2), we get,

$$i_1 = \frac{4 + 6i_3}{7} \quad ....(4)$$

Applying KVL in mesh 3 for t>0,

$$10(i_3-i_1)+20(i_3-i_2)+2\frac{di_3}{dt}=0$$

or, $$-10i_1-20i_2+30i_3+2\frac{di_3}{dt}=0 \quad \text{.......................(5)}$$

Substituting expressions of i_1 and i_2 from equations (4) and (3) to equation (5).

$$-10\left(\frac{4+6i_3}{7}\right)-20\left(\frac{10+50i_3}{70}\right)+30i_3+2\frac{di_3}{dt}=0$$

or, $$\frac{di_3}{dt}+\frac{25}{7}i_3=\frac{30}{7} \quad \text{..............(6)}$$

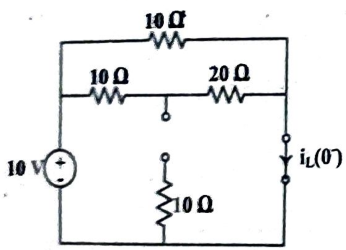
The equation (6) is first order linear differential equation whose solution is given by,

$$i_3(t)=Ke^{-\frac{25}{7}t}+\frac{30/7}{25/7}$$

or, $$i_3(t)=Ke^{-\frac{25}{7}t}+1.2 \quad \text{............(7)}$$

The above equation is the general solution and the value of K can be determined using initial conditions.

At $t=0^-$, the equivalent circuit is;



$R_{eq}=10//(10+20)=7.5\ \Omega$

$$i_L(0^-)=\frac{10}{7.5}=\frac{4}{3}\text{ A}$$

$$i_L(0^+)=\frac{4}{3}\text{ A}=i_3(0^+)$$

Writing equation (7) at $t=0^+$ and substituting $i_3(0^+)$, we get

$$\frac{4}{3}=K+1.2$$

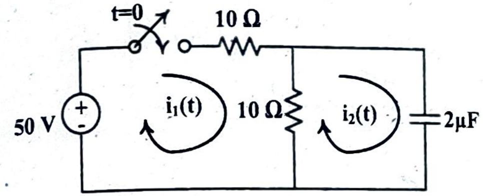
or, $$K=\frac{2}{15}$$

Substituting the value of K in general solution, we get

$$i_3(t)=\frac{2}{15}e^{-\frac{25}{7}t}+1.2\ \text{A}$$

Example 2.33

In the two mesh network shown in the figure below, the switch is closed at t = 0. Find the mesh currents $i_1(t)$ and $i_2(t)$ as shown and the capacitor voltage $v_c(t)$. **[2068 Shrawan]**



Solution:

Applying KVL in mesh 1 for t>0,

$$50=10i_1+10(i_1-i_2)$$

or, $$50=20i_1-10i_2$$

or, $$i_1=\frac{5+i_2}{2} \quad \text{...........(1)}$$

Applying KVL in mesh 2 for t>0,

$$10(i_2-i_1)+\frac{1}{2\times10^{-6}}\int i_2\,dt=0$$

Differentiating above equation w.r.t time,

$$10\frac{di_2}{dt}-10\frac{di_1}{dt}+\frac{1}{2\times10^{-6}}i_2=0 \quad \text{.....................(2)}$$

Substituting expression of i_1 from equation (1) to (2),

$$10\frac{di_2}{dt}-10\times\frac{1}{2}\frac{di_2}{dt}+5\times10^5 i_2=0$$

or, $$5\frac{di_2}{dt}+5\times10^5 i_2=0$$

or, $$\frac{di_2}{dt}+10^5 i_2=0 \quad \text{...........(3)}$$

The above equation is first order linear differential equation whose solution can be expressed as;

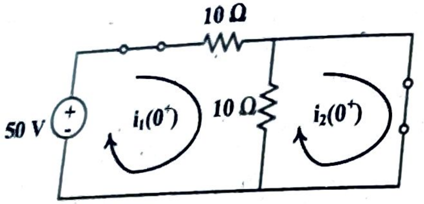
$$i_2(t)=Ke^{-10^5t} \quad \text{...........(4)}$$

The above equation gives the general solution of differential equation (3). The value of K can be determined using initial conditions.

At $t=0^-$, $v_c(0^-) = 0$ [By circuit inspection]

Then, at $t=0^+$, $v_c(0^+) = 0$

The equivalent circuit at $t=0^+$ is shown below;



$$\text{Here, } i_1(0^+) = i_2(0^+) = \frac{50}{10} = 5\ A$$

Writing equation (4) at $t=0^+$ and substituting the value of $i_2(0^+)$, we get,

$$5 = K$$

or, $K = 5$

Substituting the value of K in general solution gives the complete solution as;

$$i_2(t) = 5\,e^{-10^5 t}\ A$$

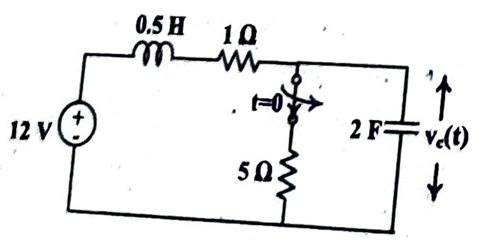
Substituting the expression of i_2 in equation (1), we get,

$$i_1(t) = \frac{5}{2} + \frac{1}{2} \times 5\,e^{-10^5 t}$$

or, $$i_1(t) = \frac{5}{2}\left(1 + e^{-10^5 t}\right)\ A$$

Example 2.34

In the following network, the switch was closed for a long time before it is being opened at t=0. Find the expression for $v_c(t)$ for $t>0$. [2068 Chaitra]



Solution:

Let i(t) be the current flowing through the circuit for $t>0$.

Then, applying KVL in the circuit for $t>0$, we get,

$$12 = 0.5\frac{di}{dt} + i + v_c$$

$$\text{Since, } i_c = C\frac{dv_c}{dt} = i$$

$$\text{Hence, } 12 = 0.5\left(2\frac{d^2v_c}{dt^2}\right) + 2\frac{dv_c}{dt} + v_c$$

or, $$\frac{d^2v_c}{dt^2} + 2\frac{dv_c}{dt} + v_c = 12 \text{ ....(1)}$$

The above equation is second order linear differential equation whose solution can be expressed as;

$$v_c(t) = v_{cN}(t) + v_{cF}(t)$$

$$\text{where, } v_{cF}(t) = \frac{12}{1} = 12$$

For $v_{cN}(t)$:

The auxiliary equation is;

$$s^2 + 2s + 1 = 0$$

or, $s = -1, -1$ (The roots are real and repeated)

So, $v_{cN}(t) = (K_1 + K_2 t)\,e^{-t}$

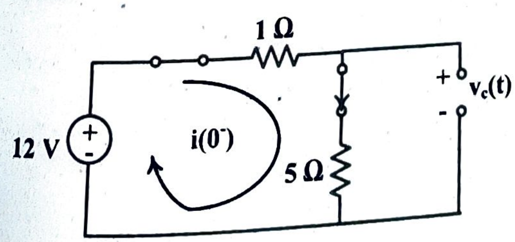
Then, $v_c(t) = v_{cN}(t) + v_{cF}(t)$

or, $v_c(t) = (K_1 + K_2 t)\,e^{-t} + 12$ ....(2)

The above equation is the general solution of the differential equation (1) and the values of K_1 and K_2 can be determined using initial conditions.

$$\frac{dv_c(t)}{dt} = -K_1 e^{-t} - K_2 t e^{-t} + K_2 e^{-t} \text{ ....(3)}$$

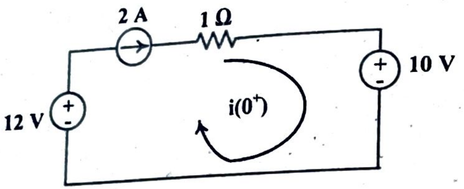
The equivalent circuit at $t=0^-$ is shown below;



Here, $i(0^-) = i_L(0^-) = 2$ A and $v_c(0^-) = 5\times2 = 10$ V

Hence, at $t=0^+$, $i_L(0^+) = 2$ A and $v_c(0^+) = 10$ V

The equivalent circuit at $t=0^+$ is shown below;



Here, $i_c(0^+)= 2$ A

or, $2\frac{dv_c}{dt}(0^+) = 2$

or, $\frac{dv_c}{dt}(0^+) = 1$ V/s

Writing equations (2) and (3) at $t=0^+$ and substituting the values of $v_c(0^+)$ and $\frac{dv_c}{dt}(0^+)$, we get,

$$10 = K_1 + 12$$

or, $K_1 = -2$

and $1 = -K_1 + K_2$

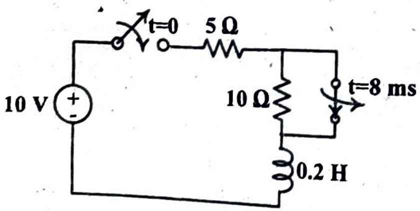
or, $K_2 = 1 + K_1 = -1$

Substituting the values of K_1 and K_2 in general solution gives the complete solution as;

$$v_c(t) = (-2 - t)\, e^{-t} + 12 \text{ V}$$

Example 2.35

In the given circuit below, switch S_1 is closed at $t = 0$ and after 8 ms, the switch S_2 is opened. Find the complete expression for current in the interval $0 < t < 8$ ms and $t > 8$ ms.



Solution:

Let i(t) be the current flowing through the circuit at any time t.

Applying KVL in the circuit for $0 < t < 8$ ms,

$$10 = 5\, i(t) + 0.2\frac{di(t)}{dt}$$

or, $\frac{di(t)}{dt} + 25\, i(t) = 50$ ....(1)

The above equation is first order linear differential equation whose solution is given by;

$$i(t) = K\, e^{-25t} + \frac{50}{25}$$

or, $i(t) = K\, e^{-25t} + 2$ ....(2)

The equation (2) is the general solution of differential equation (1) in which the value of K can be determined using initial conditions.

At $t = 0^-$, $i_L(0^-) = 0$. [By circuit inspection]

Then, at $t = 0^+$, $i_L(0^+) = 0 = i(0^+)$

Writing equation (2) at $t = 0^+$ and substituting the value of $i(0^+)$, we get

$$0 = K + 2$$

or, $K = -2$

Substituting value of K in equation (2), we get the complete solution as;

$$i(t) = -2\, e^{-25t} + 2$$

or, $\mathbf{i(t) = 2(1 - e^{-25t})\ A}$

At t = 8 ms,

$$i(8 \text{ ms}) = 0.3625 \text{ A}$$

Again, applying KVL in the circuit for t > 8 ms,

$$10 = 15\, i(t) + 0.2\frac{di(t)}{dt}$$

or, $\frac{di(t)}{dt} + 75\, i(t) = 50$ ....(3)

The above equation is first order linear differential equation whose solution is given by;

$$i(t) = K_1\, e^{-75t} + \frac{50}{75}$$

or, $i(t) = K_1\, e^{-75t} + \frac{2}{3}$ ....(4)

The equation (4) is the general solution of differential equation (3) in which the value of K_1 can be determined using value at t=8 ms.

At t = 8 ms,

$$0.3625 = K_1\, e^{-75\times8\times10^{-3}} + \frac{2}{3}$$

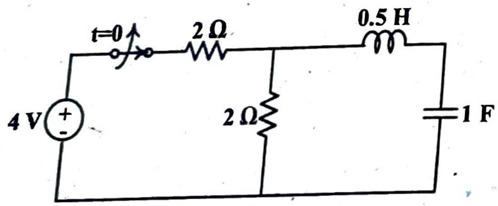
or, $K_1 = -0.554$

Substituting the value of K_1 in equation (4) gives the complete solution as;

$$\mathbf{i(t) = -0.554\, e^{-75t} + \frac{2}{3}\ A}$$

Example 2.36

The circuit shown below is in steady state with switch closed. The switch is opened at t =0. Find the current through inductor for t > 0.



Solution:

Let i(t) be the current flowing through the circuit at any time t.

Then, applying KVL in the circuit for t>0, we get,

$$0 = 2\,i(t) + 0.5\frac{di(t)}{dt} + v_c(t) \quad \ldots\ldots\ldots\ldots(1)$$

$$0 = 2\,i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{1}\int_{-\infty}^{t} i(t)\,dt$$

Differentiating this equation w.r.t. time, we get

$$0 = 2\frac{di(t)}{dt} + 0.5\frac{d^2i(t)}{dt^2} + i(t)$$

or, $$\frac{d^2i(t)}{dt^2} + 4\frac{di(t)}{dt} + 2\,i(t) = 0 \quad \ldots\ldots\ldots\ldots(2)$$

The above equation is second order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

where, $i_F(t) = 0$

For $i_N(t)$:

The auxiliary equation is;

$$s^2 + 4s + 2 = 0$$

or, $s = -0.586, -3.414$ (The roots are real and unequal)

So, $i_N(t) = K_1\, e^{-0.586t} + K_2\, e^{-3.414t}$

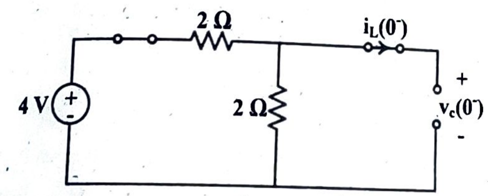
Then, $i(t) = i_N(t) + i_F(t)$

or, $$i(t) = K_1\, e^{-0.586t} + K_2\, e^{-3.414t} \quad \ldots\ldots\ldots\ldots(3)$$

The above equation is the general solution of the differential equation (2) and the values of K_1 and K_2 can be determined using initial conditions.

$$\frac{di(t)}{dt} = -0.586\,K_1\, e^{-0.586t} - 3.414\,K_2\, e^{-3.414t} \quad \ldots\ldots\ldots\ldots(4)$$

At $t=0^-$, the equivalent circuit is as follows;



Here, $i_L(0^-) = 0$

and $v_c(0^-) = 2$ V

Then, at $t=0^+$, $i_L(0^+) = i(0^+) = 0$ and $v_c(0^+) = 2$ V

Writing equation (1) at $t=0^+$, we get,

$$0 = 2\,i(0^+) + 0.5\frac{di}{dt}(0^+) + v_c(0^+)$$

or, $$\frac{di}{dt}(0^+) = -4\ \text{A/s}$$

Writing equations (3) and (4) at $t=0^+$ and substituting the values of $i(0^+)$ and $\frac{di}{dt}(0^+)$, we get,

$$0 = K_1 + K_2 \quad \ldots\ldots\ldots\ldots(a)$$

and $$-4 = -0.586\,K_1 - 3.414\,K_2 \quad \ldots\ldots\ldots\ldots(b)$$

Solving equations (a) and (b), we get,

$$K_1 = -1.414 \text{ and } K_2 = 1.414$$

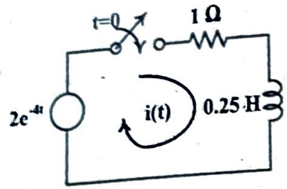
Substituting the values of K_1 and K_2 in general solution gives the complete solution as;

$$\mathbf{i(t) = -1.414\, e^{-0.586t} + 1.414\, e^{-3.414t}\ A}$$

Example 2.37

An exponential voltage $v(t) = 2e^{-4t}$ is applied at time $t = 0$ to a series R–L circuit comprising $R = 1\ \Omega$ and $L = 0.25$ H. Obtain the particular solution for the current i(t) in the circuit.

[2068 Chaitra]



Solution:

Applying KVL in the circuit for t>0,

$$2e^{-4t} = 1 \times i(t) + 0.25\frac{di(t)}{dt}$$

or, $$\frac{di(t)}{dt} + 4\,i(t) = 8e^{-4t} \text{ ....(1)}$$

The above equation is first order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

where, $i_N(t) = Ke^{-4t}$

For $i_F(t)$:

Let the trial solution be $i_F(t) = A\,t\,e^{-4t}$

Then, replacing i(t) in the differential equation (1) by $i_F(t)$, we get

$$\frac{d}{dt}\left(A\,t\,e^{-4t}\right) + 4\left(A\,t\,e^{-4t}\right) = 8e^{-4t}$$

or, $-4A\,t\,e^{-4t} + A\,e^{-4t} + 4A\,t\,e^{-4t} = 8e^{-4t}$

or, $A = 8$

So, $i_F(t) = 8\,t\,e^{-4t}$

Then, $i(t) = K\,e^{-4t} + 8\,t\,e^{-4t}$ ....(2)

The above equation is the general solution of differential equation (1) and the value of K can be determined using initial conditions.

At $t=0^-$, $i_L(0^-) = 0$ [By circuit inspection]

Then, $i_L(0^+) = 0 = i(0^+)$

Writing equation (2) at $t=0^+$ and substituting $i(0^+)$, we get

$$0 = K + 0$$

or, $K = 0$

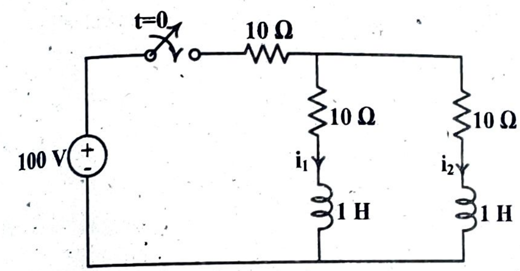
Substituting the value of K in general solution, we get the particular solution for current as;

$$i(t) = 8\,t\,e^{-4t}\ \text{A}$$

Example 2.38

In the network shown, the switch is closed at $t = 0$, with the network previously un-energized. For the element values shown in the diagram, find $i_1(t)$ and $i_2(t)$, by classical method for $t > 0$.

[2075 Ashwin]



Solution:

Since the parallel branches have equal values of impedances,

$$i_1(t) = i_2(t) = i(t)\ \text{(Say)}$$

and hence the source current $= i_1(t) + i_2(t) = 2\,i(t)$

Now, applying KVL in left mesh,

$$100 = 2i(t) \times 10 + i(t) \times 10 + 1 \times \frac{di(t)}{dt}$$

or, $$\frac{di(t)}{dt} + 30\,i(t) = 100 \text{ ....(1)}$$

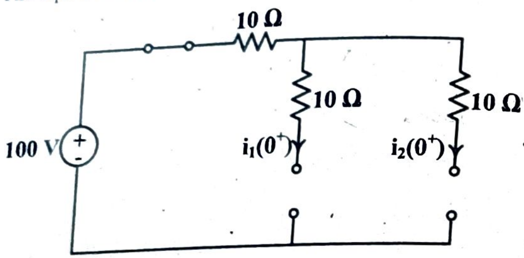
The above equation is first order linear differential equation whose solution can be expressed as;

$$i(t) = K\,e^{-30t} + \frac{100}{30}$$

or, $$i(t) = K\,e^{-30t} + \frac{10}{3} \text{ ....(2)}$$

The above equation (2) is the general solution of differential equation (1) where the value of K can be determined using initial condition.

The equivalent circuit at $t = 0^+$ is as follows;



Here, $i_1(0^+) = i_2(0^+) = i(0^+) = 0$

Writing equation (2) at $t = 0^+$ and substituting the value of $i(0^+)$, we get,

$$0 = K + \frac{10}{3}$$

or, $K = -\frac{10}{3}$

Substituting the value of K in equation (2), we get the complete solution for branch currents;

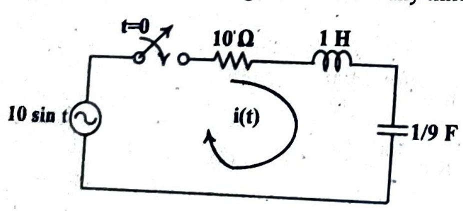
$$i(t) = i_1(t) = i_2(t) = K\,e^{-30t} + \frac{10}{3}\ \text{A}$$

Example 2.39

Find the time expression for current for $t > 0$ in RLC series circuit with $R = 10\ \Omega$, $L = 1$ H and $C = \frac{1}{9}$ F, if the circuit is supplied by $v(t) = 10 \sin t$ at $t = 0$. Assume that the capacitor and inductors are initially de-energized. Use classical method. [2075 Ashwin]

Solution:

Let, i(t) be the current through the circuit at any time t.



Applying KVL in the circuit for $t > 0$,

$$10 \sin t = 10\,i(t) + 1 \times \frac{di(t)}{dt} + v_c(t) \quad \text{............(1)}$$

or, $$10 \sin t = 10\,i(t) + 1 \times \frac{di(t)}{dt} + \frac{1}{1/9}\int_{-\infty}^{t} i(t)\,dt$$

Differentiating the above equation w.r.t. time,

$$10 \cos t = 10\frac{di(t)}{dt} + \frac{d^2i(t)}{dt^2} + 9\,i(t)$$

or, $$\frac{d^2i(t)}{dt^2} + 10\frac{di(t)}{dt} + 9\,i(t) = 10 \cos t \quad \text{...........(2)}$$

The equation (2) is second order linear differential equation whose solution can be expressed as;

$$i(t) = i_N(t) + i_F(t)$$

For $i_N(t)$:

The auxiliary equation is;

$$s^2 + 10s + 9 = 0$$

or, $s = -1, -9$ (The roots are real and unequal)

So, $i_N(t) = K_1\,e^{-t} + K_2\,e^{-9t}$

For $i_F(t)$:

Let the trial solution be $i_F(t) = A \cos t + B \sin t$

Then, replacing i(t) in differential equation (2) by $i_F(t)$, we get

$$\frac{d^2}{dt^2}(A \cos t + B \sin t) + 10\frac{d}{dt}(A \cos t + B \sin t) + 9\,(A \cos t + B \sin t) = 10 \cos t$$

or, $-A \cos t - B \sin t + 10\,(-A \sin t + B \cos t) + 9\,(A \cos t + B \sin t) = 10 \cos t$

or, $-A \cos t - B \sin t - 10A \sin t + 10B \cos t + 9A \cos t + 9B \sin t = 10 \cos t$

or, $(-10A + 8B) \sin t + (8A + 10B) \cos t = 10 \cos t$

Equating the coefficients of sin and cos terms on both sides,

$$-10A + 8B = 0 \quad \text{...............(a)}$$

$$8A + 10B = 10 \quad \text{...............(b)}$$

Solving equations (a) and (b), we get

$$A = 0.49$$

and $\quad B = 0.61$

So, $i_F(t) = 0.49 \cos t + 0.61 \sin t$

Now, $i(t) = i_N(t) + i_F(t)$

or, $\quad i(t) = K_1 e^{-t} + K_2 e^{-9t} + 0.49 \cos t + 0.61 \sin t$ ....(3)

The equation (3) is the general solution of differential equation (2) where the values of K_1 and K_2 can be determined using initial conditions.

$$\frac{di(t)}{dt} = -K_1 e^{-t} - 9K_2 e^{-9t} - 0.49 \sin t + 0.61 \cos t \quad \text{............(4)}$$

Given that inductor and capacitor are initially de-energized,

$$i_L(0^+) = i(0^+) = 0$$

and $\quad v_c(0^+) = 0$

Writing equation (1) at $t = 0^+$, we get

$$10 \sin (0) = 10\, i(0^+) + 1 \times \frac{di}{dt}(0^+) + v_c(0^+)$$

or, $\quad \frac{di}{dt}(0^+) = 0$

Writing equation (3) and (4) at $t = 0^+$ and substituting $i(0^+)$ and $\frac{di}{dt}(0^+)$, we get

$$0 = K_1 + K_2 + 0.49$$

or, $\quad K_1 + K_2 = -0.49$ ....(5)

and $\quad 0 = -K_1 - 9K_2 + 0.61$

or, $\quad K_1 + 9K_2 = 0.61$ ....(6)

Solving equations (5) and (6), we get

$$K_1 = -0.6275$$

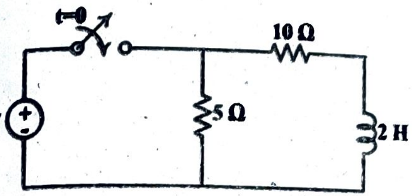
and $\quad K_2 = 0.1375$

Substituting the values of K_1 and K_2 in the general solution, we get the complete solution as follows;

$$i(t) = -0.63\, e^{-t} + 0.14\, e^{-9t} + 0.49 \cos t + 0.61 \sin t \ \text{ A}$$

Example 2.40

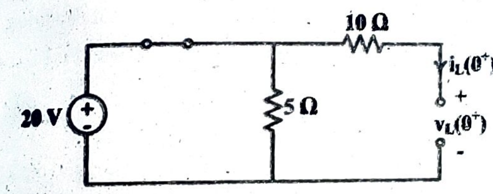
In the circuit shown in figure below, obtain an expression for voltage across the inductor if the switch is closed at t = 0 using classical method.



Solution:

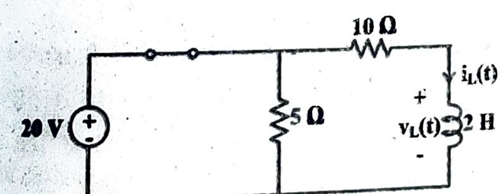
Let $i_L(t)$ and $v_L(t)$ be current through and voltage across inductor at any time t.

The equivalent circuit at $t = 0^+$ is as follows;



Here, $i_L(0^+) = 0$ $\quad$ and $\quad v_L(0^+) = 20$ V

The equivalent circuit for $t > 0$ is as follows;



Applying KVL in the outer loop for $t > 0$,

$$20 = 10\, i_L(t) + v_L(t)$$

But $\quad i_L(t) = \frac{1}{L} \int_{-\infty}^{t} v_L(t)\, dt$

Then, $20 = 10 \times \frac{1}{L} \int_{-\infty}^{t} v_L(t)\,dt + v_L(t)$

or, $20 = 5 \int_{-\infty}^{t} v_L(t)\,dt + v_L(t)$

Differentiating above equation w.r.t. time,

$$0 = 5\, v_L(t) + \frac{dv_L(t)}{dt}$$

or, $\frac{dv_L(t)}{dt} + 5\, v_L(t) = 0$ ....(1)

The equation (1) is first order linear differential equation whose solution is given by;

$$v_L(t) = K\, e^{-5t} \quad \text{...........(2)}$$

The equation (2) is the general solution of differential equation (1) where the value of K can be determined using initial condition.

Writing equation (2) at $t = 0^+$ and substituting value of $v_L(0^+)$, we get,

$$20 = K$$

or, $K = 20$

Substituting the value of K in equation (2), we get the complete solution as;

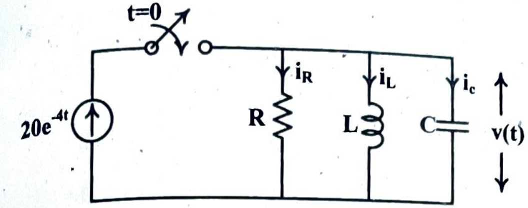
$$\mathbf{v_L(t) = 20\, e^{-5t}\ V}$$

Example 2.41

An exponential current $i(t) = 20\, e^{-4t}$ A is suddenly applied at time $t = 0$ to a parallel RLC circuit comprising of resistor $R = \frac{1}{10}\,\Omega$, inductor $L = 10$ mH and capacitor $C = 2.5\ \mu F$. Obtain the complete particular solution for voltage v(t) across the network, by Classical method. Assume zero initial current through the inductor and zero initial charge across the capacitor before application of the current.

Solution:

Let $i_R(t)$, $i_L(t)$ and $i_c(t)$ be the currents through resistor, inductor and capacitor respectively.



Given thatzero initial current through the inductor and zero initial charge across the capacitor before application of the current,

$$i_L(0^+) = 0$$

and $v_c(0^+) = 0$

Applying KCL for $t > 0$,

$$i_R(t) + i_L(t) + i_c(t) = 20\, e^{-4t}$$

or, $\frac{v(t)}{R} + i_L(t) + C\frac{dv(t)}{dt} = 20\, e^{-4t}$ ....(1)

or, $\frac{v(t)}{R} + \frac{1}{L} \int_{-\infty}^{t} v(t)\,dt + C\frac{dv(t)}{dt} = 20\, e^{-4t}$

or, $10\, v(t) + 100 \int_{-\infty}^{t} v(t)\,dt + 2.5 \times 10^{-6}\frac{dv(t)}{dt} = 20\, e^{-4t}$

Differentiating above equation w.r.t. time,

$$10\frac{dv(t)}{dt} + 100\, v(t) + 2.5 \times 10^{-6}\frac{d^2v(t)}{dt^2} = 20 \times (-4)\, e^{-4t}$$

or, $\frac{d^2v(t)}{dt^2} + 4 \times 10^6\frac{dv(t)}{dt} + 40 \times 10^6\ v(t) = -3.2 \times 10^7 e^{-4t}$

........(2)

The equation (2) is second order linear differential equation whose solution can be expressed as;

$$v(t) = v_N(t) + v_F(t)$$

For $v_N(t)$:

The auxiliary equation is;

$$s^2 + 4 \times 10^6\, s + 40 \times 10^6 = 0$$

or, $s = -10, -4 \times 10^6$ (The roots are real and unequal)

So, $v_N(t) = K_1 e^{-10t} + K_2 e^{-4\times10^{-6}t}$

For $v_F(t)$:

Let the trial solution be $v_F(t) = A e^{-4t}$

Replacing v(t) in differential equation (2) by $v_F(t)$,

$$\frac{d^2}{dt^2}(A e^{-4t}) + 4\times10^6\frac{d}{dt}(A e^{-4t}) + 40\times10^6(A e^{-4t}) = -3.2\times10^7 e^{-4t}$$

or, $16A e^{-4t} - 16\times10^6 A e^{-4t} + 40\times10^6 A e^{-4t} = -3.2\times10^7 e^{-4t}$

or, $A = -1.33$

So, $v_F(t) = -1.33\, e^{-4t}$

Then, $v(t) = v_N(t) + v_F(t)$

or, $v(t) = K_1 e^{-10t} + K_2 e^{-4\times10^{-6}t} - 1.33\, e^{-4t}$ ....(3)

The equation (3) is the general solution of differential equation (2) where the values of K_1 and K_2 can be determined using initial conditions.

$$\frac{dv(t)}{dt} = -10K_1\, e^{-10t} - 4\times10^6\, K_2\, e^{-4\times10^{-6}t} + 5.32\, e^{-4t}$$

........(4)

Writing equation (1) at $t = 0^+$, we get

$$10\, v(0^+) + i_L(0^+) + C\frac{dv}{dt}(0^+) = 20$$

or, $\frac{dv}{dt}(0^+) = 8\times10^6$ V/s

Writing equations (3) and (4) at $t = 0^+$ and substituting values of $v(0^+)$ and $\frac{dv}{dt}(0^+)$, we get

$0 = K_1 + K_2 - 1.33$

or, $K_1 + K_2 = 1.33$ ....(a)

and $8\times10^6 = -10K_1 - 4\times10^6 K_2 + 5.32$

or, $10K_1 + 4\times10^6 K_2 = (5.32 - 8\times10^6)$ ....(b)

Solving equations (a) and (b), we get

$K_1 = 3.33$ and $K_2 = -2$

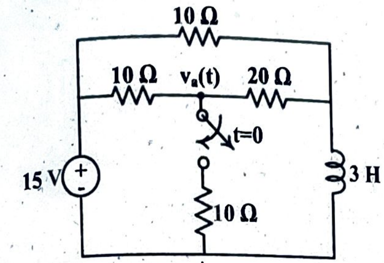
Substituting the values of K_1 and K_2 in equation (3), we get the complete solution as;

$$v(t) = 3.33\, e^{-10t} - 2\, e^{-4\times10^{-6}t} - 1.33\, e^{-4t} \text{ V}$$

Example 2.42

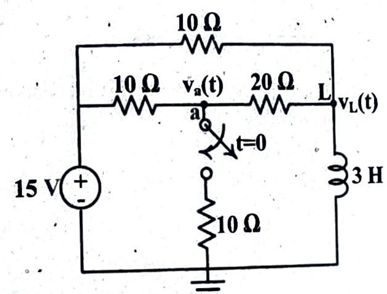
Find $v_a(t)$ for $t > 0$ in the figure below using classical method.

[2071 Chaitra]



Solution:

Assuming node 'a' having voltage $v_a(t)$ and node 'L' having voltage $v_L(t)$ for performing Nodal Analysis;



Applying KCL at node 'a' for $t > 0$,

$$\frac{v_a - 15}{10} + \frac{v_a - v_L}{20} + \frac{v_a}{10} = 0$$

or, $\frac{1}{4}v_a - \frac{1}{20}v_L = \frac{3}{2}$

or, $\frac{1}{20}v_L = \frac{1}{4}v_a - \frac{3}{2}$

or, $v_L = 5v_a - 30$ ....(1)

Applying KCL at node 'L' for t > 0,

$$\frac{v_L - v_a}{20} + \frac{v_L - 15}{10} + i_L = 0$$

or, $$\frac{v_L - v_a}{20} + \frac{v_L - 15}{10} + \frac{1}{L}\int_{-\infty}^{t} v_L\, dt = 0$$

or, $$\frac{3}{20} v_L - \frac{1}{20} v_a + \frac{1}{3}\int_{-\infty}^{t} v_L\, dt = \frac{3}{2}$$

Differentiating above equation w.r.t. time,

$$\frac{3}{20}\frac{dv_L}{dt} - \frac{1}{20}\frac{dv_a}{dt} + \frac{1}{3} v_L = 0 \quad \ldots\ldots\ldots\ldots(2)$$

From equations (1) and (2),

$$\frac{3}{20}\frac{d}{dt}(5v_a - 30) - \frac{1}{20}\frac{dv_a}{dt} + \frac{1}{3}(5v_a - 30) = 0$$

or, $$\frac{7}{10}\frac{dv_a}{dt} + \frac{5}{3} v_a = 10$$

or, $$\frac{dv_a}{dt} + \frac{50}{21} v_a = \frac{100}{7} \quad \ldots\ldots\ldots\ldots\ldots(3)$$

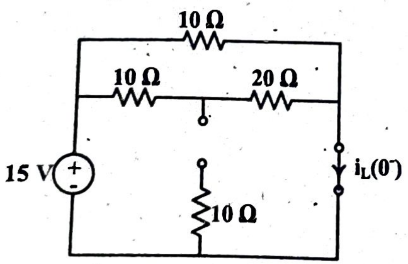
The equation (3) is first order linear differential equation whose solution can be expressed as;

$$v_a(t) = K e^{\frac{-50}{21}t} + \frac{100/7}{50/21}$$

or, $$v_a(t) = K e^{-2.38t} + 6 \quad \ldots\ldots\ldots\ldots(4)$$

The equation (4) is the general solution of the differential equation (3) where value of K can be determined using initial conditions.

