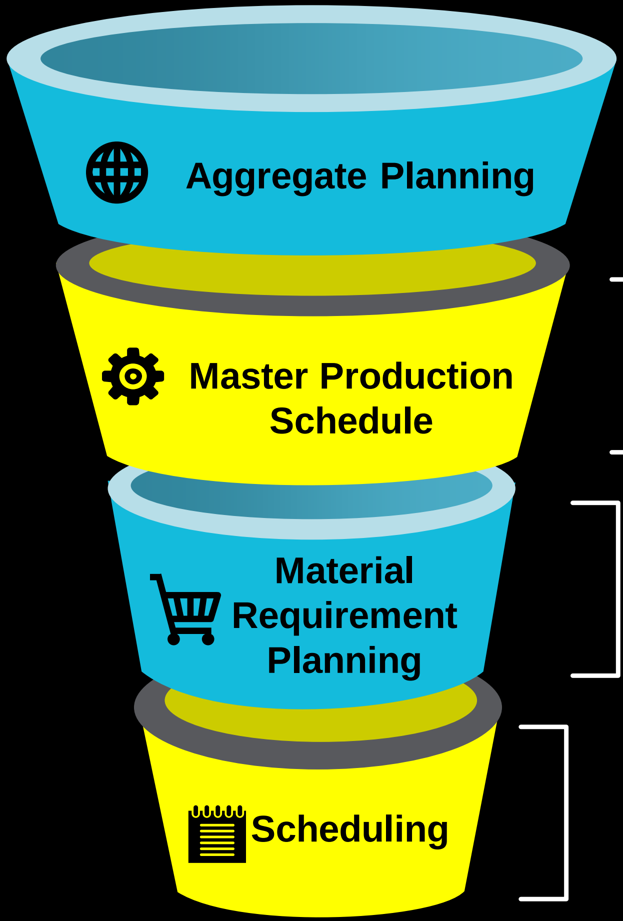
Module-5

Production Scheduling and Sequencing

Content

- 5.1 **Scheduling**: Inputs for scheduling, loading and scheduling devices, factors influencing scheduling, scheduling techniques, use of Gantt Charts and basic scheduling problems. Project scheduling by using elements of network analysis –PERT & CPM, cost analysis & crashing, resource leveling
- **5.2 Sequencing:** Product sequencing, dispatching, progress report & expediting and control. Johnson's Rule for optimal sequence of N jobs on 2 machine. Process n Jobs on 3 Machines (n/3 problem) and Jackson Algorithm. Processing of 2 Jobs on m Machine (2/m) problem



Keeping in view the estimated future demand of the final product, aggregate planning is done for long time horizon usually for 1-3 years.

Long term plan is broken down in master production schedules for shorter time period comprising of the complete information about the quantity and time of different products to be produced.

On the basis of Master production schedule, demand for the raw material, components etc (known as secondary demand) are estimated under the material requirement planning system.

Finally, very short term schedules are prepared which establish both the timing and the use of resources within a firm.

Scheduling

- Inputs for scheduling
- Loading and scheduling devices
- Factors influencing scheduling
- Scheduling techniques
- Use of Gantt Charts
- Basic scheduling problems.
- Project scheduling by using elements of network analysis
  - PERT & CPM
  - Cost analysis & crashing
  - Resource leveling

Introduction - Scheduling

- Scheduling process involves the decision making regarding the allocation of available capacity or resources. Thus, the process can be called as time-phased plans or schedule of activities.
- Scheduling is concerned with preparation of machine loads and fixation of starting and completion dates for each of the operations. Machines have to be loaded according to their capability of performing the given task and according to their capacity. Thus the duties include:
  - Loading, the machines as per their capability and capacity.
  - Determining the start and completion times for each operation.
  - To coordinate with sales department regarding delivery schedules.
- Detailed scheduling determines start times, finish times and work assignments for all jobs at each work centre. Calendar times are specified when job orders, employees, and materials (inputs), as well as job completion (outputs), should occur at each work centre. By estimating how long each job will take to complete and when it is due, schedulers can establish start and finish dates and develop the detailed schedule.

Introduction

- The production-schedules are developed by performing the following functions:
  - *Loading*
  - *Sequencing*
  - *Scheduling*
- *A production schedule is the time table that specifies the times at which the jobs in a production department will be processed on various machines.*
- The schedule gives the starting and ending times of each job on the machines on which the job has to be processed.
- Last stage of planning before production occurs.

Objectives of scheduling

- Meet customer due dates
- Minimize job lateness
- Minimize response time
- Minimize completion time
- Minimize time in the system
- Minimize overtime
- Maximize machine or labor utilization
- Minimize idle time
- Minimize work-in-process inventory

Scheduling Operations

- Process Industry
  - Linear programming
  - EOQ with non-instantaneous replenishment
- Mass Production
  - Assembly line balancing
- Project
  - Project –scheduling techniques (PERT, CPM)
- Batch Production
  - Aggregate planning
  - Master scheduling
  - Material requirements planning (MRP)
  - Capacity requirements planning (CRP)

Inputs for scheduling

1. Bill of Material (BOM) & Blue prints (Drawings) – shows kind of item required and detailed work to be carried out.
2. Master Production Schedule (MPS) – shows priority and quantity of finished products , due dates, quantity of raw materials
3. Route sheet – shows operations and machines to be scheduled and processing times
4. Inventory Records – shows availability of materials & tools and time required for procurement of items not in stock.
5. Machine-Load chart – shows the quantity of work already scheduled on various machines and the amount of spare (remaining) capacity available for use.

Loading

- Machine loading is the process of converting operation schedule into practices in conjunctions with routing.
- This process is assigning specific jobs to machines, men, or work centers based on relative priorities and capacity utilization.
- Loading ensures maximum possible utilization of productive facilities and avoid bottleneck in production.
- It‘s important to either overloading or under loading the facilities, work centers or machines to ensure maximum utilization of resources.

Loading and scheduling devices

- Loading devices
  - Machine load chart
- Scheduling devices
  - Gantt charts
  - Milestone chart
  - Progress chart
  - Networks
  - Earned values
  - Line of balance

Factors influencing scheduling

External Factors

1. Customers demand
2. Customers delivery dates
3. Stock of goods already lying with the dealers & retailers.

Internal Factors

1. Stock of finished good with firm
2. Time interval to manufacture each component, subassembly and then assembly.
3. Availability of equipments & machinery their capacity & specification.
4. Availability of materials
5. Availability of manpower
6. Manufacturing facilities – Power, MH devices, work-bench area
7. Feasibility of economic production run - EOQ

Scheduling Approaches

There are mainly two types of approaches which are commonly used:

1. ***Forward scheduling***: In forward scheduling, the scheduler schedules all activities forward in time.
   - (a) Jobs are given earliest available time slot in operation;
   - (b) Usually excessive WIP results.

2. ***Backward scheduling:*** In backward scheduling, the scheduler begins with a planned date and moves backward in time.
   - (a) Start with due date and work backward through operations reviewing lead times;
   - (b) Less WIP but must have accurate lead time.

Scheduling techniques

- Graphs-
  - Gantt Chart Job shop

Job shop

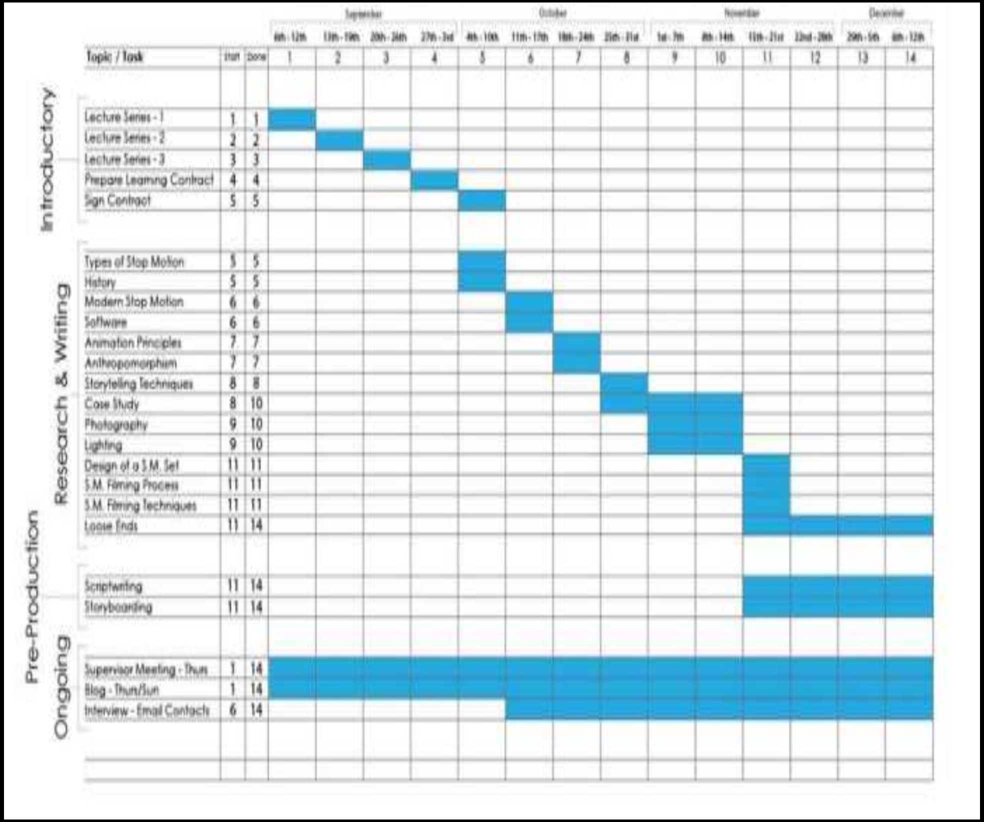
- Diagram-
  - Network diagram
  - CPM
  - PERT

PROJECTS

Gantt Charts

- A **Gantt chart** is a type of ***bar chart*** that illustrates a project schedule, named after its inventor, ***Henry Gantt***
- This chart lists the tasks to be performed on the vertical axis, and time intervals on the horizontal axis
- The *Gantt progress chart* graphically displays the current status of each job or activity relative to its scheduled completion date
- To illustrate the relationship between project activities & time
- To show the multiple project activities on one chart
- To provide a simple & easy to understand representation of project scheduling

Gantt Charts - sample



Steps to Creating a Gantt Chart

1. Determine Project start date and deadline
2. Gather all information surrounding the list of activities within a project – the Work Breakdown Structure may be useful for this
3. Determine how long each activity will take
4. Evaluate what activities are dependant on others
5. Create Graph shell including the timeline and list of activities
6. Using either Forward Scheduling or Backward Scheduling Begin to add bars ensuring to include dependencies and the full duration for each activity

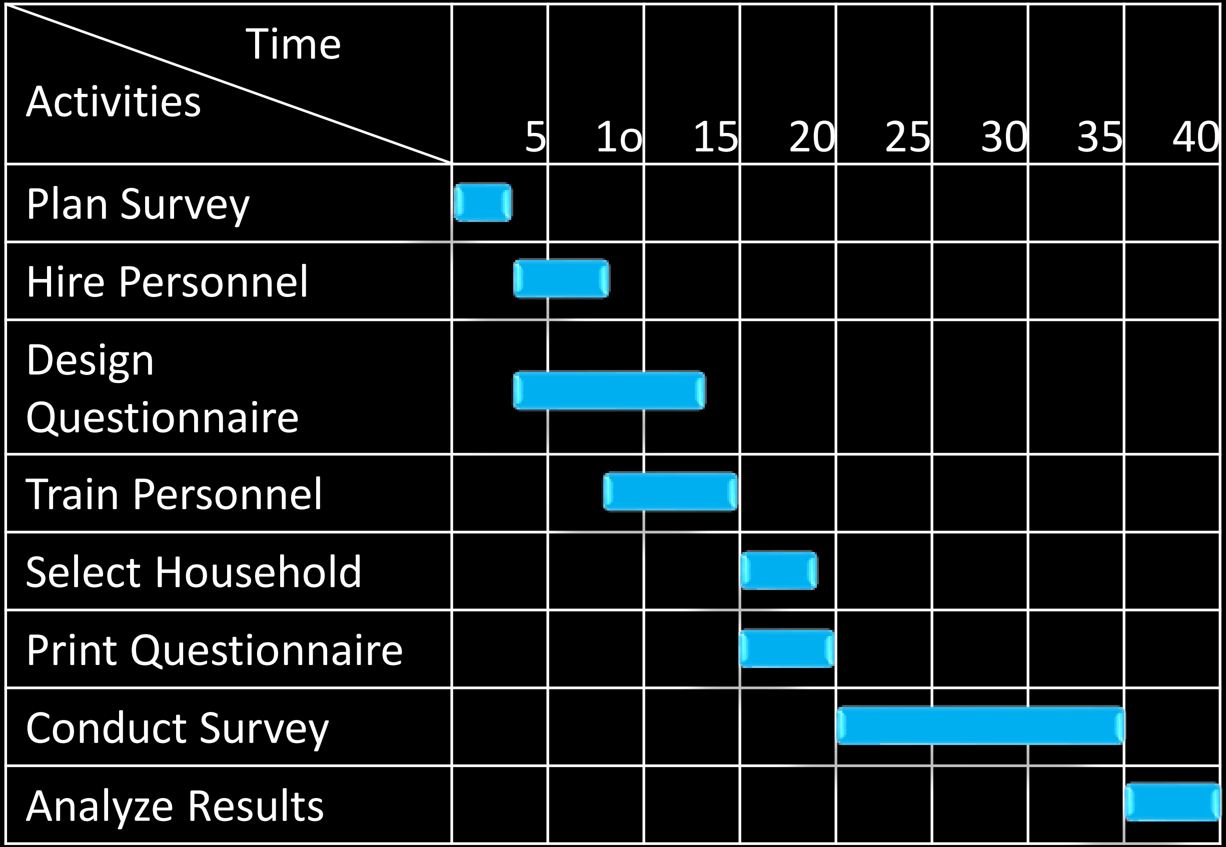
Gantt Chart- Example

Consider a project on market survey of households. Do the following for this project-

- a) Enumerate all the activities
- b) Write down the logical sequence of the activities (in terms of immediate predecessors).
- c) Get the estimate for the time required by each activity.
- d) Draw Gantt chart.

