SETS

The basic concepts of sets

Phiwokwakhe Khathwane(2022004325) and Tshepo Lekgetho(2024193605)

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MATM 1622 assignment

FORMAL DEFINITION OF SETS

What's are they?

A set is a fundamental concept in mathematics that denotes a group of unique objects collectively referred to as its members or elements.

What are they used for?

Sets are widely used in various branches of mathematics and have well-defined notation and properties. Mathematical sets are required for data organization and analysis, problem solving, decision making, and understanding of various phenomena. They are used to efficiently manage and analyse data in fields such as statistics, databases, and information retrieval.

Things to know about sets

- **Notation** - Sets are typically denoted by capital letters, such as A, B, C, etc. The elements of a set are enclosed in curly braces {} and separated by commas.

    For example:

    ❖ If we have a set A that contains the elements 1, 2, and 3, it can be written as A = {1, 2, 3}.

    ❖ Sets can also be described using set-builder notation, where a condition is given to determine the elements of the set. This notation is written as {x | condition}, where x represents the elements that satisfy the given condition, and they are separated by commas. For example:

- **Elements** - The objects or values within a set are called elements. Each element in a set is unique, and there are no duplicate elements within a set.

    For example:
    ❖ Set A contains **three** distinct elements: A = {1, 2, 3}

    ❖ Set B contains five distinct **elements**: B = {a, b, c, e, d}

- **Set Equality** - Two sets are considered equal **if and only if** they contain the same elements, regardless of the order in which the elements are listed.

    For example:

    ❖ {1, 2, 3} = {3, 2, 1} (The order doesn't matter)

- **Cardinality** - The number of elements in a set is called its cardinality. It is denoted by |A|, where A is the set.

    For example:

    ❖ If A = {1, 2, 3}, then |A| = 3.

- **Containment** - A Set A is said to be a subset of another set B if every element of A is also an element of B. This is denoted as $A \subseteq B$.
    If A is a subset of B, but A is not equal to B, it is called a proper subset, denoted as $A \subset B$.

- **Universal Set** - The universal set, often denoted as U, is the set that contains all the elements under consideration in a particular context.

- **Empty Set** - The empty set, denoted as ∅ or {}, is a set that contains no elements. It is a subset of every set.

- **Set Operations** - Sets can be combined or manipulated using various set operations, including:

  - ❖ **Union (A ∪ B):** Combines two sets to create a new set containing all distinct elements from both sets.
  - ❖ **Intersection (A ∩ B):** Creates a new set containing elements that are common to both sets.
  - ❖ **Difference (A − B):** Forms a new set with elements from the first set that are not in the second set.
  - ❖ **Complement ($\overline{A}$):** Contains all elements not in the set.

Still don't understand yet?

Let's consider the following. If set A has the elements 1,2,3,5,2 and 5, another set B which have the elements 3,5,2,4 and 6 and set C which has the elements 1 and 2.

Notation

A = {1,2,3,5}
B = {2,3,4,5,6}
C = {1,2}

- As mentioned above, in set A we will only consider **distinct** values and the order in all the sets does not matter.

Cardinality

$|A| = 4$
$|B| = 5$
$|C| = 2$

- As mentioned above, we count the number of elements in each set.

Containment

$C \subset A$ → Means that C is a proper set of A.
$A \not\subset B$ → Means A is not a subset of B.
$B \not\subset A$ → Means B is not a subset of A.
$C \not\subset B$ → Means C is not a subset of B.

Set operations

Union:

$A \cup B = \{1,2,3,4,5,6\}$
$A \cup C = \{1,2,3,5\}$

Intersection:

$A \cap B = \{2,3,5\}$
$A \cap B = \{1,2\}$

Difference:

$A - B = \{1\}$
$B - A = \{4,6\}$

$C - A = \{\}$ or $\emptyset$

- Note: The union operations and intersections are **commutative**, meaning $A \cup B = B \cup A$ and $A \cap B = B \cap A$. (There's a whole new chapter for that which is why we won't go into details).

HOPE YOU NOW UNDERSTAND SETS ☺