The equivalent circuit at $t = 0^-$ is as follows;

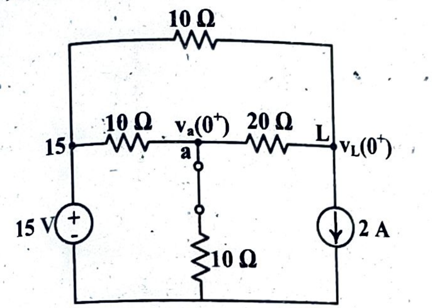


Here, the equivalent resistance, $R_{eq} = (10 + 20) \,//\, 10 = 7.5\ \Omega$

Then, $i_L(0^-) = \frac{15}{7.5} = 2$ A

At $t = 0^+$, $i_L(0^+) = i_L(0^-) = 2$ A

The equivalent circuit at $t = 0^+$ is as follows;



Applying KCL at node 'a' for $t = 0^+$,

$$\frac{v_a(0^+) - 15}{10} + \frac{v_a(0^+) - v_L(0^+)}{20} + \frac{v_a(0^+)}{10} = 0$$

or, $$\frac{1}{4} v_a(0^+) - \frac{1}{20} v_L(0^+) = \frac{3}{2} \quad \ldots\ldots\ldots\ldots(5)$$

Applying KCL at node 'L' for $t = 0^+$,

$$\frac{v_L(0^+) - v_a(0^+)}{20} + \frac{v_L(0^+) - 15}{10} + i_L(0^+) = 0$$

or, $$\frac{-1}{20} v_a(0^+) + \frac{3}{20} v_L(0^+) = \frac{-1}{2} \quad \ldots\ldots\ldots\ldots(6)$$

Solving equations (5) and (6), we get

$$v_a(0^+) = \frac{40}{7}\ \text{V} \qquad \text{and} \qquad v_L(0^+) = \frac{-10}{7}\ \text{V}$$

Writing equation (4) at $t = 0^+$ and substituting the value of $v_a(0^+)$, we get,

$$\frac{40}{7} = K + 6$$

or, $$K = \frac{-2}{7}$$

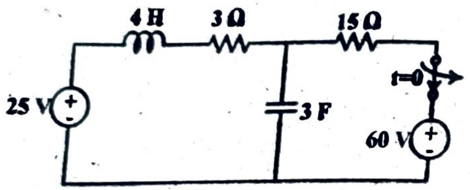
Substituting the value of K in equation (4), we get the complete solution as;

$$v_a(t) = \frac{-2}{7} e^{-2.38t} + 6\ \text{V}$$

Example 2.43

Keeping the switch closed for a long time, if the switch is opened at $t = 0$ in the circuit shown in figure below, find the expression for voltage across capacitor in the circuit using classical method of solution.

[2071 Chaitra]



Solution:

Let i(t) be the current flowing through the circuit for t>0.

Then, applying KVL in the circuit for t>0, we get,

$$25 = 4\frac{di}{dt} + 3 \times i(t) + v_c$$

Since, $i_c = C\frac{dv_c}{dt} = i(t)$

$$\text{Hence, } 25 = 4\left(3\frac{d^2v_c}{dt^2}\right) + 3\times 3\frac{dv_c}{dt} + v_c$$

$$\text{or, } \frac{d^2v_c}{dt^2} + \frac{3}{4}\frac{dv_c}{dt} + \frac{1}{12}v_c = \frac{25}{12} \quad \text{............(1)}$$

The above equation is second order linear differential equation whose solution can be expressed as;

$$v_c(t) = v_{cN}(t) + v_{cF}(t)$$

$$\text{where, } v_{cF}(t) = \frac{25/12}{1/12} = 25$$

For $v_{cN}(t)$:

The auxiliary equation is;

$$s^2 + \frac{3}{2}s + \frac{1}{12} = 0$$

or, $s = -0.136, -0.614$ (The roots are real and unequal)

So, $v_{cN}(t) = K_1 e^{-0.136t} + K_2 e^{-0.614t}$

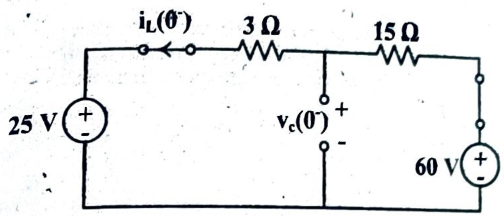
Then, $v_c(t) = v_{cN}(t) + v_{cF}(t)$

$$\text{or, } v_c(t) = K_1 e^{-0.136t} + K_2 e^{-0.614t} + 25 \quad \text{......................(2)}$$

The above equation is the general solution of the differential equation (1) and the values of K_1 and K_2 can be determined using initial conditions.

$$\frac{dv_c(t)}{dt} = -0.136\, K_1\, e^{-0.136t} - 0.614\, e^{-0.614t} \quad \text{............(3)}$$

The equivalent circuit at $t=0^-$ is shown below;



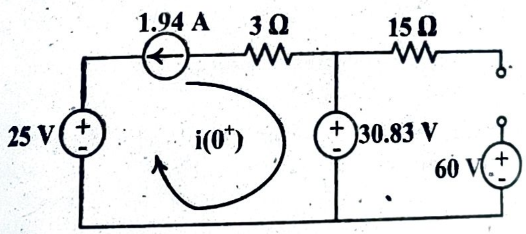
Here, $i_L(0^-) = \frac{60-25}{15+3} = 1.944$ A and

$v_c(0^-) = 60 - 15 \times 1.944$

or, $v_c(0^-) = 30.83$ V

Hence, at $t=0^+$, $i_L(0^+) = 1.944$ A and $v_c(0^+) = 30.83$ V

The equivalent circuit at $t = 0^+$ is shown below;



Here, $i_c(0^+) = i(0^+) = -1.944$ A

or, $3\times\frac{dv_c}{dt}(0^+) = -1.944$

or, $\frac{dv_c}{dt}(0^+) = -0.648$ V/s

Writing equations (2) and (3) at $t=0^+$ and substituting the values of $v_c(0^+)$ and $\frac{dv_c}{dt}(0^+)$, we get,

$30.83 = K_1 + K_2 + 25$

or, $K_1 + K_2 = 5.83$ ....(a)

and $0.648 = 0.136\, K_1 + 0.614\, K_2$ ....(b)

Solving equations (a) and (b), we get

$K_1 = 6.13$ and $K_2 = -0.303$

Substituting the values of K_1 and K_2 in general solution gives the complete solution as;

$$v_c(t) = 6.13\, e^{-0.136t} - 0.303\, e^{-0.614t} + 25 \text{ V}$$

Multiple Choice Questions

1. The continuity relation for an inductor is
   a) $v_L(0^-) = v_L(0^+)$ b) $i_L(0^-) = i_L(0^+)$
   c) $\frac{dv_L}{dt}(0^-) = \frac{dv_L}{dt}(0^+)$ d) $\frac{di_L}{dt}(0^-) = \frac{di_L}{dt}(0^+)$
2. The continuity relation for a capacitor is
   a) $v_c(0^-) = v_c(0^+)$ b) $i_c(0^-) = i_c(0^+)$
   c) $\frac{dv_c}{dt}(0^-) = \frac{dv_c}{dt}(0^+)$ d) $\frac{di_c}{dt}(0^-) = \frac{di_c}{dt}(0^+)$
3. The particular integral (P.I.) computed while solving differential equation is
   a) natural response b) forced response
   c) both of these d) none
4. Which of the following responses is independent of the circuit excitation?
   a) natural response b) forced response
   c) both of these d) none
5. Which of the following responses is dependent on the circuit excitation?
   a) natural response b) forced response
   c) both of these d) none
6. What order differential equation is obtained in series RLC circuit?
   a) first b) second
   c) third d) both a and b

ANSWERS

1.(b), 2.(a), 3.(b), 4.(a), 5.(b), 6.(b)

□□□

CHAPTER 3

Transient Analysis using Laplace Transform

3.1 LAPLACE TRANSFORM

Laplace Transform is used in Electric Circuit Theory to convert time domain function into its Laplace Transform (or s domain) counter-part.

The Laplace Transforms of some commonly used functions in Circuit Analysis are as follows.

a) $£[i(t)] = I(s)$

b) $£[v(t)] = V(s)$

c) $£[1] = \frac{1}{s}$

d) $£[e^{-at}] = \frac{1}{s+a}$

e) $£[t^n] = \frac{n!}{s^{n+1}}$, $£[t] = \frac{1}{s^2}$, $£[t^2] = \frac{1}{s^3}$

f) $£\left[\frac{di(t)}{dt}\right] = sI(s) - i(0^+)$

g) $£\left[\int_0^t i(t)\, dt\right] = \frac{I(s)}{s}$

h) $£[\sin \omega t] = \frac{\omega}{s^2 + \omega^2}$

i) $£[\cos \omega t] = \frac{s}{s^2 + \omega^2}$

Frequency Shifting Theorem:

If $£[x(t)] = X(s)$,

Then, $£[e^{-at} x(t)] = X(s+a)$

The Laplace Transforms of the following functions can be determined using Frequency Shifting Theorem.

$$£[t\, e^{-at}] = \frac{1}{(s+a)^2}$$

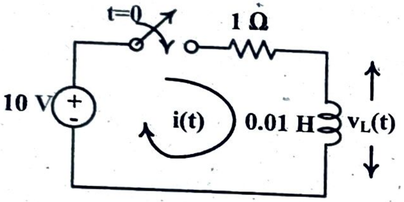
$$£[e^{-at} \sin \omega t] = \frac{\omega}{(s+a)^2 + \omega^2}$$

$$£[e^{-at} \cos \omega t] = \frac{s+a}{(s+a)^2 + \omega^2}$$

3.2 SOLVED PROBLEMS OF TRANSIENT ANALYSIS BY LAPLACE TRANSFORM METHOD

Example 3.1

A DC source of 10 V is suddenly applied at time t = 0 to a series R–L circuit comprising R = 1 Ω and L =0.01 H. Obtain the expression for current in the circuit.



Solution:

For determining initial condition:

At $t=0^-$, $i_L(0^-) = 0$ [By circuit inspection]

Then, $i_L(0^+) = 0 = i(0^+)$

Applying KVL in the circuit for t>0,

$$10 = 1\times i(t) + 0.01\frac{di(t)}{dt}$$

Taking Laplace Transform on both sides,

$$\frac{10}{s} = I(s) + 0.01[sI(s) - i(0^+)]$$

or, $$\frac{10}{s} = I(s) + 0.01\, s\, I(s)$$

or, $$I(s)\,(0.01\, s + 1) = \frac{10}{s}$$

or, $$I(s) = \frac{10}{s(0.01s + 1)}$$

or, $$I(s) = \frac{1000}{s\,(s + 100)}$$

For partial fraction expansion,

Let, $$\frac{1000}{s\,(s+100)} = \frac{A}{s} + \frac{B}{s+100}$$

Then, $1000 = A\,(s + 100) + B\, s$

Put s = 0, then

$$1000 = A \times 100$$

or, $$A = 10$$

Put s = –100, then

$$1000 = B\,(-100)$$

or, $$B = -10$$

Thus, $$I(s) = \frac{10}{s} + \frac{-10}{s+100}$$

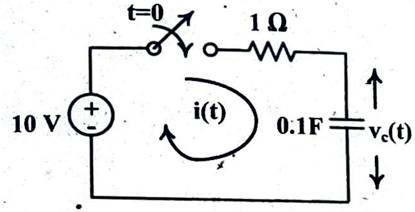
Taking Inverse Laplace Transform,

$$i(t) = 10 - 10\, e^{-100t}$$

or, $$\mathbf{i(t) = 10(1 - e^{-100t})\ A}$$

Example 3.2

A DC source of 10 V is suddenly applied at time t = 0 to a series R–C circuit comprising R = 1 Ω and C =0.1 F. Obtain the expression for current in the circuit.



Solution:

For determining initial condition:

At $t=0^-$, $v_c(0^-) = 0$ [By circuit inspection]

Then, $v_c(0^+) = 0$

Applying KVL in the circuit for t>0,

$$10 = 1\times i(t) + \frac{1}{0.1}\int_{-\infty}^{t} i(t)\, dt$$

or, $$10 = i(t) + v_c(0^+) + \frac{1}{0.1}\int_{0}^{t} i(t)\, dt$$

or, $$10 = i(t) + \frac{1}{0.1}\int_{0}^{t} i(t)\, dt$$

Taking Laplace Transform on both sides,

$$\frac{10}{s} = I(s) + 10\frac{I(s)}{s}$$

or, $$I(s)\left(1+\frac{10}{s}\right) = \frac{10}{s}$$

or, $$I(s)\left(\frac{s+10}{s}\right) = \frac{10}{s}$$

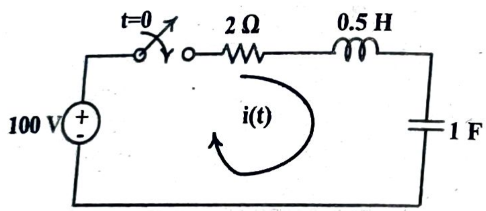
or, $$I(s) = \frac{10}{s+10}$$

Taking Inverse Laplace Transform,

$$i(t) = 10e^{-10t}\ \text{A}$$

Example 3.3

A DC source of100 V is suddenly applied at time t = 0 to a series RLC circuit comprising R = 2 Ω, L=0.5 H and C =1F. Obtain the expression for current in the circuit?



Solution:

For determining initial conditions:

At $t=0^-$, $i_L(0^-) = 0$ and $v_c(0^-) = 0$

Then, at $t=0^+$, $i_L(0^+) = i(0^+) = 0$ and $v_c(0^+) = 0$

Then, applying KVL in the circuit for t>0, we get,

$$100 = 2\,i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{1}\int_{-\infty}^{t} i(t)\,dt$$

or, $$100 = 2\,i(t) + 0.5\frac{di(t)}{dt} + v_c(0^+) + \frac{1}{1}\int_{0}^{t} i(t)\,dt$$

or, $$100 = 2\,i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{1}\int_{0}^{t} i(t)\,dt$$

Taking Laplace Transform on both sides,

$$\frac{100}{s} = 2\,I(s) + 0.5[sI(s) - i(0^+)] + \frac{I(s)}{s}$$

or, $$\frac{100}{s} = I(s)\left(2 + 0.5s + \frac{1}{s}\right).$$

or, $$I(s)\left(\frac{2s+0.5s^2+1}{s}\right) = \frac{100}{s}$$

or, $$I(s) = \frac{100}{0.5s^2+2s+1}$$

or, $$I(s) = \frac{100/0.5}{s^2+4s+2}$$

or, $$I(s) = \frac{200}{(s+0.586)\,(s+3.414)}$$

For partial fraction expansion,

Let, $$\frac{200}{(s+0.586)\,(s+3.414)} = \frac{A}{s+0.586} + \frac{B}{s+3.414}$$

Then, $200 = A\,(s + 3.414) + B\,(s + 0.586)$

Put $s = -0.586$, then

$$200 = A \times (-0.586 + 3.414)$$

or, $A = 70.72$

Put $s = -3.414$, then $200 = B\,(-3.414 + 0.586)$

or, $B = -70.72$

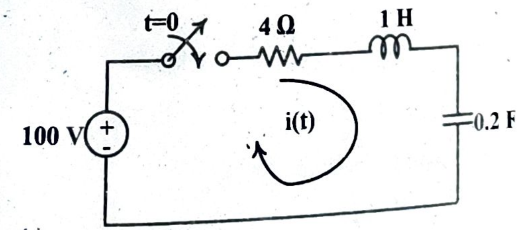
Thus, $$I(s) = \frac{70.72}{s+0.586} + \frac{-70.72}{s+3.414}$$

Taking Inverse Laplace Transform,

$$i(t) = 70.72\,e^{-0.586t} - 70.72\,e^{-3.414t}\ \text{A}$$

Example 3.4

A DC source of 100 V is suddenly applied at time t = 0 to a series RLC circuit comprising R = 4 Ω, L=1 H and C =0.2F. Obtain the expression for current in the circuit.



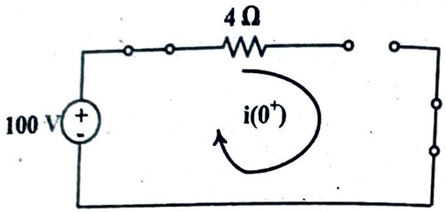
Solution:

For determining initial conditions:

At $t=0^-$, $i_L(0^-) = 0$ and $v_c(0^-) = 0$

Then, at $t=0^+$, $i_L(0^+) = 0$ and $v_c(0^+) = 0$

The equivalent circuit at $t=0^+$ is shown below;



Here, $i(0^+) = 0$

Then, applying KVL in the circuit for $t>0$, we get,

$$100 = 4\,i(t) + 1\frac{di(t)}{dt} + \frac{1}{0.2}\int_{-\infty}^{t} i(t)\,dt$$

or, $$100 = 4\,i(t) + 1\frac{di(t)}{dt} + v_c(0^+) + \frac{1}{0.2}\int_{0}^{t} i(t)\,dt$$

or, $$100 = 4\,i(t) + \frac{di(t)}{dt} + \frac{1}{0.2}\int_{0}^{t} i(t)\,dt$$

Taking Laplace Transform on both sides,

$$\frac{100}{s} = 4\,I(s) + [sI(s) - i(0^+)] + 5\frac{I(s)}{s}$$

or, $$\frac{100}{s} = I(s)\left(4 + s + \frac{5}{s}\right)$$

or, $$I(s)\left(\frac{4s + s^2 + 5}{s}\right) = \frac{100}{s}$$

or, $$I(s) = \frac{100}{s^2 + 4s + 5}$$

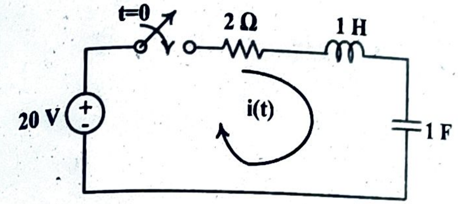
or, $$I(s) = 100\frac{1}{(s+2)^2 + 1^2}$$

Taking Inverse Laplace Transform,

$$i(t) = 100\,e^{-2t}\sin t \text{ A}$$

Example 3.5

A DC source of 20 V is suddenly applied at time t = 0 to a series RLC circuit comprising R = 2 Ω, L=1 H and C =1 F. Obtain the expression for current in the circuit.



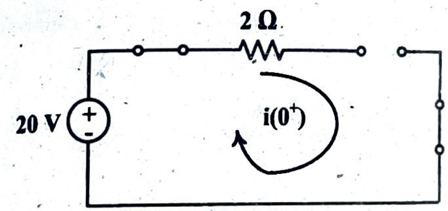
Solution:

For determining initial conditions:

At $t=0^-$, $i_L(0^-) = 0$ and $v_c(0^-) = 0$

Then, at $t=0^+$, $i_L(0^+) = 0$ and $v_c(0^+) = 0$

The equivalent circuit at $t=0^+$ is shown below;



Here, $i(0^+) = 0$

Then, applying KVL in the circuit for $t>0$, we get,

$$20 = 2\,i(t) + 1\frac{di(t)}{dt} + \frac{1}{1}\int_{-\infty}^{t} i(t)\,dt$$

or, $$20 = 2\,i(t) + \frac{di(t)}{dt} + v_c(0^+) + \frac{1}{1}\int_{0}^{t} i(t)\,dt$$

or, $$20 = 2\,i(t) + \frac{di(t)}{dt} + \frac{1}{1}\int_{0}^{t} i(t)\,dt$$

Taking Laplace Transform on both sides,

$$\frac{20}{s} = 2\,I(s) + [sI(s) - i(0^+)] + \frac{I(s)}{s}$$

or, $$\frac{20}{s} = I(s)\left(2 + s + \frac{1}{s}\right)$$

or, $$I(s)\left(\frac{2s + s^2 + 1}{s}\right) = \frac{20}{s}$$

or, $$I(s) = \frac{20}{s^2 + 2s + 1}$$

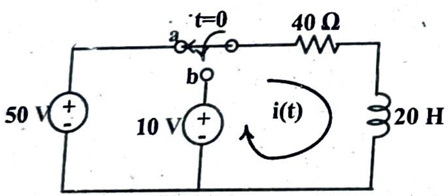
or, $$I(s) = \frac{20}{(s+1)^2}$$

Taking Inverse Laplace Transform,

$$i(t) = 20\,t\,e^{-t}\ \text{A}$$

Example 3.6

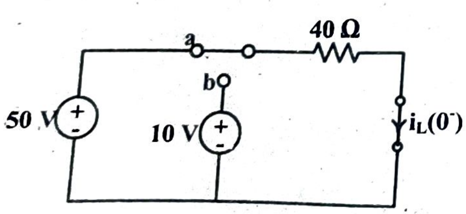
The switch in the figure below has been in position 'a' for a long time. Then, it is moved to 'b' at t=0. Obtain the expression for current i(t) for t>0. R L



Solution:

For determining initial conditions:

At $t=0^-$, the equivalent circuit is as follows;



Here, $i_L(0^-) = \frac{50}{40} = 1.25$ A

Then, $i_L(0^+) = 1.25\text{ A} = i(0^+)$

Applying KVL in the circuit for t>0,

$$10 = 40 \times i(t) + 20\frac{di(t)}{dt}$$

Taking Laplace Transform on both sides,

$$\frac{10}{s} = 40\,I(s) + 20[sI(s) - i(0^+)]$$

or, $$\frac{10}{s} = 40\,I(s) + 20\,s\,I(s) - 20\times1.25$$

or, $$I(s)\,(20\,s + 40) = \frac{10}{s} + 25$$

or, $$I(s)\,(20\,s + 40) = \frac{10 + 25s}{s}$$

or, $$I(s) = \frac{10 + 25s}{s(20s + 40)}$$

or, $$I(s) = \frac{0.5 + 1.25s}{s\,(s+2)}$$

For partial fraction expansion,

Let, $$\frac{0.5 + 1.25s}{s\,(s+2)} = \frac{A}{s} + \frac{B}{s+2}$$

Then, $0.5 + 1.25s = A\,(s+2) + B\,s$

Put s = 0, then $0.5 = A \times 2$

or, $A = 0.25$

Put s = –2, then $0.5 + 1.25\,(-2) = B\,(-2)$

or, $B = 1$

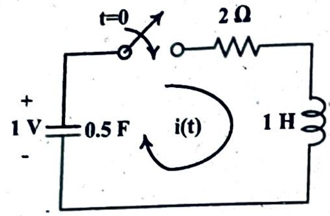
Thus, $$I(s) = \frac{0.25}{s} + \frac{1}{s+2}$$

Taking Inverse Laplace Transform,

$$i(t) = 0.25 + e^{-2t}\ \text{A}$$

Example 3.7

The capacitor shown in the circuit below is charged initially to a voltage of 1 V with polarity as indicated in the figure. Find the expression for i(t).

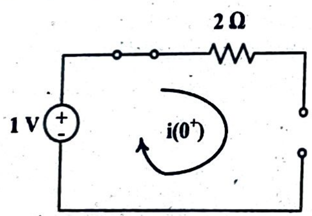


Solution:

For determining initial conditions:

At $t=0^+$, $v_c(0^+) = -1$ V [Negative sign is introduced because polarity of the initial voltage across capacitor does not conform with the given direction of current]

The equivalent circuit at $t=0^+$ is shown below;



Here, $i(0^+) = 0$

Applying KVL in the circuit for t>0, we get,

$$0 = 2\,i(t) + 1\frac{di(t)}{dt} + \frac{1}{0.5}\int_{-\infty}^{t} i(t)\,dt$$

or, $$0 = 2\,i(t) + 1\frac{di(t)}{dt} + v_c(0^+) + \frac{1}{0.5}\int_{0}^{t} i(t)\,dt$$

or, $$0 = 2\,i(t) + \frac{di(t)}{dt} - 1 + \frac{1}{0.5}\int_{0}^{t} i(t)\,dt$$

or, $$1 = 2\,i(t) + \frac{di(t)}{dt} + \frac{1}{0.5}\int_{0}^{t} i(t)\,dt$$

Taking Laplace Transform on both sides,

$$\frac{1}{s} = 2\,I(s) + [sI(s) - i(0^+)] + 2\frac{I(s)}{s}$$

or, $$\frac{1}{s} = I(s)\left(2 + s + \frac{2}{s}\right)$$

or, $$I(s)\left(\frac{2s + s^2 + 2}{s}\right) = \frac{1}{s}$$

or, $$I(s) = \frac{1}{s^2 + 2s + 2}$$

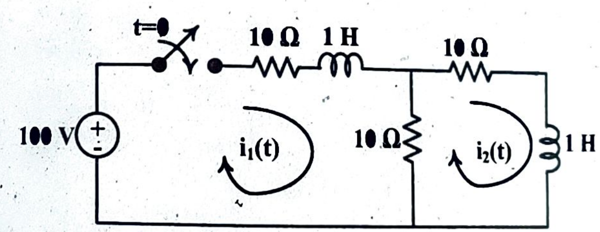
or, $$I(s) = \frac{1}{(s+1)^2 + 1^2}$$

Taking Inverse Laplace Transform,

$$i(t) = e^{-t} \sin t \text{ A}$$

Example 3.8

In the network shown in the figure below, the switch is closed at t=0. With the network parameter values shown, find the expression for $i_1(t)$ and $i_2(t)$ if the network is un–energized before the switch is closed.



Solution:

Since the circuit is un-energized before switch is closed at t = 0,

$i_1(0^+) = i_1(0^+) = 0$

Applying KVL in mesh 1 for t>0,

$$100 = 10\,i_1 + 10\,(i_1 - i_2) + 1\frac{di_1}{dt}$$

or, $$100 = 20\,i_1 - 10\,i_2 + \frac{di_1}{dt}$$

Taking Laplace Transform on both sides,

$$\frac{100}{s} = 20\,I_1(s) - 10\,I_2(s) + [sI_1(s) - i_1(0^+)]$$

or, $$\frac{100}{s} = (s+20)\,I_1(s) - 10\,I_2(s) \ldots\ldots\ldots\ldots(1)$$

Applying KVL in mesh 2 for t>0,

$$-10\,(i_1 - i_2) + 10\,i_2 + 1\,\frac{di_2}{dt} = 0$$

or, $$-10\,i_1 + 20\,i_2 + \frac{di_2}{dt} = 0$$

Taking Laplace Transform on both sides,

$$-10\,I_1(s) + 20\,I_2(s) + [sI_2(s) - i_2(0^+)] = 0$$

or, $$-10\,I_1(s) + (s+20)\,I_2(s) = 0 \ldots\ldots\ldots\ldots(2)$$

Writing equations (1) and (2) in matrix form, we get

$$\begin{bmatrix} s+20 & -10 \\ -10 & s+20 \end{bmatrix}\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{100}{s} \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$I_1(s) = \frac{\Delta_1}{\Delta} \text{ and } I_2(s) = \frac{\Delta_2}{\Delta}$$

Where,

$$\Delta = \begin{vmatrix} s+20 & -10 \\ -10 & s+20 \end{vmatrix}; \Delta_1 = \begin{vmatrix} \frac{100}{s} & -10 \\ 0 & s+20 \end{vmatrix};$$

and $$\Delta_2 = \begin{vmatrix} s+20 & \frac{100}{s} \\ -10 & 0 \end{vmatrix}$$

Therefore, $\Delta = (s+20)^2 - 100 = s^2 + 40s + 300$ ✓

$$\Delta_1 = \frac{100}{s}(s+20) \text{ and } \Delta_2 = \frac{1000}{s}$$

So, $$I_1(s) = \frac{100(s+20)}{s(s^2+40s+300)} = \frac{100(s+20)}{s(s+10)(s+30)}$$

For partial fraction expansion,

Let, $$\frac{100(s+20)}{s(s+10)(s+30)} = \frac{A}{s} + \frac{B}{s+10} + \frac{C}{s+30}$$

Then, $100(s+20) = A\,(s+10)\,(s+30) + B\,s\,(s+30) + C\,s\,(s+10)$

Put $s = 0$, then $100 \times 20 = A \times 10 \times 30$

or, $$A = \frac{20}{3}$$

Put $s = -10$, then $100 \times 10 = B \times (-10) \times 20$

or, $B = -5$

Put $s = -30$, then $100 \times (-10) = C \times (-30) \times (-20)$

or, $$C = \frac{-5}{3}$$

Thus, $$I_1(s) = \frac{20/3}{s} + \frac{-5}{s+10} + \frac{-5/3}{s+30}$$

Taking Inverse Laplace Transform,

$$i_1(t) = \frac{20}{3} - 5\,e^{-10t} - \frac{5}{3}\,e^{-30t} \text{ A}$$

Again, $$I_2(s) = \frac{1000}{s(s^2+40s+300)} = \frac{1000}{s(s+10)(s+30)}$$

For partial fraction expansion,

Let, $$\frac{1000}{s(s+10)(s+30)} = \frac{D}{s} + \frac{E}{s+10} + \frac{F}{s+30}$$

Then, $1000 = D\,(s+10)\,(s+30) + E\,s\,(s+30) + F\,s\,(s+10)$

Put $s = 0$, then $1000 = D \times 10 \times 30$

or, $$D = \frac{10}{3}$$

Put $s = -10$, then $1000 = E \times (-10) \times 20$

or, $E = -5$

Put $s = -30$, then $1000 = F \times (-30) \times (-20)$

or, $$F = \frac{5}{3}$$

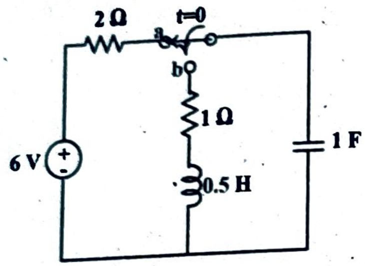
Thus, $$I_2(s) = \frac{10/3}{s} + \frac{-5}{s+10} + \frac{5/3}{s+30}$$

Taking Inverse Laplace Transform,

$$i_2(t) = \frac{10}{3} - 5\,e^{-10t} + \frac{5}{3}\,e^{-30t} \text{ A}$$

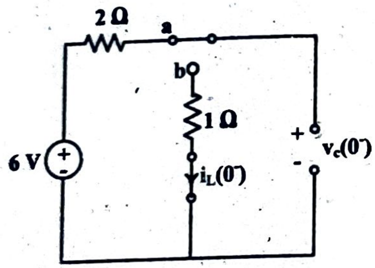
Example 3.9

The switch in the figure below has been in position 'a' for a long time. Then, it is moved to 'b' at t=0. Obtain the expression for voltage across capacitor for t>0.



Solution:

The equivalent circuit at $t=0^-$ is shown below;



Here, $i_L(0^-) = 0$ and $v_c(0^-) = 6$ V

Hence, at $t=0^+$, $i_L(0^+) = i(0^+) = 0$ and $v_c(0^+) = 6$ V

Then, applying KVL in the circuit for $t>0$, we get,

$$0 = 1 \times i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{1}\int_{-\infty}^{t} i(t)\,dt$$

or, $$0 = i(t) + 0.5\frac{di(t)}{dt} + v_c(0^+) + \frac{1}{1}\int_{0}^{t} i(t)\,dt$$

or, $$i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{1}\int_{0}^{t} i(t)\,dt = -6$$

Taking Laplace Transform on both sides,

$$I(s) + 0.5\,[sI(s) - i(0^+)] + \frac{I(s)}{s} = \frac{-6}{s}$$

or, $$I(s)\left(1 + 0.5\,s + \frac{1}{s}\right) = \frac{-6}{s}$$

or, $$I(s)\left(\frac{s + 0.5\,s^2 + 1}{s}\right) = \frac{-6}{s}$$

or, $$I(s) = \frac{-6}{0.5s^2 + s + 1}$$

or, $$I(s) = \frac{-12}{s^2 + 2s + 2} \quad \text{............(1)}$$

Also, we know,

$$i_c(t) = i(t) = C\frac{dv_c(t)}{dt}$$

Taking Laplace Transform on both sides,

$$I(s) = 1 \times [sV_c(s) - v_c(0^+)]$$

or, $$I(s) = sV_c(s) - 6$$

or, $$V_c(s) = \frac{I(s) + 6}{s} \quad \text{............(2)}$$

From equations (1) and (2),

$$V_c(s) = \frac{\frac{-12}{s^2 + 2s + 2} + 6}{s}$$

or, $$V_c(s) = \frac{6s^2 + 12s}{s(s^2 + 2s + 2)}$$

or, $$V_c(s) = \frac{6s + 12}{s^2 + 2s + 2}$$

or, $$V_c(s) = 6\left[\frac{s + 2}{s^2 + 2s + 2}\right]$$

or, $$V_c(s) = 6\left[\frac{s + 1}{(s + 1)^2 + 1^2} + \frac{1}{(s + 1)^2 + 1^2}\right]$$

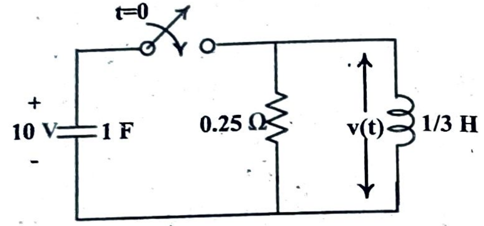
Taking Inverse Laplace Transform on both sides,

$$v_c(t) = 6\,(e^{-t}\cos t + e^{-t}\sin t)$$

or, $$v_c(t) = 6\,e^{-t}\,(\cos t + \sin t)\ \text{V}$$

Example 3.10

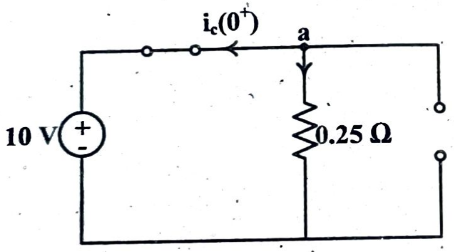
Figure below shows a parallel circuit where capacitor has an initial voltage of 10 V with polarity indicated in the figure. The switch is closed at t=0. Find v(t) for t>0.



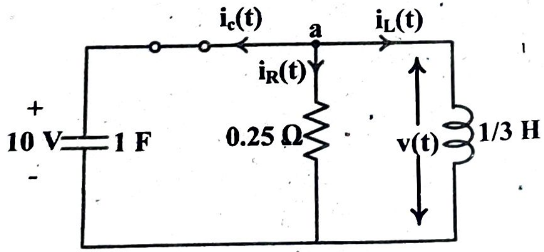
Solution:

At $t=0^+$, $i_L(0^+) = 0$ and $v_c(0^+) = v(0^+) = 10$ V

The equivalent circuit at $t=0^+$ is shown below;



Let $i_c(t)$, $i_L(t)$ and $i_R(t)$ be the current flowing through capacitor, inductor and resistor respectively for time t>0 with directions as indicated in the figure below.