Gantt Chart- Solution

Activity	Description	Immediate Predecessor	Duration (days)
A	Plan Survey	-	3
B	Hire Personnel	A	5
C	Design Questionnaire	A	10
D	Train Personnel	B	7
E	Select Household	C,D	4
F	Print Questionnaire	C	5
G	Conduct Survey	E,F	15
H	Analyze Results	G	5



Advantages of Gantt Chart

- A useful tool for displaying time-based information within a project
- Very simple to create
- They provide a useful overview of project activities, a good starting point for project planning
- The charts are widely used and understood
- There exists several PC software packages that allow you to build Gantt Charts

Limitations of Gantt Chart

- It does not help identifying the key activities which are important for project completion in time.
- It does not permit identification of important stages in the completion of the project.
- The time (duration) estimates of the activities are made well in advance, and therefore, are likely to involve considerable amount of uncertainty. But uncertainties cannot be taken into account in scheduling activities using a Gantt chart.
- Utility of this chart becomes limited, especially in projects having a large number of activities.
- The Gantt Chart does not explain the reasoning behind the chosen duration of each activity
- The Gantt Chart is very difficult to update when changes to the project plan take place. This makes it time consuming and results in long-term planning being very difficult
- Gantt Charts encourage a one-step approach to planning – this prevents flexibility in project planning
- As Gantt Charts are difficult to update manually, they can often become obsolete
- The charts do not consider project costs or resources.

Basic scheduling problems

- Bar Chart
- Network analysis consisting of three different closely resembling methods.
  namely :
  - Precedence diagram method (PDM)
  - Critical Path Method (CPM)
  - Program evaluation and review technique (PERT)
- Line of Balance (LOB)

Project

What is a project?

Project is an interrelated set of activities that has a definite starting and ending point resulting in to an unique product. Few examples of project are-

- Constructing a bridge, dam, highway or building
- Producing an aero plane, missile or rocket
- Introducing a new product
- Installation of a large computer system
- Construction of a ship
- Maintenance of major equipment/plant
- Commissioning of a power plant
- Conducting national election

Steps in Project scheduling

Managing a project (regardless of its size and complexity) requires identifying every activity to be undertaken and planning- when each activity must begin and end in order to complete the overall project in time. Typically all projects involve the following steps:

- Description of the project
- Development of network diagram
- Insertion of time of starting/ending of each activity
- Analysis of the network diagram.
- Development of the project plan
- Exception of the project
- Periodically assessment of the progress of project

Project scheduling

- Network analysis
  - CPM - Critical Path Method
  - PERT – Program Evaluation & Review Technique
- Crashing

Terminologies used in network diagram

- **Activity:** An activity means work/job. it is a time consuming process. it is represented by an arrow (→) in the network diagram as Tail → head

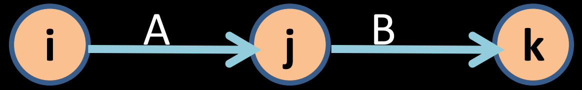
"A" is called an Activity

- **Event :** An event is a specific instant of time → marks the "start" and "end" of an activity

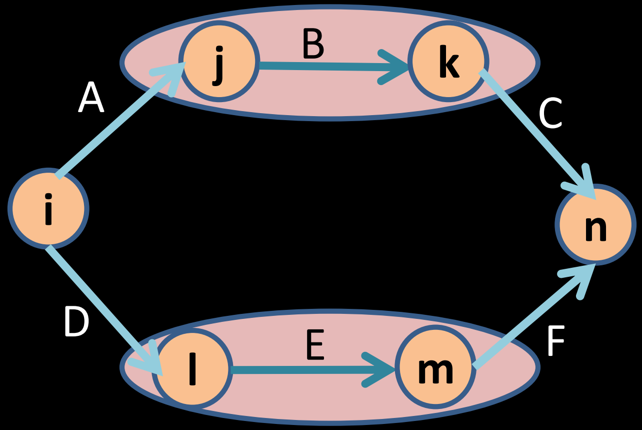
Tail Event   Head Event

- ***Duration (d)*** : Duration is the estimated or actual time required to complete a task or an activity.
- ***Predecessor activity***: Activities that must be completed immediately prior to the start of another activity are called predecessor activities.
- ***Successor activity*** : Activities that cannot be started until one or more of other activities are completed but immediately succeed them are called successor activities.
- ***Dummy activity***: An activity which does not consume any resource but merely depicts the dependence of one activity on other is called dummy activity. It is introduced in a network when two or more parallel activities have the same start and finish nodes. Shown by dotted arrow line.

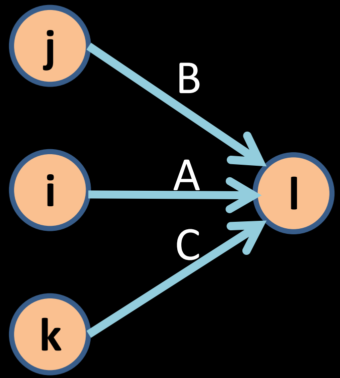
- Serial activities flow from one to the next



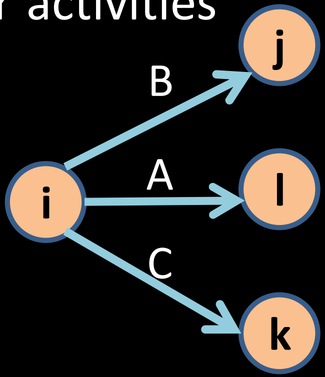
- Concurrent activities are accomplished together.



- Merge activities have two or more immediate predecessor



- Burst activities have two or more successor activities



- ***Total project time:*** Time to complete the project. In other words, it is the duration of critical path.
- ***Earliest start time (ESi)***: It is the earliest possible time at which an activity can start. it is calculated by moving from 1st to last event in the network diagram.
- ***Latest start time (LS)*** : It is the latest possible time by which an activity can start.
- ***Earliest finish time (EFj)*** : It is the earliest possible time at which an activity can finished/end.
- ***Latest finish time (LFi)*** : It is the last event time of the head event. It is calculated by moving backward in the network diagram.
- ***Float/slack*** : Slack is with reference to an event. Float is with reference to an activity.
- ***Total float*** : (Latest finish time- Earliest start time) – Activity duration
- ***Free float*** : (Earliest finish time- Earliest start time) – Activity duration.
- ***Critical path***: the longest possible continuous pathway taken from the initial event to the terminal event. It determines the total calendar time required for the project; and, therefore, any time delays along the critical path will delay the reaching of the terminal event by at least the same amount.
- ***Critical activity***: An activity that has total float equal to zero. An activity with zero float is not necessarily on the critical path since its path may not be the longest.

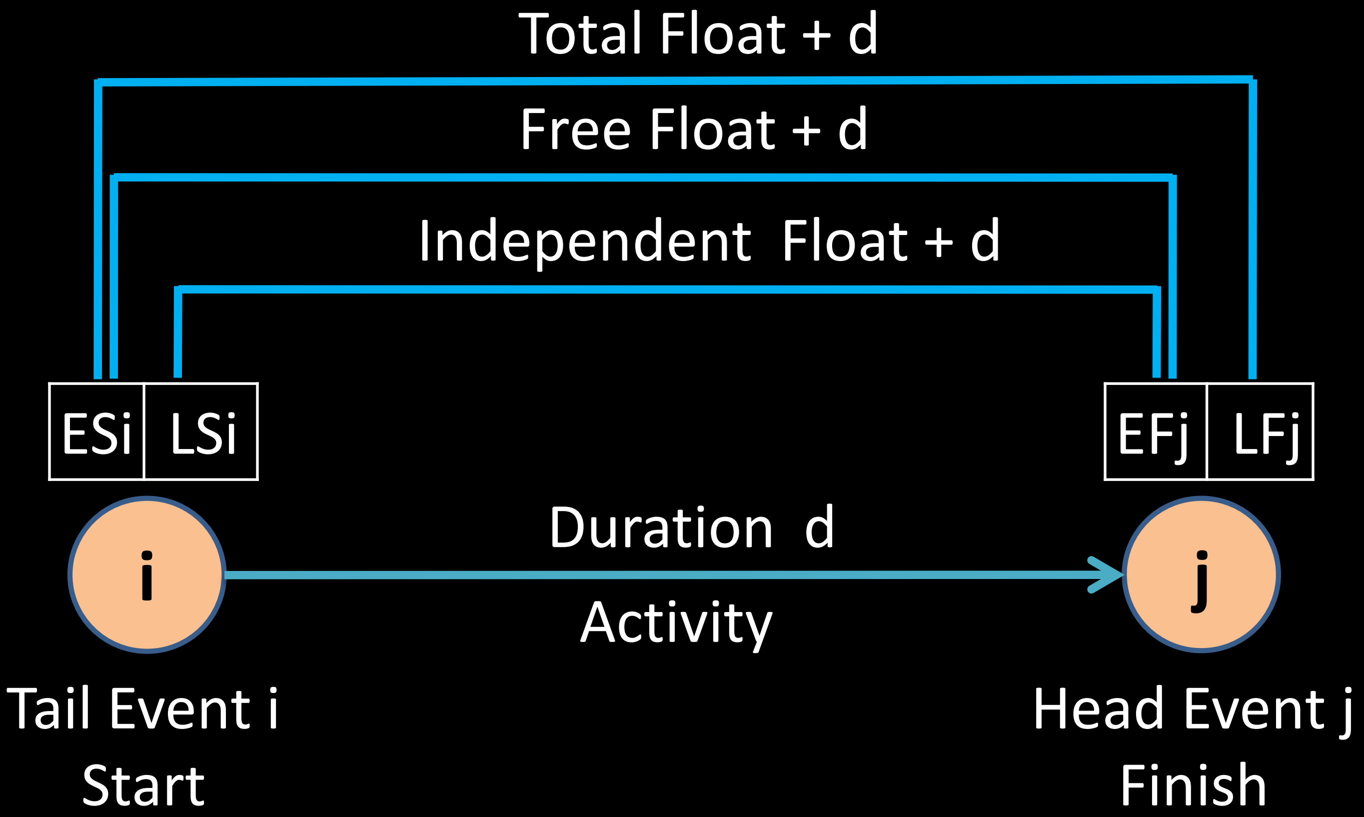
Float/Slack

- It may be observed that for every critical activity in a network, the earliest start and latest start time are the same.
- This is so since the critical activities cannot be scheduled later than their earliest schedule time without delaying the total project duration, they do not have any flexibility in scheduling.
- Since there exists only one path through the network that is the longest i.e critical path , the other paths must either be equal or shorter.
- However, non-critical activities do have some flexibility i.e. these activities can be delayed for some time without affecting the project duration.
- Therefore, there are activities that can be completed before the time when they are actually needed.
- The time between the scheduled completion date and the required date to meet critical path is referred as the slack/float time.
- The activities on the critical path have zero slack/float time, i.e. any delay in these activities will delay the project completion.

- ***Float or Slack = (Time available for completion of activity)***
  – (Time necessary to complete the same)
  = (LST – EST) or (LFT – EFT)
- This flexibility is termed as ***slack in case of an event*** and as ***float in case of an activity***
- ***Slack is used with PERT and Float is used with CPM***
- The terms "slack" and "float" are often used interchangeably. However, the essential difference between the terms is that slack is typically associated with inactivity, while float is associated with activity. Slack time allows an activity to start later than originally planned, while float time allows an activity to take longer than originally planned.
- The activities on the critical path have zero slack/float time, i.e. any delay in these activities will delay the project completion.
- The use of slack/float time provides better resource scheduling.
- It is also used as warning sign i.e. if available slack begins to decrease then activity is taking longer than anticipated.

