Regression Analysis

In [25]:

```
import os
#os.environ['KMP_DUPLICATE_LIB_OK']='True'
import pandas as pd
import numpy as np
import statsmodels.api as sm
import statsmodels.formula.api as smf
from statsmodels.stats.outliers_influence import variance_inflation_factor as vi
f
import scipy.stats as stats
import matplotlib.cm as cm
from IPython.display import display
from mpl_toolkits.mplot3d import Axes3D
from sklearn.feature_selection import f_regression
from statsmodels.stats.anova import anova_lm
import matplotlib.pyplot as plt
```

In [26]:

```
mileage = pd.read_csv('data_reshaped.csv')
gdp=pd.read_csv('GDP.csv')
US_EMP = pd.read_csv('US_EMP_RATE.csv')
```

In [27]:

```
US_EMP = US_EMP.iloc[:-1,:]
US_EMP.columns = ['date1','EMP']
US_POP = pd.read_csv('US_POP.csv')
US_POP = US_POP.iloc[:-1,:]
US_POP.columns = ['date2','POP']
```

In [28]:

```
len(mileage)==len(gdp)==len(US_EMP)==len(US_POP) == 60
```

Out[28]:

```
True
```

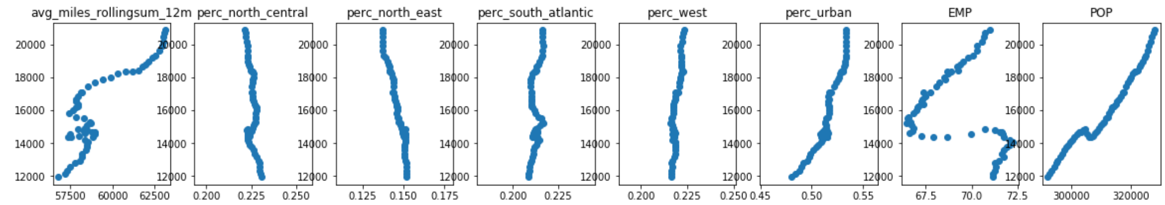
In [29]:

```
data = pd.concat([mileage,gdp,US_EMP,US_POP],axis=1)
data = data.drop(columns = ['DATE','date2','date1','quarter'])
features = data.drop(columns = ['GDP'])
```

See relationship between the response and predictors

In [30]:

```
plt.figure(figsize=(20, 3))
for i,col in enumerate(features.columns):
    plt.subplot(1,len(features.columns),i+1)
    plt.scatter(features[col],data.GDP)
    plt.title(col)
```



Correlation between the response and predictors

In [31]:

```
import seaborn as sns
correlation_matrix = data.corr().round(2)
# annot = True to print the values inside the square
plt.figure(figsize = (16,5))
sns.heatmap(data=correlation_matrix, annot=True)
```

Out[31]:

```
<matplotlib.axes._subplots.AxesSubplot at 0x1c2383ab00>
```

Full Model

In [33]:

```
reg_M1 = smf.ols('GDP~avg_miles_rollingsum_12m + perc_north_central + \
                  perc_north_east + perc_south_atlantic + perc_west + \
                  perc_urban + EMP + POP',data=data).fit()
reg_M1.summary()
```

Out[33]:

OLS Regression Results

Dep. Variable:	GDP	**R-squared:**	0.997
Model:	OLS	**Adj. R-squared:**	0.997
Method:	Least Squares	**F-statistic:**	2505.
Date:	Sun, 13 Oct 2019	**Prob (F-statistic):**	2.26e-63
Time:	20:17:20	**Log-Likelihood:**	-372.29
No. Observations:	60	**AIC:**	762.6
Df Residuals:	51	**BIC:**	781.4
Df Model:	8		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-1.011e+05	1.66e+04	-6.075	0.000	-1.34e+05	-6.77e+04
avg_miles_rollingsum_12m	-0.1009	0.048	-2.120	0.039	-0.196	-0.005
perc_north_central	1.061e+04	1.85e+04	0.572	0.570	-2.66e+04	4.78e+04
perc_north_east	5112.3242	3.05e+04	0.168	0.867	-5.6e+04	6.63e+04
perc_south_atlantic	5.525e+04	1.96e+04	2.822	0.007	1.59e+04	9.46e+04
perc_west	6.735e+04	3.37e+04	1.999	0.051	-303.523	1.35e+05
perc_urban	-1.833e+04	9329.298	-1.964	0.055	-3.71e+04	403.630
EMP	297.2237	27.069	10.980	0.000	242.880	351.568
POP	0.2645	0.021	12.881	0.000	0.223	0.306

Omnibus:	0.424	**Durbin-Watson:**	0.811
Prob(Omnibus):	0.809	**Jarque-Bera (JB):**	0.575
Skew:	-0.159	**Prob(JB):**	0.750
Kurtosis:	2.642	**Cond. No.**	8.01e+08

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 8.01e+08. This might indicate that there are
strong multicollinearity or other numerical problems.