Then, applying KCL at node 'a' in the circuit for t>0, we get,

$$i_c(t) + i_R(t) + i_L(t) = 0$$

or, $$C\frac{dv_c(t)}{dt} + \frac{v_R(t)}{R} + \frac{1}{L}\int_{-\infty}^{t} v_L(t)\,dt = 0$$

But voltages across all the components are equal as they are in parallel,

So, $$1\frac{dv(t)}{dt} + \frac{v(t)}{0.25} + \frac{1}{1/3}\int_{-\infty}^{t} v(t)\,dt = 0$$

Taking Laplace Transform on both sides,

$$4\,V(s) + [sV(s) - v(0^+)] + \frac{3V(s)}{s} = 0$$

or, $$V(s)\left(4 + s + \frac{3}{s}\right) = 10$$

or, $$V(s)\left(\frac{4s + s^2 + 3}{s}\right) = 10$$

or, $$V(s) = \frac{10s}{s^2 + 4s + 3}$$

or, $$V(s) = \frac{10s}{(s+1)(s+3)}$$

For partial fraction expansion,

Let, $$\frac{10s}{(s+1)\cdot(s+3)} = \frac{A}{s+1} + \frac{B}{s+3}$$

Then, $10s = A(s+3) + B(s+1)$

Put $s = -1$, then $10 \times (-1) = A \times (-1+3)$

or, $A = -5$

Put $s = -3$, then $10 \times (-3) = B \times (-3+1)$

or, $B = 15$

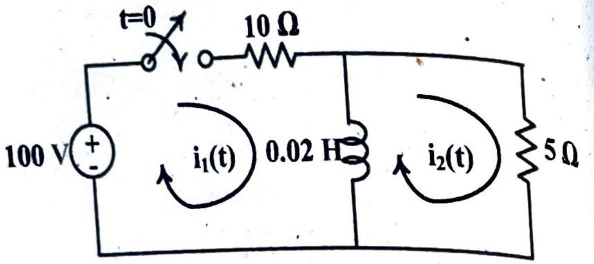
Thus, $$V(s) = \frac{-5}{s+1} + \frac{15}{s+3}$$

Taking Inverse Laplace Transform,

$$v(t) = -5\,e^{-t} + 15\,e^{-3t} \text{ V}$$

Example 3.11

In the two mesh network shown in the figure below, find the currents which result when the switched is closed.



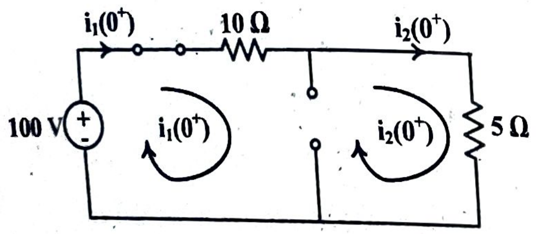
Solution:

Since the circuit is un-energized before the switch is closed,

$i_L(0^-) = 0$

Then, $i_L(0^+) = 0$

The equivalent circuit at $t = 0^+$ is as follows;



Here, $i_1(0^+) = i_2(0^+) = \dfrac{100}{10+5} = 6.67$ A

Applying KVL in mesh 1 for $t > 0$,

$$100 = 10\, i_1 + 0.02 \frac{d}{dt}(i_1 - i_2)$$

or, $$100 = 10\, i_1 + 0.02 \frac{di_1}{dt} - 0.02 \frac{di_2}{dt}$$

Taking Laplace Transform on both sides,

$$\frac{100}{s} = 10\, I_1(s) + 0.02[sI_1(s) - i_1(0^+)] - 0.02[sI_2(s) - i_2(0^+)]$$

or, $$\frac{100}{s} = 10\, I_1(s) + 0.02[sI_1(s) - 6.67] - 0.02[sI_2(s) - 6.67]$$

or, $$10\, I_1(s) + 0.02sI_1(s) - 0.02sI_2(s) = \frac{100}{s}$$

or, $$(0.02s + 10)\, I_1(s) - 0.02s\, I_2(s) = \frac{100}{s}$$

or, $$(s + 500)\, I_1(s) - s\, I_2(s) = \frac{5000}{s} \quad \text{..................} (1)$$

Applying KVL in outer loop for $t > 0$,

$$10\, i_1 + 5\, i_2 = 100$$

Taking Laplace Transform on both sides,

$$10\, I_1(s) + 5\, I_2(s) = \frac{100}{s} \quad \text{..................} (2)$$

Writing equations (1) and (2) in matrix form, we get

$$\begin{bmatrix} s+500 & -s \\ 10 & 5 \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \dfrac{5000}{s} \\ \dfrac{100}{s} \end{bmatrix}$$

By Cramer's rule,

$$I_1(s) = \frac{\Delta_1}{\Delta} \text{ and } I_2(s) = \frac{\Delta_2}{\Delta}$$

Where, $\Delta = \begin{vmatrix} s+500 & -s \\ 10 & 5 \end{vmatrix}$; $\Delta_1 = \begin{vmatrix} \dfrac{5000}{s} & -s \\ \dfrac{100}{s} & 5 \end{vmatrix}$;

and $\Delta_2 = \begin{vmatrix} s+500 & \dfrac{5000}{s} \\ 10 & \dfrac{100}{s} \end{vmatrix}$

Therefore, $\Delta = 5 \times (s + 500) + s \times 10 = 15s + 2500$

$$\Delta_1 = 100\left(\frac{s+250}{s}\right) \text{ and } \Delta_2 = 100$$

So, $$I_1(s) = \frac{100(s+250)}{15s(s+166.67)} = \frac{\frac{20}{3}(s+250)}{s(s+166.67)}$$

For partial fraction expansion,

Let, $$\frac{\frac{20}{3}(s+250)}{s(s+166.67)} = \frac{A}{s} + \frac{B}{s+166.67}$$

Then, $\frac{20}{3}(s + 250) = A(s + 166.67) + B\,s$

Put s = 0, then $\frac{20}{3} \times 250 = A \times 166.67$

or, $A = 10$

Put s = –166.67, then $\frac{20}{3} \times (-166.67 + 250) = B \times (-166.67)$

or, $B = -3.33$

Thus, $I_1(s) = \frac{10}{s} + \frac{-3.33}{s + 166.67}$

Taking Inverse Laplace Transform,

$$i_1(t) = 10 - 3.33\,e^{-166.67t}\ \text{A}$$

Now, $I_2(s) = \frac{100}{15(s + 166.67)}$

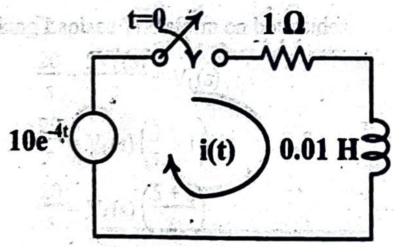
or, $I_2(s) = \frac{20/3}{(s + 166.67)}$

Taking Inverse Laplace Transform,

$$i_2(t) = 6.66\,e^{-166.67t}\ \text{A}$$

Example 3.12

An exponential source $V(t) = 10e^{-4t}$ is suddenly applied at time $t = 0$ to a series R–L circuit comprising $R = 1\ \Omega$ and $L = 0.01$ H. Obtain the expression for the current i(t) in the circuit.



Solution:

At $t = 0^-$, $i_L(0^-) = 0$ [By circuit inspection]

Then, $i_L(0^+) = 0 = i(0^+)$

Applying KVL in the circuit for t>0,

$$10\,e^{-4t} = 1 \times i(t) + 0.01\frac{di(t)}{dt}$$

Taking Laplace Transform on both sides,

$$\frac{10}{s+4} = I(s) + 0.01[sI(s) - i(0^+)]$$

or, $\frac{10}{s+4} = I(s) + 0.01\,s\,I(s)$

or, $I(s)\,(0.01\,s + 1) = \frac{10}{s+4}$

or, $I(s) = \frac{10}{(s+4)(0.01s+1)}$

or, $I(s) = \frac{1000}{(s+4)(s+100)}$

For partial fraction expansion,

Let, $\frac{1000}{(s+4)(s+100)} = \frac{A}{s+4} + \frac{B}{s+100}$

Then, $1000 = A(s + 100) + B(s + 4)$

Put s = –4, then $1000 = A \times (-4 + 100)$

or, $A = 10.41$

Put s = –100, then $1000 = B(-100 + 4)$

or, $B = -10.41$

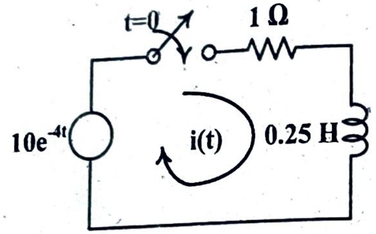
Thus, $I(s) = \frac{10.41}{s+4} + \frac{-10.41}{s+100}$

Taking Inverse Laplace Transform,

$$i(t) = 10.41\,e^{-4t} - 10.41e^{-100t}\ \text{A}$$

Example 3.13

An exponential source $V(t) = 10e^{-4t}$ is suddenly applied at time $t = 0$ to a series R–L circuit comprising $R = 1\ \Omega$ and $L = 0.25$ H. Obtain the expression for the current i(t) in the circuit.



Solution:

At $t=0^-$, $i_L(0^-) = 0$ [By circuit inspection]

Then, $i_L(0^+) = 0 = i(0^+)$

Applying KVL in the circuit for t>0,

$$10\,e^{-4t} = 1\times i(t) + 0.25\frac{di(t)}{dt}$$

Taking Laplace Transform on both sides,

$$\frac{10}{s+4} = I(s) + 0.25[sI(s) - i(0^+)]$$

or, $$\frac{10}{s+4} = I(s) + 0.25\ s\ I(s)$$

or, $$I(s)\ (\,0.25\ s + 1) = \frac{10}{s+4}$$

or, $$I(s) = \frac{10}{(s+4)(0.25s+1)}$$

or, $$I(s) = \frac{10/.25}{(s+4)(s+4)}$$

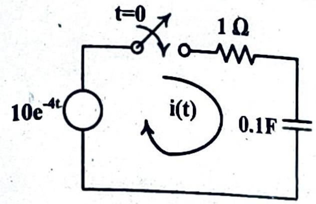
or, $$I(s) = \frac{40}{(s+4)^2}$$

Taking Inverse Laplace Transform,

$$i(t) = 40\ t\ e^{-4t}\ \text{A}$$

Example 3.14

An exponential source $V(t) = 10e^{-4t}$ is suddenly applied at time $t = 0$ to a series R–C circuit comprising $R = 1\ \Omega$ and $C = 0.1$F. Obtain the expression for the current i(t) in the circuit.



Solution:

At $t=0^-$, $v_c(0^-) = 0$ [By circuit inspection]

Then, $v_c(0^+) = 0$

Applying KVL in the circuit for t>0,

$$10\,e^{-4t} = 1\times i(t) + \frac{1}{0.1}\int_{-\infty}^{t} i(t)\,dt$$

or, $$10\,e^{-4t} = 1\times i(t) + v_c(0^+) + \frac{1}{0.1}\int_{0}^{t} i(t)\,dt$$

Taking Laplace Transform on both sides,

$$\frac{10}{s+4} = I(s) + 10\,\frac{I(s)}{s}$$

or, $$\frac{10}{s+4} = I(s)\left(1 + \frac{10}{s}\right)$$

or, $$I(s)\left(\frac{s+10}{s}\right) = \frac{10}{s+4}$$

or, $$I(s) = \frac{10s}{(s+4)(s+10)}$$

For partial fraction expansion,

Let, $$\frac{10s}{(s+4)(s+10)} = \frac{A}{s+4} + \frac{B}{s+10}$$

Then, $10s = A\,(s+10) + B\,(s+4)$

Put $s = -4$, then $10\times(-4) = A\times(-4+10)$

or, $A = -6.67$

Put s = –10, then 10 × (–10) = B (–10 + 4)

or, B = 16.67

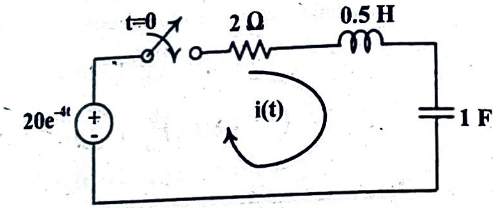
Thus, $I(s) = \dfrac{-6.67}{s+4} + \dfrac{16.67}{s+10}$

Taking Inverse Laplace Transform,

$$i(t) = -6.67\, e^{-4t} + 16.67\, e^{-10t} \text{ A}$$

Example 3.15

An exponential source $V(t) = 20e^{-4t}$ is suddenly applied at time $t = 0$ to a series RLC circuit comprising $R = 2\ \Omega$, $L=0.5$ H and $C=1$F. Obtain the expression for the current i(t) in the circuit.

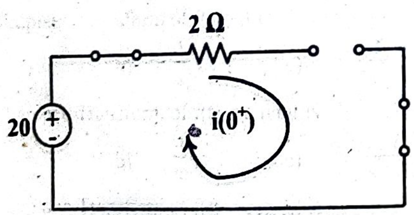


Solution:

At $t=0^-$, $i_L(0^-) = 0$ and $v_c(0^-) = 0$

Then, at $t=0^+$, $i_L(0^+) = 0$ and $v_c(0^+) = 0$

The equivalent circuit at $t=0^+$ is shown below;



Here, $i(0^+) = 0$

Applying KVL in the circuit for t>0, we get,

$$20\,e^{-4t} = 2\,i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{1}\int_{-\infty}^{t} i(t)\,dt$$

or, $$20\,e^{-4t} = 2\,i(t) + 0.5\frac{di(t)}{dt} + v_c(0^+) + \frac{1}{1}\int_{0}^{t} i(t)\,dt$$

or, $$20\,e^{-4t} = 2\,i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{1}\int_{0}^{t} i(t)\,dt$$

Taking Laplace Transform on both sides,

$$\frac{20}{s+4} = 2\,I(s) + 0.5[sI(s) - i(0^+)] + \frac{I(s)}{s}$$

or, $$\frac{20}{s+4} = I(s)\left(2 + 0.5s + \frac{1}{s}\right)$$

or, $$I(s)\left(\frac{2s + 0.5s^2 + 1}{s}\right) = \frac{20}{s+4}$$

or, $$I(s) = \frac{20s}{(s+4)(0.5s^2 + 2s + 1)}$$

or, $$I(s) = \frac{20s/0.5}{(s+4)(s^2 + 4s + 2)}$$

or, $$I(s) = \frac{40s}{(s+0.586)(s+3.414)(s+4)}$$

For partial fraction expansion,

Let, $$\frac{40s}{(s+0.586)(s+3.414)(s+4)} = \frac{A}{s+0.586} + \frac{B}{s+3.414} + \frac{C}{s+4}$$

Then, 40s = A (s + 3.414)(s + 4) + B (s + 0.586)(s + 4) + C (s + 0.586)(s + 3.414)

Put s = –0.586, then 40 × (–0.586) = A × (–0.586 + 3.414) × (–0.586 + 4)

or, A = –2.43

Put s = –3.414, then 40 × (–3.414) = B × (–3.414 + 0.586) × (–3.414 + 4)

or, B = 82.43

Put s = –4, then 40 × (–4) = C × (–4 + 0.586) × (–4 + 3.414)

or, C = –80

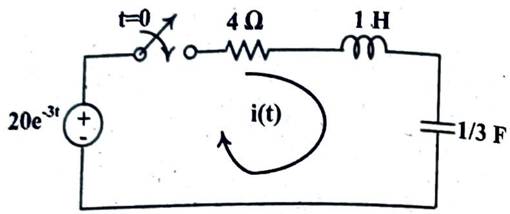
Thus, $I(s) = \dfrac{-2.43}{s+0.586} + \dfrac{82.43}{s+3.414} + \dfrac{-80}{s+4}$

Taking Inverse Laplace Transform,

$$i(t) = -2.43\, e^{-0.586t} + 82.43\, e^{-3.414t} - 80\, e^{-4t} \text{ A}$$

Example 3.16

An exponential source V(t) = $20e^{-3t}$ is suddenly applied at time t = 0 to a series RLC circuit comprising R = 4 Ω , L= 1 H and C=1/3F. Obtain the expression for the current i(t) in the circuit.

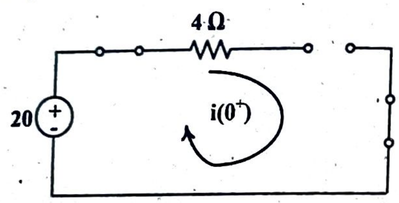


Solution:

At $t=0^-$, $i_L(0^-) = 0$ and $v_c(0^-) = 0$

Then, at $t=0^+$, $i_L(0^+) = 0$ and $v_c(0^+) = 0$

The equivalent circuit at $t=0^+$ is shown below;



Here, $i(0^+) = 0$

Applying KVL in the circuit for t>0, we get,

$$20\,e^{-3t} = 4\,i(t) + 1\frac{di(t)}{dt} + \frac{1}{1/3}\int_{-\infty}^{t} i(t)\,dt$$

or, $$20\,e^{-3t} = 4\,i(t) + 1\frac{di(t)}{dt} + v_c(0^+) + \frac{1}{1/3}\int_{0}^{t} i(t)\,dt$$

or, $$20\,e^{-3t} = 4\,i(t) + \frac{di(t)}{dt} + 3\int_{0}^{t} i(t)\,dt$$

Taking Laplace Transform on both sides,

$$\frac{20}{s+3} = 4\,I(s) + [sI(s) - i(0^+)] + \frac{3I(s)}{s}$$

or, $$\frac{20}{s+3} = I(s)\left(4 + s + \frac{3}{s}\right)$$

or, $$I(s)\left(\frac{4s + s^2 + 3}{s}\right) = \frac{20}{s+3}$$

or, $$I(s) = \frac{20s}{(s+3)(s^2+4s+3)}$$

or, $$I(s) = \frac{20s}{(s+3)(s+1)(s+3)}$$

or, $$I(s) = \frac{20s}{(s+1)(s+3)^2}$$

For partial fraction expansion,

Let, $$\frac{20s}{(s+1)(s+3)^2} = \frac{A}{s+1} + \frac{B}{s+3} + \frac{C}{(s+3)^2}$$

Then, $20s = A\,(s+3)^2 + B\,(s+1)(s+3) + C\,(s+1)$

Put s = –1, then $20 \times (-1) = A \times (-1+3)^2$

or, $A = -5$

Put s = –3, then $20 \times (-3) = C \times (-3+1)$

or, $C = 30$

Put s = 0, then $0 = 9A + 3B + C$

or, $B = 5$

Thus, $$I(s) = \frac{-5}{s+1} + \frac{5}{s+3} + \frac{30}{(s+3)^2}$$

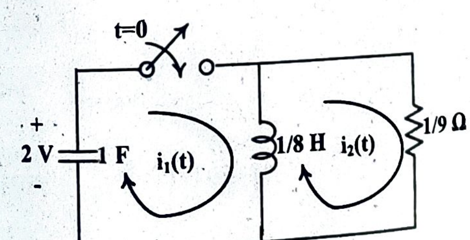
Taking Inverse Laplace Transform,

$$i(t) = -5\,e^{-t} + 5\,e^{-3t} + 30\,t\,e^{-3t}\ \text{A}$$

Example 3.17

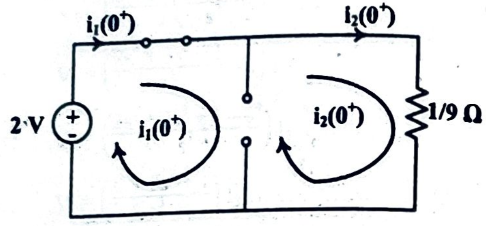
Imp.

For the circuit shown in the figure below, find the mesh currents i_1 and i_2 for t>0.



Solution:

The equivalent circuit at $t = 0^+$ is;



Here, $v_c(0^+) = -2$ V

and $\quad i_1(0^+) = i_2(0^+) = \dfrac{2}{1/9} = 18$ A

Applying KVL in outer loop for $t > 0$,

$$\frac{1}{1}\int_{-\infty}^{t} i_1(t)\,dt + \frac{1}{9}i_2(t) = 0$$

or, $$v_c(0^+) + \frac{1}{1}\int_{0}^{t} i_1(t)\,dt + \frac{1}{9}i_2(t) = 0$$

or, $$\frac{1}{1}\int_{0}^{t} i_1(t)\,dt + \frac{1}{9}i_2(t) = 2$$

Taking Laplace Transform on both sides,

$$\frac{1}{s}I_1(s) + \frac{1}{9}I_2(s) = \frac{2}{s} \quad \text{............(1)}$$

Applying KVL in right mesh for $t > 0$,

$$\frac{1}{8}\frac{d}{dt}(i_2(t) - i_1(t)) + \frac{1}{9}i_2(t) = 0$$

or, $$\frac{1}{8}\frac{di_2(t)}{dt} - \frac{1}{8}\frac{di_1(t)}{dt} + \frac{1}{9}i_2(t) = 0$$

Taking Laplace Transform on both sides,

$$\frac{1}{8}[sI_2(s) - i_2(0^+)] - \frac{1}{8}[sI_1(s) - i_1(0^+)] + \frac{1}{9}I_2(s) = 0$$

or, $$\frac{1}{8}[sI_2(s) - 18] - \frac{1}{8}[sI_1(s) - 18] + \frac{1}{9}I_2(s) = 0$$

or, $$\frac{-s}{8}I_1(s) + \left(\frac{s}{8} + \frac{1}{9}\right)I_2(s) = 0 \quad \text{............(2)}$$

Writing equations (1) and (2) in matrix form, we get

$$\begin{bmatrix} \frac{1}{s} & \frac{1}{9} \\ \frac{-s}{8} & \left(\frac{s}{8} + \frac{1}{9}\right) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} \frac{2}{s} \\ 0 \end{bmatrix}$$

By Cramer's rule,

$$I_1(s) = \frac{\Delta_1}{\Delta} \text{ and } I_2(s) = \frac{\Delta_2}{\Delta}$$

Where, $\Delta = \begin{vmatrix} \frac{1}{s} & \frac{1}{9} \\ \frac{-s}{8} & \left(\frac{s}{8} + \frac{1}{9}\right) \end{vmatrix}$; $\Delta_1 = \begin{vmatrix} \frac{2}{s} & \frac{1}{9} \\ 0 & \left(\frac{s}{8} + \frac{1}{9}\right) \end{vmatrix}$; and $\Delta_2 = \begin{vmatrix} \frac{1}{s} & \frac{2}{s} \\ \frac{-s}{8} & 0 \end{vmatrix}$

Therefore, $\Delta = \dfrac{s^2 + 9s + 8}{72s}$

$$\Delta_1 = \frac{s + 8/9}{4s} \text{ and } \Delta_2 = \frac{1}{4}$$

So, $\quad I_1(s) = \dfrac{18(s + 8/9)}{(s + 1)(s + 8)}$

For partial fraction expansion,

Let, $\dfrac{18(s + 8/9)}{(s + 1)(s + 8)} = \dfrac{A}{s + 1} + \dfrac{B}{s + 8}$

Then, $18(s + 8/9) = A(s + 8) + B(s + 1)$

Put $s = -1$, then $18 \times \left(-1 + \dfrac{8}{9}\right) = A \times (-1 + 8)$

or, $\quad A = -0.28$

Put $s = -8$, then $18 \times \left(-8 + \dfrac{8}{9}\right) = B \times (-8 + 1)$

or, $\quad B = 18.28$

Thus, $I_1(s) = \dfrac{-0.28}{s + 1} + \dfrac{18.28}{s + 8}$

Taking Inverse Laplace Transform,

$$\mathbf{i_1(t) = -0.28\,e^{-t} + 18.28\,e^{-8t}\ A}$$

Again, $I_2(s) = \dfrac{18s}{(s + 1)(s + 8)}$

For partial fraction expansion,

Let, $\dfrac{18s}{(s + 1)(s + 8)} = \dfrac{C}{s + 1} + \dfrac{D}{s + 8}$

Then, $18s = C(s + 8) + D(s + 1)$

Put $s = -1$, then $18 \times (-1) = C \times (-1 + 8)$

or, $C = -2.57$

Put $s = -8$, then $18 \times (-8) = D \times (-8 + 1)$

or, $D = 20.57$

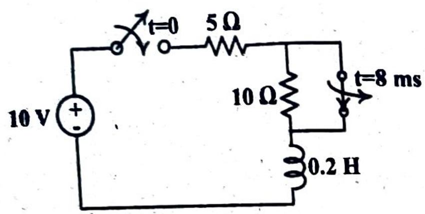
Thus, $I_2(s) = \dfrac{-2.57}{s+1} + \dfrac{20.57}{s+8}$

Taking Inverse Laplace Transform,

$$i_2(t) = -2.57\, e^{-t} + 20.57\, e^{-8t} \text{ A}$$

Example 3.18

In the given circuit below, switch S_1 is closed at $t = 0$ and after 8 ms, the switch S_2 is opened. Find the complete expression for current in the interval $0 < t < 8$ ms and $t > 8$ ms. [2068 Shrawan]



Solution:

Let i(t) be the current flowing through the circuit at any time t.

At $t=0^-$, $i_L(0^-) = 0$. [By circuit inspection]

Then, at $t=0^+$, $i_L(0^+) = 0 = i(0^+)$

Applying KVL in the circuit for $0 < t < 8$ ms,

$$10 = 5\, i(t) + 0.2 \frac{di(t)}{dt}$$

or, $\dfrac{di(t)}{dt} + 25\, i(t) = 50$

Taking Laplace Transform on both sides,

$$sI(s) - i(0^+) + 25I(s) = \frac{50}{s}$$

or, $sI(s) + 25I(s) = \dfrac{50}{s}$

or, $I(s)(s + 25) = \dfrac{50}{s}$

or, $I(s) = \dfrac{50}{s(s + 25)}$

Using partial fraction expansion,

$$\frac{50}{s(s+25)} = \frac{2}{s} - \frac{2}{s+25}$$

So, $I(s) = \dfrac{2}{s} - \dfrac{2}{s+25}$

Taking Inverse Laplace Transform,

$i(t) = 2 - 2\, e^{-25t}$

or, $\mathbf{i(t) = 2(1 - e^{-25t})\ A;\ 0 < t < 8\ ms}$

At t = 8 ms,

$i(8\ \text{ms}) = 0.3625$ A

For $t > 8$ ms, let $t' = t - 8 \times 10^{-3}$ such that t' varies from 0 to ∞.

Then, $i(t' = 0) = i(t = 8 \times 10^{-3}) = 0.3625$ A

Again, applying KVL in the circuit for $t > 8$ ms,

$$10 = 15\, i(t') + 0.2 \frac{di(t')}{dt}$$

or, $\dfrac{di(t')}{dt} + 75\, i(t') = 50$

Taking Laplace Transform on both sides,

$$sI(s) - i(t' = 0^+) + 75I(s) = \frac{50}{s}$$

or, $sI(s) - 0.3625 + 75I(s) = \dfrac{50}{s}$

or, $I(s)(s + 75) = \dfrac{50}{s} + 0.3625$

or, $I(s) = \dfrac{50}{s(s+75)} + \dfrac{0.3625}{s+75}$

Using partial fraction expansion,

$$\frac{50}{s(s+75)} = \frac{2/3}{s} - \frac{2/3}{s+75}$$

So, $I(s) = \dfrac{2/3}{s} - \dfrac{2/3}{s+75} + \dfrac{0.3625}{s+75}$

Taking Inverse Laplace Transform,

$$i(t') = \frac{2}{3} - \frac{2}{3} e^{-75t'} + 0.6325 e^{-75t'}$$

or, $i(t') = \dfrac{2}{3} - 0.3042\, e^{-75t'}$

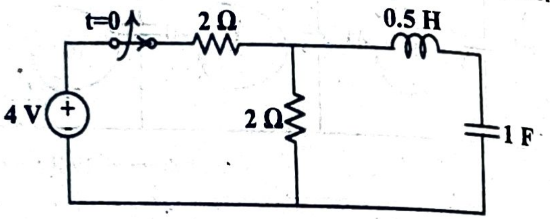
or, $i(t) = \dfrac{2}{3} - 0.3042\, e^{-75(t - 8 \times 10^{-3})}$

or, $i(t) = \frac{2}{3} - 0.3042\, e^{75 \times 8 \times 10^{-3}}\, e^{-75t'}$

$$i(t) = \frac{2}{3} - 0.554\, e^{-75t}\ \text{A};\ t > 8\ \text{ms}$$

Example 3.19

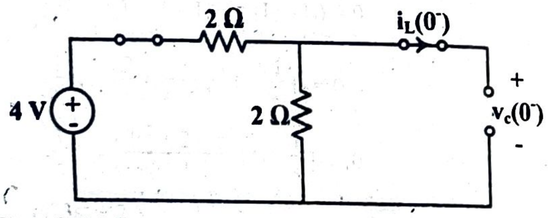
The circuit shown below is in steady state with switch closed. The switch is opened at t =0. Find the current through inductor for t > 0.
[2068 Shrawan]



Solution:

Let i(t) be the current flowing through the circuit at any time t.

At $t=0^-$, the equivalent circuit is as follows;



Here, $i_L(0^-) = 0$

and $v_c(0^-) = 2$ V

Then, at $t=0^+$; $i_L(0^+) = i(0^+) = 0$ and $v_c(0^+) = 2$ V

Now, applying KVL in the circuit for t>0, we get,

t> Switch open

$$0 = 2\, i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{1}\int_{-\infty}^{t} i(t)\, dt$$

or, $$0 = 2\, i(t) + 0.5\frac{di(t)}{dt} + v_c(0^+) + \frac{1}{1}\int_{0}^{t} i(t)\, dt$$

or, $$-2 = 2\, i(t) + 0.5\frac{di(t)}{dt} + \frac{1}{1}\int_{0}^{t} i(t)\, dt$$

Taking Laplace Transform on both sides,

$$\frac{-2}{s} = 2\, I(s) + 0.5[sI(s) - i(0^+)] + \frac{I(s)}{s}$$

or, $$\frac{-2}{s} = I(s)\left(2 + 0.5s + \frac{1}{s}\right)$$

or, $$I(s)\left(\frac{2s + 0.5s^2 + 1}{s}\right) = \frac{-2}{s}$$

or, $$I(s) = \frac{-2}{0.5s^2 + 2s + 1}$$

or, $$I(s) = \frac{-2/0.5}{s^2 + 4s + 2}$$

or, $$I(s) = \frac{-4}{(s + 0.586)(s + 3.414)}$$

For partial fraction expansion,

Let, $$\frac{-4}{(s + 0.586)(s + 3.414)} = \frac{A}{s + 0.586} + \frac{B}{s + 3.414}$$

Then, $-4 = A\,(s + 3.414) + B\,(s + 0.586)$

Put $s = -0.586$, then

$-4 = A \times (-0.586 + 3.414)$

or, $A = -1.414$

Put $s = -3.414$, then $-4 = B\,(-3.414 + 0.586)$

or, $B = 1.414$

Thus, $$I(s) = \frac{-1.414}{s + 0.586} + \frac{1.414}{s + 3.414}$$

Taking Inverse Laplace Transform,

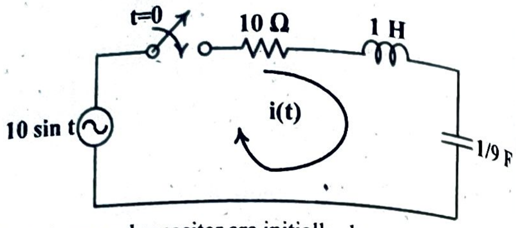
$$i(t) = -1.414\, e^{-9.586t} + 1.414\, e^{-3.414t}\ \text{A}$$

Example 3.20

Find the time expression for current for t > 0 in RLC series circuit with R = 10 Ω, L = 1 H and C = $\frac{1}{9}$ F, if the circuit is supplied by v(t) = 10 sin t at t = 0. Assume that the capacitor and inductors are initially de-energized. Use Laplace Transform method.

Solution:

Let, i(t) be the current through the circuit at any time t.



Given that inductor and capacitor are initially de-energized,

$$i_L(0^+) = i(0^+) = 0 \quad \text{and} \quad v_c(0^+) = 0$$

Applying KVL in the circuit for t > 0,

$$10 \sin t = 10\, i(t) + 1 \times \frac{di(t)}{dt} + \frac{1}{1/9} \int_{-\infty}^{t} i(t)\, dt$$

or, $$10 \sin t = 10\, i(t) + \frac{di(t)}{dt} + v_c(0^+) + \frac{1}{1/9} \int_{0}^{t} i(t)\, dt$$

or, $$10 \sin t = 10\, i(t) + \frac{di(t)}{dt} + 9 \int_{0}^{t} i(t)\, dt$$

Taking Laplace Transform on both sides,

$$\frac{10}{s^2+1} = 10\, I(s) + [sI(s) - i(0^+)] + \frac{9I(s)}{s}$$

or, $$\frac{10}{s^2+1} = I(s)\left(10 + s + \frac{9}{s}\right)$$

or, $$I(s)\left(\frac{10s + s^2 + 9}{s}\right) = \frac{10}{s^2+1}$$

or, $$I(s) = \frac{10s}{(s^2 + 10s + 9)(s^2 + 1)}$$

or, $$I(s) = \frac{10s}{(s+1)(s+9)(s^2+1)}$$

For partial fraction expansion,

Let, $$\frac{10s}{(s+1)(s+9)(s^2+1)} = \frac{A}{s+1} + \frac{B}{s+9} + \frac{Cs + D}{s^2+1}$$

Then, $10s = A\,(s+9)(s^2+1) + B\,(s+1)(s^2+1) + (Cs + D)(s+1)(s+9)$

Put $s = -1$, then $10 \times (-1) = A \times 8 \times 2$

or, $A = -0.625$

Put $s = -9$, then $10 \times (-9) = B \times (-8) \times 82$

or, $B = 0.14$

Put $s = 0$, then $0 = 9A + B + 9D$

or, $D = 0.61$

Equating coefficient of s^3 on both sides,

$$0 = A + B + C$$

or, $C = 0.485$

Thus, $$I(s) = \frac{-0.625}{s+1} + \frac{0.14}{s+9} + \frac{0.485s + 0.61}{s^2+1}$$

or, $$I(s) = \frac{-0.625}{s+1} + \frac{0.14}{s+9} + \frac{0.485(s + 1.26)}{s^2+1}$$

or, $$I(s) = \frac{-0.625}{s+1} + \frac{0.14}{s+9} + 0.485\left[\frac{s}{s^2+1} + 1.26 \times \frac{1}{s^2+1}\right]$$

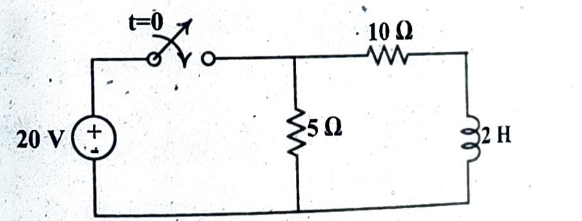
Taking Inverse Laplace Transform,

$$i(t) = -0.625\, e^{-t} + 0.14\, e^{-9t} + 0.485[\cos t + 1.26 \sin t]$$

or, $$\mathbf{i(t) = -0.625\, e^{-t} + 0.14\, e^{-9t} + 0.485 \cos t + 0.61 \sin t \;\; A}$$

Example 3.21

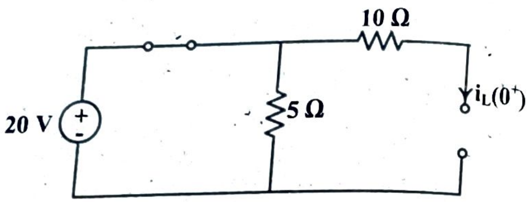
In the circuit shown in figure below, obtain an expression for voltage across the inductor if the switch is closed at t = 0 using Laplace Transform method. [2075 Ashwin]



Solution:

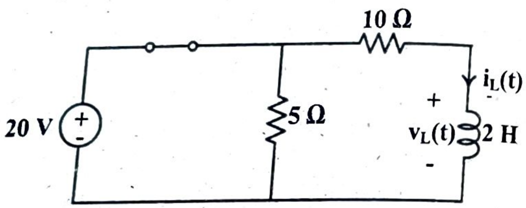
Let $i_L(t)$ and $v_L(t)$ be current through and voltage across inductor at any time t.