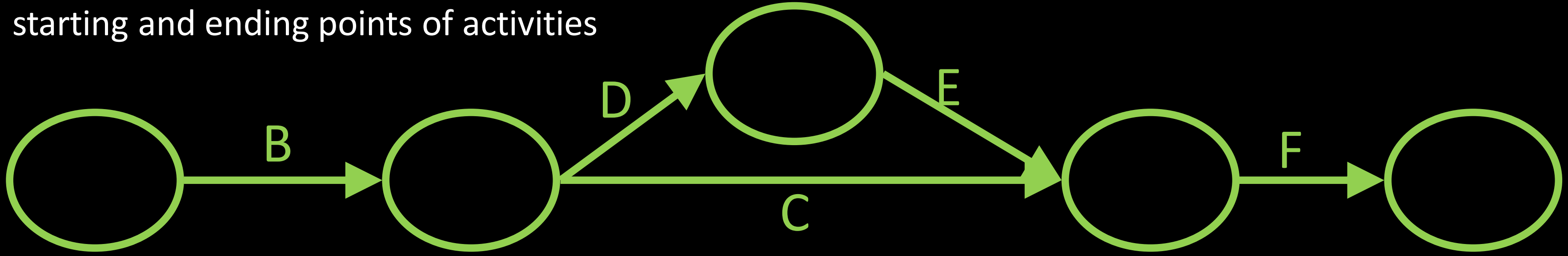
- Positive slack would indicate ahead of schedule; negative slack would indicate behind schedule; and zero slack would indicate on schedule.
- Thus, the difference between the length of a given path and the length of the critical path is known as slack/float. Knowing where slack/float is located helps managers to prioritize allocation of scarce resources and direct their efforts to control activities.
- Hence , ***float*** or ***slack*** is a measure of the excess time and resources available to complete a task.
- It is the amount of time that a project task can be delayed without causing a delay in any subsequent tasks (***free float***) or the whole project (***total float***).
- ***Independent Float*** : It is the amount of time by which the start of an activity can be delayed without affecting earliest start time of an immediate following activities assuming that the preceding activity has finished at its latest finish time.

  = (EST of head event – LST of tail event) – Activity duration.

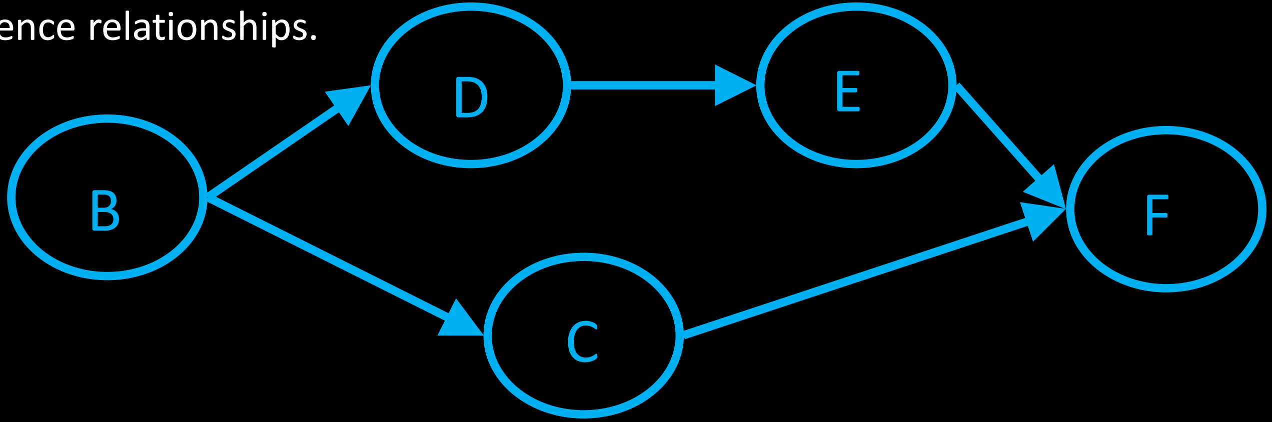

Total Float + d
Free Float + d
Independent Float + d
ESi
LSi
EFj
LFj
Duration d
i
j
Activity
Tail Event i
Start
Head Event j
Finish

AOA vs. AON

Activities On Arc (AOA)- Arrows are used to represent activities or tasks. Nodes represent starting and ending points of activities



Activities On Node (AON)- Nodes are used to represent activities or tasks, while arrows represent precedence relationships.



Forward Pass

- Forward pass determines the earliest times (ES- Early Start) each activity can begin and the earliest it can be completed (EF- Early Finish).
- There are three steps for applying the forward pass:
  - Add all activity times along each path as we move through the network (ES + d = EF)
  - Carry the EF time to the activity nodes immediately succeeding the recently completed node. That EF becomes the ES of the next node, unless the succeeding node is a merge point
  - At a merge point, the largest preceding EF becomes the ES for that node (because the earliest the successor can begin is when all preceding activities have been completed)

Backward Pass

- The goal of the backward pass is to determine each activity's Late Start (LS) and Late Finish (LF) times.
- There are three steps for applying the backward pass:
    - Subtract activity times along each path through the network (LF – d = LS).
    - Carry back the LS time to the activity nodes immediately preceding the successor node. That LS becomes the LF of the next node, unless the preceding node is a burst point.
    - In the case of a burst point, the smallest succeeding LS becomes the LF for that node (because the latest the predecessor can finish is when any one of the successor activities should start)

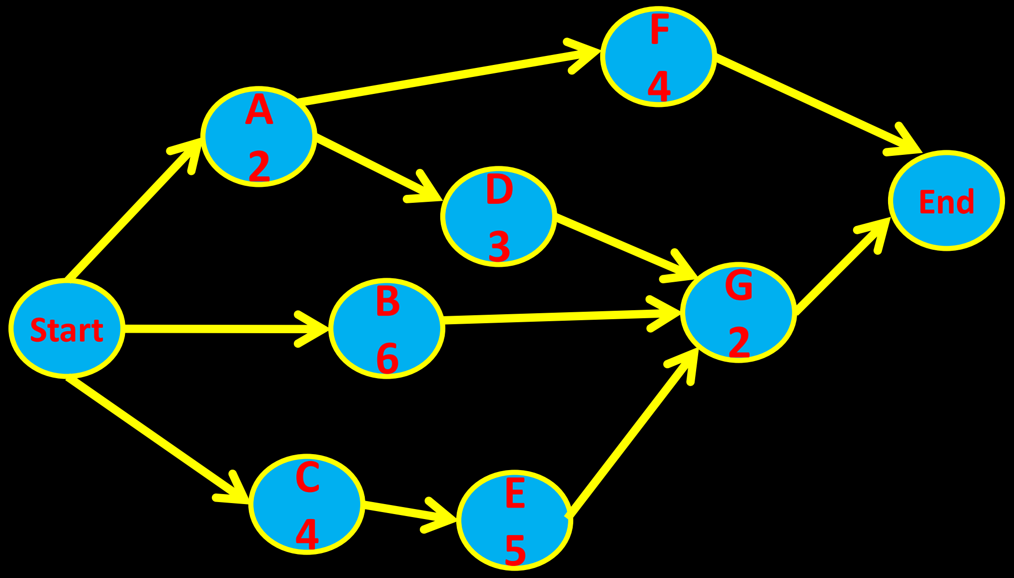
Project scheduling - network analysis

Guidelines for Network Construction-

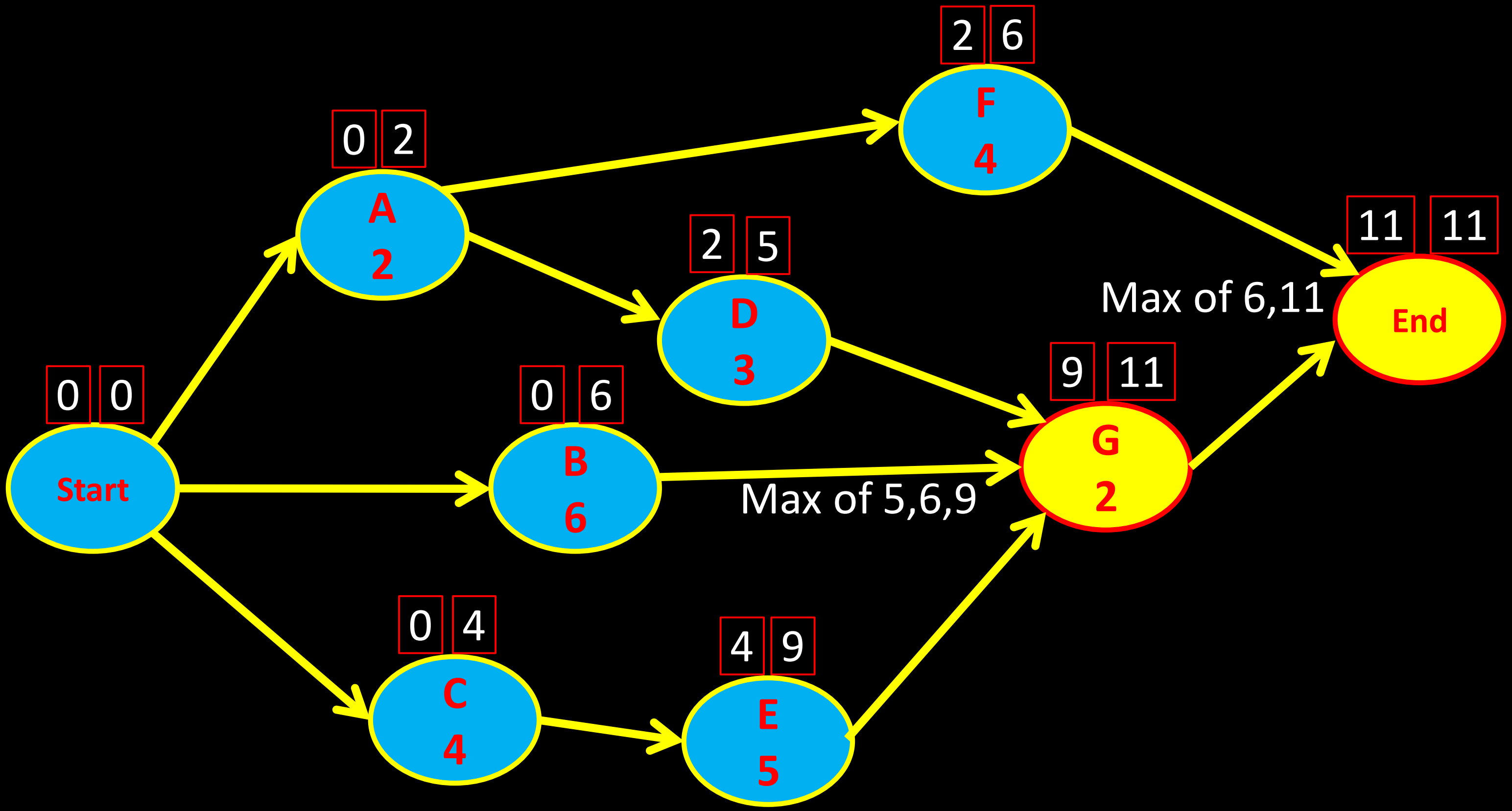
- The network should have a unique starting node (tail event) and unique completion node (head event).
- No activity should be represented by more than one arrow (→) in the network.
- No two activities should have the same starting node and same ending node.
- The flow of the diagram should be from left to right.
- Dummy activity is an imaginary activity indicating precedence relationship only. duration of dummy activity is zero.
- The length of arrow bear no relationship to activity time.
- The arrow in a network identifies the logical condition of dependence.
- The direction of arrow indicates the direction of work flow.
- All networks are constructed logically or based on the principal of dependency.
- No event can be reached in a project before the completion of precedence activity.
- Every activity in the network should be completed to reach the objective.
- No set of activities should form a circular loop. Arrows should not be crossed unless it is completely unavoidable.
- Arrows should be kept straight & not curved or bent.
- Angle between arrows should as large as possible.
- Each activity must have a tail or head event..
- Once the diagram is complete the nodes should be numbered from left to right. It should then be possible to address each activity uniquely by its tail & head event.