Reduced Model

In [34]:

```
features2 = features.drop(columns=['perc_north_east','perc_north_east'])
```

In [35]:

```
reg_M2 = smf.ols('GDP~avg_miles_rollingsum_12m + perc_south_atlantic + perc_west
+ \
                perc_urban + EMP + POP',data=data).fit()
```

In [36]:

```
reg_M2.summary()
```

Out[36]:

OLS Regression Results

Dep. Variable:	GDP	**R-squared:**	0.997
Model:	OLS	**Adj. R-squared:**	0.997
Method:	Least Squares	**F-statistic:**	3448.
Date:	Sun, 13 Oct 2019	**Prob (F-statistic):**	7.72e-67
Time:	20:17:27	**Log-Likelihood:**	-372.49
No. Observations:	60	**AIC:**	759.0
Df Residuals:	53	**BIC:**	773.6
Df Model:	6		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
Intercept	-9.522e+04	7259.016	-13.118	0.000	-1.1e+05	-8.07e+04
avg_miles_rollingsum_12m	-0.0959	0.045	-2.136	0.037	-0.186	-0.006
perc_south_atlantic	5.21e+04	1.79e+04	2.914	0.005	1.62e+04	8.8e+04
perc_west	6.689e+04	3.24e+04	2.068	0.044	1999.845	1.32e+05
perc_urban	-2.034e+04	5129.333	-3.964	0.000	-3.06e+04	-1e+04
EMP	288.0799	19.587	14.708	0.000	248.794	327.366
POP	0.2626	0.008	33.602	0.000	0.247	0.278

Omnibus:	1.073	**Durbin-Watson:**	0.803
Prob(Omnibus):	0.585	**Jarque-Bera (JB):**	1.100
Skew:	-0.216	**Prob(JB):**	0.577
Kurtosis:	2.497	**Cond. No.**	6.99e+08

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 6.99e+08. This might indicate that there are
strong multicollinearity or other numerical problems.

Multicollinearity

In [37]:

```
[vif(np.array(features2),i) for i in range(1,len(features2.columns))]
```

Out[37]:

```
[29003.525591546262,
 18901.71434688995,
 101389.48686298347,
 49777.60035428292,
 7869.024549142253,
 23646.53179038305]
```

In [38]:

```
from statsmodels.stats.diagnostic import het_breuschpagan
bp_test = het_breuschpagan(reg_M2.resid, reg_M2.model.exog)
labels = ['LM Statistic', 'LM-Test p-value', 'F-Statistic', 'F-Test p-value']
print(dict(zip(labels, bp_test)))
```

```
{'LM Statistic': 12.04268347554496, 'LM-Test p-value': 0.06102333452
9280264, 'F-Statistic': 2.218160753699983, 'F-Test p-value': 0.05543
935852191886}
```

Serial Correlation

In [39]:

```
bg_test=sm.stats.diagnostic.acorr_breusch_godfrey(reg_M2)
labels = ['LM Statistic', 'LM-Test p-value', 'F-Statistic', 'F-Test p-value']
print(dict(zip(labels, bg_test)))
```

```
{'LM Statistic': 28.239651583435375, 'LM-Test p-value': 0.0016524026
139669256, 'F-Statistic': 3.8233365773028156, 'F-Test p-value': 0.00
09555603321952383}
```

Normality Check

In [40]:

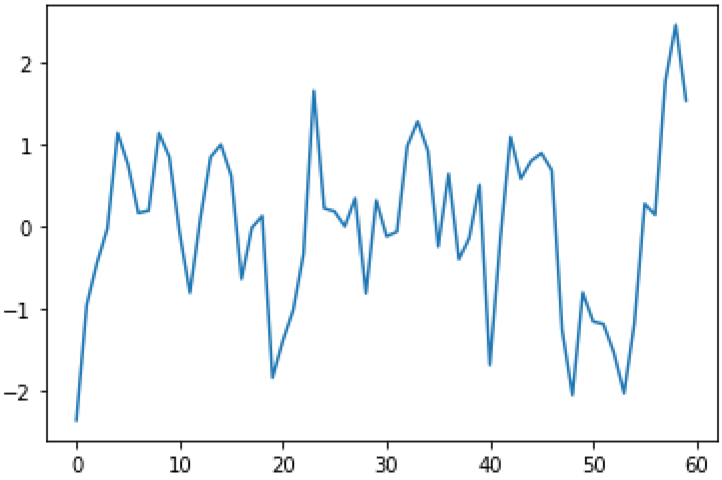
```
infl = reg_M2.get_influence()
```

In [41]:

```
plt.plot(data.index,infl.resid_studentized_external)
```

Out[41]:

```
[<matplotlib.lines.Line2D at 0x1c21554470>]
```



Influential Points

In [42]:

```
n = len(data)
p = len(features2.columns) + 1
```

In [43]:

```
inflsum = infl.summary_frame()
reg_dffits=inflsum.dffits
dffits_thresh = 2*np.sqrt((p+1)/(n-p-1))
atyp_dffits = np.abs(reg_dffits) > dffits_thresh
a_dffits_ind = set(data.index[atyp_dffits])
a_dffits_ind
```

Out[43]:

```
{0, 23, 58}
```

F-Test

In [58]:

```
## F-test : reduced vs full model
## reg_M2 : reduced model (i.e. null hypothesis), reg_M1 : full model (ie altern
ate hypothesis)
F_stat = ((reg_M2.ssr - reg_M1.ssr) / (reg_M2.df_resid - reg_M1.df_resid) / (reg
_M1.ssr / reg_M1.df_resid))
F_stat
```

Out[58]:

```
0.16600064173028078
```

In [61]:

```
## p - value is 0.15 >> 0.05, hence we donot have enough information to reject t
he null hypothesis
## hence we accept the reduced model
import scipy.stats as stats
alpha = 0.05
p_value = stats.f.cdf(F_stat, reg_M2.df_resid - reg_M1.df_resid, reg_M1.df_resid
)
p_value
```

Out[61]:

```
0.1524984894387115
```