The equivalent circuit at $t = 0^+$ is as follows;



Here, $i_L(0^+) = 0$

The equivalent circuit for $t > 0$ is as follows;



Applying KVL in the outer loop for $t > 0$,

$$20 = 10\, i_L(t) + v_L(t)$$

But $\quad i_L(t) = \frac{1}{L}\int_{-\infty}^{t} v_L(t)\, dt = i_L(0^+) + \frac{1}{L}\int_{0}^{t} v_L(t)\, dt = \frac{1}{L}\int_{0}^{t} v_L(t)\, dt$

Then, $20 = 10 \times \frac{1}{2}\int_{0}^{t} v_L(t)\, dt + v_L(t)$

or, $\quad 20 = 5\int_{0}^{t} v_L(t)\, dt + v_L(t)$

Taking Laplace Transform on both sides,

$$\frac{20}{s} = \frac{5V_L(s)}{s} + V_L(s)$$

or, $\quad \frac{20}{s} = V_L(s)\left(\frac{5}{s} + 1\right)$

or, $\quad \frac{20}{s} = V_L(s)\left(\frac{5+s}{s}\right)$

or, $\quad V_L(s) = \frac{20}{s+5}$

Taking Inverse Laplace Transform on both sides,

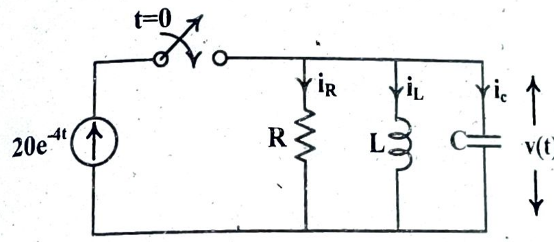
$$v_L(t) = 20\, e^{-5t}\ \text{V}$$

Example 3.22

An exponential current $i(t) = 20\, e^{-4t}$ A is suddenly applied at time $t = 0$ to a parallel RLC circuit comprising of resistor $R = \frac{1}{10}\,\Omega$, inductor $L = 10$ mH and capacitor $C = 2.5\ \mu F$. Obtain the complete particular solution for voltage v(t) across the network, by Laplace transform method. Assume zero initial current through the inductor and zero initial charge across the capacitor before application of the current. **[2075 Ashwin]**

Solution:

Let $i_R(t)$, $i_L(t)$ and $i_c(t)$ be the currents through resistor, inductor and capacitor respectively.



Given thatzero initial current through the inductor and zero initial charge across the capacitor before application of the current,

$$i_L(0^+) = 0 \qquad \text{and} \qquad v_c(0^+) = 0$$

Applying KCL for $t > 0$,

$$i_R(t) + i_L(t) + i_c(t) = 20\, e^{-4t}$$

or, $\quad \frac{v(t)}{R} + \frac{1}{L}\int_{-\infty}^{t} v(t)\, dt + C\frac{dv(t)}{dt} = 20\, e^{-4t}$

or, $\quad 10\, v(t) + i_L(0^+) + \frac{1}{L}\int_{0}^{t} v(t)\, dt + 2.5 \times 10^{-6}\frac{dv(t)}{dt} = 20\, e^{-4t}$

or, $\quad 10\, v(t) + 100\int_{0}^{t} v(t)\, dt + 2.5 \times 10^{-6}\frac{dv(t)}{dt} = 20\, e^{-4t}$

Taking Laplace Transform on both sides,

$$10\,V(s) + \frac{100\,V(s)}{s} + 2.5 \times 10^{-6}\,[sV(s) - v(0^+)] = \frac{20}{s+4}$$

or, $$10\,V(s) + \frac{100\,V(s)}{s} + 2.5 \times 10^{-6}\,sV(s) = \frac{20}{s+4}$$

or, $$V(s)\left(10 + \frac{100}{s} + 2.5 \times 10^{-6}\,s\right) = \frac{20}{s+4}$$

or, $$V(s)\left(\frac{2.5 \times 10^{-6}\,s^2 + 10s + 100}{s}\right) = \frac{20}{s+4}$$

or, $$V(s) = \frac{20s}{(s+4)(2.5 \times 10^{-6}\,s^2 + 10s + 100)}$$

or, $$V(s) = \frac{8 \times 10^6 s}{(s+4)(s^2 + 4 \times 10^6\,s + 40 \times 10^6)}$$

or, $$V(s) = \frac{8 \times 10^6 s}{(s+4)(s+10)(s+4 \times 10^6)}$$

For partial fraction expansion,

Let,

$$\frac{8 \times 10^6 s}{(s+4)(s+10)(s+4 \times 10^6)} = \frac{A}{s+4} + \frac{B}{s+10} + \frac{C}{s+4 \times 10^6}$$

Then,

$$A = \left[\frac{8 \times 10^6 s}{(s+10)(s+4 \times 10^6)}\right]_{s=-4} = -1.33$$

$$B = \left[\frac{8 \times 10^6 s}{(s+4)(s+4 \times 10^6)}\right]_{s=-10} = 3.33$$

$$C = \left[\frac{8 \times 10^6 s}{(s+4)(s+10)}\right]_{s=-4\times 10^{-6}} = -2$$

Now,

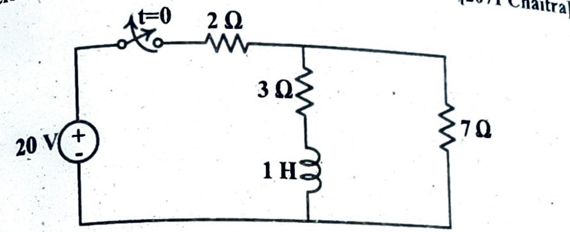
$$V(s) = \frac{-1.33}{s+4} + \frac{3.33}{s+10} + \frac{-2}{s+4 \times 10^6}$$

Taking Inverse Laplace Transform,

$$v(t) = -1.33\,e^{-4t} + 3.33\,e^{-10t} - 2\,e^{-4\times 10^{-6}\,t}\ \text{V}$$

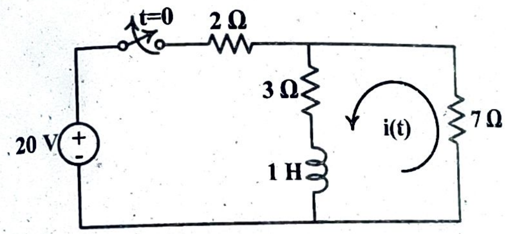
Example 3.23

Using Laplace Transform method, find the current through and voltage across inductor for t > 0 in the circuit shown in the figure below. **[2071 Chaitra]**



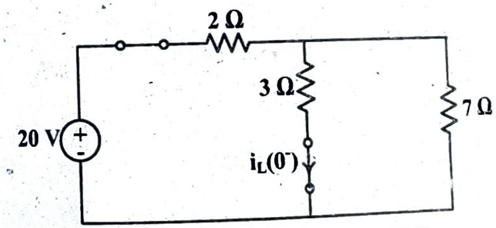
Solution:

Let the current flowing through the circuit be i(t) for t > 0 having direction as shown below;



For determining initial condition:

The equivalent circuit at $t=0^-$ is;



Here, equivalent resistance = $2 + 3 \,//\, 7 = 4.1\ \Omega$

Then, source current = $\frac{20}{4.1} = 4.878$ A

Using current divider rule,

$$i_L(0^-) = \frac{7}{7+3} \times 4.878$$

or, $$i_L(0^-) = 3.415\ \text{A}$$

Then, $i_L(0^+) = 3.415\text{ A} = i(0^+)$

Applying KVL in the circuit for $t > 0$,

$$10\times i(t) + 1\frac{di(t)}{dt} = 0$$

Taking Laplace Transform on both sides,

$$10\,I(s) + [sI(s) - i(0^+)] = 0$$

or, $(s + 10)\,I(s) = 3.415$

or, $I(s) = \dfrac{3.415}{s + 10}$

Taking Inverse Laplace Transform,

$$\mathbf{i(t) = 3.415\,e^{-10t}\ A}$$

This is the required expression for current through the inductor.

For the expression of voltage across inductor,

$$v_L(t) = L\frac{di(t)}{dt}$$

or, $v_L(t) = 1\times\dfrac{d}{dt}\left(3.415\,e^{-10t}\right)$

or, $\mathbf{v_L(t) = -34.15\,e^{-10t}\ V}$

Multiple Choice Questions

1. The Laplace Transform of $f(t) = e^{-at}$ is

   a) $\dfrac{1}{s-a}$ b) $\dfrac{1}{s+a}$ c) $\dfrac{1}{s}$ d) 1

2. The Laplace Transform of $f(t) = t$ is

   a) 1 b) $\dfrac{1}{s}$ c) $\dfrac{1}{s^2}$ d) $\dfrac{1}{s^3}$

3. The Laplace Transform of $f(t) = \sin\omega t$ is

   a) $\dfrac{1}{s}$ b) $\dfrac{1}{\omega}$ c) $\dfrac{\omega}{s^2+\omega^2}$ d) $\dfrac{s}{s^2+\omega^2}$

4. The Laplace Transform of $f(t) = \cos\omega t$ is

   a) $\dfrac{1}{s}$ b) $\dfrac{1}{\omega}$ c) $\dfrac{\omega}{s^2+\omega^2}$ b) $\dfrac{s}{s^2+\omega^2}$

ANSWERS

1.(b), 2.(c), 3.(c), 4.(d)

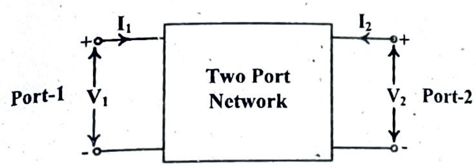
□□□

CHAPTER 4

Network Function and its Frequency Response

4.1 NETWORK FUNCTION

Generally, the electrical networks are composed of two ports, one input port and the other output port. The figure below shows the black-box model of two port network showing its input and output voltages and currents.



Here, V_1 and I_1 are voltage and current of port-1 with polarity and direction as shown in the figure. Similarly, V_2 and I_2 are voltage and current of port-2 with polarity and direction as shown in the figure.

The ratio of Laplace Transform of any two of these four variables is referred to as Network Function. The Network Function may be classified into two categories; Driving Point Functions and Transfer Functions.

Driving Point Functions involve the ratio of two different variables of the same port. If port-1 is assumed to be input port and port-2 the output port, the following four ratios or driving point functions exist.

a) Driving Point Input Impedance:

$$Z_{11} = \frac{V_1}{I_1}$$

b) Driving Point Output Impedance:

$$Z_{22} = \frac{V_2}{I_2}$$

c) Driving Point Input Admittance:

$$Y_{11} = \frac{I_1}{V_1}$$

d) Driving Point Output Admittance:

$$Y_{22} = \frac{I_2}{V_2}$$

Transfer Functions involve the ratio of two different variables of the different port. If port-1 is assumed to be input port and port-2 the output port, the following ratios or transfer functions exist:

a) Forward Transfer Impedance:

$$Z_{21} = \frac{V_2}{I_1}$$

z = V/I

b) Reverse Transfer Impedance:

$$Z_{12} = \frac{V_1}{I_2}$$

c) Forward Transfer Admittance:

Y = V/I

$$Y_{21} = \frac{I_2}{V_1}$$

d) Reverse Transfer Admittance:

$$Y_{12} = \frac{I_1}{V_2}$$

e) Forward Voltage Ratio Transfer Function (or Voltage Gain):

$$G_{21} = \frac{V_2}{V_1}$$

f) Reverse Voltage Ratio Transfer Function:

$$G_{12} = \frac{V_1}{V_2}$$

g) Forward Current Ratio Transfer Function (or Current Gain):

$$\alpha_{21} = \frac{I_2}{I_1}$$

h) Reverse Current Ratio Transfer Function:

$$\alpha_{12} = \frac{I_1}{I_2}$$

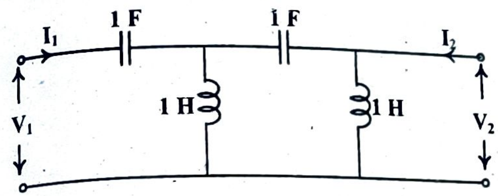
The table below shows the impedance functions of circuit elements used for computing network functions.

Circuit Elements	Impedance function Z(s)
Resistance (R) ; in Ω	R
Inductance (L) ; in H	sL
Capacitance (C) ; in F	$\frac{1}{sC}$

4.2 SOLVED PROBLEMS OF NETWORK FUNCTIONS

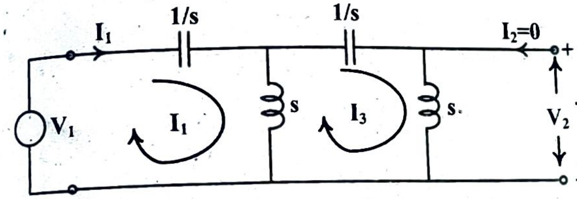
Example 4.1

Obtain the forward voltage ratio transfer function of the network shown in the figure below.



Solution:

Representing the given circuit in s-domain and adding a voltage source in port 1 and leaving port 2 as it is, we get the following circuit.



Let I_3 be the mesh current of mesh 3.

Applying KVL in mesh 1,

$$V_1 = \frac{1}{s} I_1 + s\,(I_1 - I_3)$$

G21

or, $$V_1 = \left(\frac{1}{s} + s\right) I_1 - s\,I_3$$

or, $$V_1 = \frac{s+1}{s} I_1 - s\,I_3 \;\ldots\ldots\ldots\ldots(1)$$

Applying KVL in mesh 3,

$$s\,(I_3 - I_1) + \frac{1}{s} I_3 + s\,I_3 = 0$$

or, $$-s\,I_1 + \left(\frac{1}{s} + 2s\right) I_3 = 0$$

or, $$s\,I_1 = \frac{2s^2+1}{s} I_3$$

or, $$I_1 = \frac{2s^2+1}{s^2} I_3 \;\ldots\ldots\ldots\ldots(2)$$

From equations (1) and (2),

$$V_1 = \left(\frac{s+1}{s}\right)\left(\frac{2s^2+1}{s^2}\right) I_3 - s\, I_3$$

or, $$V_1 = \left(\frac{2s^4+3s^2+1}{s^3} - s\right) I_3$$

or, $$V_1 = \left(\frac{2s^4+3s^2+1-s^4}{s^3}\right) I_3$$

or, $$V_1 = \left(\frac{s^4+3s^2+1}{s^3}\right) I_3 \quad \text{.........................(3)}$$

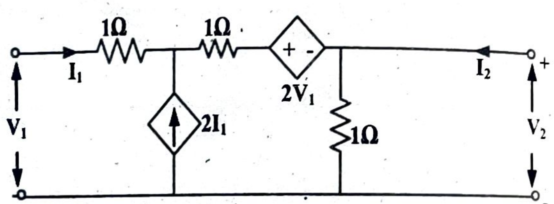
Again, $V_2 = s\, I_3$............(4)

Dividing equation (4) by (3), we get,

$$G_{21} = \frac{V_2}{V_1} = \frac{s^4}{s^4+3s^2+1}$$

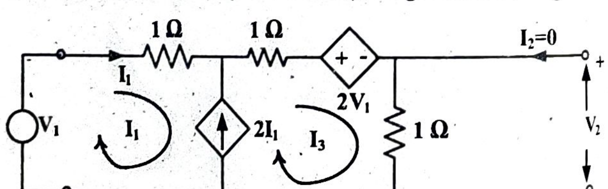
Example 4.2

For the network shown in figure below, compute forward voltage ratio transfer function.



Solution:

Adding a voltage source in port 1, leaving port 2 as it is and assuming mesh current I_3 in mesh 3 (middle mesh), we get the following circuit.



Applying KVL in super mesh 1 & 3,

$$V_1 - 2V_1 = 1\times I_1 + 1\times I_3 + 1\times I_3$$

or, $$-V_1 = I_1 + 2\, I_3$$

or, $$V_1 = -I_1 - 2I_3 \quad \text{............(1)}$$

In the common branch of mesh 1 and mesh 3,

$$I_3 - I_1 = 2I_1$$

or, $$3I_1 = I_3$$

or, $$I_1 = \frac{1}{3} I_3 \quad \text{............(2)}$$

From equations (1) and (2),

$$V_1 = -\frac{1}{3} I_3 - 2I_3$$

or, $$V_1 = -\frac{7}{3} I_3 \quad \text{............(3)}$$

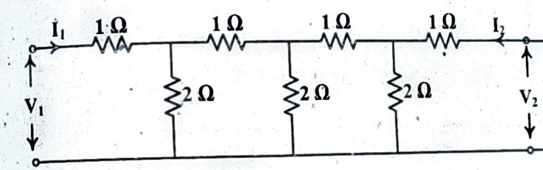
Also, $V_2 = 1\times I_3$ ....(4)

Dividing equation (4) by (3),

$$G_{21} = \frac{V_2}{V_1} = -\frac{3}{7}$$

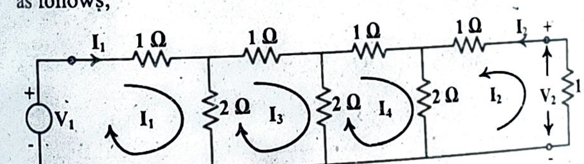
Example 4.3

For the resistor network given below, determine Z_{11}, G_{21}, Z_{21}, Y_{21} and α_{21}.



Solution:

Adding a voltage source in port 1, leaving port 2 as it is and assuming mesh currents I_3 and I_4 in mesh 3 and 4 respectively, we get the circuit as follows;



Applying KVL in mesh 1,

$$V_1 = 1\times I_1 + 2\times(I_1 - I_3)$$

or, $$V_1 = 3I_1 - 2I_3 \quad \text{..............(1)}$$

Applying KVL in mesh 2,

$$2 \times (I_2 + I_4) + 1 \times I_2 + 1 \times I_2 = 0$$

or, $2I_4 + 4I_2 = 0$

or, $I_4 = -2I_2$ ....(2)

Applying KVL in mesh 3,

$$2 \times (I_3 - I_1) + 1 \times I_3 + 2 \times (I_3 - I_4) = 0$$

or, $-2I_1 + 5I_3 - 2I_4 = 0$

or, $5I_3 = 2I_1 + 2I_4$

or, $I_3 = \frac{2}{5} I_1 + \frac{2}{5} I_4$ ....(3)

From equations (2) and (3),

$$I_3 = \frac{2}{5} I_1 + \frac{2}{5}(-2I_2)$$

or, $I_3 = \frac{2}{5} I_1 - \frac{4}{5} I_2$ ....(4)

Applying KVL in mesh 4,

$$2 \times (I_4 - I_3) + 1 \times I_4 + 2 \times (I_4 + I_2) = 0$$

or, $-2I_3 + 2I_2 + 5I_4 = 0$ ....(5)

From equations (2), (4) and (5),

$$-2\left(\frac{2}{5} I_1 - \frac{4}{5} I_2\right) + 2I_2 + 5(-2I_2) = 0$$

or, $-\frac{4}{5} I_1 + \frac{8}{5} I_2 + 2I_2 - 10I_2 = 0$

or, $-4I_1 = 32I_2$

or, $I_2 = \frac{-1}{8} I_1$ ....(6)

So, $\alpha_{21} = \frac{I_2}{I_1} = \frac{-1}{8}$

From equations (1), (4) and (6),

$$V_1 = 3I_1 - 2\left(\frac{2}{5} I_1 - \frac{4}{5} I_2\right)$$

or, $V_1 = 3I_1 - \frac{4}{5} I_1 + \frac{8}{5} I_2$

or, $V_1 = \frac{11}{5} I_1 + \frac{8}{5}\left(\frac{-1}{8} I_1\right)$

or, $V_1 = 2\,I_1$ ....(7)

So, $Z_{11} = \frac{V_1}{I_1} = 2$

Again, $V_2 = -1 \times I_2 = -1 \times \frac{-1}{8} I_1 = \frac{1}{8} I_1$ ....(8)

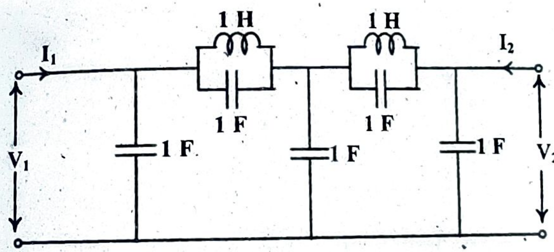
So, $Z_{21} = \frac{V_2}{I_1} = \frac{1}{8}$

Now, $G_{21} = \frac{V_2}{V_1} = \frac{\frac{1}{8} I_1}{2I_1} = \frac{1}{16}$

And, $Y_{21} = \frac{I_2}{V_1} = \frac{\frac{-1}{8} I_1}{2I_1} = -\frac{1}{16}$

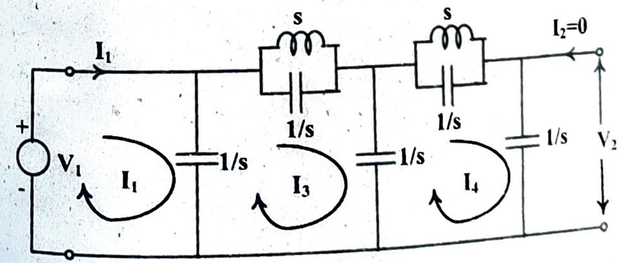
Example 4.4

For the network shown in figure below, compute forward voltage ratio transfer function.



Solution:

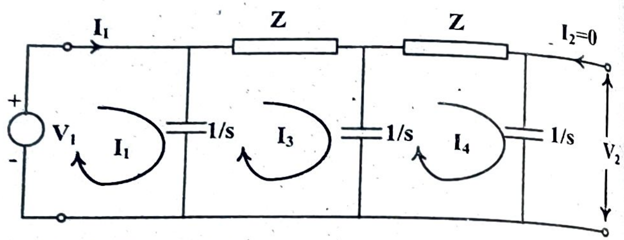
Representing the given circuit in s-domain, adding a voltage source in port 1, leaving port 2 as it is and assuming mesh currents I_3 and I_4 in mesh 3 and 4 respectively, we get the following circuit.



Representing the parallel combination of 1 H inductor and 1 F capacitor by impedance Z,

$$Z = \frac{s \times 1/s}{s + 1/s} = \frac{s}{s^2 + 1}$$

Then, the above circuit becomes as follows;



Applying KVL in mesh 1,

$$V_1 = \frac{1}{s} I_1 - \frac{1}{s} I_3 \quad \text{............(1)}$$

Applying KVL in mesh 3,

$$\frac{1}{s}(I_3 - I_1) + Z \times I_3 + \frac{1}{s}(I_3 - I_4) = 0$$

or, $$-\frac{1}{s} I_1 + \left(\frac{2}{s} + \frac{s}{s^2+1}\right) I_3 - \frac{1}{s} I_4 = 0$$

or, $$-\frac{1}{s} I_1 + \left(\frac{2s^2 + 2 + s^2}{s(s^2+1)}\right) I_3 - \frac{1}{s} I_4 = 0$$

or, $$\left(\frac{3s^2+2}{s(s^2+1)}\right) I_3 = \frac{1}{s}(I_1 + I_4)$$

or, $$I_3 = \left(\frac{s^2+1}{3s^2+2}\right)(I_1 + I_4) \quad \text{...........(2)}$$

Applying KVL in mesh 4,

$$\frac{1}{s}(I_4 - I_3) + Z \times I_4 + \frac{1}{s} I_4 = 0$$

or, $$\frac{1}{s} I_3 = \left(\frac{2}{s} + \frac{s}{s^2+1}\right) I_4$$

or, $$\frac{1}{s} I_3 = \left(\frac{3s^2+2}{s(s^2+1)}\right) I_4$$

or, $$I_3 = \left(\frac{3s^2+2}{s^2+1}\right) I_4 \quad \text{...........(3)}$$

Equating equations (2) and (3),

$$\left(\frac{s^2+1}{3s^2+2}\right)(I_1 + I_4) = \left(\frac{3s^2+2}{s^2+1}\right) I_4$$

or, $$I_1 + I_4 = \left(\frac{3s^2+2}{s^2+1}\right)^2 I_4$$

or, $$I_1 = \frac{(3s^2+2)^2 - (s^2+1)^2}{(s^2+1)^2} I_4$$

or, $$I_1 = \frac{8s^4 + 10s^2 + 3}{(s^2+1)^2} I_4 \quad \text{...........(4)}$$

From equations (1), (3) and (4),

$$V_1 = \frac{1}{s}\left(\frac{8s^4 + 10s^2 + 3}{(s^2+1)^2}\right) I_4 - \frac{1}{s}\left(\frac{3s^2+2}{s^2+1}\right) I_4$$

or, $$V_1 = \frac{8s^4 + 10s^2 + 3 - (3s^2+2)(s^2+1)}{s(s^2+1)^2} I_4$$

or, $$V_1 = \frac{8s^4 + 10s^2 + 3 - 3s^4 - 5s^2 - 2}{s(s^2+1)^2} I_4$$

or, $$V_1 = \frac{5s^4 + 5s^2 + 1}{s(s^2+1)^2} I_4 \quad \text{..............(5)}$$

Also, $V_2 = \frac{1}{s} I_4$ ....(6)

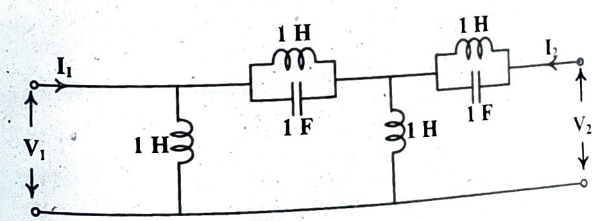
Dividing equation (6) by (5),

$$G_{21} = \frac{V_2}{V_1} = \frac{\frac{1}{s}}{\frac{5s^4 + 5s^2 + 1}{s(s^2+1)^2}} = \frac{(s^2+1)^2}{5s^4 + 5s^2 + 1}$$

which is the required forward voltage ratio transfer function.

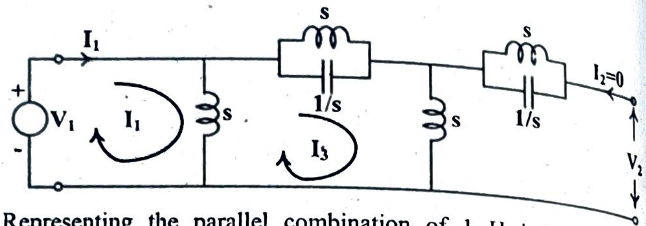
Example 4.5

Find the forward voltage ratio transfer function $G_{21}(s)$ in the circuit shown in the figure below. [2068 Shrawan]



Solution:

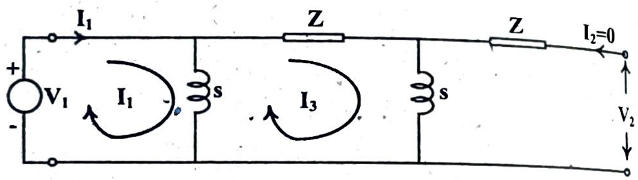
Representing the given circuit in s–domain, adding a voltage source in port 1, leaving port 2 as it is and assuming mesh current I_3 in mesh 3, we get the following circuit.



Representing the parallel combination of 1 H inductor and 1 F capacitor by impedance Z,

$$Z = \frac{s \times 1/s}{s + 1/s} = \frac{s}{s^2 + 1}$$

Then, the above circuit becomes as follows;



Applying KVL in mesh 1,

$$V_1 = sI_1 - sI_3 \quad \text{............(1)}$$

Applying KVL in mesh 3,

$$s(I_3 - I_1) + sI_3 + ZI_3 = 0$$

or, $$-sI_1 + \left(2s + \frac{s}{s^2 + 1}\right) I_3 = 0$$

or, $$sI_1 = \left(\frac{2s^3 + 3s}{s(s^2 + 1)}\right) I_3$$

or, $$I_1 = \left(\frac{2s^2 + 3}{s^2 + 1}\right) I_3 \quad \text{..............(2)}$$

Substituting expression of I_1 from equation (2) to (1),

$$V_1 = s\left(\frac{2s^2 + 3}{s^2 + 1}\right) I_3 - sI_3$$

or, $$V_1 = \frac{s(s^2 + 2)}{(s^2 + 1)} I_3 \quad \text{..............(3)}$$

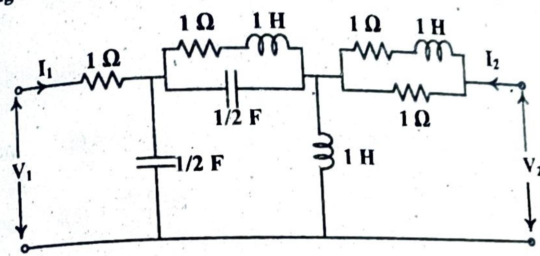
Also, $V_2 = sI_3$ ....(4)

Dividing equations (4) by (3),

$$G_{21} = \frac{V_2}{V_1} = \frac{s^2 + 1}{s^2 + 2}$$

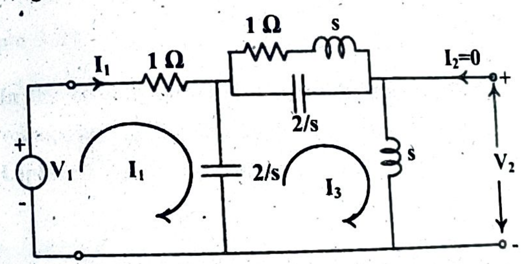
Example 4.6

Find the forward voltage ratio transfer function $G_{21}(s)$ and forward transfer admittance $Y_{21}(s)$ in the circuit shown in the figure below. [2074 Chaitra]

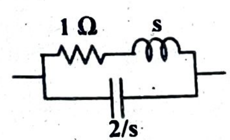


Solution:

Representing the given circuit in s-domain, adding a voltage source in port 1, leaving port 2 as it is and assuming mesh current I_3 in mesh 3, we get the following circuit.

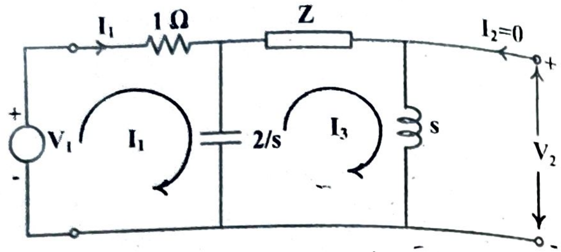


Representing the following branch by Z;



$$Z = (s + 1) // \frac{2}{s} = \frac{(s + 1)\frac{2}{s}}{s + 1 + \frac{2}{s}} = \frac{2(s + 1)}{s^2 + s + 2}$$

Now, the circuit gets reduced as follows;



Applying KVL in mesh 1,

$$V_1 = 1 \times I_1 + \frac{2}{s} \times (I_1 - I_3)$$

or, $$V_1 = \left(1 + \frac{2}{s}\right) I_1 - \frac{2}{s} I_3 \quad \text{.........(1)}$$

Applying KVL in mesh 3,

$$\frac{2}{s} \times (I_3 - I_1) + Z \times I_3 + s \times I_3 = 0$$

or, $$-\frac{2}{s} I_1 + \left[\frac{2}{s} + \frac{2(s+1)}{s^2+s+2} + s\right] I_3 = 0$$

or, $$\frac{2}{s} I_1 = \frac{2s^2 + 2s + 4 + 2s^2 + 2s + s^2(s^2+s+2)}{s(s^2+s+2)} I_3$$

or, $$I_1 = \frac{s^4 + s^3 + 6s^2 + 4s + 4}{2(s^2+s+2)} I_3 \quad \text{...........(2)}$$

Substituting the expression of I_1 from equation (2) to (1),

$$V_1 = \left(1 + \frac{2}{s}\right)\left[\frac{s^4 + s^3 + 6s^2 + 4s + 4}{2(s^2+s+2)}\right] I_3 - \frac{2}{s} I_3$$

or, $$V_1 = \frac{(s+2)(s^4 + s^3 + 6s^2 + 4s + 4) - 4(s^2+s+2)}{2s(s^2+s+2)} I_3 \quad \text{..........(3)}$$

Also, $V_2 = s\, I_3$ ....(4)

Dividing equation (4) by (3), we get voltage ratio transfer function as;

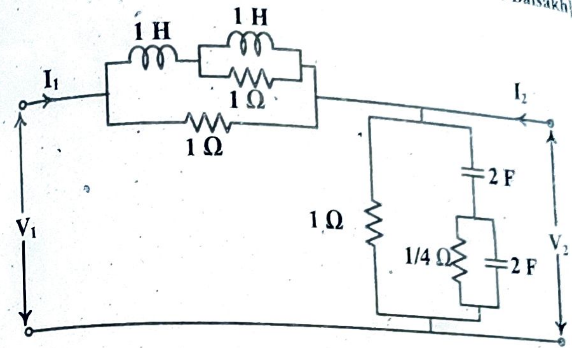
$$G_{21} = \frac{V_2}{V_1} = \frac{s}{\dfrac{(s+2)(s^4 + s^3 + 6s^2 + 4s + 4) - 4(s^2+s+2)}{2s(s^2+s+2)}}$$

or, $$G_{21} = \frac{2s(s^2+s+2)}{s^4+3s^3+8s^2+12s+8}$$

Example 4.7

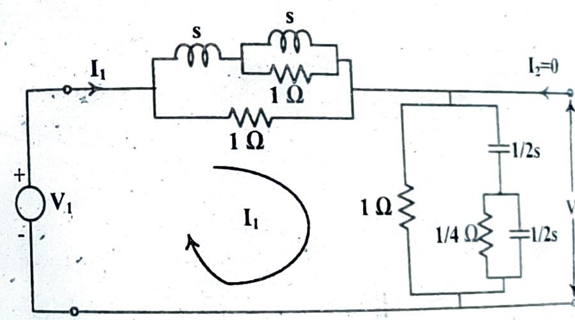
For the two port network shown in the figure below, find the driving point impedance of port one and the voltage ratio transfer function.

[2068 Baisakh]

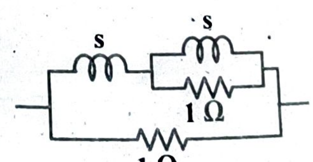


Solution:

Representing the given circuit in s–domain and adding a voltage source in port 1 and leaving port 2 as it is, we get the following circuit.