Example- AON

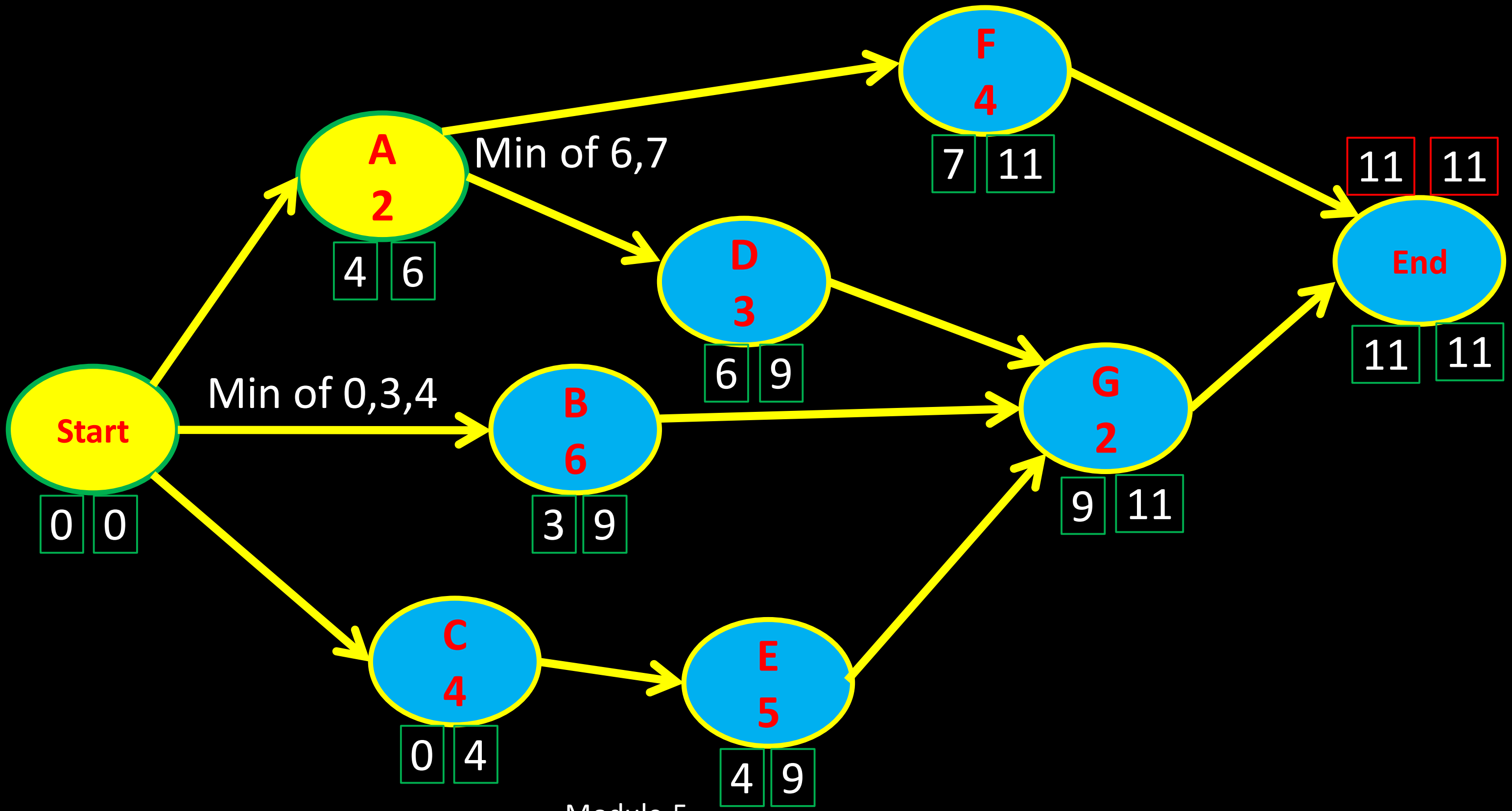
Activity	Predecessor	Duration
A	-	2
B	-	6
C	-	4
D	A	3
E	C	5
F	A	4
G	B,D,E	2



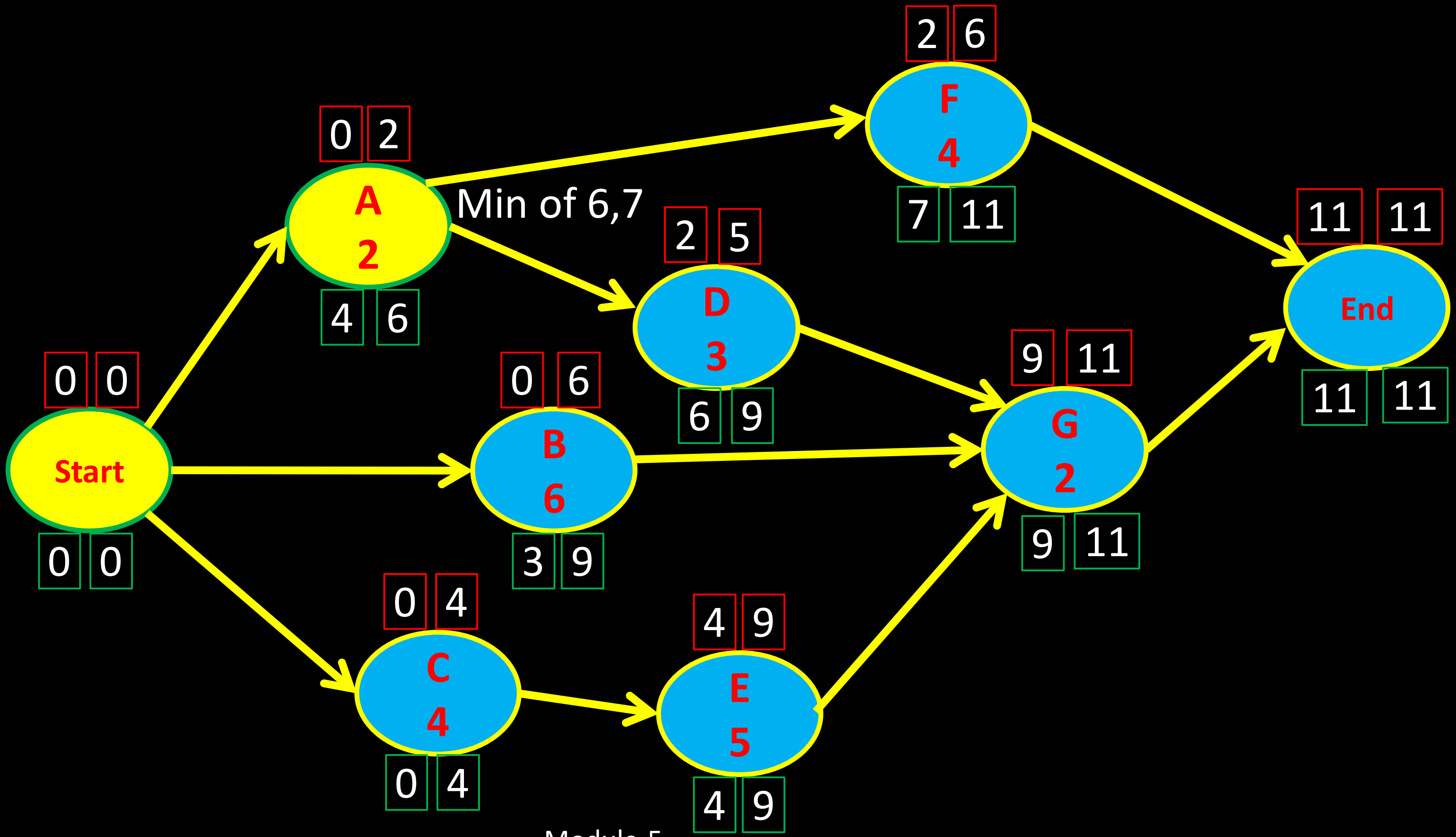
Example- Forward Pass



Example- Backward Pass



Example- Summary



Example- EST,EFT,LST,LFT

EST	DR	EFT
Task Name		
LST	TF	LFT

EST = Earliest start time

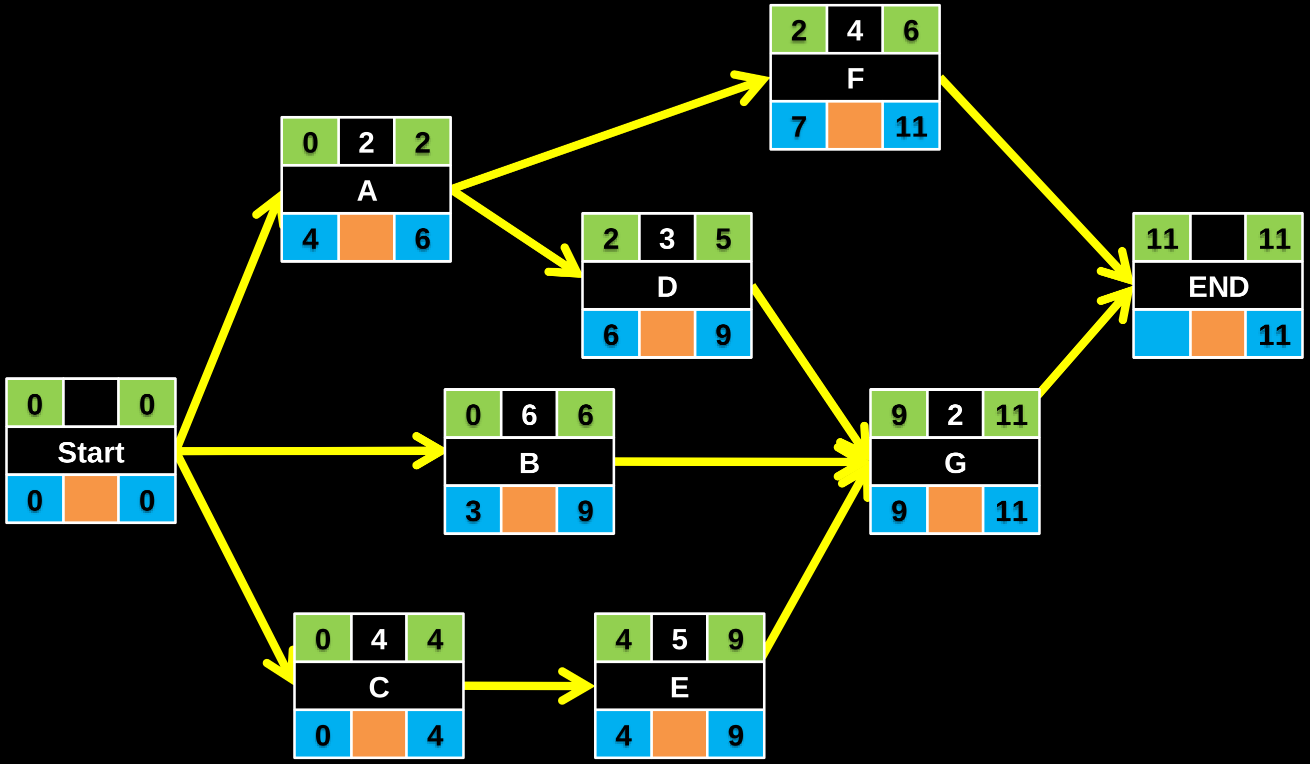
EFT = Earliest finish time

LST = Latest start time

LFT = Latest finish time

DR = Duration

TF = Total Float



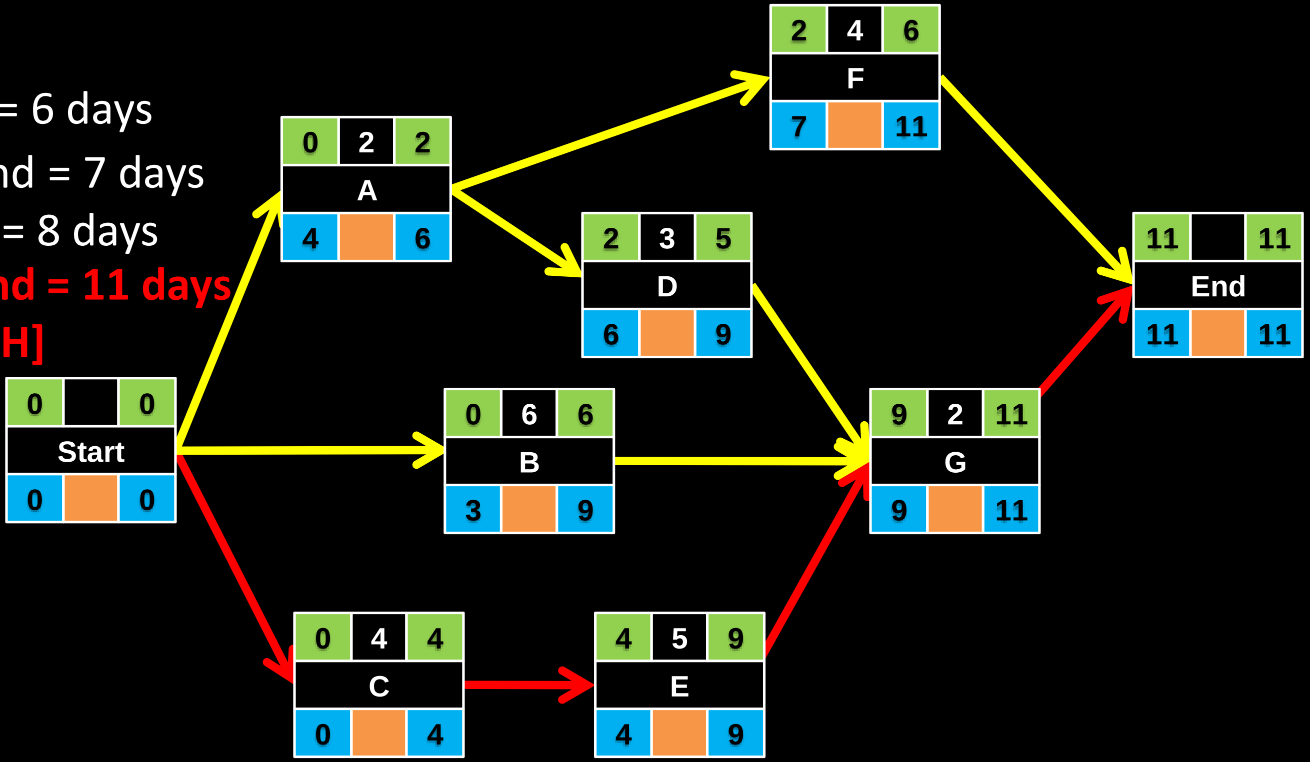
CPM- Critical Path Method

Path 1-: Start-A-F-End = 6 days

Path 2-: Start-A-D-G-End = 7 days

Path 3-: Start-B-G-End = 8 days

Path 4-: Start-C-E-G-End = 11 days [CRITICAL PATH]



SLACK Calculation

Always slack is calculated for AON network

Activity	Duration	EST	EFT	LST	LFT	SLACK (ST-ST or FT-FT)
A	2	0	2	4	6	4
B	6	0	6	3	9	3
C	4	0	4	0	4	0
D	3	2	5	6	9	4
E	5	4	9	4	9	0
F	4	2	6	7	11	5
G	2	9	11	9	11	0

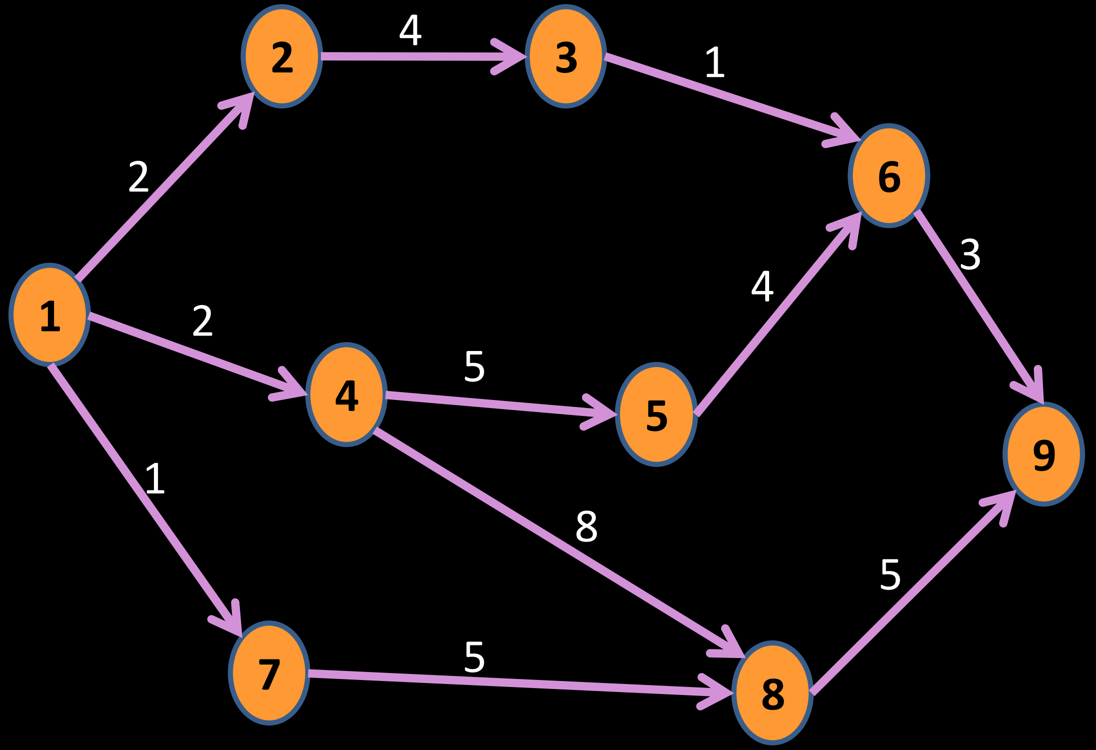
Example- AOA

A project schedule has the characteristics shown in table. Construct the network and locate the critical path. Calculate the various time estimates and floats.

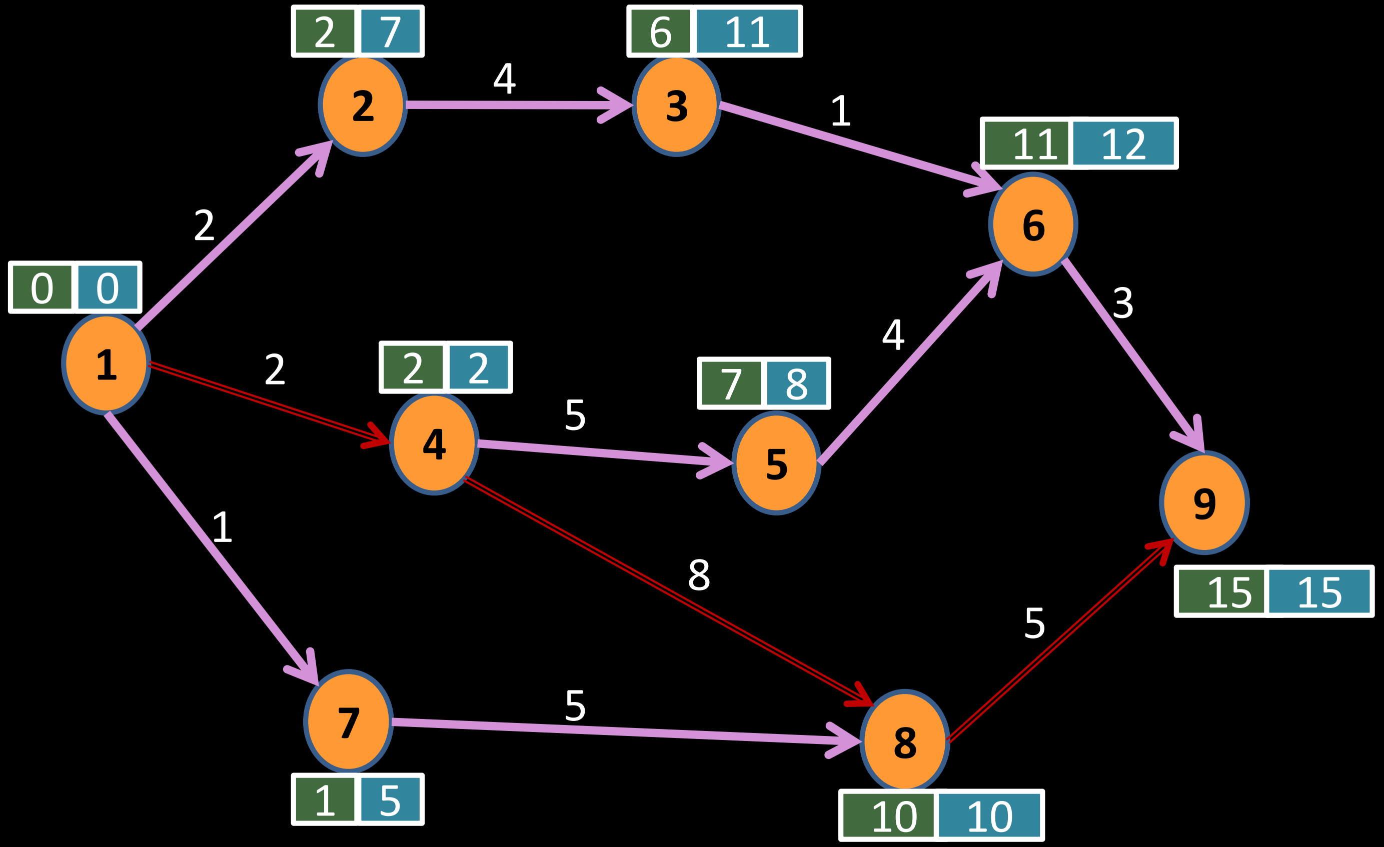
Activity	Duration
1-2	2
1-4	2
1-7	1
2-3	4
3-6	1
4-5	5
4-8	8
5-6	4
6-9	3
7-8	5
8-9	5

Solution- AOA

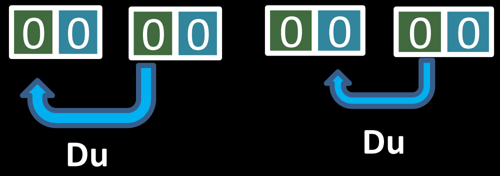
Activity	Duration
1-2	2
1-4	2
1-7	1
2-3	4
3-6	1
4-5	5
4-8	8
5-6	4
6-9	3
7-8	5
8-9	5

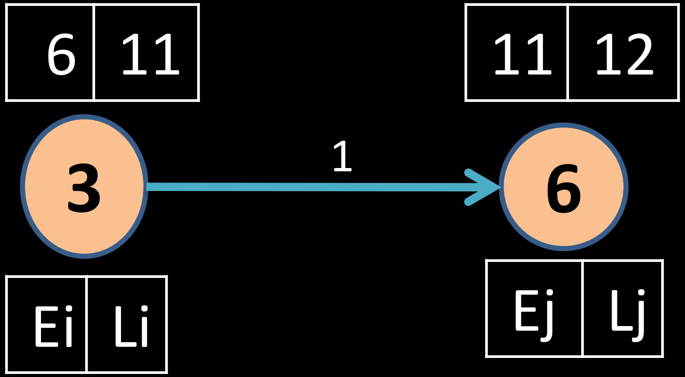


- Earliest and Latest time calculation
- Critical path – 1-4-8-9
- Project Duration= 15 days



Activity	Duration	EST	EFT (EST+ Du)	LST (LFT-Du)	LFT	Total float	Free float FF	Independent float
1-2	2	0	2	5	7	5	0	0
1-4	2	0	2	0	2	0	0	0
1-7	1	0	1	4	5	4	0	0
2-3	4	2	6	7	11	5	0	-5
3-6	1	6	7	11	12	5	4	-1
4-5	5	2	7	3	8	1	0	-1
4-8	8	2	10	2	10	0	0	0
5-6	4	7	11	8	12	1	0	-1
6-9	3	11	14	12	15	1	1	0
7-8	5	1	6	5	10	4	4	0
8-9	5	10	15	10	15	0	0	0





- Total Float = (Lj-Ei)-Dij
  = (12-6)-1= 5
- Free Float = (Ej-Ei)- Dij
  = (11-6)-1 = 4
- Independent Float = (Ej-Li)-Dij= (11-11)-1
  = -1

Advantages of CPM

- It shows the graphical view of the project.
- It discovers and makes dependencies visible.
- It helps in project planning, scheduling, and controlling.
- It helps in contingency planning.
- It shows the critical path, and identifies critical activities requiring special attention.
- It helps you assign the float to activities and flexibility to float activities.
- It shows you where you need to take action to bring project back on track.
- To shorten the planned critical path of a project by:
  - Pruning (Trimming and cutting) critical path activities
  - “Fast tracking" (To do more things in the same time in order to finish a job earlier than normal or planned.)
  - “Crashing the critical path" (Shortening the durations of critical path activities by adding resources).

Limitations of CPM

- Because the critical path method is an optimal planning tool, it always assumes that all resources are available for the project at all times.
- It does not consider resource dependencies.
- There are chances of misusing float or slack.
- Less attention on non-critical activities, though sometimes they may also become critical activities.
- Projects based on the critical path often fail to be completed within the approved time duration.

PERT

PERT planning involves the following steps:

1. Identify the specific activities and milestones.
2. Determine the proper sequence of the activities.
3. Construct a network diagram.
4. Estimate the time required for each activity.
5. Determine the critical path.
6. Update the PERT chart as the project progresses.