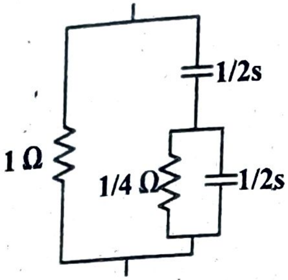


Representing the following branch by Z_1;



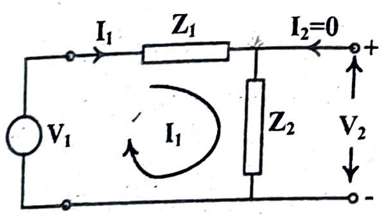
$$Z_1 = [s + s\,//\,1]\,//\,1 = \left[s + \frac{s\times 1}{s+1}\right]//\,1 = \frac{s^2+2s}{s+1}\,//\,1 = \frac{s^2+2s}{s^2+3s+1}$$

Representing the following branch by Z_2;



$$Z_2 = \left[\left(\frac{1}{4}\,//\,\frac{1}{2s}\right) + \frac{1}{2s}\right]//\,1 = \left[\frac{1}{2(s+2)} + \frac{1}{2s}\right]//\,1 = \frac{s+1}{s^2+3s+1}$$

Now, the circuit gets reduces as follows;



Applying KVL in the circuit,

$$V_1 = (Z_1 + Z_2)\, I_1$$

or, $\dfrac{V_1}{I_1} = Z_{11} = Z_1 + Z_2 = \dfrac{s^2+2s}{s^2+3s+1} + \dfrac{s+1}{s^2+3s+1} = 1$; which is the required driving point input impedance of port one.

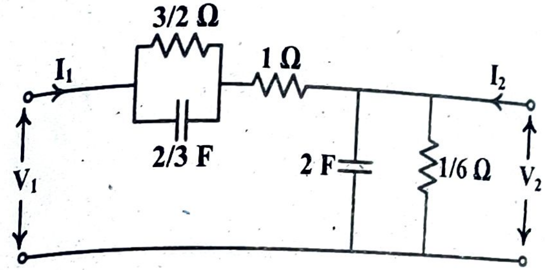
Also, $V_2 = Z_2\, I_1$

Hence, $G_{21} = \dfrac{V_2}{V_1} = \dfrac{Z_2}{Z_1+Z_2} = \dfrac{\dfrac{s+1}{s^2+3s+1}}{\dfrac{s^2+2s}{s^2+3s+1} + \dfrac{s+1}{s^2+3s+1}} = \dfrac{s+1}{s^2+3s+1}$;

which is the required voltage ratio transfer function.

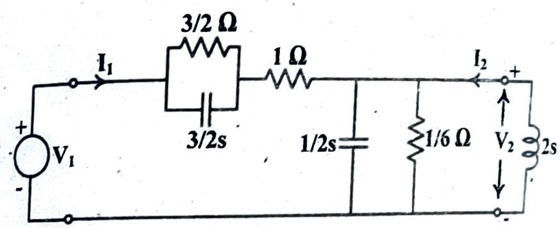
Example 4.8

Find the voltage ratio transfer function of the two port network shown in the figure below, if the port 2 is terminated with 2 H inductor. [2075 Ashwin]



Solution:

Representing circuit in s-domain, adding source in port 1 and terminating port 2 by 2 H inductor, we get the circuit as follows;



Denoting series combination of 1 Ω resistor and parallel combination of 3/2 Ω and 2/3 F by Z_1;

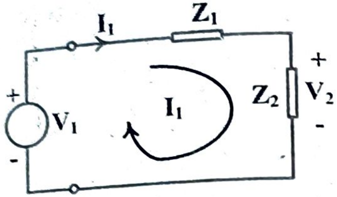
$$Z_1 = \left(\frac{3}{2}\,//\,\frac{3}{2s}\right) + 1 = \frac{\frac{3}{2}\times\frac{3}{2s}}{\frac{3}{2}+\frac{3}{2s}} + 1 = \frac{2s+5}{2(s+1)}$$

Denoting parallel combination of 2F capacitor, 1/6 Ω resistor and 2 H inductor by Z_2;

$$\frac{1}{Z_2} = \frac{1}{1/2s} + \frac{1}{1/6} + \frac{1}{2s} = 2s + 6 + \frac{1}{2s} = \frac{4s^2+12s+1}{2s}$$

or, $$Z_2 = \frac{2s}{4s^2+12s+1}$$

Now, the circuit becomes as follows;



Using Voltage Divider Rule in the above series circuit,

$$V_2 = \frac{Z_2}{Z_1 + Z_2} V_1$$

$$\text{or,} \quad \frac{V_2}{V_1} = \frac{Z_2}{Z_1 + Z_2} = \frac{\dfrac{2s}{4s^2 + 12s + 1}}{\dfrac{2s + 5}{2(s + 1)} + \dfrac{2s}{4s^2 + 12s + 1}}$$

$$\text{or,} \quad G_{21} = \frac{2s \times 2(s + 1)}{(2s + 5)(4s^2 + 12s + 1) + 2s \times 2(s + 1)}$$

$$\text{or,} \quad G_{21} = \frac{4s(s + 1)}{8s^3 + 48s^2 + 66s + 5}$$

which is the required voltage ratio transfer function.

4.3 FREQUENCY RESPONSE OF NETWORKS

The response of the network when the input frequency ω is changed over a certain range is called frequency response of the network.

Let's consider a network function N(s). If we consider the case of sinusoidal steady state only then we can replace s by $j\omega$. This $N(j\omega)$ is a complex frequency and can be written in rectangular co-ordinates as below.

$$N(j\omega) = R(\omega) + j\,X(\omega).$$

where $R(\omega)$ is the real part of the network function $N(j\omega)$ while $X(\omega)$ is the imaginary part.

The network function $N(j\omega)$ can also be written in polar form as below:

$$N(j\omega) = |N(j\omega)|\, e^{j\phi(\omega)}$$

where $|N(j\omega)|$ is the magnitude of the network function $N(j\omega)$ and $\phi(\omega)$ is its phase angle.

4.4 BODE PLOT

The scientist H.W. Bode suggested a specific method to obtain the frequency response in which logarithmic values are used. In general, Bode plot consists of two plots:

a) Magnitude plot in which decibel values of $|N(j\omega)|$ are plotted against logarithmic values of ω.

b) Phase plot in which $\phi(\omega)$ in degrees are plotted against logarithmic values of ω.

4.5 SOLVED PROBLEMS OF BODE PLOTS

Example 4.9

For the network function given below draw the asymptotic bode-plot.

$$N(s) = \frac{210(s^2 + 45s + 200)}{s(s + 20)(s^2 + 80s + 700)}$$ **[2070 Ashad]**

Solution:

The given network function is;

$$N(s) = \frac{210(s + 5)(s + 40)}{s(s + 20)(s + 10)(s + 70)}$$

$$\text{or,} \quad N(s) = \frac{210 \times 5 \times 40 \left(1 + \frac{s}{5}\right)\left(1 + \frac{s}{40}\right)}{s \times 20 \times 10 \times 70 \times \left(1 + \frac{s}{20}\right)\left(1 + \frac{s}{10}\right)\left(1 + \frac{s}{70}\right)}$$

$$\text{or,} \quad N(s) = \frac{3\left(1 + \frac{s}{5}\right)\left(1 + \frac{s}{40}\right)}{s\left(1 + \frac{s}{20}\right)\left(1 + \frac{s}{10}\right)\left(1 + \frac{s}{70}\right)}$$

Replacing s by $j\omega$, we get

$$N(j\omega) = \frac{3\left(1 + \frac{j\omega}{5}\right)\left(1 + \frac{j\omega}{40}\right)}{j\omega\left(1 + \frac{j\omega}{20}\right)\left(1 + \frac{j\omega}{10}\right)\left(1 + \frac{j\omega}{70}\right)}$$

The corner frequencies are: 5(+), 10(–), 20(–), 40(+), 70(–)

Starting frequency = 0.1 rad/s

Magnitude plot:

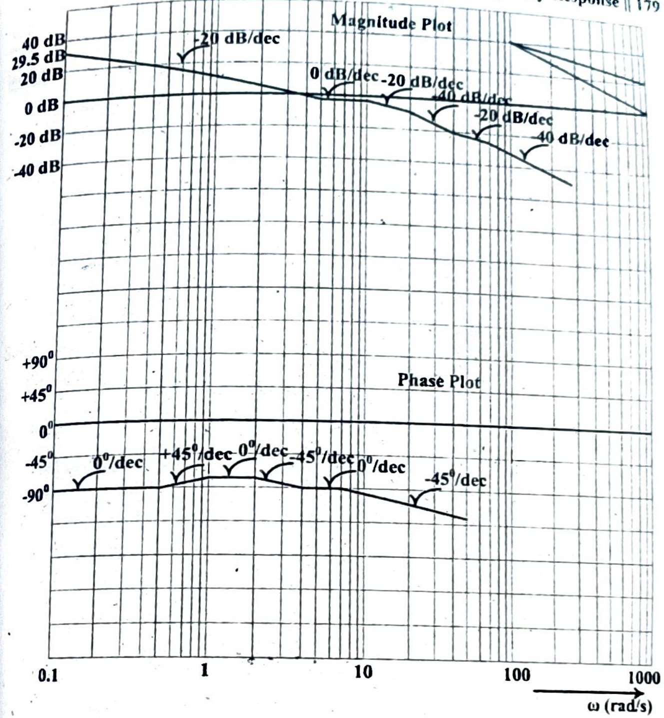
Starting magnitude = $20 \log \left|\frac{3}{j\omega}\right|_{\omega=0.1} = 20 \log\left(\frac{3}{0.1}\right) = 29.542$ dB

Factors	Corner frequency	Individual Slope	Final Slope
$3[j\omega]^{-1}$	low(–)	–20dB/decade	–20dB/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	+20dB/decade	0dB/decade
$\left[1+\frac{j\omega}{10}\right]^{-1}$	10(–)	–20dB/decade	–20dB/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(–)	–20dB/decade	–40dB/decade
$\left[1+\frac{j\omega}{40}\right]^{+1}$	40(+)	+20dB/decade	–20dB/decade
$\left[1+\frac{j\omega}{70}\right]^{-1}$	70(–)	–20dB/decade	–40dB/decade

Phase plot:

Starting phase = –90°

Factors	Corner frequency	Effective frequency	Individual Slope	Final Slope
$3[j\omega]^{-1}$	low(–)	low(–)	0°/decade	0°/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	0.5(+)	+45°/decade	+45°/decade
$\left[1+\frac{j\omega}{10}\right]^{-1}$	10(–)	1(–)	–45°/decade	0°/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(–)	2(–)	–45°/decade	–45°/decade
$\left[1+\frac{j\omega}{40}\right]^{+1}$	40(+)	4(+)	+45°/decade	0°/decade
$\left[1+\frac{j\omega}{70}\right]^{-1}$	70(–)	7(–)	–45°/decade	–45°/decade



Example 4.10

Sketch the asymptotic Bode plots for the transfer function given by;

$$G(s) = \frac{20(s+5)}{s(s^2+2s+10)(s^2+21s+20)}$$ **[2068 Shrawan]**

Solution:

The given network function is;

$$G(s) = \frac{20\ (s+5)}{s\ (s^2+2s+10)\ (s^2+21s+20)}$$

or, $$G(s) = \frac{20\ (s+5)}{s\ (s+1)\ (s+20)\ (s^2+21s+20)}$$

$$\text{or,}\quad G(s) = \frac{20 \times 5\left(1 + \frac{s}{5}\right)}{s \times 20 \times 10 \times (1+s)\left(1 + \frac{s}{20}\right)\left(1 + \frac{2s}{10} + \frac{s^2}{10}\right)}$$

$$\text{or,}\quad G(s) = \frac{0.5\left(1 + \frac{s}{5}\right)}{s\,(1+s)\left(1 + \frac{s}{20}\right)\left(1 + \frac{2s}{10} + \frac{s^2}{10}\right)}$$

Replacing s by jω, we get

$$G(j\omega) = \frac{0.5\left(1 + \frac{j\omega}{5}\right)}{j\omega\,(1+j\omega)\left(1 + \frac{j\omega}{20}\right)\left(1 + \frac{2j\omega}{10} + \frac{(j\omega)^2}{10}\right)}$$

The corner frequencies are:

$$1(-), \sqrt{10} = 3.16(—), 5(+), 20(-)$$

Starting frequency = 0.1 rad/s

Magnitude plot:

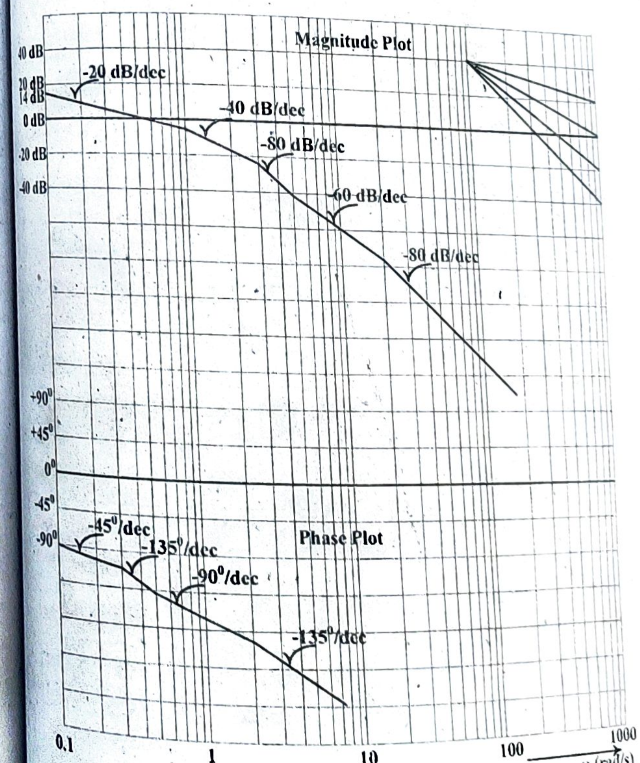
Starting magnitude $= 20\log\left|\frac{0.5}{j\omega}\right|_{\omega=0.1} = 20\log\left(\frac{0.5}{0.1}\right) = 14$ dB

Factors	Corner frequency	Individual Slope	Final Slope
$0.5[j\omega]^{-1}$	low(–)	–20dB/decade	–20dB/decade
$[1+j\omega]^{-1}$	1(–)	–20dB/decade	–40dB/decade
$\left[1+\frac{2j\omega}{10}+\frac{(j\omega)^2}{10}\right]^{-1}$	3.16(—)	–40dB/decade	–80dB/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	+20dB/decade	–60dB/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(–)	–20dB/decade	–80dB/decade

Phase plot:

Starting phase = –90°

Factors	Corner frequency	Effective frequency	Individual Slope	Final Slope
$0.5[j\omega]^{-1}$	low(–)	low(–)	0°/decade	0°/decade
$[1+j\omega]^{-1}$	1(–)	0.1(–)	–45°/decade	–45°/decade
$\left[1+\frac{2j\omega}{10}+\frac{(j\omega)^2}{10}\right]^{-1}$	3.16(—)	0.316(—)	–90°/decade	–135°/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	0.5(+)	+45°/decade	–90°/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(–)	2(–)	–45°/decade	–135°/decade



Example 4.11

Sketch the asymptotic Bode plots for the transfer function given by;

$$G(s) = \frac{10s(s+3)}{(s+1)(s^2+2s+16)}$$ [2067 Ashad]

Solution:

The given network function is;

$$G(s) = \frac{10s(s+3)}{(s+1)(s^2+2s+16)}$$

or, $$G(s) = \frac{10 \times 3\, s\left(1+\frac{s}{3}\right)}{16 \times (1+s)\left(1+\frac{2s}{16}+\frac{s^2}{16}\right)}$$

or, $$G(s) = \frac{1.875\, s\left(1+\frac{s}{3}\right)}{(1+s)\left(1+\frac{2s}{16}+\frac{s^2}{16}\right)}$$

Replacing s by jω, we get

$$G(j\omega) = \frac{1.875\,(j\omega)\left(1+\frac{j\omega}{3}\right)}{(1+j\omega)\left(1+\frac{2j\omega}{16}+\frac{(j\omega)^2}{16}\right)}$$

The corner frequencies are: 1(−), 3(+), $\sqrt{16} = 4$(—)

Starting frequency = 0.1 rad/s

Magnitude plot:

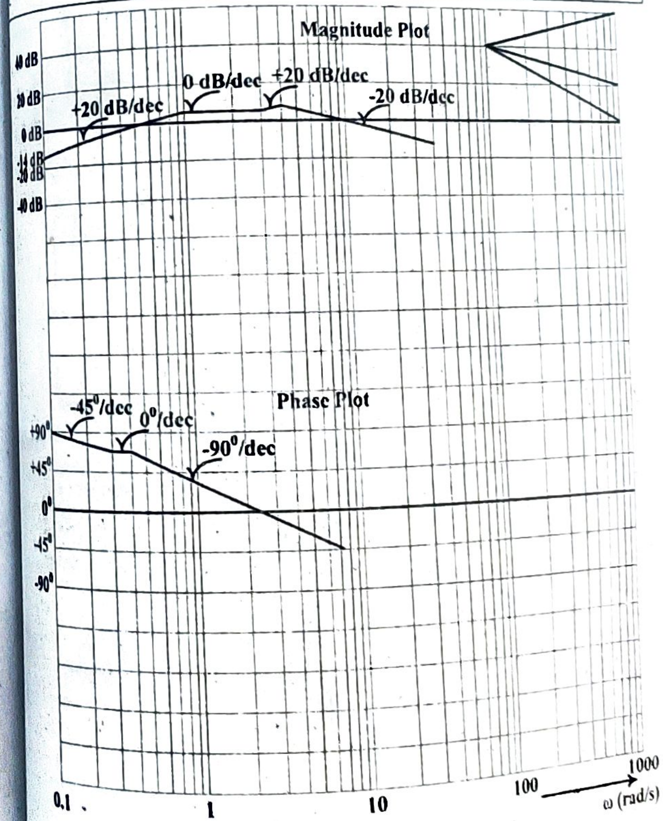
Starting magnitude = $20 \log |1.875j\omega|_{\omega=0.1} = 20 \log (1.875 \times 0.1) =$ −14.54 dB

Factors	Corner frequency	Individual Slope	Final Slope
$1.875[j\omega]^{+1}$	low(+)	+20dB/decade	+20dB/decade
$[1+j\omega]^{-1}$	1(−)	−20dB/decade	0 dB/decade
$\left[1+\frac{j\omega}{3}\right]^{+1}$	3(+)	+20dB/decade	+20dB/decade
$\left[1+\frac{2j\omega}{16}+\frac{(j\omega)^2}{16}\right]^{-1}$	4(—)	−40dB/decade	−20dB/decade

Phase plot:

Starting phase = +90°

Factors	Corner frequency	Effective frequency	Individual Slope	Final Slope
$1.875[j\omega]^{+1}$	low(+)	low(+)	0°/decade	0°/decade
$[1+j\omega]^{-1}$	1(−)	0.1(−)	−45°/decade	−45°/decade
$\left[1+\frac{j\omega}{3}\right]^{+1}$	3(+)	0.3(+)	+45°/decade	0°/decade
$\left[1+\frac{2j\omega}{16}+\frac{(j\omega)^2}{16}\right]^{-1}$	4(−)	0.4(−)	−90°/decade	−90°/decade



Example 4.12

For the network function given below draw the asymptotic bode plot.

$$N(s) = \frac{10(s+10)}{s(s+40)(s^2+5s+4)}$$

[2068 Baisakh]

Solution:

The given network function is;

$$N(s) = \frac{10(s+10)}{s(s+40)(s^2+5s+4)}$$

or, $$N(s) = \frac{10(s+10)}{s(s+1)(s+4)(s+40)}$$

or, $$N(s) = \frac{10 \times 10\left(1+\frac{s}{10}\right)}{s \times 40 \times 4 \times (1+s)\left(1+\frac{s}{4}\right)\left(1+\frac{s}{40}\right)}$$

or, $$N(s) = \frac{0.625\left(1+\frac{s}{10}\right)}{s\,(1+s)\left(1+\frac{s}{4}\right)\left(1+\frac{s}{40}\right)}$$

Replacing s by jω, we get

$$N(j\omega) = \frac{0.625\left(1+\frac{j\omega}{10}\right)}{j\omega\,(1+j\omega)\left(1+\frac{j\omega}{4}\right)\left(1+\frac{j\omega}{40}\right)}$$

The corner frequencies are: 1(–), 4(–), 10(+), 40(–)

Starting frequency = 0.1 rad/s

Magnitude plot:

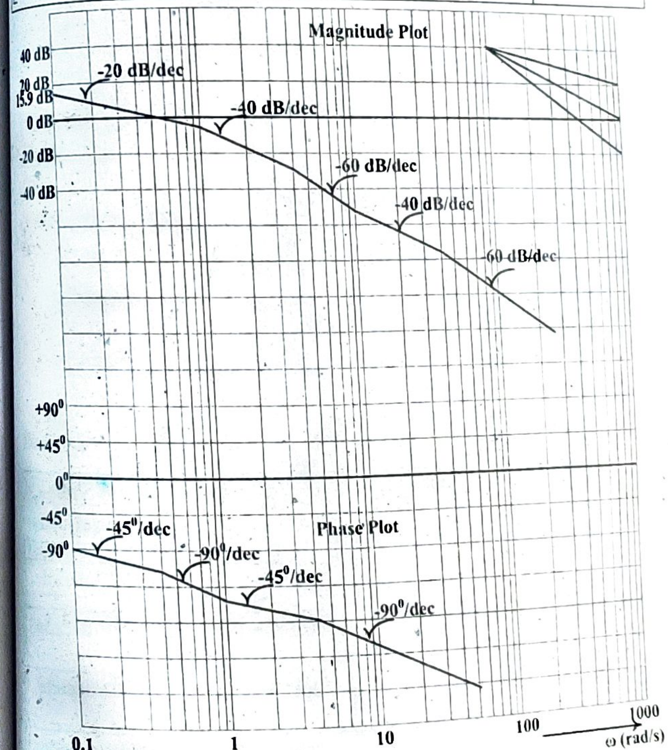
Starting magnitude $= 20\log\left|\frac{0.625}{j\omega}\right|_{\omega=0.1} = 20\log\left(\frac{0.625}{0.1}\right) = 15.92\text{ dB}$

Factors	Corner frequency	Individual Slope	Final Slope
$0.625[j\omega]^{-1}$	low(–)	–20dB/decade	–20dB/decade
$[1+j\omega]^{-1}$	1(–)	–20dB/decade	–40dB/decade
$\left[1+\frac{j\omega}{4}\right]^{-1}$	4(–)	–20dB/decade	–60dB/decade
$\left[1+\frac{j\omega}{10}\right]^{+1}$	10(+)	+20dB/decade	–40dB/decade
$\left[1+\frac{j\omega}{40}\right]^{-1}$	40(–)	–20dB/decade	–60dB/decade

Phase plot:

Starting phase = –90°

Factors	Corner frequency	Effective frequency	Individual Slope	Final Slope
$0.625[j\omega]^{-1}$	low(–)	low(–)	0°/decade	0°/decade
$[1+j\omega]^{-1}$	1(–)	0.1(–)	–45°/decade	–45°/decade
$\left[1+\frac{j\omega}{4}\right]^{-1}$	4(–)	0.4(–)	–45°/decade	–90°/decade
$\left[1+\frac{j\omega}{10}\right]^{+1}$	10(+)	1(+)	+45°/decade	–45°/decade
$\left[1+\frac{j\omega}{40}\right]^{-1}$	40(–)	4(–)	–45°/decade	–90°/decade



Example 4.13

For the network function given below draw the asymptotic bode-plot.

$$G(s) = \frac{20(s+5)}{s^2(s+20)(s^2+41s+40)}$$ [2064 Falgun]

Solution:

The given network function is;

$$G(s) = \frac{20(s+5)}{s^2(s+20)(s^2+41s+40)}$$

or, $$G(s) = \frac{20(s+5)}{s^2(s+1)(s+20)(s+40)}$$

or, $$G(s) = \frac{20\times 5\left(1+\frac{s}{5}\right)}{s^2\times 20\times 40\times(1+s)\left(1+\frac{s}{20}\right)\left(1+\frac{s}{40}\right)}$$

or, $$G(s) = \frac{0.125\left(1+\frac{s}{5}\right)}{s^2(1+s)\left(1+\frac{s}{20}\right)\left(1+\frac{s}{40}\right)}$$

Replacing s by jω, we get

$$G(j\omega) = \frac{0.125\left(1+\frac{j\omega}{5}\right)}{(j\omega)^2(1+j\omega)\left(1+\frac{j\omega}{20}\right)\left(1+\frac{j\omega}{40}\right)}$$

The corner frequencies are:

1(–), 5(+), 20(–), 40(–)

Starting frequency = 0.1 rad/s

Magnitude plot:

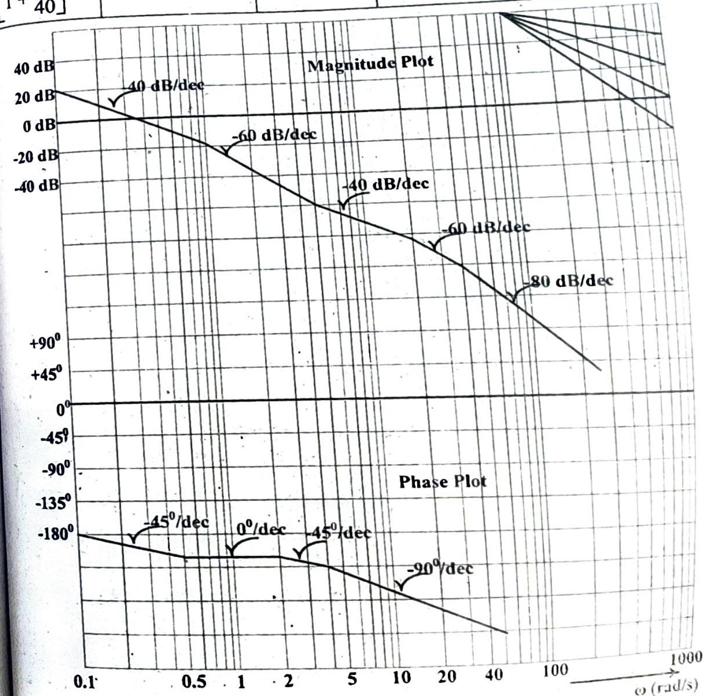
Starting magnitude = $20\log\left|\frac{0.125}{(j\omega)^2}\right|_{\omega=0.1} = 20\log\left(\frac{0.125}{(0.1)^2}\right) = 21.94$ dB

Factors	Corner frequency	Individual Slope	Final Slope
$0.125[j\omega]^{-2}$	low(–)	–40dB/decade	–40dB/decade
$[1+j\omega]^{-1}$	1(–)	–20dB/decade	–60dB/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	+20dB/decade	–40dB/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(–)	–20dB/decade	–60dB/decade
$\left[1+\frac{j\omega}{40}\right]^{-1}$	40(–)	–20dB/decade	–80dB/decade

Phase plot:

Starting phase = –180°

Factors	Corner frequency	Effective frequency	Individual Slope	Final Slope
$0.125[j\omega]^{-2}$	low(–)	low(–)	0°/decade	0°/decade
$[1+j\omega]^{-1}$	1(–)	0.1(–)	–45°/decade	–45°/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	0.5(+)	+45°/decade	0°/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(–)	2(–)	–45°/decade	–45°/decade
$\left[1+\frac{j\omega}{40}\right]^{-1}$	40(–)	4(–)	–45°/decade	–90°/decade



Example 4.14

For the network function given below draw the asymptotic bode-plots.

$$G(s) = \frac{20(s+5)}{(s+10)(s^2+21s+20)}$$ [2066 Bhadra]

Solution:

The given network function is;

$$G(s) = \frac{20(s+5)}{(s+10)(s^2+21s+20)}$$

or, $$G(s) = \frac{20(s+5)}{(s+1)(s+10)(s+20)}$$

or, $$G(s) = \frac{20 \times 5\left(1+\frac{s}{5}\right)}{20 \times 10 \times (1+s)\left(1+\frac{s}{10}\right)\left(1+\frac{s}{20}\right)}$$

or, $$G(s) = \frac{0.5\left(1+\frac{s}{5}\right)}{(1+s)\left(1+\frac{s}{10}\right)\left(1+\frac{s}{20}\right)}$$

Replacing s by jω, we get

$$G(j\omega) = \frac{0.5\left(1+\frac{j\omega}{5}\right)}{(1+j\omega)\left(1+\frac{j\omega}{10}\right)\left(1+\frac{j\omega}{20}\right)}$$

The corner frequencies are:

1(–), 5(+), 10(–), 20(–)

Starting frequency = 0.1 rad/s

Magnitude plot:

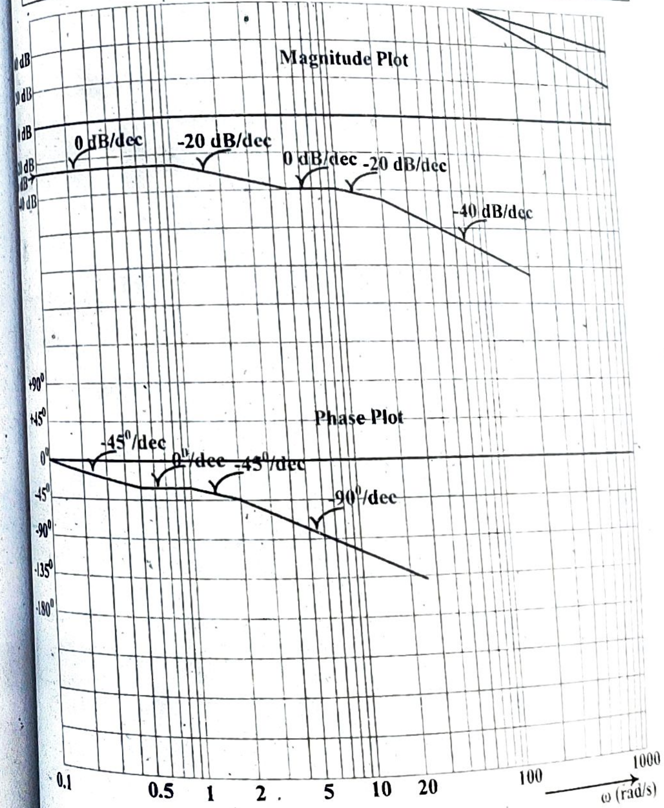
Starting magnitude = 20 log |0.5| = – 6.02 dB

Factors	Corner frequency	Individual Slope	Final Slope
0.5	low	0dB/decade	0dB/decade
$[1+j\omega]^{-1}$	1(–)	–20dB/decade	–20dB/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	+20dB/decade	0dB/decade
$\left[1+\frac{j\omega}{10}\right]^{-1}$	10(–)	–20dB/decade	–20dB/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(–)	–20dB/decade	–40dB/decade

Phase plot:

Starting phase = 0°

Factors	Corner frequency	Effective frequency	Individual Slope	Final Slope
0.5	low	low	0°/decade	0°/decade
$[1+j\omega]^{-1}$	1(–)	0.1(–)	–45°/decade	–45°/decade
$\left[1+\frac{j\omega}{5}\right]^{+1}$	5(+)	0.5(+)	+45°/decade	0°/decade
$\left[1+\frac{j\omega}{10}\right]^{-1}$	10(–)	1(–)	–45°/decade	–45°/decade
$\left[1+\frac{j\omega}{20}\right]^{-1}$	20(–)	2(–)	–45°/decade	–90°/decade



Multiple Choice Questions

1. The impedance function of inductance (L) is
   a) L b) sL
   c) $\frac{1}{sL}$ d) none of these
2. The impedance function of capacitance (C) is
   a) C b) sC
   c) $\frac{1}{sC}$ d) none of these
3. The impedance function of resistance (R) is
   a) R b) sR
   c) $\frac{1}{sR}$ d) none of these
4. Bode plot is used to represent
   a) Time Response b) Frequency Response
   c) Both of these d) none
5. In any network function N(s), if s is equal to anyone of the zeros, then N(s) =
   a) 1 b) 2
   c) 0 d) ∞
6. In any network function N(s), if s is equal to anyone of the poles, then N(s) =
   a) 1 b) 2
   c) 0 d) ∞

ANSWERS

1.(b), 2.(c), 3.(a), 4.(b), 5.(a), 6.(d)

❑❑❑

CHAPTER 5

Fourier Series

5.1 INTRODUCTION

Fourier Series is a way of representing any periodic signal as weighted sum of large (possibly infinite) number of sinusoidal signals of different frequencies which are integral multiple of frequency of original signal. These weighting components are referred to as Fourier Coefficients.

For any periodic signal x(t) with time period T, the Fourier Series can be expressed in **trigonometric form** as;

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

where, a_0, a_n and b_n are Fourier Coefficients and can be computed as follows;

$$a_0 = \frac{2}{T}\int_0^T x(t)\, dt$$

$$a_n = \frac{2}{T}\int_0^T x(t) \cos n\omega t\, dt$$

$$b_n = \frac{2}{T}\int_0^T x(t) \sin n\omega t\, dt$$

If the independent variable is given as ωt, then the coefficients are computed as follows;

$$a_0 = \frac{1}{\pi}\int_0^{2\pi} x(t)\, d\omega t$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} x(t)\cos n\omega t\, d\omega t$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} x(t)\sin n\omega t\, d\omega t$$

Note: The upper and lower limits of integration while computing Fourier Series Coefficients can be any value but their difference should be equal to one period.

In exponential form, the Fourier Series can be expressed as;

$$x(t) = \sum_{n=-\infty}^{\infty} A_n e^{jn\omega t}$$

$$\text{where, } A_n = \frac{1}{T}\int_0^{T} x(t)\, e^{-jn\omega t}\, dt$$

$$\text{and} \quad A_0 = \frac{1}{T}\int_0^{T} x(t)\, dt$$

If the independent variable is given as ωt, then

$$A_n = \frac{1}{2\pi}\int_0^{2\pi} x(t)\, e^{-jn\omega t}\, d\omega t$$

$$\text{and} \quad A_0 = \frac{1}{2\pi}\int_0^{2\pi} x(t)\, d\omega t$$

5.2 EVEN AND ODD SIGNALS

The even and odd signals are the signals which satisfy particular symmetry relations.

For a signal x(t) to be an even signal,

$$x(-t) = x(t)$$

Geometrically, the graph of an even signal is symmetric with respect to the y-axis, meaning that its graph remains unchanged after reflection about the y-axis.

For an even signal, the trigonometric Fourier coefficient $b_n=0$.

For a signal x(t) to be an odd signal,

$$x(-t) = -x(t)$$

Geometrically, the graph of an odd signal has rotational symmetry with respect to the origin, meaning that its graph remains unchanged after rotation of 180 degrees about the origin.

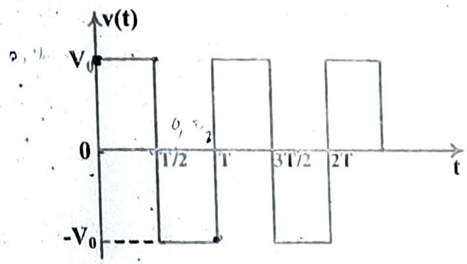
For an odd signal, the trigonometric Fourier coefficient $a_n=0$.