Activity Time

- ***Optimistic time***: The minimum possible time required to accomplish an activity (to) or a path (O), assuming everything proceeds better than is normally expected
- ***Pessimistic time*: T**he maximum possible time required to accomplish an activity (tp) or a path (P), assuming everything goes wrong (but excluding major catastrophes).
- ***Most likely time*: T**he best estimate of the time required to accomplish an activity (tm) or a path (M), assuming everything proceeds as normal.
- ***Expected time***: The best estimate of the time required to accomplish an activity (te) or a path (TE), accounting for the fact that things don't always proceed as normal (the implication being that the expected time is the average time the task would require if the task were repeated on a number of occasions over an extended period of time.

$$te=(to+4tm+tp)\div 6$$

PERT- Probability to complete project

- In order to find out the project duration, the expected task time for each activity is to be determined using the formula

$$te = \frac{to + 4tm + tp}{6}$$

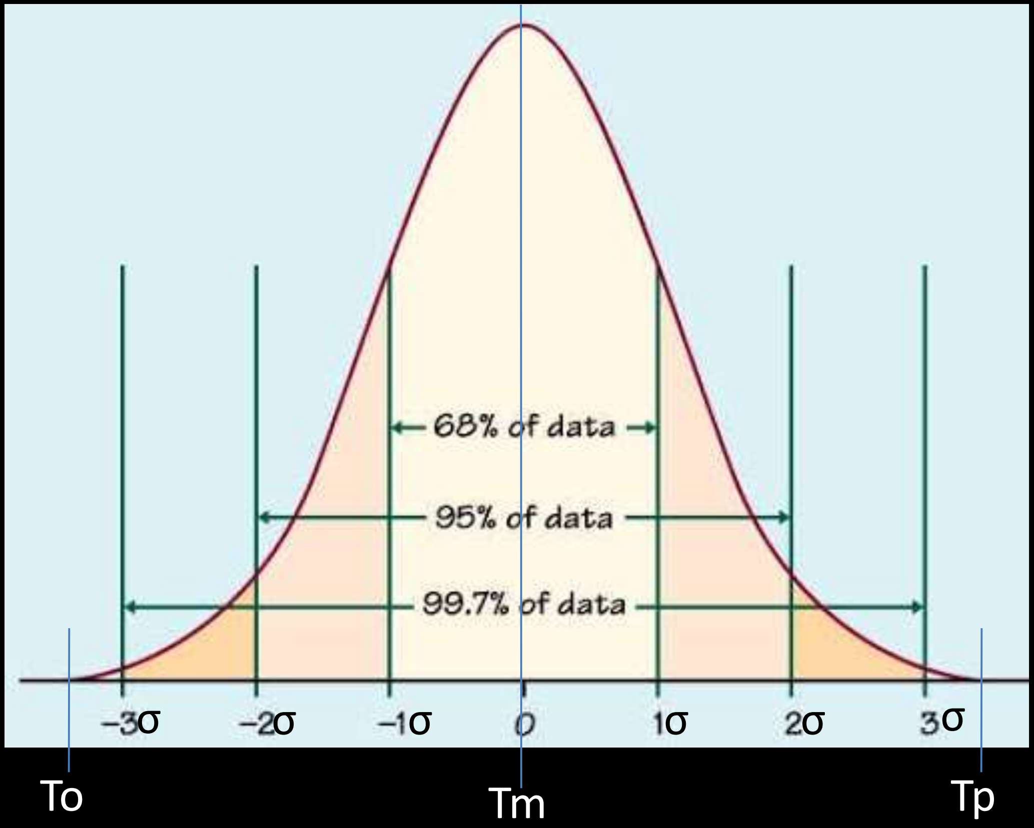
In order to find out the probability of completion in a given day, standard deviation (σ) for the critical path is to be determined using the formula

$$\sigma^2 = \left[\frac{tp\text{-}to}{6}\right]^2$$

- We assume that the distribution for the total duration of project is normal. Then the std normal variate for desired time(scheduled Time) D is given by

$$Z = \frac{\text{Scheduled Time (D)- Expected Time (Te)}}{\sigma} = \quad Z = \frac{D - Te}{\sigma}$$

Normal Distribution



- We know that the project duration for Critical Path (by network construction), we call it Te.
- We also know to calculate the SD for the Critical Path.
- We are to find the probability of completing the project at a certain duration we call this D.

Standard Normal Probabilities

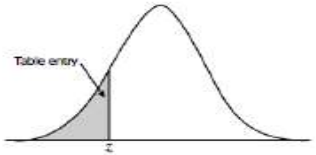


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

Standard Normal Probabilities

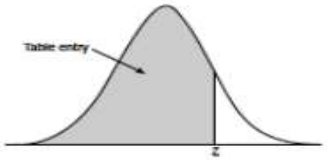


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Z-Probability

Z(+)	Probability (Pr)(%)	Z(–)	Probability (Pr)(%)
0	50.0	0	50.0
+0.1	53.98	–0.1	46.02
+0.2	57.93	–0.2	42.07
+0.3	61.79	–0.3	38.21
+0.4	65.54	–0.4	34.46
+0.5	69.15	–0.5	30.85
+0.6	72.57	–0.6	27.43
+0.7	75.80	–0.7	24.20
+0.8	78.81	–0.8	21.19
+0.9	81.59	–0.9	18.41
+1.0	84.13	–1.0	15.87
+1.1	86.43	–1.1	13.57
+1.2	88.49	–1.2	11.51
+1.3	90.32	–1.3	9.68
+1.4	91.92	–1.4	8.08
+1.5	93.32	–1.5	6.68
+1.6	94.52	–1.6	5.48
+1.7	95.54	–1.7	4.46
+1.8	96.41	–1.8	3.59
+1.9	97.13	–1.9	2.87
+2.0	97.72	–2.0	2.28
+2.1	98.21	–2.1	1.79
+2.2	98.61	–2.2	1.39
+2.3	98.93	–2.3	1.07
+2.4	99.18	–2.4	0.82
+2.5	99.38	–2.5	0.62
+2.6	99.53	–2.6	0.47
+2.7	99.65	–2.7	0.35
+2.8	99.74	–2.8	0.26
+2.9	99.81	–2.9	9.19
+3.0	99.87	–3.0	0.13

Example of PERT

A project consists of the following activities and time estimates.

Activity	Optimistic time (to),day	Pessimistic time (tp), day	Most likely time (tm), day
1-2	3	15	6
1-3	2	14	5
1-4	6	30	12
2-5	2	8	5
2-6	5	17	11
3-6	3	15	6
4-7	3	27	9
5-7	1	7	4
6-7	2	8	5

a) What is the project duration?

b) What is the probability that the project will be completed in 27 days?

Solution of PERT

- In order to find out the project duration, the expected task time for each activity is to be determined using the formula

$$te = \frac{to + 4tm + tp}{6}$$

In order to find out the probability of completion in a given day, standard deviation (σ) for the critical path is to be determined using the formula

$$\sigma^2 = \left[\frac{tp - to}{6} \right]^2$$

So the required data are presented in the table of next slide

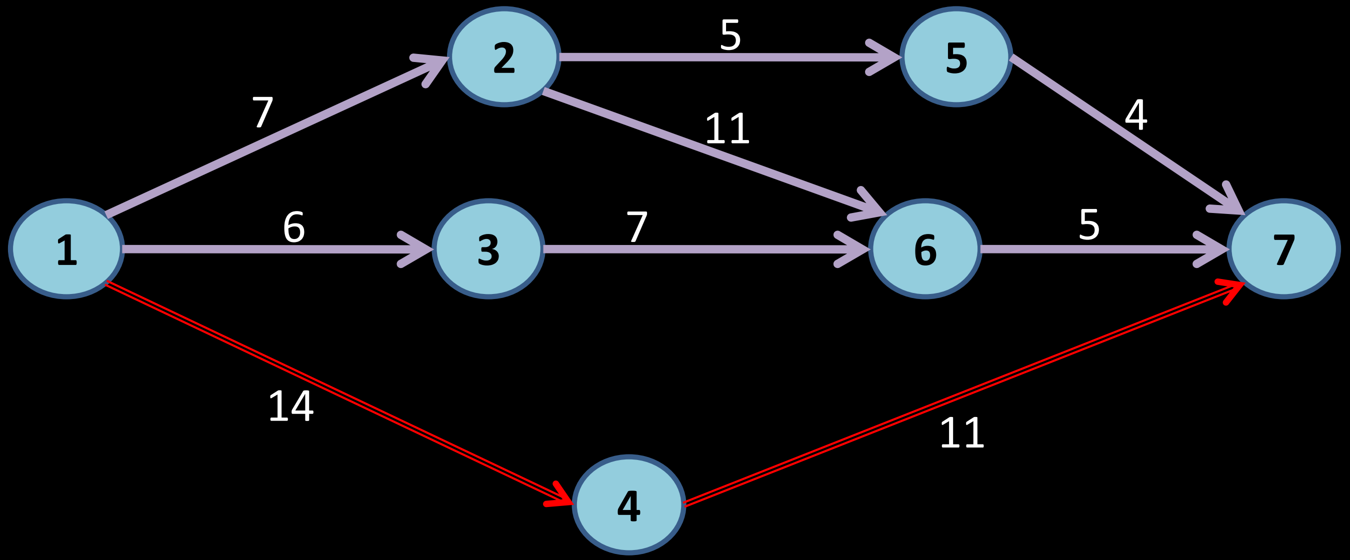
Solution of PERT

So the required data are presented in the Following

Activity	to	tp	tm	te	σ^2
1-2	3	15	6	7	4
1-3	2	14	5	6	4
1-4	6	30	12	14	16
2-5	2	8	5	5	1
2-6	5	17	11	11	6
3-6	3	15	6	7	4
4-7	3	27	9	11	16
5-7	1	7	4	4	1
6-7	2	8	5	5	1

Solution of PERT

Using the expected time (te) value, the network is drawn as shown below.



Path:
1-2-5-7=7+5+4=16
1-2-6-7=7+11+5=23
1-3-6-7=6+7+5=18
1-4-7=14+11=25
Hence, **Critical path is 1-4-7=25 days**

Solution-PERT

a) 1-4-7 is critical path. The duration of project

$= 14+11= 25$ days.

b) The sum of variances along the critical path

$= 16+16 = 32$.

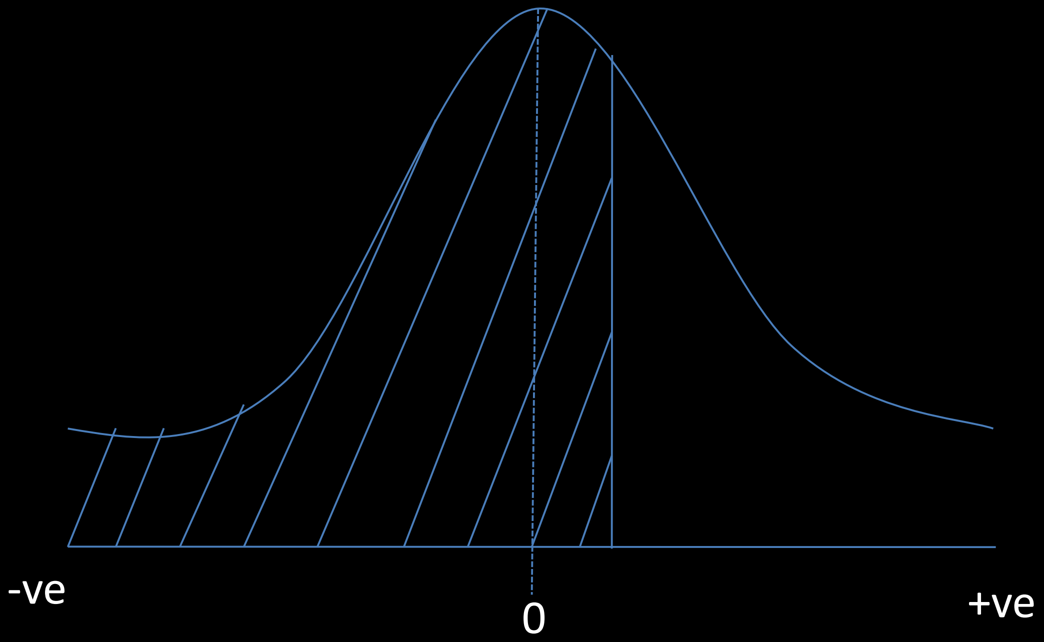
Now the standard deviation, $\sigma = \sqrt{\sigma^2} = \sqrt{32} = 5.656$

Expected project duration, Te = 25 days

Solution-PERT

- Given D = 27 day.
- With reference to Normal Distribution Curve

$$Z = \frac{D - Te}{\sigma} = \frac{27 - 25}{5.656} = 0.35$$



Solution-PERT

- From Normal Distribution Table, for Z = 0.35, the fraction of (shadow) area = 0.637 i.e., 63.68%
- So probability of completion of project in 27 days is 63.68%
- [Note: If probability of completion of project will be given and the due date (D) will be asked to find out, then backward calculation can give the result].

Advantages of PERT

- PERT (Program Evaluation and Review Technique) compels managers to plan, which helps them see how the pieces fit together.
- Each subordinate manger has to plan the events for which he or she is responsible.
- It concentrates on critical elements that may need correction.
- It makes possible a kind of forward-looking control.
- The network system with its subsystems enables managers to aim reports and pressure for action at the right spot and level in the organization structure at the right time.

Limitations of PERT

- Because of its emphasis on “activity-time” to its operation, PERT is not useful when no reasonable estimates of time schedule can be made.
- Another disadvantage has been its emphasis on time only but not on costs.

Differences bet PERT & CPM

Sl No.	PERT	CPM
1.	PERT is a probabilistic model with uncertainty in activity duration. activity duration is calculated from to, tp & tm by relation te = (to+4tm+tp)/6	CPM is a deterministic model with well known activity duration
2.	It is an event oriented approach	It is an activity oriented approach
3.	PERT terminology uses word like network diagram event and slack	CPM terminology sues word like arrow diagram nodes and floats
4.	The use of dummy activity is required for representing the proper sequencing	No dummy activity
5.	PERT basically does not demarcate between critical and noncritical activity	CPM maks the critical activity
6.	PERT is applied in projects where resources are always available	CPM in applied to projects where minimum overall cost is the prime importance.
7.	PERT is suitable in defense project & R &D, where activity time can't be readily predicted	7. Suitable for plant maintenance, civil construction projects etc. where activity duration is known.

Cost Analysis

- Generally costs are divided into two
  - direct costs
  - indirect costs
- Direct costs- Direct costs are associated with the physical construction of the project including
  - Materials,
  - Equipment,
  - Labor and
  - Subcontractor (if exists)
- Indirect costs - They are generally broken down into two categories:
  - head office overhead and
  - general conditions (project or site overhead)

Crashing

- ***Crashing an activity*** refers to taking special costly measures to reduce the duration of an activity below its normal value.
- These special measures might include using overtime, hiring additional temporary help, using special time-saving materials, obtaining special equipment, etc.
- ***Crashing the project*** refers to crashing a number of activities in order to reduce the duration of the project below its normal value.

Crashing

- Crashing a project means the process of accelerating an activity or multiple activities to shorten the overall duration of a project.
- By adding additional people, equipment, or man- hours, a project manager can shorten an activity’s duration.
- If the activity affected is critical, the project will be shortened as well.
- Activities are crashed for different reasons:
  - An activity may need to be completed by a specific date for contractual reasons.
  - Some activities can be accomplished more economically during a certain time of the year, encouraging managers to accelerate preceding activities.

Crashing Steps

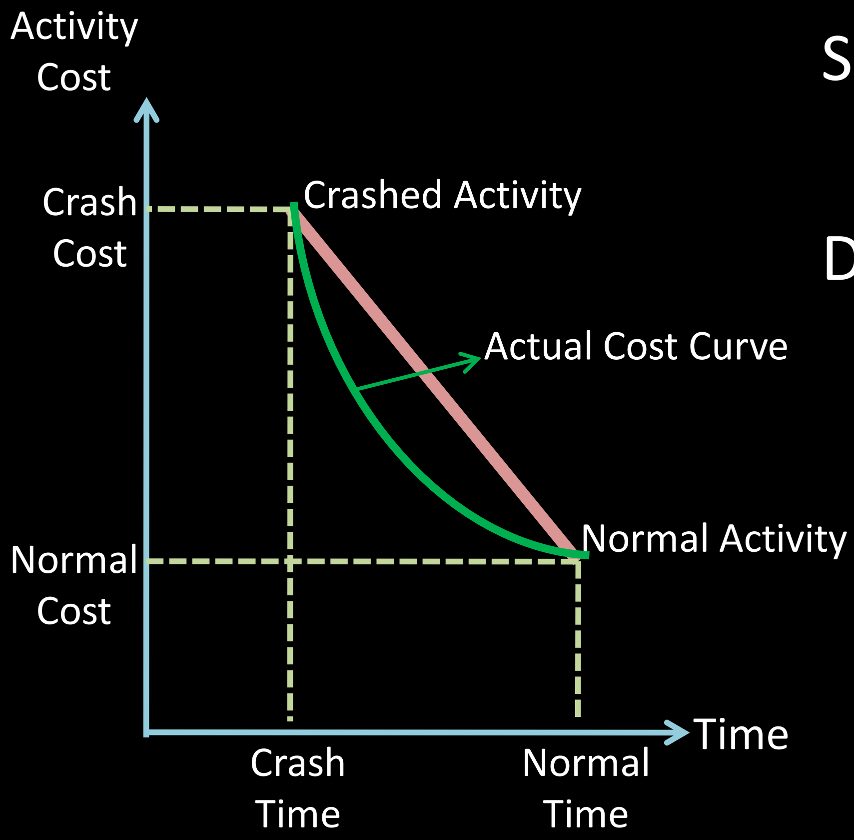
Crashing of a network can be done in following steps:

- **1. Critical path:**
  - Find the normal critical path and identify the critical activity.
- **2. Cost slope:**
  - Calculate the cost slope for the different activities by using the formula.
  - Cost slope = [Crash cost-Normal cost]/[Normal time- Crash time]
  - The cost slope indicates the extra cost required to expedite an activity per unit time.
- **3. Ranking:**
  - Rank the activities in the ascending order of cost slope . The activity having the minimum cost slope have to be crashed first, crash the selected activity to its minimum duration.

Crashing Steps

- **4. Crashing:**
  - Crash the activities in the critical path as per the ranking i.e., activity having lower cost slope would be crashed first to the maximum extent possible. Calculate the new direct cost by cumulative adding the cost or crashing to the normal cost.
- **5. Parallel crashing:**
  - As the critical path duration is reduced by the crashing in step 3, other paths also become critical, i.e., we get parallel critical path. This means that project duration can be reduced duly by simultaneous crashing or activity on the parallel critical path.
- **6. Total cost:**
  - Crashing as per steps 3 and 4, one reaches a point when further crashing is either not possible or does not result in the reduction or crashing or project duration. For the different project durations total cost is found up to total cost is got by adding corresponding fixed cost to the direct cost, and the direct cost is got by adding the expediting crashing cost commutative to the normal cost.

Cost Slop



$$\text{Slop} = \frac{\text{Crash Cost-Normal Cost}}{\text{Normal Time-Crash Time}}$$

Direct Cost =

Direct cost of previous Crashing

+

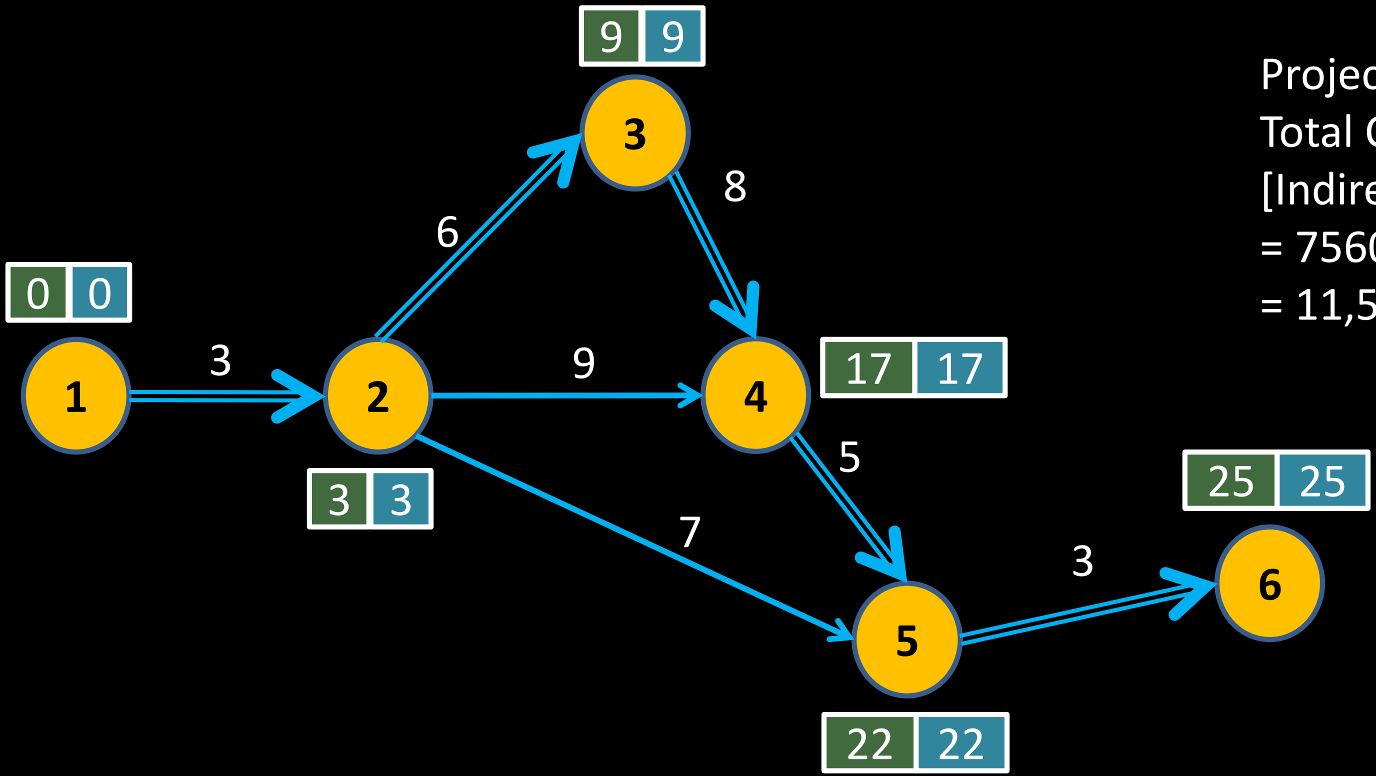
[Crashing Cost slop X Crashing Time] of current crashing

Crashing- Example

The following table gives data on normal cost and time, crash cost and time for a project. The indirect cost is Rs. 160/day. Determine optimum project duration and corresponding minimum cost.

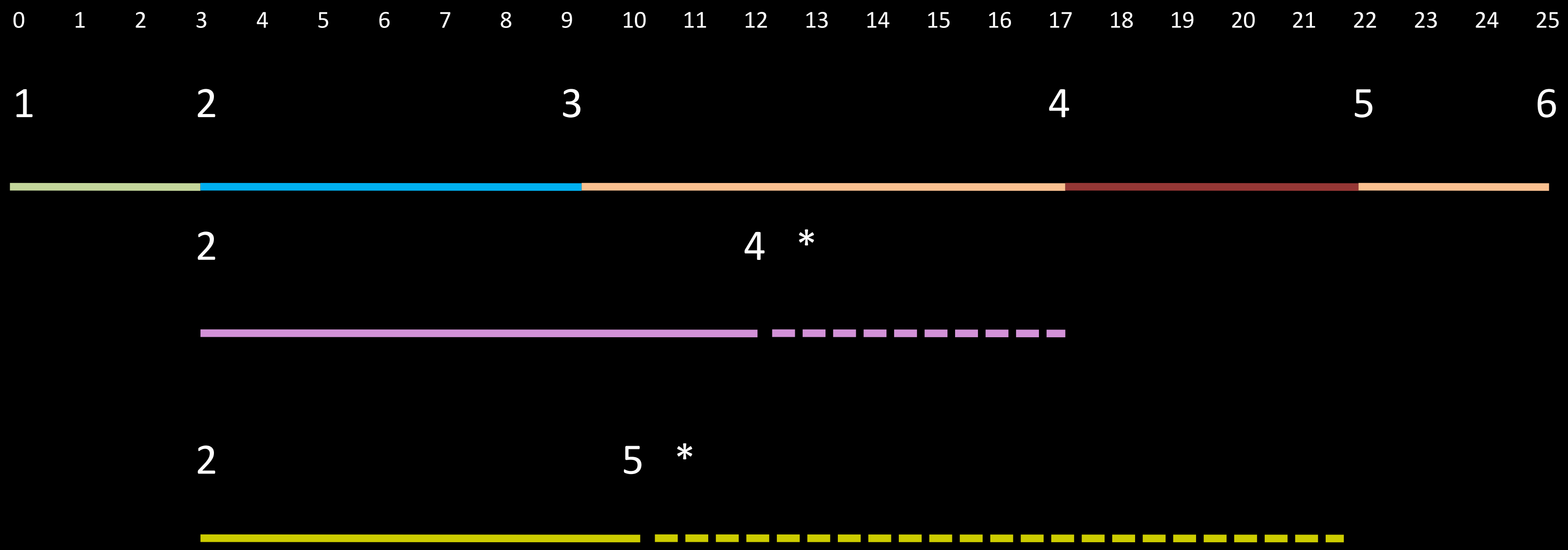
Activity	Normal		Crash	
	Time	Cost	Time	Cost
1-2	3	360	2	400
2-3	6	1440	4	1620
2-4	9	2160	5	2380
2-5	7	1120	5	1600
3-4	8	400	4	800
4-5	5	1600	3	1770
5-6	3	480	2	760

Network Diagram



Project Duration-25
Total Cost= Direct Cost + [Indirect Cost X Duration]
= 7560 + [160 X 25]
= 11,560/-