Note: Every signal need not be an even or an odd signal. Some of the signals may be neither even nor odd.

5.3 SOLVED PROBLEMS OF FOURIER SERIES

Example 5.1

Find the trigonometric Fourier Series of the waveform shown in the figure below. Also plot the line spectra.



Solution:

The given signal can be represented over a period in equation form as;

$$v(t) = \begin{cases} V_0 & \text{for } 0 < t < \frac{T}{2} \\ -V_0 & \text{for } \frac{T}{2} < t < T \end{cases}$$

The trigonometric Fourier series of the given signal can be expressed as;

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

where, $a_0 = \frac{2}{T}\int_0^T v(t)\,dt$

$$= \frac{2}{T}\left\{\int_0^{T/2} V_0\,dt + \int_{T/2}^{T} -V_0\,dt\right\}$$

$$= \frac{2}{T}\left\{[V_0 t]_0^{T/2} + [-V_0 t]_{T/2}^{T}\right\}$$

$$= \frac{2}{T}\left\{V_0\frac{T}{2} - V_0\left(T - \frac{T}{2}\right)\right\} = 0$$

Since the given signal is an odd signal, $a_0 = a_n = 0$

$$b_n = \frac{2}{T}\int_0^T v(t)\sin n\omega t\,dt$$

$$= \frac{2}{T}\left\{\int_0^{T/2} V_0 \sin n\omega t\,dt + \int_{T/2}^{T} -V_0 \sin n\omega t\,dt\right\}$$

$$= \frac{2}{T}\left\{V_0\left[\frac{-\cos n\omega t}{n\omega}\right]_0^{T/2} - V_0\left[\frac{-\cos n\omega t}{n\omega}\right]_{T/2}^{T}\right\}$$

$$= \frac{2V_0}{Tn\omega}\left\{\left[-\cos n\omega\frac{T}{2} + \cos 0\right] - \left[-\cos n\omega T + \cos n\omega\frac{T}{2}\right]\right\}$$

But $\omega = \frac{2\pi}{T}$

Then, $b_n = \frac{V_0}{n\pi}\{[-\cos n\pi + \cos 0] - [-\cos 2n\pi + \cos n\pi]\}$

$$= \frac{2V_0}{n\pi}[1 - (-1)^n]$$

For even n, $b_n = 0$

For odd n, $b_n = \frac{4V_0}{n\pi}$

Now, the required Fourier series of the given signal is;

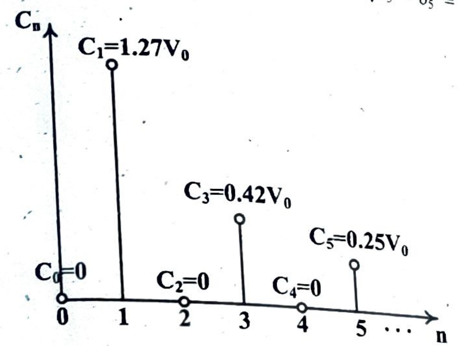
$$v(t) = \sum_{n=1}^{\infty}\left(\frac{4V_0}{n\pi}\right)\sin n\omega t \;\; ; \text{n is odd}$$

$$= \frac{4V_0}{\pi}\sin\omega t + \frac{4V_0}{3\pi}\sin 3\omega t + \frac{4V_0}{5\pi}\sin 5\omega t + \ldots\ldots\ldots$$

For line spectra,

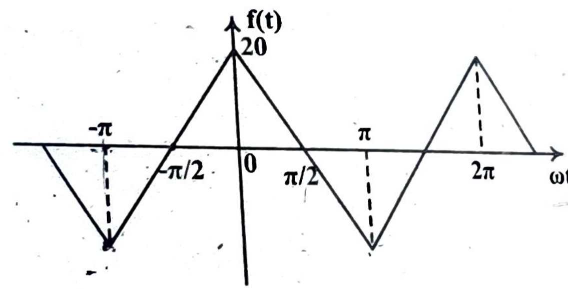
$C_0 = \frac{a_0}{2} = 0$, $C_1 = \sqrt{a_1^2 + b_1^2} = 1.27V_0$, $C_2 = \sqrt{a_2^2 + b_2^2} = 0$,

$C_3 = \sqrt{a_3^2 + b_3^2} = 0.42V_0$, $C_4 = \sqrt{a_4^2 + b_4^2} = 0$, $C_5 = \sqrt{a_5^2 + b_5^2} = 0.25V_0$



Example 5.2

Find the trigonometric Fourier Series of the waveform shown in the figure below. Also plot the line spectra. **[2073 Shrawan]**



Solution:

The given signal can be represented over a period in equation form as;

$$f(t) = \begin{cases} \frac{40}{\pi}\omega t + 20 & \text{for } -\pi < \omega t < 0 \\ \frac{-40}{\pi}\omega t + 20 & \text{for } 0 < \omega t < \pi \end{cases}$$

The trigonometric Fourier series of the given signal can be expressed as;

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

where,
$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0}\left(\frac{40}{\pi}\omega t + 20\right) d\omega t + \int_{0}^{\pi}\left(\frac{-40}{\pi}\omega t + 20\right) d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\frac{40}{\pi}\frac{(\omega t)^2}{2} + 20(\omega t)\right]_{-\pi}^{0} + \left[\frac{-40}{\pi}\frac{(\omega t)^2}{2} + 20(\omega t)\right]_{0}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\left[-\frac{40}{\pi}\frac{(-\pi)^2}{2} - 20(-\pi)\right] + \left[\frac{-40}{\pi}\frac{\pi^2}{2} + 20\pi\right]\right\} = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(t)\cos n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_{-\pi}^{0}\left(\frac{40}{\pi}\omega t + 20\right)\cos n\omega t\, d\omega t + \int_{0}^{\pi}\left(\frac{-40}{\pi}\omega t + 20\right)\cos n\omega t\, d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{40}{\pi}\omega t + 20\right)\left(\frac{\sin n\omega t}{n}\right) - \left(\frac{40}{\pi}\right)\left(\frac{-\cos n\omega t}{n^2}\right)\right]_{-\pi}^{0}\right.$$
$$\left. + \left[\left(\frac{-40}{\pi}\omega t + 20\right)\left(\frac{\sin n\omega t}{n}\right) - \left(\frac{-40}{\pi}\right)\left(\frac{-\cos n\omega t}{n^2}\right)\right]_{0}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{40}{n^2\pi}(\cos 0 - \cos n\pi) - \frac{40}{n^2\pi}(\cos n\pi - \cos 0)\right\}$$

$$= \frac{80}{n^2\pi^2}[1 - (-1)^n]$$

For even n, $a_n = 0$

For odd n, $a_n = \frac{160}{n^2\pi^2}$

Since the given signal is an even signal, $b_n = 0$

Now, the required Fourier series of the given signal is;

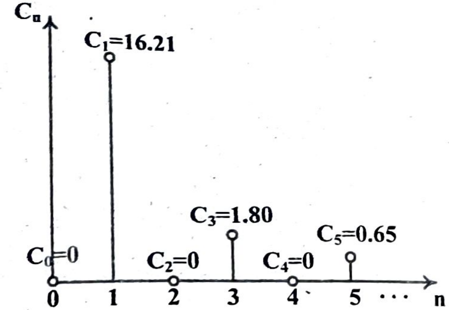
$$f(t) = \sum_{n=1}^{\infty}\left(\frac{160}{n^2\pi^2}\right)\cos n\omega t \quad ; \text{n is odd}$$

$$= \frac{160}{\pi^2}\left[\cos \omega t + \frac{1}{9}\cos 3\omega t + \frac{1}{25}\cos 5\omega t + \dots\dots\dots\right]$$

For line spectra,

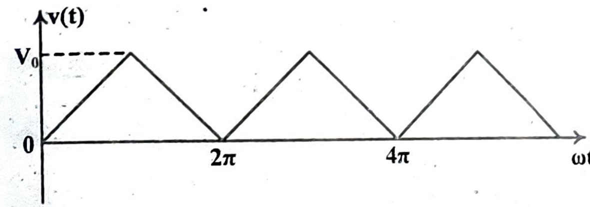
$C_0 = \frac{a_0}{2} = 0,\ C_1 = \sqrt{a_1^2 + b_1^2} = 16.21,\ C_2 = \sqrt{a_2^2 + b_2^2} = 0,$

$C_3 = \sqrt{a_3^2 + b_3^2} = 1.80,\ C_4 = \sqrt{a_4^2 + b_4^2} = 0,\ C_5 = \sqrt{a_5^2 + b_5^2} = 0.65$



Example 5.3

Find the trigonometric Fourier Series of the waveform shown in the figure below. Also plot the line spectra.



Solution:

The given signal can be represented over a period in equation form as;

$$v(t) = \begin{cases} \frac{V_0}{\pi}\omega t & \text{for } 0 < \omega t < \pi \\ \frac{-V_0}{\pi}\omega t + 2V_0 & \text{for } \pi < \omega t < 2\pi \end{cases}$$

The trigonometric Fourier series of the given signal can be expressed as;

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

where, $a_0 = \frac{1}{\pi}\int_0^{2\pi} v(t)\, d\omega t$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{V_0}{\pi}\omega t\right) d\omega t + \int_{\pi}^{2\pi}\left(\frac{-V_0}{\pi}\omega t + 2V_0\right) d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\frac{V_0}{\pi}\frac{(\omega t)^2}{2}\right]_0^{\pi} + \left[\frac{-V_0}{\pi}\frac{(\omega t)^2}{2} + 2V_0(\omega t)\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\left[\frac{V_0}{\pi}\frac{\pi^2}{2}\right] + \left[\frac{-V_0}{\pi}\frac{3\pi^2}{2} + 2V_0\pi\right]\right\}$$

$$= V_0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\cos n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{V_0}{\pi}\omega t\right)\cos n\omega t\, d\omega t + \int_{\pi}^{2\pi}\left(\frac{-V_0}{\pi}\omega t + 2V_0\right)\cos n\omega t\, d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{V_0}{\pi}\omega t\right)\left(\frac{\sin n\omega t}{n}\right) - \left(\frac{V_0}{\pi}\right)\left(\frac{-\cos n\omega t}{n^2}\right)\right]_0^{\pi}\right.$$

$$\left.+\left[\left(\frac{-V_0}{\pi}\omega t + 2V_0\right)\left(\frac{\sin n\omega t}{n}\right) - \left(\frac{-V_0}{\pi}\right)\left(\frac{-\cos n\omega t}{n^2}\right)\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{V_0}{n^2\pi}(\cos n\pi - \cos 0) - \frac{V_0}{n^2\pi}(\cos 2n\pi - \cos n\pi)\right\}$$

$$= \frac{2V_0}{n^2\pi^2}[(-1)^n - 1]$$

For even n, $a_n = 0$

For odd n, $a_n = \frac{-4V_0}{n^2\pi^2}$

Since the given signal is an even signal, $b_n = 0$

Now, the required Fourier series of the given signal is;

$$v(t) = \frac{V_0}{2} + \sum_{n=1}^{\infty}\left(\frac{-4V_0}{n^2\pi^2}\right)\cos n\omega t \text{ ; n is odd}$$

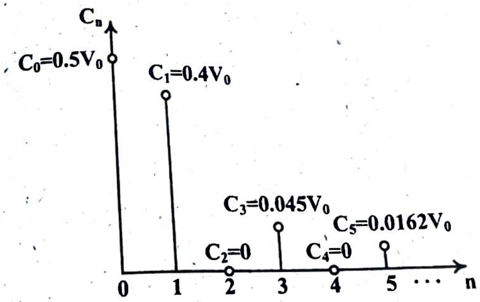
$$= \frac{V_0}{2} - \frac{4V_0}{\pi^2}\left[\cos\omega t + \frac{1}{9}\cos 3\omega t + \frac{1}{25}\cos 5\omega t + \dots\dots\dots\right]$$

For line spectra,

$C_0 = \frac{a_0}{2} = 0.5V_0$, $C_1 = \sqrt{a_1^2 + b_1^2} = 0.405V_0$, $C_2 = \sqrt{a_2^2 + b_2^2} = 0$,

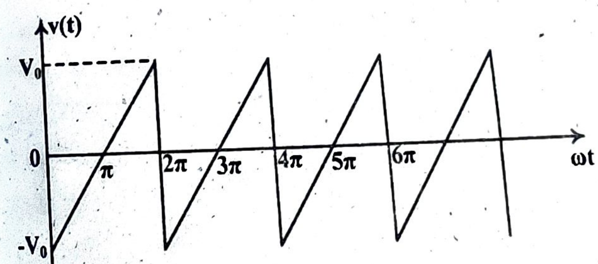
$C_3 = \sqrt{a_3^2 + b_3^2} = 0.045V_0$,

$C_4 = \sqrt{a_4^2 + b_4^2} = 0$, $C_5 = \sqrt{a_5^2 + b_5^2} = 0.0162V_0$



Example 5.4

Find the trigonometric Fourier Series of the waveform shown in the figure below. Also plot the line spectra.



Solution:

The given signal can be represented over a period in equation form as;

$$v(t) = \frac{V_0}{\pi}\omega t - V_0 \qquad \text{for } 0 < \omega t < 2\pi$$

The trigonometric Fourier series of the given signal can be expressed as;

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

Since the given signal is an odd signal, $a_0 = a_n = 0$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\sin n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{V_0}{\pi}\omega t - V_0\right)\sin n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{V_0}{\pi}\omega t - V_0\right)\left(\frac{-\cos n\omega t}{n}\right) - \left(\frac{V_0}{\pi}\right)\left(\frac{-\sin n\omega t}{n^2}\right)\right]_0^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\left(\frac{V_0}{\pi}(2\pi) - V_0\right)\frac{-\cos 2n\pi}{n} - \left(\frac{V_0}{\pi}\times 0 - V_0\right)\frac{-\cos 0}{n}\right\}$$

$$= -\frac{2V_0}{n\pi}$$

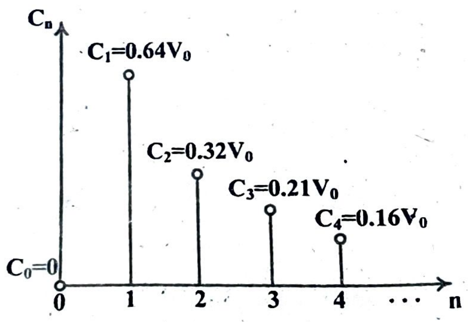
Now, the required Fourier series of the given signal is;

$$v(t) = \sum_{n=1}^{\infty}\left(\frac{-2V_0}{n\pi}\right)\sin n\omega t$$

For line spectra,

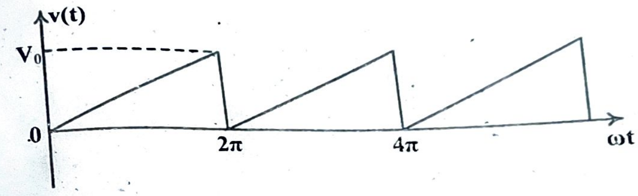
$C_0 = \frac{a_0}{2} = 0$, $C_1 = \sqrt{a_1^2 + b_1^2} = 0.64V_0$, $C_2 = \sqrt{a_2^2 + b_2^2} = 0.32V_0$,

$C_3 = \sqrt{a_3^2 + b_3^2} = 0.21V_0$, $C_4 = \sqrt{a_4^2 + b_4^2} = 0.16V_0$



Example 5.5

Find the trigonometric Fourier Series of the waveform shown in the figure below. Also plot the line spectra.



Solution:

The given signal can be represented over a period in equation form as;

$$v(t) = \frac{V_0}{2\pi}\omega t \qquad \text{for } 0 < \omega t < 2\pi$$

The trigonometric Fourier series of the given signal can be expressed as;

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos n\omega t + b_n \sin n\omega t)$$

where, $a_0 = \frac{1}{\pi}\int_0^{2\pi} v(t)\, d\omega t$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{V_0}{2\pi}\omega t\right) d\omega t$$

$$= \frac{1}{\pi}\left[\frac{V_0}{2\pi}\frac{(\omega t)^2}{2}\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{V_0}{2\pi}\frac{4\pi^2}{2}\right]$$

$$= V_0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\cos n\omega t \, d\omega t$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{V_0}{2\pi}\omega t\right)\cos n\omega t \, d\omega t$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{V_0}{2\pi}\omega t\right)\left(\frac{\sin n\omega t}{n}\right) - \left(\frac{V_0}{2\pi}\right)\left(\frac{-\cos n\omega t}{n^2}\right)\right]_0^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{V_0}{2n^2\pi}(\cos 2n\pi - \cos 0)\right\}$$

$$= 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\sin n\omega t \, d\omega t$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{V_0}{2\pi}\omega t\right)\sin n\omega t \, d\omega t$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{V_0}{2\pi}\omega t\right)\left(\frac{-\cos n\omega t}{n}\right) - \left(\frac{V_0}{2\pi}\right)\left(\frac{-\sin n\omega t}{n^2}\right)\right]_0^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{V_0}{2\pi}(2\pi)\frac{-\cos 2n\pi}{n}\right\}$$

$$= -\frac{V_0}{n\pi}$$

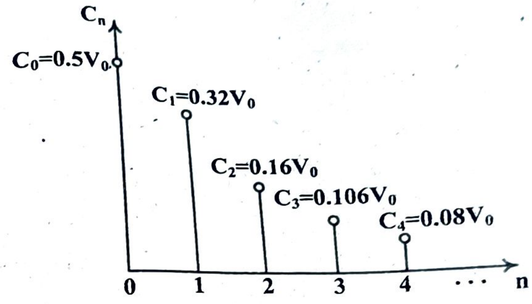
Now, the required Fourier series of the given signal is;

$$v(t) = \frac{V_0}{2} + \sum_{n=1}^{\infty}\left(\frac{-V_0}{n\pi}\right)\sin n\omega t$$

For line spectra,

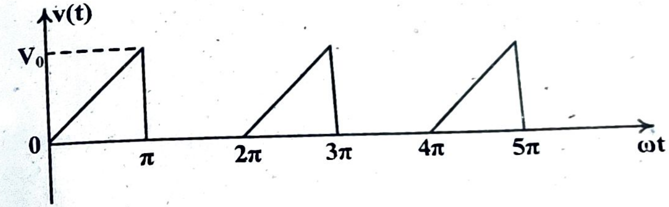
$C_0 = \frac{a_0}{2} = 0.5V_0$, $C_1 = \sqrt{a_1^2 + b_1^2} = 0.32V_0$, $C_2 = \sqrt{a_2^2 + b_2^2} = 0.16V_0$,

$C_3 = \sqrt{a_3^2 + b_3^2} = 0.106V_0$, $C_4 = \sqrt{a_4^2 + b_4^2} = 0.08V_0$



Example 5.6

Find the trigonometric Fourier Series of the waveform shown in the figure below. Also plot the line spectra.



Solution:

The given signal can be represented over a period in equation form as;

$$v(t) = \begin{cases} \frac{V_0}{\pi}\omega t & \text{for } 0 < \omega t < \pi \\ 0 & \text{for } \pi < \omega t < 2\pi \end{cases}$$

The trigonometric Fourier series of the given signal can be expressed as;

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

where, $a_0 = \frac{1}{\pi}\int_0^{2\pi} v(t)\, d\omega t = \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{V_0}{\pi}\omega t\right) d\omega t + \int_{\pi}^{2\pi} 0\, d\omega t\right\}$

$$= \frac{1}{\pi}\left[\frac{V_0}{\pi}\frac{(\omega t)^2}{2}\right]_0^{\pi} = \frac{1}{\pi}\left[\frac{V_0}{\pi}\frac{\pi^2}{2}\right]$$

$$= \frac{V_0}{2}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\cos n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{V_0}{\pi}\omega t\right)\cos n\omega t\, d\omega t + \int_{\pi}^{2\pi} 0 \times \cos n\omega t\, d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{V_0}{\pi}\omega t\right)\left(\frac{\sin n\omega t}{n}\right) - \left(\frac{V_0}{\pi}\right)\left(\frac{-\cos n\omega t}{n^2}\right)\right]_0^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{V_0}{n^2\pi}(\cos n\pi - \cos 0)\right\}$$

$$= \frac{V_0}{n^2\pi^2}[(-1)^n - 1]$$

For even n, $a_n = 0$

For odd n, $a_n = \frac{-2V_0}{n^2\pi^2}$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\sin n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{V_0}{\pi}\omega t\right)\sin n\omega t\, d\omega t + \int_{\pi}^{2\pi} 0 \times \sin n\omega t\, d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{V_0}{\pi}\omega t\right)\left(\frac{-\cos n\omega t}{n}\right) - \left(\frac{V_0}{\pi}\right)\left(\frac{-\sin n\omega t}{n^2}\right)\right]_0^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{V_0}{\pi}(\pi)\frac{-\cos n\pi}{n}\right\}$$

$$= -\frac{V_0}{n\pi}(-1)^n$$

For even n, $b_n = -\frac{V_0}{n\pi}$

For odd n, $b_n = \frac{V_0}{n\pi}$

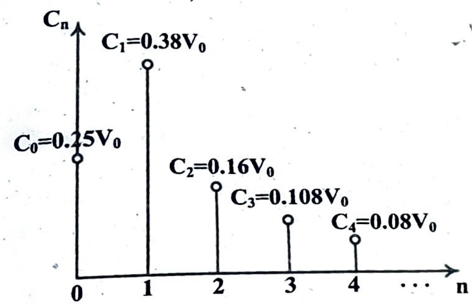
Now, the required Fourier series of the given signal is;

$$v(t) = \frac{V_0}{4} + \sum_{n=1}^{\infty}\left(\frac{V_0}{n^2\pi^2}[(-1)^n - 1]\right)\cos n\omega t + \left(\frac{-V_0}{n\pi}(-1)^n\right)\sin n\omega t$$

For line spectra,

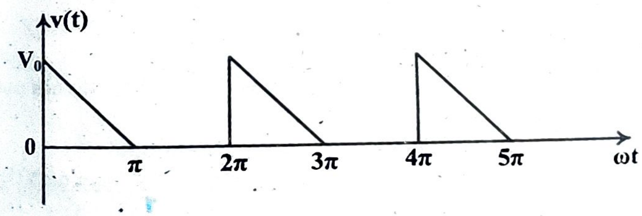
$C_0 = \frac{a_0}{2} = 0.25V_0$, $C_1 = \sqrt{a_1^2 + b_1^2} = 0.38V_0$, $C_2 = \sqrt{a_2^2 + b_2^2} = 0.16V_0$,

$C_3 = \sqrt{a_3^2 + b_3^2} = 0.108V_0$, $C_4 = \sqrt{a_4^2 + b_4^2} = 0.08V_0$



Example 5.7

Find the trigonometric Fourier Series of the waveform shown in the figure below. Also plot the line spectra.



Solution:

The given signal can be represented over a period in equation form as;

$$v(t) = \begin{cases} \frac{-V_0}{\pi}\omega t + V_0 & \text{for } 0 < \omega t < \pi \\ 0 & \text{for } \pi < \omega t < 2\pi \end{cases}$$

The trigonometric Fourier series of the given signal can be expressed as;

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

$$\text{where, } a_0 = \frac{1}{\pi}\int_0^{2\pi} v(t)\, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{-V_0}{\pi}\omega t + V_0\right) d\omega t + \int_{\pi}^{2\pi} 0\, d\omega t\right\}$$

$$= \frac{1}{\pi}\left[\frac{-V_0}{\pi}\frac{(\omega t)^2}{2} + V_0(\omega t)\right]_0^{\pi}$$

$$= \frac{1}{\pi}\left[\frac{-V_0}{\pi}\frac{\pi^2}{2} + V_0\pi\right]$$

$$= \frac{V_0}{2}$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v(t) \cos n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{-V_0}{\pi}\omega t + V_0\right)\cos n\omega t\, d\omega t + \int_{\pi}^{2\pi} 0 \times \cos n\omega t\, d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{-V_0}{\pi}\omega t + V_0\right)\left(\frac{\sin n\omega t}{n}\right) - \left(\frac{-V_0}{\pi}\right)\left(\frac{-\cos n\omega t}{n^2}\right)\right]_0^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{-\frac{V_0}{n^2\pi}(\cos n\pi - \cos 0)\right\}$$

$$= \frac{V_0}{n^2\pi^2}[1-(-1)^n]$$

For even n, $a_n = 0$

For odd n, $a_n = \frac{2V_0}{n^2\pi^2}$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v(t) \sin n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{-V_0}{\pi}\omega t + V_0\right)\sin n\omega t\, d\omega t + \int_{\pi}^{2\pi} 0 \times \sin n\omega t\, d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{-V_0}{\pi}\omega t + V_0\right)\left(\frac{-\cos n\omega t}{n}\right) - \left(\frac{-V_0}{\pi}\right)\left(\frac{-\sin n\omega t}{n^2}\right)\right]_0^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\left(\frac{-V_0}{\pi}\pi + V_0\right)\frac{-\cos n\pi}{n} - \left(\frac{-V_0}{\pi}\times 0 + V_0\right)\frac{-\cos 0}{n}\right\}$$

$$= \frac{V_0}{n\pi}$$

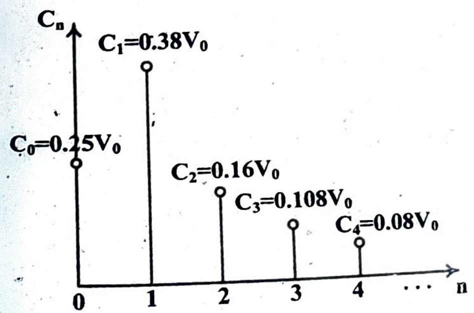
Now, the required Fourier series of the given signal is;

$$\mathbf{v(t)} = \frac{V_0}{4} + \sum_{n=1}^{\infty}\left(\frac{V_0}{n^2\pi^2}[1-(-1)^n]\right)\cos n\omega t + \left(\frac{V_0}{n\pi}\right)\sin n\omega t$$

<u>For line spectra,</u>

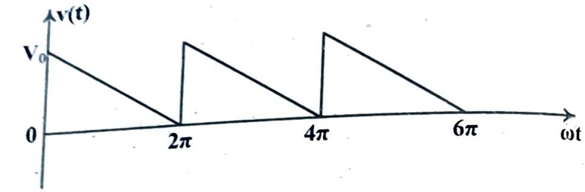
$C_0 = \frac{a_0}{2} = 0.25V_0$, $C_1 = \sqrt{a_1^2 + b_1^2} = 0.38V_0$, $C_2 = \sqrt{a_2^2 + b_2^2} = 0.16V_0$,

$C_3 = \sqrt{a_3^2 + b_3^2} = 0.108V_0$, $C_4 = \sqrt{a_4^2 + b_4^2} = 0.08V_0$



Example 5.8

Find the trigonometric Fourier Series of the waveform shown in the figure below. Also plot the line spectra.



Solution:

The given signal can be represented over a period in equation form as;

$$v(t) = -\frac{V_0}{2\pi}\omega t + V_0 \qquad \text{for } 0 < \omega t < 2\pi$$

The trigonometric Fourier series of the given signal can be expressed as;

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

$$\text{where, } a_0 = \frac{1}{\pi}\int_0^{2\pi} v(t)\, d\omega t$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{-V_0}{2\pi}\omega t + V_0\right) d\omega t$$

$$= \frac{1}{\pi}\left[\frac{-V_0}{2\pi}\frac{(\omega t)^2}{2} + V_0(\omega t)\right]_0^{2\pi}$$

$$= \frac{1}{\pi}\left[\frac{-V_0}{2\pi}\frac{4\pi^2}{2} + V_0(2\pi)\right]$$

$$= V_0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\cos n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{-V_0}{2\pi}\omega t + V_0\right)\cos n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{-V_0}{2\pi}\omega t + V_0\right)\left(\frac{\sin n\omega t}{n}\right) - \left(\frac{-V_0}{2\pi}\right)\left(\frac{-\cos n\omega t}{n^2}\right)\right]_0^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{-\frac{V_0}{2n^2\pi}(\cos 2n\pi - \cos 0)\right\}$$

$$= 0$$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\sin n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\int_0^{2\pi}\left(\frac{-V_0}{2\pi}\omega t + V_0\right)\sin n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{-V_0}{2\pi}\omega t + V_0\right)\left(\frac{-\cos n\omega t}{n}\right) - \left(\frac{-V_0}{2\pi}\right)\left(\frac{-\sin n\omega t}{n^2}\right)\right]_0^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{-V_0}{2\pi}(2\pi)\frac{-\cos 2n\pi}{n}\right\}$$

$$= \frac{V_0}{n\pi}$$

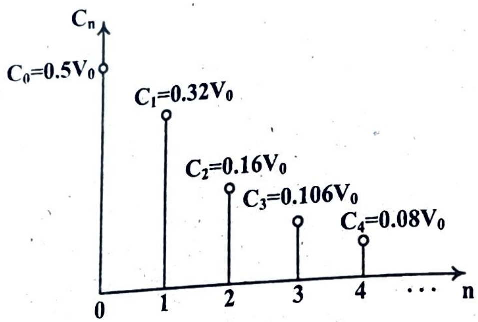
Now, the required Fourier series of the given signal is;

$$v(t) = \frac{V_0}{2} + \sum_{n=1}^{\infty}\left(\frac{V_0}{n\pi}\right)\sin n\omega t$$

<u>For line spectra,</u>

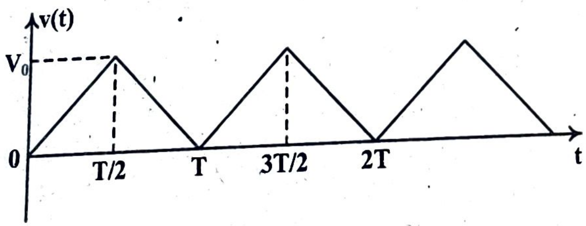
$C_0 = \frac{a_0}{2} = 0.5V_0$, $C_1 = \sqrt{a_1^2 + b_1^2} = 0.32V_0$, $C_2 = \sqrt{a_2^2 + b_2^2} = 0.16V_0$,

$C_3 = \sqrt{a_3^2 + b_3^2} = 0.106V_0$, $C_4 = \sqrt{a_4^2 + b_4^2} = 0.08V_0$



Example 5.9

Find the trigonometric Fourier Series of the waveform shown in the figure below. Also plot the line spectra.



Solution:

The given signal can be represented over a period in equation form as;

$$v(t) = \begin{cases} \frac{2V_0}{T}t & \text{for } 0 < t < \frac{T}{2} \\ \frac{-2V_0}{T}t + 2V_0 & \text{for } \frac{T}{2} < t < T \end{cases}$$

The trigonometric Fourier series of the given signal can be expressed as;

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

where, $a_0 = \frac{2}{T}\int_0^T v(t)\,dt$

$$= \frac{2}{T}\left\{\int_0^{T/2}\left(\frac{2V_0}{T}t\right)dt + \int_{T/2}^{T}\left(\frac{-2V_0}{T}t + 2V_0\right)dt\right\}$$

$$= \frac{2}{T}\left\{\left[\frac{2V_0}{T}\frac{t^2}{2}\right]_0^{T/2} + \left[\frac{-2V_0}{T}\frac{t^2}{2} + 2V_0(t)\right]_{T/2}^{T}\right\}$$

$$= \frac{2}{T}\left\{\left[\frac{2V_0}{T}\frac{(T/2)^2}{2}\right] + \left[\frac{-2V_0}{T}\frac{3T^2/4}{2} + 2V_0(T/2)\right]\right\}$$

$$= V_0$$

$$a_n = \frac{2}{T}\int_0^T v(t)\cos n\omega t\,dt$$

$$= \frac{2}{T}\left\{\int_0^{T/2}\left(\frac{2V_0}{T}t\right)\cos n\omega t\,dt + \int_{T/2}^{T}\left(\frac{-2V_0}{T}t + 2V_0\right)\cos n\omega t\,dt\right\}$$

$$= \frac{2}{T}\left[\left[\left(\frac{2V_0}{T}t\right)\left(\frac{\sin n\omega t}{n\omega}\right) - \left(\frac{2V_0}{T}\right)\left(\frac{-\cos n\omega t}{(n\omega)^2}\right)\right]_0^{T/2}\right.$$
$$\left. + \left[\left(\frac{-2V_0}{T}t + 2V_0\right)\left(\frac{\sin n\omega t}{n\omega}\right) - \left(\frac{-2V_0}{T}\right)\left(\frac{-\cos n\omega t}{(n\omega)^2}\right)\right]_{T/2}^{T}\right\}$$

Since $\omega = \frac{2\pi}{T}$,

$$a_n = \frac{2}{T}\left\{\frac{V_0T}{2n^2\pi^2}(\cos n\pi - \cos 0) - \frac{V_0T}{2n^2\pi^2}(\cos 2n\pi - \cos n\pi)\right\}$$

$$= \frac{2V_0}{n^2\pi^2}[(-1)^n - 1]$$

For even n, $a_n = 0$

For odd n, $a_n = \frac{-4V_0}{n^2\pi^2}$

Since the given signal is an even signal, $b_n = 0$

Now, the required Fourier series of the given signal is;

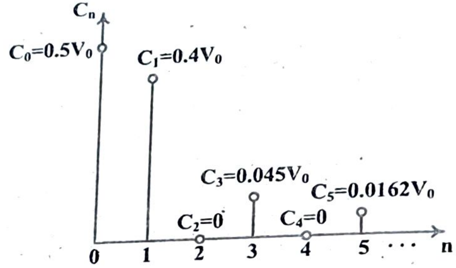
$$v(t) = \frac{V_0}{2} + \sum_{n=1}^{\infty}\left(\frac{-4V_0}{n^2\pi^2}\right)\cos n\omega t \text{ ; n is odd}$$

$$= \frac{V_0}{2} - \frac{4V_0}{\pi^2}\left[\cos \omega t + \frac{1}{9}\cos 3\omega t + \frac{1}{25}\cos 5\omega t + \dots\dots\dots\right]$$

For line spectra,

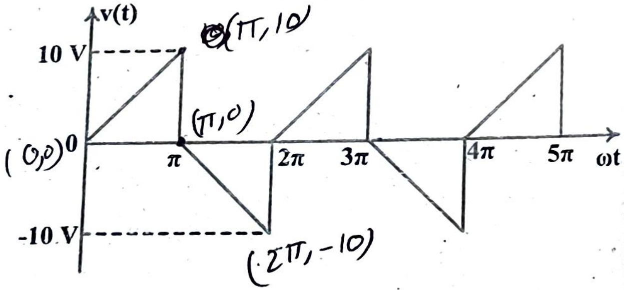
$$C_0 = \frac{a_0}{2} = 0.5V_0,\ C_1 = \sqrt{a_1^2 + b_1^2} = 0.405V_0,\ C_2 = \sqrt{a_2^2 + b_2^2} = 0,$$

$$C_3 = \sqrt{a_3^2 + b_3^2} = 0.045V_0,\ C_4 = \sqrt{a_4^2 + b_4^2} = 0,\ C_5 = \sqrt{a_5^2 + b_5^2} = 0.0162V_0$$



Example 5.10 R

Find the trigonometric Fourier Series of the waveform shown in the figure below. Also plot the line spectra. **[2073 Chaitra]**



Solution:

The given signal can be represented over a period in equation form as;

$$v(t) = \begin{cases} \dfrac{10}{\pi}\omega t & \text{for } 0 < \omega t < \pi \\ \dfrac{-10}{\pi}\omega t + 10 & \text{for } \pi < \omega t < 2\pi \end{cases}$$

The trigonometric Fourier series of the given signal can be expressed as;

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

$$\text{where, } a_0 = \frac{1}{\pi}\int_0^{2\pi} v(t)\, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{10}{\pi}\omega t\right) d\omega t + \int_{\pi}^{2\pi}\left(\frac{-10}{\pi}\omega t + 10\right) d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\frac{10}{\pi}\frac{(\omega t)^2}{2}\right]_0^{\pi} + \left[\frac{-10}{\pi}\frac{(\omega t)^2}{2} + 10(\omega t)\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\left[\frac{10}{\pi}\frac{\pi^2}{2}\right] + \left[\frac{-10}{\pi}\frac{3\pi^2}{2} + 10\pi\right]\right\}$$

$$= 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v(t) \cos n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{10}{\pi}\omega t\right) \cos n\omega t\, d\omega t + \int_{\pi}^{2\pi}\left(\frac{-10}{\pi}\omega t + 10\right) \cos n\omega t\, d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{10}{\pi}\omega t\right)\cancel{\left(\frac{\sin n\omega t}{n}\right)}^{0} - \left(\frac{10}{\pi}\right)\left(\frac{-\cos n\omega t}{n^2}\right)\right]_0^{\pi}\right.$$

$$\left. + \left[\left(\frac{-10}{\pi}\omega t + 10\right)\cancel{\left(\frac{\sin n\omega t}{n}\right)}^{0} - \left(\frac{-10}{\pi}\right)\left(\frac{-\cos n\omega t}{n^2}\right)\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{10}{n^2\pi}(\cos n\pi - \cos 0) - \frac{10}{n^2\pi}(\cos 2n\pi - \cos n\pi)\right\}$$

$$= \frac{20}{n^2\pi^2}[(-1)^n - 1]$$

For even n, $a_n = 0$

For odd n, $a_n = \dfrac{-40}{n^2\pi^2}$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\sin n\omega t \, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{10}{\pi}\omega t\right)\sin n\omega t\, d\omega t + \int_{\pi}^{2\pi}\left(\frac{-10}{\pi}\omega t + 10\right)\sin n\omega t\, d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{10}{\pi}\omega t\right)\left(\frac{-\cos n\omega t}{n}\right) - \left(\frac{10}{\pi}\right)\left(\frac{-\sin n\omega t}{n^2}\right)\right]_0^{\pi}\right.$$

$$\left. + \left[\left(\frac{-10}{\pi}\omega t + 10\right)\left(\frac{-\cos n\omega t}{n}\right) - \left(\frac{-10}{\pi}\right)\left(\frac{-\sin n\omega t}{n^2}\right)\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{10}{\pi}(\pi)\frac{-\cos n\pi}{n} + \left(\frac{-10}{\pi}\times 2\pi + 10\right)\frac{-\cos 2n\pi}{n} - \left(\frac{-10}{\pi}\times\pi + 10\right)\frac{-\cos n\pi}{n}\right\}$$

$$= \frac{10}{n\pi}\left[1 - (-1)^n\right]$$

For even n, $b_n = 0$

For odd n, $b_n = \dfrac{20}{n\pi}$

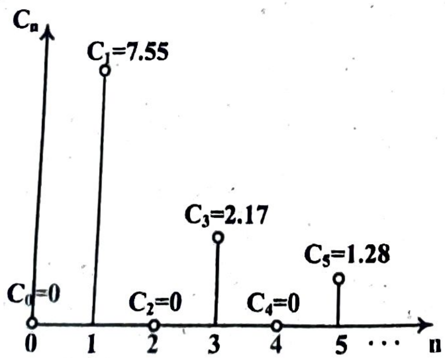
Now, the required Fourier series of the given signal is;

$$v(t) = \sum_{n=1}^{\infty}\left(\frac{-40}{n^2\pi^2}\right)\cos n\omega t + \left(\frac{20}{n\pi}\right)\sin n\omega t \text{ ; n is odd}$$

For line spectra,

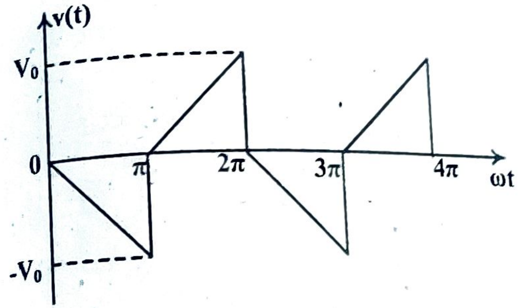
$C_0 = \frac{a_0}{2} = 0$, $C_1 = \sqrt{a_1^2 + b_1^2} = 7.55$, $C_2 = \sqrt{a_2^2 + b_2^2} = 0$,

$C_3 = \sqrt{a_3^2 + b_3^2} = 2.17$, $C_4 = \sqrt{a_4^2 + b_4^2} = 0$, $C_5 = \sqrt{a_5^2 + b_5^2} = 1.28$



Example 5.11

Find the trigonometric Fourier Series of the waveform shown in the figure below. Also plot the line spectra. [2072 Chaitra]



Solution:

The given signal can be represented over a period in equation form as;

$$v(t) = \begin{cases} \dfrac{-V_0}{\pi}\omega t & \text{for } 0 < \omega t < \pi \\ \dfrac{V_0}{\pi}\omega t - V_0 & \text{for } \pi < \omega t < 2\pi \end{cases}$$

The trigonometric Fourier series of the given signal can be expressed as;

$$v(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n \cos n\omega t + b_n \sin n\omega t)$$

where, $a_0 = \dfrac{1}{\pi}\displaystyle\int_0^{2\pi} v(t)\, d\omega t$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{-V_0}{\pi}\omega t\right)d\omega t + \int_{\pi}^{2\pi}\left(\frac{V_0}{\pi}\omega t - V_0\right)d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\frac{-V_0}{\pi}\frac{(\omega t)^2}{2}\right]_0^{\pi} + \left[\frac{V_0}{\pi}\frac{(\omega t)^2}{2} - V_0(\omega t)\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\left[\frac{-V_0}{\pi}\frac{\pi^2}{2}\right] + \left[\frac{V_0}{\pi}\frac{3\pi^2}{2} - V_0\pi\right]\right\}$$

$$= 0$$

$$a_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\cos n\omega t \, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{-V_0}{\pi}\omega t\right)\cos n\omega t\, d\omega t + \int_{\pi}^{2\pi}\left(\frac{V_0}{\pi}\omega t - V_0\right)\cos n\omega t\, d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{-V_0}{\pi}\omega t\right)\left(\frac{\sin n\omega t}{n}\right) - \left(\frac{-V_0}{\pi}\right)\left(\frac{-\cos n\omega t}{n^2}\right)\right]_0^{\pi} + \left[\left(\frac{V_0}{\pi}\omega t - V_0\right)\left(\frac{\sin n\omega t}{n}\right) - \left(\frac{V_0}{\pi}\right)\left(\frac{-\cos n\omega t}{n^2}\right)\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{-\frac{V_0}{n^2\pi}(\cos n\pi - \cos 0) + \frac{V_0}{n^2\pi}(\cos 2n\pi - \cos n\pi)\right\}$$

$$= \frac{2V_0}{n^2\pi^2}\left[1 - (-1)^n\right]$$

For even n, $a_n = 0$

For odd n, $a_n = \frac{4V_0}{n^2\pi^2}$

$$b_n = \frac{1}{\pi}\int_0^{2\pi} v(t)\sin n\omega t\, d\omega t$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi}\left(\frac{-V_0}{\pi}\omega t\right)\sin n\omega t\, d\omega t + \int_{\pi}^{2\pi}\left(\frac{V_0}{\pi}\omega t - V_0\right)\sin n\omega t\, d\omega t\right\}$$

$$= \frac{1}{\pi}\left\{\left[\left(\frac{-V_0}{\pi}\omega t\right)\left(\frac{-\cos n\omega t}{n}\right) - \left(\frac{-V_0}{\pi}\right)\left(\frac{-\sin n\omega t}{n^2}\right)\right]_0^{\pi} + \left[\left(\frac{V_0}{\pi}\omega t - V_0\right)\left(\frac{-\cos n\omega t}{n}\right) - \left(\frac{V_0}{\pi}\right)\left(\frac{-\sin n\omega t}{n^2}\right)\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{\pi}\left\{\frac{-V_0}{\pi}(\pi)\frac{-\cos n\pi}{n} + \left(\frac{V_0}{\pi}\times 2\pi - V_0\right)\frac{-\cos 2n\pi}{n} - \left(\frac{V_0}{\pi}\times\pi - V_0\right)\frac{-\cos n\pi}{n}\right\}$$

$$= \frac{V_0}{n\pi}\left[(-1)^n - 1\right]$$

For even n, $b_n = 0$

For odd n, $b_n = \frac{-2V_0}{n\pi}$

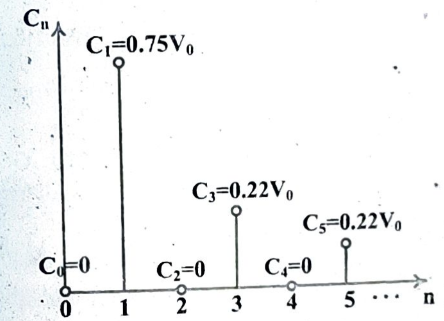
Now, the required Fourier series of the given signal is;

$$v(t) = \sum_{n=1}^{\infty}\left(\frac{4V_0}{n^2\pi^2}\right)\cos n\omega t + \left(\frac{-2V_0}{n\pi}\right)\sin n\omega t \text{ ; n is odd}$$

For line spectra,

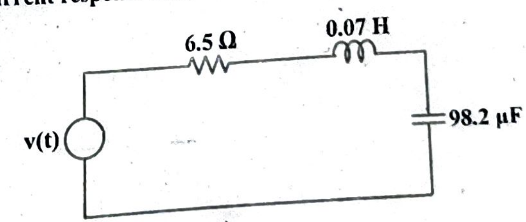
$C_0 = \frac{a_0}{2} = 0$, $C_1 = \sqrt{a_1^2 + b_1^2} = 0.75V_0$, $C_2 = \sqrt{a_2^2 + b_2^2} = 0$,

$C_3 = \sqrt{a_3^2 + b_3^2} = 0.22V_0$, $C_4 = \sqrt{a_4^2 + b_4^2} = 0$, $C_5 = \sqrt{a_5^2 + b_5^2} = 0.13V_0$



Example 5.12

The network of figure shown below has an applied voltage of v(t) = (40 sin ωt + 80 sin 3ωt) Volts where ω=500 rad/s. Find the current response and hence the average power.



Solution:

The given voltage signal, v(t) = (40 sin ωt + 80 sin 3ωt) consists of fundamental component and third harmonic. So, impedance will be different for different frequency components.

For fundamental component,

$$Z_1 = R + j(X_{L1} - X_{C1}) \quad \text{or,} \quad Z_1 = 6.5 + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\text{or,} \quad Z_1 = 6.5 + j\left(500\times0.07 - \frac{1}{500\times98.2\times10^{-6}}\right)$$

$$\text{or,} \quad Z_1 = 6.5 + j14.633 \quad \text{or,} \quad Z_1 = 16.01\angle66.05^\circ\,\Omega$$

For third harmonic,

$$Z_3 = R + j(X_{L3} - X_{C3}) \quad \text{or,} \quad Z_3 = 6.5 + j\left(3\omega L - \frac{1}{3\omega C}\right)$$

$$\text{or,} \quad Z_3 = 6.5 + j\left(3\times500\times0.07 - \frac{1}{3\times500\times98.2\times10^{-6}}\right)$$

$$\text{or,} \quad Z_3 = 6.5 + j98.21 \quad \text{or,} \quad Z_3 = 98.43\angle86.21^\circ\,\Omega$$

Now, the current response can be expressed as;

$$i(t) = \frac{40}{|Z_1|}\sin(\omega t - 66.05^\circ) + \frac{80}{|Z_2|}\sin(3\omega t - 86.21^\circ)$$

$$\text{or,} \quad \mathbf{i(t) = 2.5\sin(\omega t - 66.05^\circ) + 0.81\sin(3\omega t - 86.21^\circ)\ A}$$

Let, I_1 and I_3 denote the RMS value of fundamental and third harmonic component of current.

Then, the equivalent RMS value of current,

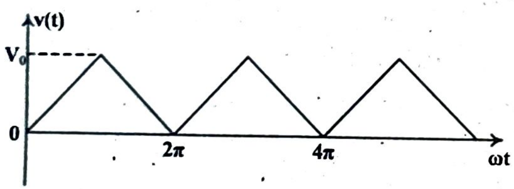
$$I_{eq} = \sqrt{I_1^2 + I_2^2} = \sqrt{\left(\frac{2.5}{\sqrt{2}}\right)^2 + \left(\frac{0.81}{\sqrt{2}}\right)^2} = 1.858\ A$$

Now, the average power is given by;

$$P = I_{eq}^2 \times R = (1.858)^2 \times 6.5 = \mathbf{22.44\ W}$$

Example 5.13

Find the exponential Fourier Series of the waveform shown in the figure below. [2072 Kartik]



Solution:

The given signal can be represented over a period in equation form as;

$$v(t) = \begin{cases} \dfrac{V_0}{\pi}\omega t & \text{for } 0 < \omega t < \pi \\ \dfrac{-V_0}{\pi}\omega t + 2V_0 & \text{for } \pi < \omega t < 2\pi \end{cases}$$

The exponential Fourier series of the given signal can be expressed as

$$v(t) = \sum_{n=-\infty}^{\infty} A_n e^{jn\omega t}$$

$$\text{where, } A_n = \frac{1}{2\pi}\int_0^{2\pi} v(t)\,e^{-jn\omega t}\,d\omega t$$

$$= \frac{1}{2\pi}\left\{\int_0^{\pi}\left(\frac{V_0}{\pi}\omega t\right)e^{-jn\omega t}\,d\omega t + \int_{\pi}^{2\pi}\left(\frac{-V_0}{\pi}\omega t + 2V_0\right)e^{-jn\omega t}\,d\omega t\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\left(\frac{V_0}{\pi}\omega t\right)\left(\frac{e^{-jn\omega t}}{-jn}\right) - \left(\frac{V_0}{\pi}\right)\left(\frac{e^{-jn\omega t}}{(-jn)^2}\right)\right]_0^{\pi}\right.$$

$$\left. + \left[\left(\frac{-V_0}{\pi}\omega t + 2V_0\right)\left(\frac{e^{-jn\omega t}}{-jn}\right) - \left(\frac{-V_0}{\pi}\right)\left(\frac{e^{-jn\omega t}}{(-jn)^2}\right)\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\left(\frac{V_0}{\pi}.\pi\right)\left(\frac{e^{-jn\pi}}{-jn}\right) - \left(\frac{V_0}{\pi}\right)\left(\frac{e^{-jn\pi}}{(-jn)^2}\right) - \left(\frac{V_0}{\pi}.0\right)\left(\frac{e^{-j0}}{-jn}\right) + \frac{V_0}{\pi}\left(\frac{e^{-j0}}{(-jn)^2}\right)\right]\right.$$

$$\left. + \left[\left(\frac{-V_0}{\pi}.2\pi + 2V_0\right)\frac{e^{-jn2\pi}}{-jn} - \left(\frac{-V_0}{\pi}\right)\left(\frac{e^{-jn2\pi}}{(-jn)^2}\right) - \left(-\frac{V_0}{\pi}.\pi + 2V_0\right)\left(\frac{e^{-jn\pi}}{-jn}\right) + \left(-\frac{V_0}{\pi}\right)\cdot\left(\frac{e^{-jn\pi}}{(-jn)^2}\right)\right]\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{jV_0(-1)^n}{n} + \frac{V_0}{\pi n^2}(-1)^n - \frac{V_0}{\pi n^2} - \frac{V_0}{\pi n^2} - \frac{jV_0}{n}(-1)^n + \frac{V_0}{\pi n^2}(-1)^n\right\}$$

$$= \frac{2}{2\pi}\left\{\frac{V_0}{\pi n^2}[(-1)^n - 1]\right\}$$

$$= \frac{V_0}{n^2\pi^2}[(-1)^n - 1]$$

But the expression is undefined for n = 0.

$$\text{So,} \quad A_0 = \frac{1}{2\pi}\int_0^{2\pi} v(t)\,d\omega t = \frac{1}{2\pi}\left\{\int_0^{\pi}\left(\frac{V_0}{\pi}\omega t\right)d\omega t + \int_{\pi}^{2\pi}\left(\frac{-V_0}{\pi}\omega t + 2V_0\right)d\omega t\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{V_0}{\pi}\frac{(\omega t)^2}{2}\right]_0^{\pi} + \left[\frac{-V_0}{\pi}\frac{(\omega t)^2}{2} + 2V_0(\omega t)\right]_{\pi}^{2\pi}\right\}$$

$$= \frac{1}{2\pi}\left\{\left[\frac{V_0}{\pi}\frac{\pi^2}{2}\right] + \left[\frac{-V_0}{\pi}\frac{3\pi^2}{2} + 2V_0\pi\right]\right\} = \frac{V_0}{2}$$

Hence, the exponential Fourier Series of the given signal is;

$$v(t) = \ldots + A_{-3}e^{-j3\omega t} + A_{-2}e^{-j2\omega t} + A_{-1}e^{-j\omega t} + A_0 + A_1e^{j\omega t} + A_2e^{j2\omega t} + A_3e^{j3\omega t} + \ldots$$

$$\text{or,} \quad \mathbf{v(t) = \ldots + \left(\frac{-2V_0}{9\pi^2}\right)e^{-j3\omega t} + \left(\frac{-2V_0}{\pi^2}\right)e^{-j\omega t}}$$

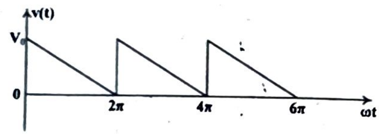
$$\mathbf{+ \frac{V_0}{2} + \left(\frac{-2V_0}{\pi^2}\right)e^{j\omega t} + \left(\frac{-2V_0}{9\pi^2}\right)e^{j3\omega t} + \ldots}$$

Multiple Choice Questions

1. Which of the following is an even function of 't'?
   a) t^2 b) $t^2 - 4t$
   c) $\sin(2t) + 3t$ d) $t^3 + 6$

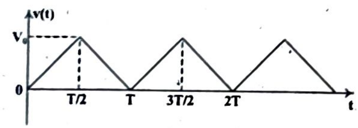
2. A "periodic function" is given by a function which
   a) has a period of $T = 2\pi$ b) satisfies $f(t + T) = f(t)$
   c) satisfies $f(t + T) = -f(t)$ d) has a period of $T = \pi$

3. The following signal is



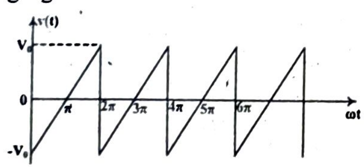
   a) an even signal b) an odd signal
   c) neither even nor odd signal d) both even and odd signal

4. The following signal is



   a) an even signal b) an odd signal
   c) neither even nor odd signal d) both even and odd signal

5. The following signal is



   a) an even signal b) an odd signal
   c) neither even nor odd signal d) both even and odd signal

ANSWERS

1.(a), 2.(b), 3.(c), 4.(a), 5.(b)

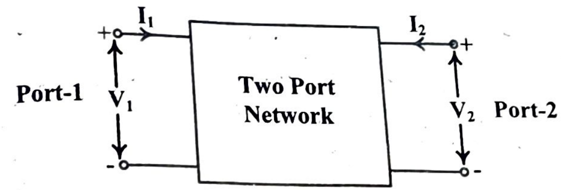
□□□

CHAPTER 6

Two Port Parameters of Networks

1 INTRODUCTION

Generally, the electrical networks are composed of two ports, one put port and the other output port. The figure below shows the black-box odel of two port network showing its input and output voltages and rrents.



Here, V_1 and I_1 are voltage and current of port-1 with polarity and irection as shown in the figure. Similarly, V_2 and I_2 are voltage and current f port-2 with polarity and direction as shown in the figure.

Out of these four variables, if we consider two variables as ndependent and remaining variables as dependent variables, then six ifferent combinations of parameters are formed which are listed in the table elow.

Name of parameters	*Matrix Equation*
Z-parameters or Open Circuit Impedance parameters	$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$
Y-parameters or Short Circuit Admittance parameters	$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$
T or Transmission or Chain parameters	$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$
T' or Inverse Transmission parameters	$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$
h or hybrid parameters	$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$
g or inverse hybrid parameters	$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$

6.2 CONDITION FOR RECIPROCITY

A two port network is said to be reciprocal, if the ratio of the excitation to response is invariant to an interchange of the positions of the excitation and response in the network. Networks containing resistors, inductors and capacitors are generally reciprocal. Networks that additionally have dependent sources are generally non-reciprocal.

The conditions of reciprocity in terms of various parameters are:

a) $Z_{12} = Z_{21}$

b) $Y_{12} = Y_{21}$

c) $AD - BC = 1$

d) $A'D' - B'C' = 1$

e) $h_{12} = -h_{21}$

f) $g_{12} = -g_{21}$

6.3 CONDITION FOR SYMMETRICITY

A two port network is said to be symmetrical if the ports can be interchanged without changing the port voltage and currents.

The conditions of symmetricity in terms of various parameters are:

a) $Z_{11} = Z_{22}$

b) $Y_{11} = Y_{22}$

c) $A = D$

d) $A' = D'$

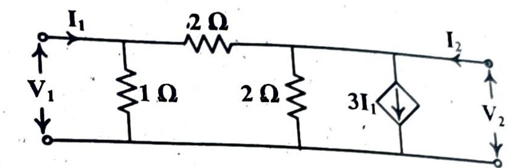
e) $h_{11}h_{22} - h_{12}h_{21} = 1$

f) $g_{11}g_{22} - g_{12}g_{21} = 1$

6.4 SOLVED PROBLEMS OF TWO PORT NETWORK PARAMETERS

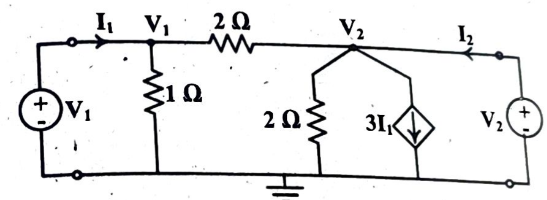
Example 6.1

Find Z-parameters and Y-parameters of the network shown in the figure below.



Solution:

Adding source on both the ports and re-arranging the circuit as follows to apply Nodal Analysis;



Applying KCL at node 1,

$$I_1 = \frac{V_1 - V_2}{2} + \frac{V_1 - 0}{1}$$

or, $$I_1 = \frac{3}{2}V_1 - \frac{1}{2}V_2 \quad \text{..........(1)}$$

Applying KCL at node 2,

$$I_2 = 3\,I_1 + \frac{V_2 - V_1}{2} + \frac{V_2 - 0}{2}$$

or, $$I_2 = 3\,I_1 + V_2 - \frac{1}{2}V_1 \quad \text{..........(2)}$$

Substituting expression of I_1 from equation (1) to (2),

$$I_2 = 3\left(\frac{3}{2}V_1 - \frac{1}{2}V_2\right) + V_2 - \frac{1}{2}V_1$$

or, $$I_2 = 4\,V_1 - \frac{1}{2}V_2 \quad \text{..........(3)}$$

Writing equations (1) and (3) in matrix form, we get

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 3/2 & -1/2 \\ 4 & -1/2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{............(4)}$$

We know, the equation of Y–parameters in matrix form as;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{............(5)}$$

Comparing equations (4) and (5), we get

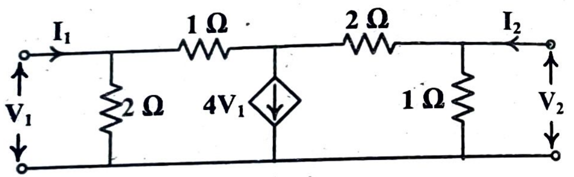
$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 3/2 & -1/2 \\ 4 & -1/2 \end{bmatrix}$$

For Z–parameters,

$$[Z] = [Y]^{-1} = \begin{bmatrix} 3/2 & -1/2 \\ 4 & -1/2 \end{bmatrix}^{-1} = \begin{bmatrix} -2/5 & 2/5 \\ -16/5 & 6/5 \end{bmatrix}$$

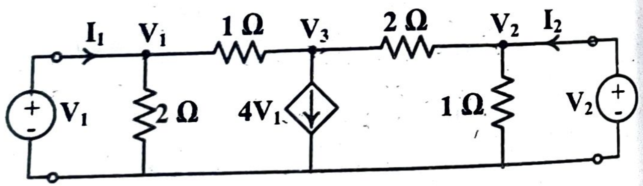
Example 6.2

Find Z–parameters of the network shown in the figure below.



Solution:

Adding source on both the ports and performing Nodal Analysis;



Applying KCL at node 1,

$$I_1 = \frac{V_1 - V_3}{1} + \frac{V_1 - 0}{2}$$

or, $$I_1 = \frac{3}{2}V_1 - V_3 \quad \text{............(1)}$$

Applying KCL at node 2,

$$I_2 = \frac{V_2 - V_3}{2} + \frac{V_2 - 0}{1}$$

or, $$I_2 = \frac{3}{2}V_2 - \frac{1}{2}V_3 \quad \text{............(2)}$$

Applying KCL at node 3,

$$4V_1 + \frac{V_3 - V_1}{1} + \frac{V_3 - V_2}{2} = 0$$

or, $$3V_1 - \frac{1}{2}V_2 + \frac{3}{2}V_3 = 0$$

or, $$V_3 = -2V_1 + \frac{1}{3}V_2 \quad \text{..................(3)}$$

Substituting expression of V_3 from equation (3) to (1),

$$I_1 = \frac{3}{2}V_1 - \left(-2V_1 + \frac{1}{3}V_2\right)$$

or, $$I_1 = \frac{7}{2}V_1 - \frac{1}{3}V_2 \quad \text{............(4)}$$

Substituting expression of V_3 from equation (3) to (2),

$$I_2 = \frac{3}{2}V_2 - \frac{1}{2}\left(-2V_1 + \frac{1}{3}V_2\right)$$

or, $$I_2 = V_1 + \frac{4}{3}V_2 \quad \text{............(5)}$$

Writing equations (4) and (5) in matrix form, we get

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 7/2 & -1/3 \\ 1 & 4/3 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{............(6)}$$

We know, the equation of Y–parameters in matrix form as;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{............(7)}$$

Comparing equations (6) and (7), we get

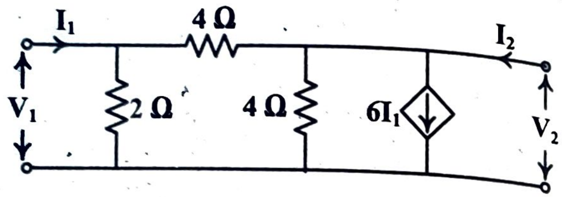
$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 7/2 & -1/3 \\ 1 & 4/3 \end{bmatrix}$$

For Z–parameters,

$$[Z] = [Y]^{-1} = \begin{bmatrix} 7/2 & -1/3 \\ 1 & 4/3 \end{bmatrix}^{-1} = \begin{bmatrix} 4/15 & 1/15 \\ -1/5 & 7/10 \end{bmatrix}$$

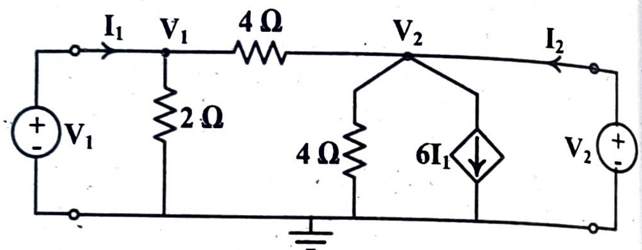
Example 6.3

Find Z–parameters of the network shown in the figure below.



Solution:

Adding source on both the ports and re–arranging the circuit as follows apply Nodal Analysis;



Applying KCL at node 1,

$$I_1 = \frac{V_1 - V_2}{4} + \frac{V_1 - 0}{2}$$

or, $I_1 = \frac{3}{4}V_1 - \frac{1}{4}V_2$ ....(1)

Applying KCL at node 2,

$$I_2 = 6\,I_1 + \frac{V_2 - V_1}{4} + \frac{V_2 - 0}{4}$$

or, $I_2 = 6\,I_1 + \frac{1}{2}V_2 - \frac{1}{4}V_1$ ....(2)

Substituting expression of I_1 from equation (1) to (2),

$$I_2 = 6\left(\frac{3}{4}V_1 - \frac{1}{4}V_2\right) + \frac{1}{2}V_2 - \frac{1}{4}V_1$$

or, $I_2 = \frac{17}{4}V_1 - V_2$ ....(3)

Writing equations (1) and (3) in matrix form, we get

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 3/4 & -1/4 \\ 17/4 & -1 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{............(4)}$$

We know, the equation of Y–parameters in matrix form as;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{............(5)}$$

Comparing equations (4) and (5), we get

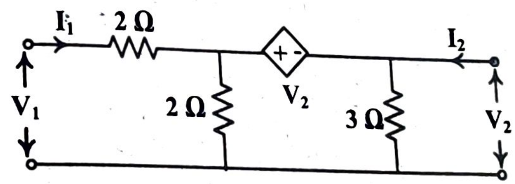
$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 3/4 & -1/4 \\ 17/4 & -1 \end{bmatrix}$$

For Z–parameters,

$$[Z] = [Y]^{-1} = \begin{bmatrix} 3/4 & -1/4 \\ 17/4 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} -3.2 & 0.8 \\ -13.6 & 2.4 \end{bmatrix}$$

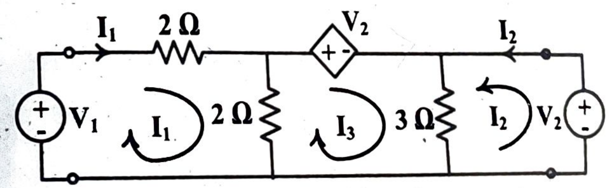
Example 6.4

Find Z and Y parameters of the network shown in the figure below.



Solution:

Adding source on both the ports and performing Mesh Analysis;



Applying KVLin mesh 1,

$$V_1 = 2I_1 + 2(I_1 - I_3)$$

or, $V_1 = 4I_1 - 2I_3$ ....(1)

Applying KVL in mesh 2,

$$V_2 = 3(I_2 + I_3)$$

or, $V_2 = 3I_2 + 3I_3$ ....(2)

Applying KVL in mesh 3,

$$-V_2 = 3(I_3 + I_2) + 2(I_3 - I_1)$$

or, $V_2 = 2I_1 - 3I_2 - 5I_3$ ....(3)

Equating equations (2) and (3),

$$3I_2 + 3I_3 = 2I_1 - 3I_2 - 5I_3$$

or, $8I_3 = 2I_1 - 6I_2$

or, $I_3 = \frac{1}{4}I_1 - \frac{3}{4}I_2$ ....(4)

Substituting expression of I_3 from equation (4) to (1),

$$V_1 = 4I_1 - 2(\frac{1}{4}I_1 - \frac{3}{4}I_2)$$

or, $V_1 = \frac{7}{2}I_1 + \frac{3}{2}I_2$ ....(5)

Substituting expression of I_3 from equation (4) to (2),

$$V_2 = 3I_2 + 3(\frac{1}{4}I_1 - \frac{3}{4}I_2)$$

or, $V_2 = \frac{3}{4}I_1 + \frac{3}{4}I_2$ ....(6)

Writing equations (5) and (6) in matrix form, we get

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 7/2 & 3/2 \\ 3/4 & 3/4 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{............(7)}$$

We know, the equation of Z–parameters in matrix form as;

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{............(8)}$$

Comparing equations (7) and (8), we get

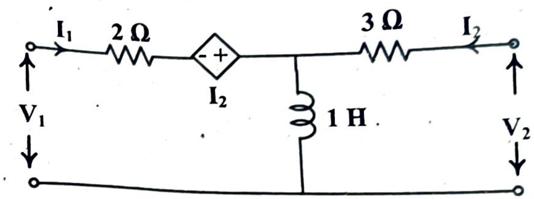
$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 7/2 & 3/2 \\ 3/4 & 3/4 \end{bmatrix}$$

For Y–parameters,

$$[Y] = [Z]^{-1} = \begin{bmatrix} 7/2 & 3/2 \\ 3/4 & 3/4 \end{bmatrix}^{-1} = \begin{bmatrix} 1/2 & -1 \\ -1/2 & 7/3 \end{bmatrix}$$

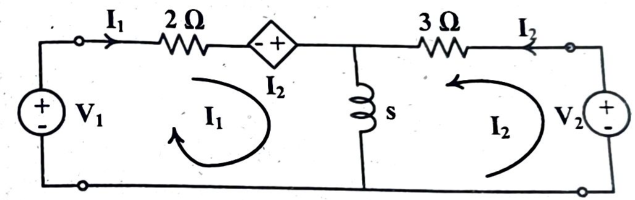
Example 6.5

Find Z–parameters of the network shown in the figure below. Also check the symmetricity and reciprocity of the network.



Solution:

Representing the circuit in s–domain, adding source on both the ports and performing Mesh Analysis;



Applying KVL in mesh 1,

$$V_1 + I_2 = 2I_1 + s \times (I_1 + I_2)$$

or, $V_1 = (s + 2)\, I_1 + (s - 1)\, I_2$ ....(1)

Applying KVL in mesh 2,

$$V_2 = 3I_2 + s \times (I_2 + I_1)$$

or, $V_2 = sI_1 + (s + 3)I_2$ ....(2)

Writing equations (1) and (2) in matrix form, we get

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} s+2 & s-1 \\ s & s+3 \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{............(3)}$$

We know, the equation of Z–parameters in matrix form as;

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{............(4)}$$

Comparing equations (3) and (4), we get

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} s+2 & s-1 \\ s & s+3 \end{bmatrix}$$

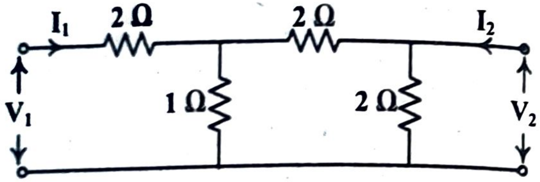
For symmetricity and reciprocity:

Since $Z_{11} \neq Z_{22}$, the given two port network is **not symmetrical**.

Since $Z_{12} \neq Z_{21}$, the given two port network is **not reciprocal**.

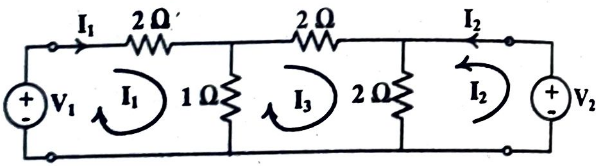
Example 6.6

For the network shown in figure, calculate Z, Y and T–parameters. Also check the symmetricity and reciprocity of the network.



Solution:

Adding source on both the ports and performing Mesh Analysis;



Applying KVL in mesh 1,

$$V_1 = 2I_1 + 1 \times (I_1 - I_3)$$

or, $V_1 = 3I_1 - I_3$ ....(1)

Applying KVL in mesh 2,

$$V_2 = 2(I_2 + I_3)$$

or, $V_2 = 2I_2 + 2I_3$ ....(2)

Applying KVL in mesh 3,

$$0 = 2(I_3 + I_2) + 1 \times (I_3 - I_1) + 2I_3$$

or, $-I_1 + 2I_2 + 5I_3 = 0$

or, $5I_3 = I_1 - 2I_2$

or, $I_3 = \frac{1}{5}I_1 - \frac{2}{5}I_2$ ....(3)

Substituting expression of I_3 from equation (3) to (1),

$$V_1 = 3I_1 - \left(\frac{1}{5}I_1 - \frac{2}{5}I_2\right)$$

or, $V_1 = \frac{14}{5}I_1 + \frac{2}{5}I_2$ ....(4)

Substituting expression of I_3 from equation (3) to (2),

$$V_2 = 2I_2 + 2\left(\frac{1}{5}I_1 - \frac{2}{5}I_2\right)$$

or, $V_2 = \frac{2}{5}I_1 + \frac{6}{5}I_2$ ....(5)

Writing equations (4) and (5) in matrix form, we get

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 14/5 & 2/5 \\ 2/5 & 6/5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{............(6)}$$

We know, the equation of Z–parameters in matrix form as;

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{............(7)}$$

Comparing equations (6) and (7), we get

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 14/5 & 2/5 \\ 2/5 & 6/5 \end{bmatrix}$$

For Y–parameters,

$$[Y] = [Z]^{-1} = \begin{bmatrix} 14/5 & 2/5 \\ 2/5 & 6/5 \end{bmatrix}^{-1} = \begin{bmatrix} 3/8 & -1/8 \\ -1/8 & 7/8 \end{bmatrix}$$

For Transmission parameters, we know;

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{............(8)}$$

From equation (5),

$$V_2 = \frac{2}{5}I_1 + \frac{6}{5}I_2$$

or, $\frac{2}{5}I_1 = V_2 - \frac{6}{5}I_2$

or, $I_1 = \frac{5}{2}V_2 - 3I_2$ ....(9)

Substituting expression of I_1 from equation (9) to (4), we get

$$V_1 = \frac{14}{5}\left(\frac{5}{2}V_2 - 3I_2\right) + \frac{2}{5}I_2$$

or, $V_1 = 7V_2 - 8I_2$ ....(10)

Writing equations (9) and (10) in matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5/2 & 3 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{............(11)}$$

Comparing equations (8) and (11), we get T–parameters matrix as;

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 7 & 8 \\ 5/2 & 3 \end{bmatrix}$$

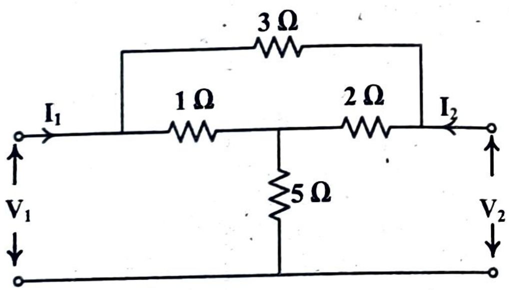
For symmetricity and reciprocity:

Since $Z_{11} \neq Z_{22}$, the given two port network is **not symmetrical**.

Since $Z_{12} = Z_{21} = 2/5$, the given two port network is **reciprocal**.

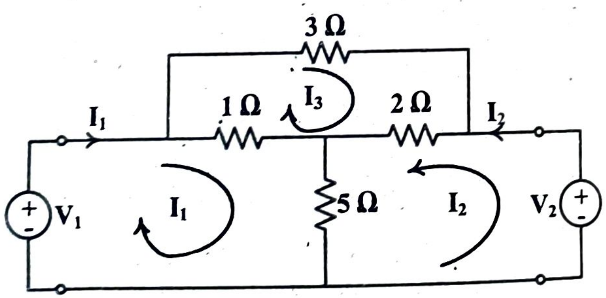
Example 6.7

Obtain Z and Y parameters of the two port network shown in the figure below. Also check the symmetricity and reciprocity of the network.



Solution:

Adding source on both the ports and performing Mesh Analysis;



Applying KVL in mesh 1,

$$V_1 = 1 \times (I_1 - I_3) + 5 \times (I_1 + I_2)$$

or, $$V_1 = 6I_1 + 5I_2 - I_3 \text{ ....(1)}$$

Applying KVL in mesh 2,

$$V_2 = 2(I_2 + I_3) + 5(I_2 + I_1)$$

or, $$V_2 = 5I_1 + 7I_2 + 2I_3 \text{ ....(2)}$$

Applying KVL in mesh 3,

$$0 = 2(I_3 + I_2) + 1 \times (I_3 - I_1) + 3I_3$$

or, $$-I_1 + 2I_2 + 6I_3 = 0$$

or, $$6I_3 = I_1 - 2I_2$$

or, $$I_3 = \frac{1}{6}I_1 - \frac{1}{3}I_2 \text{ ....(3)}$$

Substituting expression of I_3 from equation (3) to (1),

$$V_1 = 6I_1 + 5I_2 - \left(\frac{1}{6}I_1 - \frac{1}{3}I_2\right)$$

or, $$V_1 = \frac{35}{6}I_1 + \frac{16}{3}I_2 \text{ ....(4)}$$

Substituting expression of I_3 from equation (3) to (2),

$$V_2 = 5I_1 + 7I_2 + 2\left(\frac{1}{6}I_1 - \frac{1}{3}I_2\right)$$

or, $$V_2 = \frac{16}{3}I_1 + \frac{19}{3}I_2 \text{ ....(5)}$$

Writing equations (4) and (5) in matrix form, we get

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 35/6 & 16/3 \\ 16/3 & 19/3 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{............(6)}$$

We know, the equation of Z–parameters in matrix form as;

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{............(7)}$$

Comparing equations (6) and (7), we get

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 35/6 & 16/3 \\ 16/3 & 19/3 \end{bmatrix}$$

For Y–parameters,

$$[Y] = [Z]^{-1} = \begin{bmatrix} 35/6 & 16/3 \\ 16/3 & 19/3 \end{bmatrix}^{-1} = \begin{bmatrix} 38/51 & -32/51 \\ -32/51 & 35/51 \end{bmatrix}$$

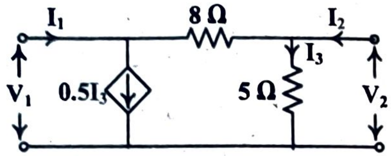
For symmetricity and reciprocity:

Since $Z_{11} \neq Z_{22}$, the given two port network is **not symmetrical**.

Since $Z_{12} = Z_{21} = 16/3$, the given two port network is **reciprocal**.

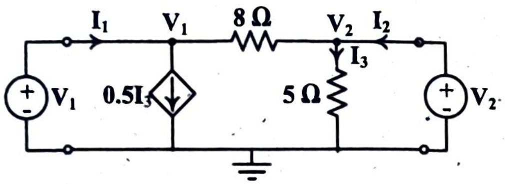
Example 6.8

Obtain the Z parameters of the network shown in the figure below.



Solution:

Adding source on both the ports and performing Nodal Analysis;



Applying KCL at node 1,

$$I_1 = \frac{V_1 - V_2}{8} + 0.5I_3$$

But $I_3 = \frac{V_2}{5}$

So, $I_1 = \frac{V_1 - V_2}{8} + 0.5\times\frac{V_2}{5}$

or, $I_1 = \frac{1}{8}V_1 - \frac{1}{40}V_2$ ....(1)

Applying KCL at node 2,

$$I_2 = \frac{V_2}{5} + \frac{V_2 - V_1}{8}$$

or, $I_2 = -\frac{1}{8}V_1 + \frac{13}{40}V_2$ ....(2)

Writing equations (1) and (2) in matrix form, we get

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 1/8 & -1/40 \\ -1/8 & 13/40 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{............(3)}$$

We know, the equation of Y–parameters in matrix form as;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{............(4)}$$

Comparing equations (3) and (4), we get

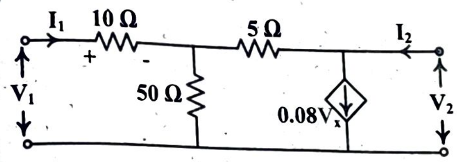
$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 1/8 & -1/40 \\ -1/8 & 13/40 \end{bmatrix}$$

For Z–parameters,

$$[Z] = [Y]^{-1} = \begin{bmatrix} 1/8 & -1/40 \\ -1/8 & 13/40 \end{bmatrix}^{-1} = \begin{bmatrix} 26/3 & 2/3 \\ 10/3 & 10/3 \end{bmatrix}$$

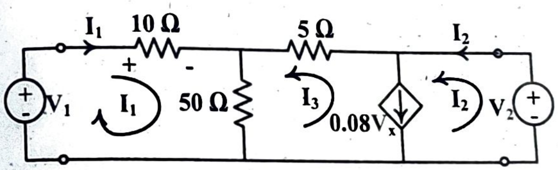
Example 6.9

Find the open circuit impedance parameters and transmission parameters of the network shown in the figure below.



Solution:

Adding source on both the ports and performing Mesh Analysis;



Applying KVL in mesh 1,

$$V_1 = 10I_1 + 50 \times (I_1 + I_3)$$

or, $V_1 = 60I_1 + 50I_3$ ....(1)

Applying KVL in super–mesh 2 & 3,

$$V_2 = 5I_3 + 50(I_1 + I_3)$$

or, $V_2 = 50I_1 + 55I_3$ ....(2)

In the common branch between mesh 2 and mesh 3,

$$I_2 - I_3 = 0.08V_x$$

But $V_x = 10I_1$

So, $I_2 - I_3 = 0.08(10I_1)$

or, $I_3 = I_2 - 0.8I_1$ ....(3)

Substituting expression of I_3 from equation (3) to (1),

$$V_1 = 60I_1 + 50(I_2 - 0.8I_1)$$

or, $V_1 = 20I_1 + 50I_2$ ....(4)

Substituting expression of I_3 from equation (3) to (2),

$$V_2 = 50I_1 + 55(I_2 - 0.8I_1)$$

or, $V_2 = 6I_1 + 55I_2$ ....(5)

Writing equations (4) and (5) in matrix form, we get

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 & 50 \\ 6 & 55 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{............(6)}$$

We know, the equation of Z–parameters in matrix form as;

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{............(7)}$$

Comparing equations (6) and (7), we get

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 20 & 50 \\ 6 & 55 \end{bmatrix}$$

For Transmission parameters, we know;

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{............(8)}$$

From equation (5),

$$V_2 = 6I_1 + 55I_2$$

or, $6I_1 = V_2 - 55I_2$

or, $I_1 = \frac{1}{6}V_2 - \frac{55}{6}I_2$ ....(9)

Substituting expression of I_1 from equation (9) to (4), we get

$$V_1 = 20\left(\frac{1}{6}V_2 - \frac{55}{6}I_2\right) + 50I_2$$

or, $V_1 = \frac{10}{3}V_2 - \frac{400}{3}I_2$ ....(10)

Writing equations (9) and (10) in matrix form,

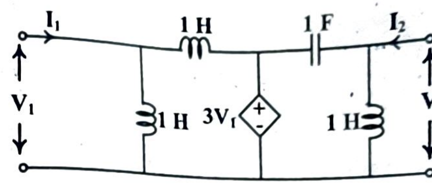
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 10/3 & 400/3 \\ 1/6 & 55/6 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{............(11)}$$

Comparing equations (8) and (11), we get T–parameters matrix as;

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 10/3 & 400/3 \\ 1/6 & 55/6 \end{bmatrix}$$

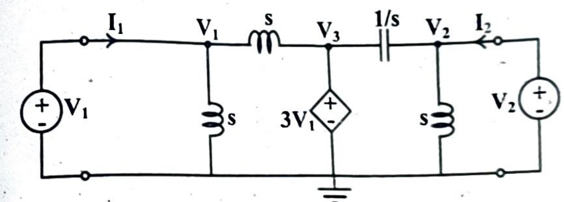
Example 6.10

Find the Z–parameters of the network shown in the figure below. Also check its reciprocity and symmetricity. **[2068 Chaitra]**



Solution:

Representing the circuit in s–domain, adding source on both the ports and performing Nodal Analysis;



Applying KCL at node 1,

$$I_1 = \frac{V_1 - V_3}{s} + \frac{V_1 - 0}{s}$$

or, $I_1 = \frac{2}{s}V_1 - \frac{1}{s}V_3$ ....(1)

Applying KCL at node 2,

$$I_2 = \frac{V_2 - V_3}{1/s} + \frac{V_2 - 0}{s}$$

or, $I_2 = \frac{s^2 + 1}{s}V_2 - s\,V_3$ ....(2)

At node 3, $V_3 = 3V_1$ ....(3)

Substituting expression of V_3 from equation (3) to (1),

$$I_1 = \frac{2}{s}V_1 - \frac{1}{s}(3V_1)$$

or, $I_1 = \frac{-1}{s}V_1$

or, $V_1 = -s\,I_1$ ....(4)

Substituting expression of V_3 from equation (3) to (2),

$$I_2 = \frac{s^2+1}{s} V_2 - s\,(3V_1)$$

or, $$I_2 = \frac{s^2+1}{s} V_2 - 3s\,V_1 \;\ldots\ldots\ldots(5)$$

Substituting expression of V_3 from equation (4) to (5),

$$I_2 = \frac{s^2+1}{s} V_2 - 3s\,(-s\,I_1)$$

or, $$I_2 = \frac{s^2+1}{s} V_2 + 3s^2 I_1$$

or, $$\frac{s^2+1}{s} V_2 = -3s^2 I_1 + I_2$$

or, $$V_2 = \left(\frac{-3s^3}{s^2+1}\right) I_1 + \left(\frac{s}{s^2+1}\right) I_2 \;\ldots\ldots\ldots(6)$$

Writing equations (4) and (6) in matrix form, we get

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -s & 0 \\ \frac{-3s^3}{s^2+1} & \frac{s}{s^2+1} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \;\ldots\ldots\ldots(7)$$

We know, the equation of Z–parameters in matrix form as;

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \;\ldots\ldots\ldots(8)$$

Comparing equations (7) and (8), we get

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} -s & 0 \\ \frac{-3s^3}{s^2+1} & \frac{s}{s^2+1} \end{bmatrix}$$

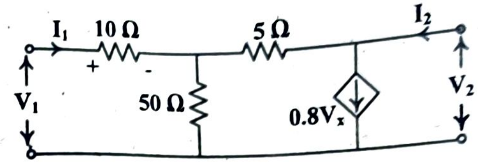
For symmetricity and reciprocity:

Since $Z_{11} \neq Z_{22}$, the given two port network is **not symmetrical.**

Since $Z_{12} \neq Z_{21}$, the given two port network is **not reciprocal.**

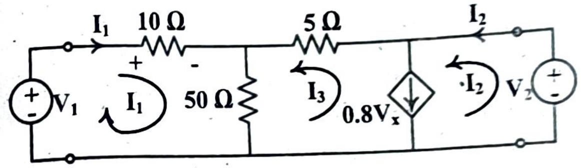
Example 6.11

Find the Z parameters and T parameters of the two port network shown in the figure below. **[2071 Shrawan]**



Solution:

Adding source on both the ports and performing Mesh Analysis:



Applying KVL in mesh 1,

$$V_1 = 10I_1 + 50 \times (I_1 + I_3)$$

or, $$V_1 = 60I_1 + 50I_3 \;\ldots\ldots\ldots(1)$$

Applying KVL in super–mesh 2 & 3,

$$V_2 = 5I_3 + 50(I_1 + I_3)$$

or, $$V_2 = 50I_1 + 55I_3 \;\ldots\ldots\ldots(2)$$

In the common branch between mesh 2 and mesh 3,

$$I_2 - I_3 = 0.8V_x$$

But $V_x = 10I_1$

So, $I_2 - I_3 = 0.8(10I_1)$

or, $$I_3 = I_2 - 8I_1 \;\ldots\ldots\ldots(3)$$

Substituting expression of I_3 from equation (3) to (1),

$$V_1 = 60I_1 + 50(I_2 - 8I_1)$$

or, $$V_1 = -340I_1 + 50I_2 \;\ldots\ldots\ldots(4)$$

Substituting expression of I_3 from equation (3) to (2),

$$V_2 = 50I_1 + 55(I_2 - 8I_1)$$

or, $$V_2 = -390I_1 + 55I_2 \;\ldots\ldots\ldots(5)$$

Writing equations (4) and (5) in matrix form, we get

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -340 & 50 \\ -390 & 55 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{............(6)}$$

We know, the equation of Z–parameters in matrix form as;

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \text{............(7)}$$

Comparing equations (6) and (7), we get

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} -340 & 50 \\ -390 & 55 \end{bmatrix}$$

For Transmission parameters, we know;

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{............(8)}$$

From equation (5),

$$V_2 = -390I_1 + 55I_2$$

or, $$390I_1 = -V_2 + 55I_2$$

or, $$I_1 = \frac{-1}{390}V_2 + \frac{11}{78}I_2 \text{......................(9)}$$

Substituting expression of I_1 from equation (9) to (4), we get

$$V_1 = -340\left(\frac{-1}{390}V_2 + \frac{11}{78}I_2\right) + 50I_2$$

or, $$V_1 = \frac{34}{39}V_2 + \frac{80}{39}I_2 \text{............(10)}$$

Writing equations (9) and (10) in matrix form,

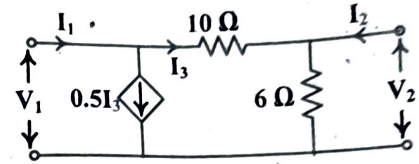
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 34/39 & -80/39 \\ -1/390 & -11/78 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{............(11)}$$

Comparing equations (8) and (11), we get T–parameters matrix as;

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 34/39 & -80/39 \\ -1/390 & -11/78 \end{bmatrix}$$

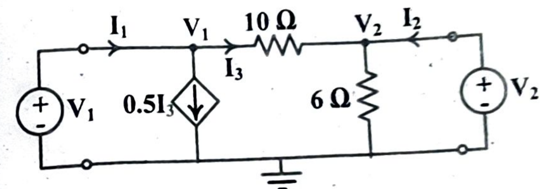
Example 6.12

For the two port network shown in the figure below, find transmission parameters and Y parameters. [2073 Shrawan]



Solution:

Adding source on both the ports and performing Nodal Analysis;



Applying KCL at node 1,

$$I_1 = \frac{V_1 - V_2}{10} + 0.5I_3$$

But $$I_3 = \frac{V_1 - V_2}{10}$$

So, $$I_1 = \frac{V_1 - V_2}{10} + 0.5 \times \frac{V_1 - V_2}{10}$$

or, $$I_1 = \frac{3}{20}V_1 - \frac{3}{20}V_2 \text{............(1)}$$

Applying KCL at node 2,

$$I_2 = \frac{V_2}{6} + \frac{V_2 - V_1}{10}$$

or, $$I_2 = -\frac{1}{10}V_1 + \frac{4}{15}V_2 \text{............(2)}$$

Writing equations (1) and (2) in matrix form, we get

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 3/20 & -3/20 \\ -1/10 & 4/15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{............(3)}$$

We know, the equation of Y–parameters in matrix form as;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{............(4)}$$

Comparing equations (3) and (4), we get

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 3/20 & -3/20 \\ -1/10 & 4/15 \end{bmatrix}$$

For Transmission parameters, we know;

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad \text{............(5)}$$

From equation (2),

$$I_2 = -\frac{1}{10}V_1 + \frac{4}{15}V_2 \qquad \text{or,} \qquad \frac{1}{10}V_1 = \frac{4}{15}V_2 - I_2$$

$$\text{or,} \qquad V_1 = \frac{8}{3}V_2 - 10I_2 \quad \text{......................(6)}$$

Substituting expression of V_1 from equation (6) to (1), we get

$$I_1 = \frac{3}{20}\left(\frac{8}{3}V_2 - 10I_2\right) - \frac{3}{20}V_2$$

$$\text{or,} \qquad I_1 = \frac{1}{4}V_2 - \frac{3}{2}I_2 \quad \text{..........(7)}$$

Writing equations (6) and (7) in matrix form,

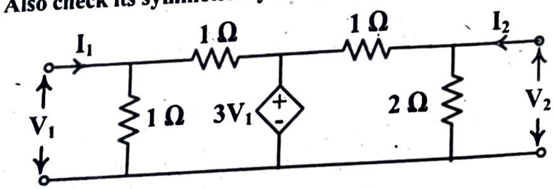
$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 8/3 & 10 \\ 1/4 & 3/2 \end{bmatrix}\begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \quad \text{............(8)}$$

Comparing equations (5) and (8), we get T–parameters matrix as;

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 8/3 & 10 \\ 1/4 & 3/2 \end{bmatrix}$$

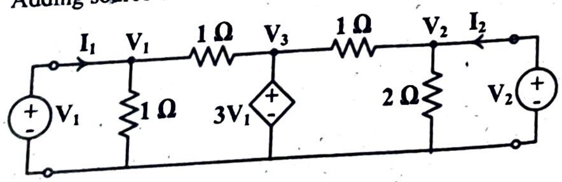
Example 6.13

Find Z and g parameters of the network shown in the figure below. Also check its symmetricity and reciprocity. [2068 Shrawan]



Solution:

Adding source on both the ports and performing Nodal Analysis;



Applying KCL at node 1,

$$I_1 = \frac{V_1 - V_3}{1} + \frac{V_1 - 0}{1} \qquad \text{or,} \qquad I_1 = 2V_1 - V_3 \quad \text{............(1)}$$

Applying KCL at node 2,

$$I_2 = \frac{V_2 - V_3}{1} + \frac{V_2 - 0}{2} \qquad \text{or,} \qquad I_2 = \frac{3}{2}V_2 - V_3 \quad \text{............(2)}$$

At node 3,

$$V_3 = 3V_1 \quad \text{............(3)}$$

Substituting expression of V_3 from equation (3) to (1),

$$I_1 = 2V_1 - (3V_1) \qquad \text{or,} \qquad I_1 = -V_1 \quad \text{............(4)}$$

Substituting expression of V_3 from equation (3) to (2),

$$I_2 = \frac{3}{2}V_2 - (3V_1) \qquad \text{or,} \qquad I_2 = -3V_1 + \frac{3}{2}V_2 \quad \text{..........(5)}$$

Writing equations (4) and (5) in matrix form, we get

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -3 & 3/2 \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{............(6)}$$

We know, the equation of Y–parameters in matrix form as;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \quad \text{............(7)}$$

Comparing equations (6) and (7), we get

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ -3 & 3/2 \end{bmatrix}$$

For Z–parameters,

$$[Z] = [Y]^{-1} = \begin{bmatrix} -1 & 0 \\ -3 & 3/2 \end{bmatrix}^{-1} = \begin{bmatrix} -1 & 0 \\ -2 & 2/3 \end{bmatrix}$$

For g–parameters, we know;

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix}\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \quad \text{............(8)}$$

From equation (5),

$$I_2 = -3V_1 + \frac{3}{2}V_2$$

$$\text{or,} \qquad \frac{3}{2}V_2 = 3V_1 + I_2$$

$$\text{or,} \qquad V_2 = 2V_1 + \frac{2}{3}I_2 \quad \text{............(9)}$$

Writing equations (4) and (9) in matrix form,

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 2 & 2/3 \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix} \quad \text{.........(10)}$$

Comparing equations (8) and (10), we get g–parameters matrix as;

$$\begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 2 & 2/3 \end{bmatrix}$$

For symmetricity and reciprocity:

Since $Z_{11} \neq Z_{22}$, the given two port network is **not symmetrical**.

Since $Z_{12} \neq Z_{21}$, the given two port network is **not reciprocal**.

Example 6.14

The Y–parameters of two Two Port Networks (TPNs) are $\begin{bmatrix} 1/4 & -5/4 \\ -1/4 & -3/4 \end{bmatrix}$ and $\begin{bmatrix} 1/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$. If these two TPNs are connected in series, what will be the equivalent Transmission parameters of the combination?

Solution:

Let the two TPNs be TPN_a and TPN_b.

Then, $[Y_a] = \begin{bmatrix} 1/4 & -5/4 \\ -1/4 & -3/4 \end{bmatrix}$ and $[Y_b] = \begin{bmatrix} 1/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}$

When two TPNs are connected in series, the equivalent TPN will have Z–parameters matrix equal to the sum of individual TPNs, i.e. if [Z] is the Z–parameters matrix of the combination, then

$$[Z] = [Z_a] + [Z_b]$$

But $\quad [Z_a] = [Y_a]^{-1} = \begin{bmatrix} 1/4 & -5/4 \\ -1/4 & -3/4 \end{bmatrix}^{-1} = \begin{bmatrix} 3/2 & -5/2 \\ -1/2 & -1/2 \end{bmatrix}$

and $\quad [Z_b] = [Y_b]^{-1} = \begin{bmatrix} 1/3 & -1/3 \\ -1/3 & 2/3 \end{bmatrix}^{-1} = \begin{bmatrix} 6 & 3 \\ 3 & 3 \end{bmatrix}$

So, $\quad [Z] = \begin{bmatrix} 3/2 & -5/2 \\ -1/2 & -1/2 \end{bmatrix} + \begin{bmatrix} 6 & 3 \\ 3 & 3 \end{bmatrix} = \begin{bmatrix} 15/2 & 1/2 \\ 5/2 & 5/2 \end{bmatrix}$

Let, V_1 and V_2 be the voltages and I_1 and I_2 be the currents of the equivalent combination network,

Then,

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 15/2 & 1/2 \\ 5/2 & 5/2 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

i.e. $\quad V_1 = \frac{15}{2} I_1 + \frac{1}{2} I_2 \text{ ....(1)}$

and $\quad V_2 = \frac{5}{2} I_1 + \frac{5}{2} I_2 \text{ ....(2)}$

For Transmission parameters, we know;

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{ ....(3)}$$

From equation (2),

$$V_2 = \frac{5}{2} I_1 + \frac{5}{2} I_2 \qquad \text{or,} \qquad \frac{5}{2} I_1 = V_2 - \frac{5}{2} I_2$$

or, $\quad I_1 = \frac{2}{5} V_2 - I_2 \text{ ....(4)}$

Substituting expression of I_1 from equation (4) to equation (1),

$$V_1 = \frac{15}{2}\left(\frac{2}{5} V_2 - I_2\right) + \frac{1}{2} I_2$$

or, $\quad V_1 = 3V_2 - 7I_2 \text{ ....(5)}$

Writing equations (5) and (4) in matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2/5 & 1 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{ ....(6)}$$

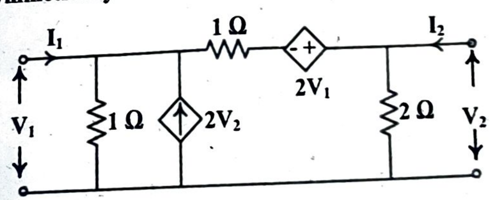
Comparing equations (3) and (6), we get

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 3 & 7 \\ 2/5 & 1 \end{bmatrix}$$

which is the required equivalent T–parameters of the combination.

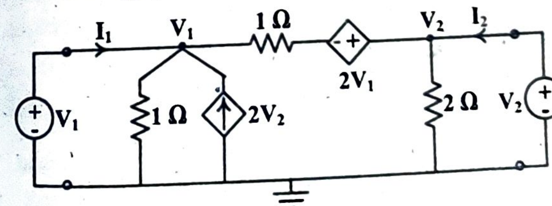
Example 6.15

Find transmission and admittance parameters for the two port network shown in the figure below. Also check its reciprocity and symmetricity. **[2074 Chaitra]**



Solution:

Adding source on both the ports and re–arranging the circuit as follows to apply Nodal Analysis;



Applying KCL at node 1,

$$I_1 + 2V_2 = \frac{V_1 - V_2 + 2V_1}{1} + \frac{V_1 - 0}{1}$$

or, $I_1 = 4V_1 - 3V_2$ ....(1)

Applying KCL at node 2,

$$I_2 = \frac{V_2 - V_1 - 2V_1}{1} + \frac{V_2 - 0}{2}$$

or, $I_2 = -3V_1 + \frac{3}{2}V_2$ ....(2)

Writing equations (1) and (2) in matrix form, we get

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -3 & 3/2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{............(3)}$$

We know, the equation of Y–parameters in matrix form as;

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} \text{............(4)}$$

Comparing equations (3) and (4), we get

$$[Y] = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ -3 & 3/2 \end{bmatrix}$$

For Transmission parameters, we know;

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{............(5)}$$

From equation (2),

$$I_2 = -3V_1 + \frac{3}{2}V_2 \qquad \text{or,} \qquad 3V_1 = \frac{3}{2}V_2 - I_2$$

or, $V_1 = \frac{1}{2}V_2 - \frac{1}{3}I_2$ ....(6)

Substituting expression of V_1 from equation (6) to (1), we get

$$I_1 = 4\left(\frac{1}{2}V_2 - \frac{1}{3}I_2\right) - 3V_2 \quad \text{or,} \quad I_1 = -V_2 - \frac{4}{3}I_2 \text{............(7)}$$

Writing equations (6) and (7) in matrix form,

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} 1/2 & 1/3 \\ -1 & 4/3 \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} \text{............(8)}$$

Comparing equations (5) and (8), we get T–parameters matrix as;

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1/2 & 1/3 \\ -1 & 4/3 \end{bmatrix}$$

For symmetricity and reciprocity:

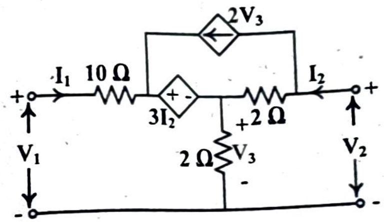
Since $Y_{11} \neq Y_{22}$, the given two port network is **not symmetrical.**

Since $Y_{12} = Y_{21} = -3$, the given two port network is **reciprocal.**

Example 6.16

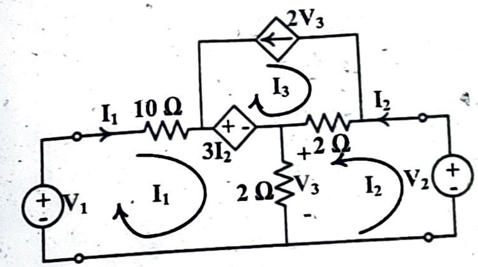
Find the Z–parameters and hence T' –parameters for the network shown in figure below and check if network is symmetrical.

[2075 Ashwin]



Solution:

Adding source on both the ports and performing Mesh Analysis;



Applying KVL in mesh 1,

$$V_1 - 3I_2 = 10I_1 + 2(I_1 + I_2)$$

or, $V_1 = 12I_1 + 5I_2$ ....(1)

Applying KVL in mesh 2,

$$V_2 = 2(I_2 + I_3) + 2(I_2 + I_1)$$

or, $V_2 = 2I_1 + 4I_2 + 2I_3$ ....(2)

In the outer branch of mesh 3, $I_3 = -2V_3$

But $V_3 = 2(I_1 + I_2)$

So, $I_3 = -2 \times 2(I_1 + I_2)$

or, $I_3 = -4I_1 - 4I_2$ ....(3)

Substituting expression of I_3 from equation (3) to (2),

$$V_2 = 2I_1 + 4I_2 + 2(-4I_1 - 4I_2)$$

or, $V_2 = -6I_1 - 4I_2$ ....(4)

Writing equations (1) and (4) in matrix form, we get

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ -6 & -4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{............(5)}$$

We know, the equation of Z–parameters in matrix form as;

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \text{............(6)}$$

Comparing equations (5) and (6), we get

$$[Z] = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 12 & 5 \\ -6 & -4 \end{bmatrix}$$

For T' –parameters, we know;

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} \quad \text{............(7)}$$

From equation (1),

$$V_1 = 12I_1 + 5I_2$$

or, $$5I_2 = V_1 - 12I_1$$

or, $$I_2 = \frac{1}{5}V_1 - \frac{12}{5}I_1 \quad \text{........................(8)}$$

Substituting expression of I_2 from equation (8) to (4), we get

$$V_2 = -6I_1 - 4\left(\frac{1}{5}V_1 - \frac{12}{5}I_1\right)$$

or, $$V_2 = \frac{-4}{5}V_1 + \frac{18}{5}I_1 \quad \text{...........(9)}$$

Writing equations (9) and (8) in matrix form,

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} -4/5 & -18/5 \\ 1/5 & 12/5 \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} \quad \text{...........(10)}$$

Comparing equations (8) and (11), we get T'–parameters matrix as;

$$\begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} = \begin{bmatrix} -4/5 & -18/5 \\ 1/5 & 12/5 \end{bmatrix}$$

For symmetricity:

Since $Z_{11} \neq Z_{22}$, the given two port network is **not symmetrical.**

Multiple Choice Questions

1. The Two Port Network without any source in their branches is called
   a) Active network b) Passive network
   c) both d) none
2. While determining Z-parameters of a Two Port Network, which set of variables are independent variables?
   a) V_1, V_2 b) I_1, I_2
   c) V_1, I_1 d) V_2, I_2
3. While determining Z-parameters of a Two Port Network, which set of variables are dependent variables?
   a) V_1, V_2 b) I_1, I_2
   c) V_1, I_1 d) V_2, I_2
4. While determining Y-parameters of a Two Port Network, which set of variables are independent variables?
   a) V_1, V_2 b) I_1, I_2
   c) V_1, I_1 d) V_2, I_2
5. While determining Y-parameters of a Two Port Network, which set of variables are dependent variables?
   a) V_1, V_2 b) I_1, I_2
   c) V_1, I_1 d) V_2, I_2
6. While determining T-parameters of a Two Port Network, which set of variables are independent variables?
   a) V_1, V_2 b) I_1, I_2
   c) V_1, I_1 d) V_2, I_2
7. While determining T-parameters of a Two Port Network, which set of variables are dependent variables?
   a) V_1, V_2 b) I_1, I_2
   c) V_1, I_1 d) V_2, I_2
8. For a Two Port Network to be reciprocal,
   a) $Z_{12} = Z_{21}$ b) $Y_{12} = Y_{21}$
   c) $AD - BC = 1$ d) all of the above
9. For a Two Port Network to be symmetrical,
   a) $Z_{12} = Z_{21}$ b) $Y_{12} = Y_{21}$
   c) $A = D$ d) all of the above

ANSWERS

1.(b), 2.(b), 3.(a), 4.(a), 5.(b), 6.(d), 7.(c), 8.(d), 9.(c)

□□□