Time Projection Line Diagram



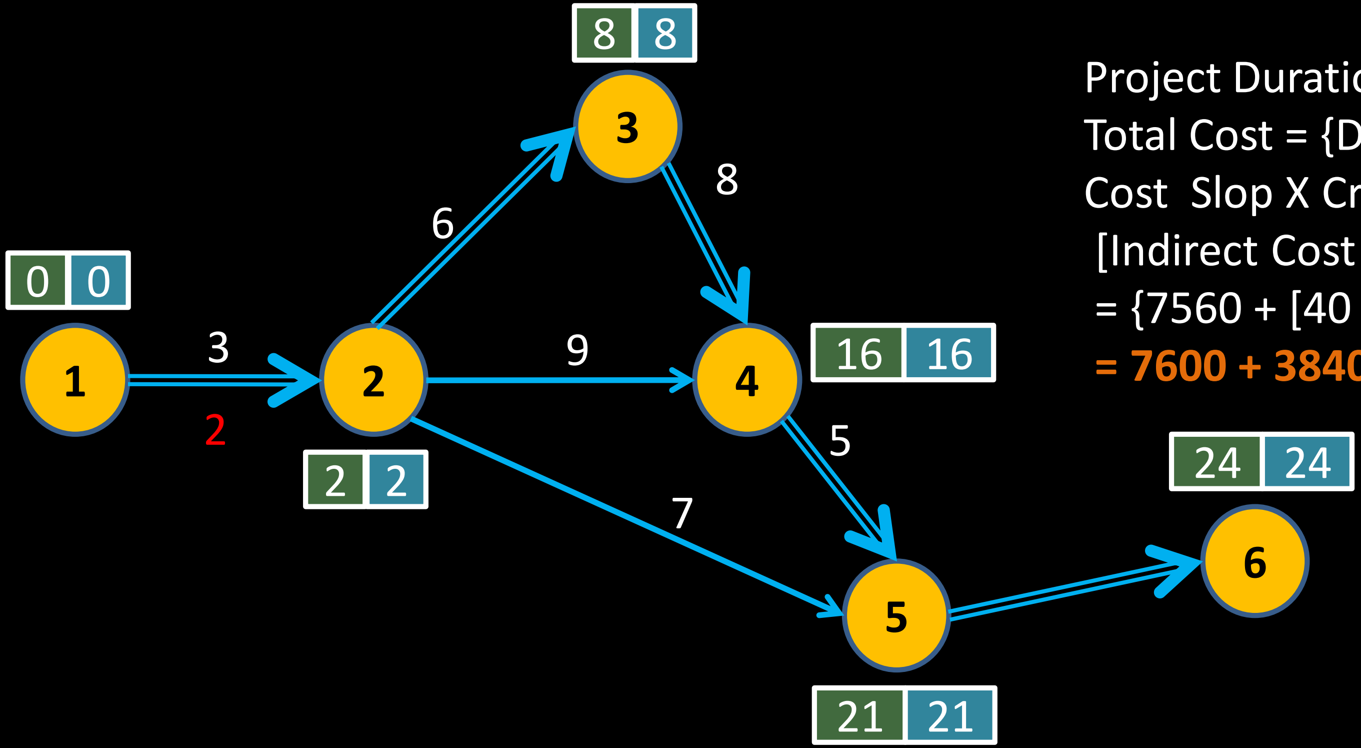
Crashing- Solution

Activity	Normal		Crash		Slop=ΔC/ΔT	
	Time	Cost	Time	Cost		
1-2	3	360	2	400	40	1
2-3	6	1440	4	1620	90	3
2-4	9	2160	5	2380	55	
2-5	7	1120	5	1600	240	
3-4	8	400	4	800	100	4
4-5	5	1600	3	1770	85	2
5-6	3	480	2	760	280	5

Total Normal Cost = 7560

Crashing- Solution

1. Crashing Activity 1-2: As min. slop is 40 for activity 1-2, we will Crash it first



Project Duration-24

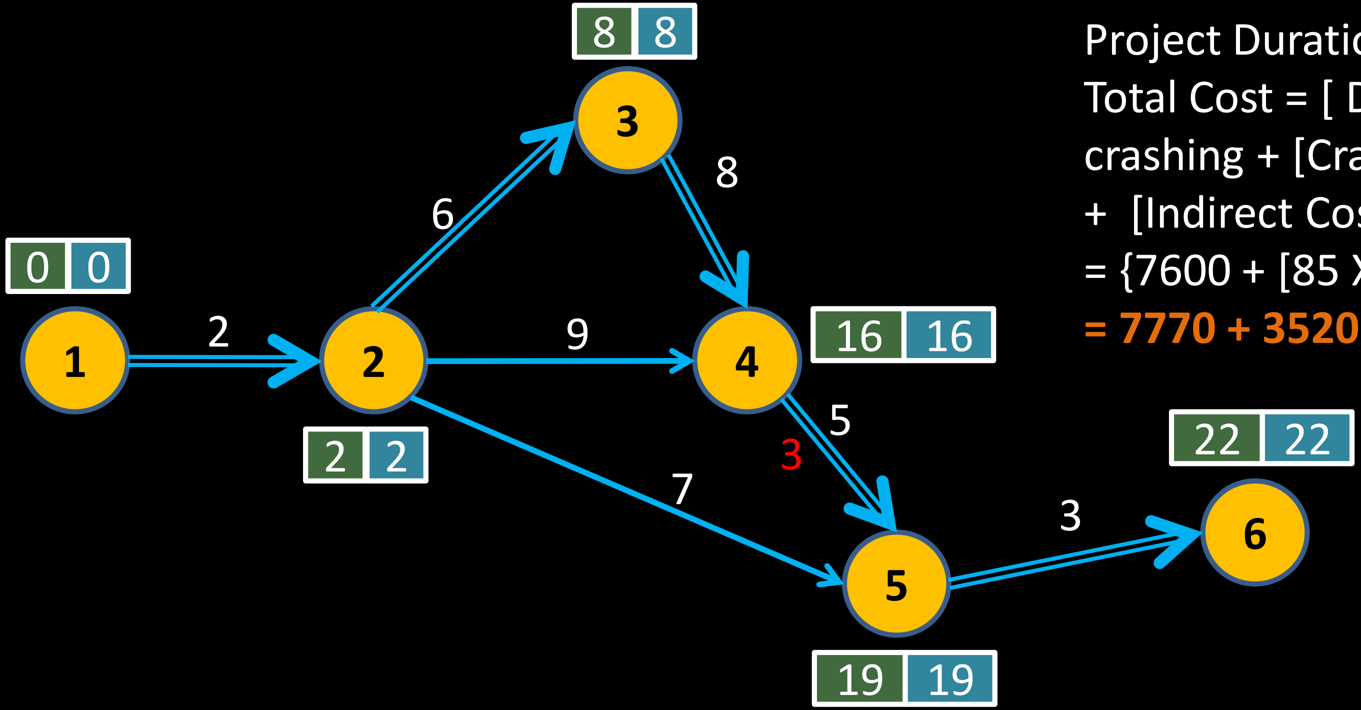
Total Cost = {Direct Cost + [Crash Cost Slop X Crash Duration]} + [Indirect Cost X Duration]

= {7560 + [40 X 1] } + [160 X 24]

= 7600 + 3840 = 11,440/-

Crashing- Solution

2. Crashing Activity 4-5: As next min. slop is 85 for activity 4-5, we will Crash it now



Project Duration-22

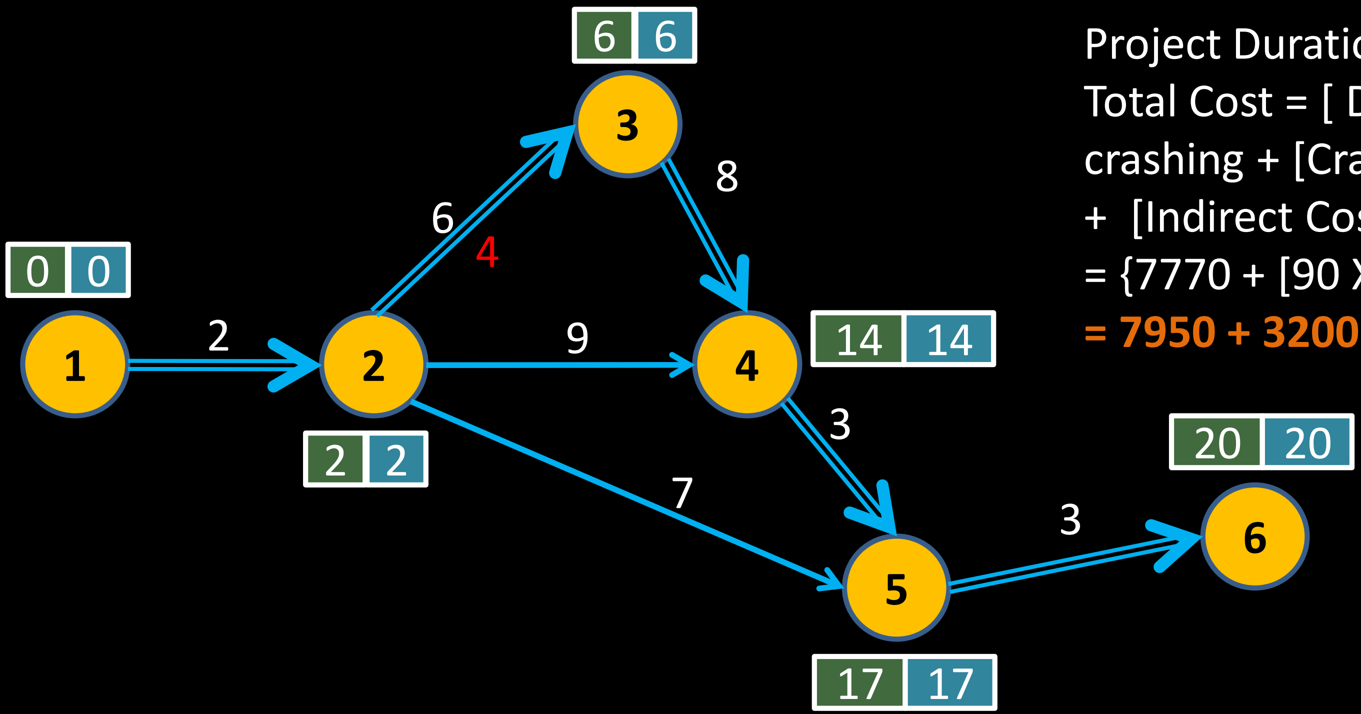
Total Cost = [Direct Cost of previous crashing + [Crash Cost X Crash time]} + [Indirect Cost X Duration]

= {7600 + [85 X 2]} + [160 X 22]

= 7770 + 3520 = 11,290/-

Crashing- Solution

3. Crashing Activity 2-3: As next min. slop is 90 for activity 2-3, we will Crash it now



Project Duration-20

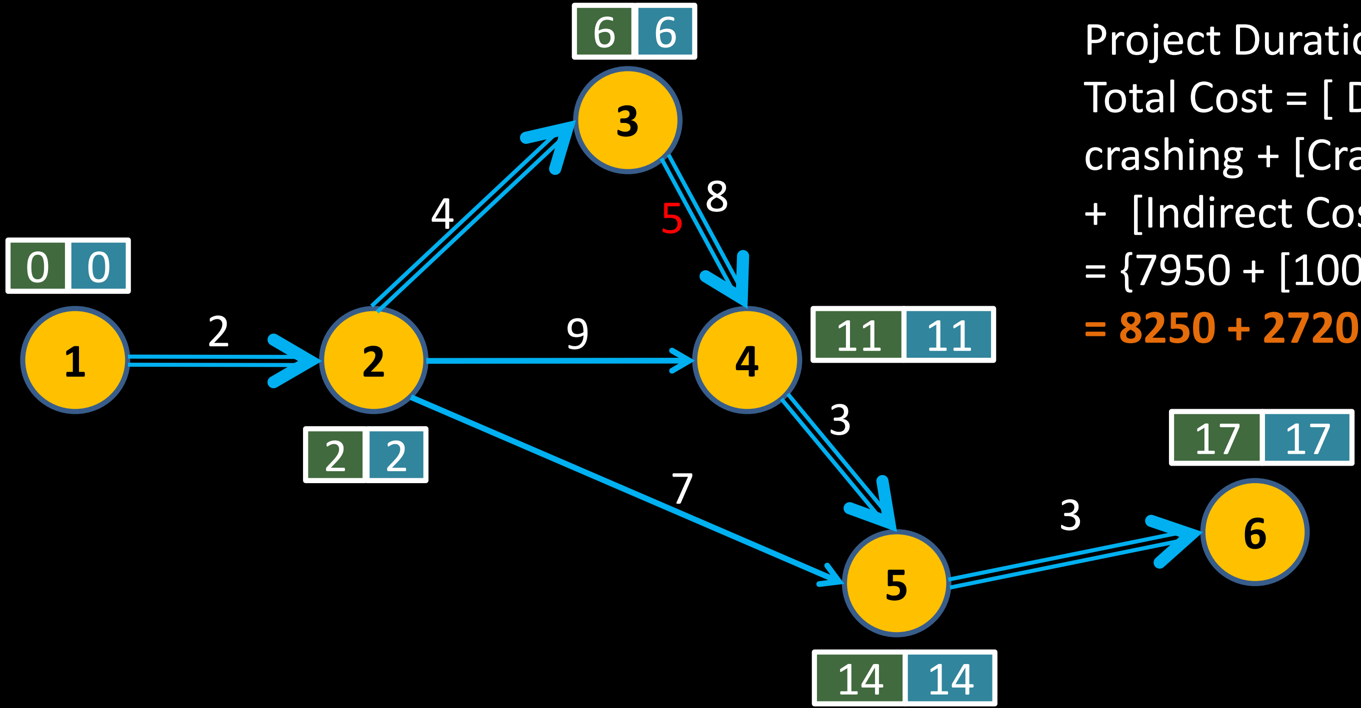
Total Cost = [Direct Cost of previous crashing + [Crash Cost X Crash time]} + [Indirect Cost X Duration]

= {7770 + [90 X 2]} + [160 X 20]

= 7950 + 3200 = 11,150/-

Crashing- Solution

4. Crashing Activity 3-4: As next min. slop is 100 for activity 3-4, we will Crash it now. If we crash 3-4 fully by 4 days then CP will be changed. Therefore to maintain original CP containing 3-4, we will crash it by 3 days only



Project Duration-17

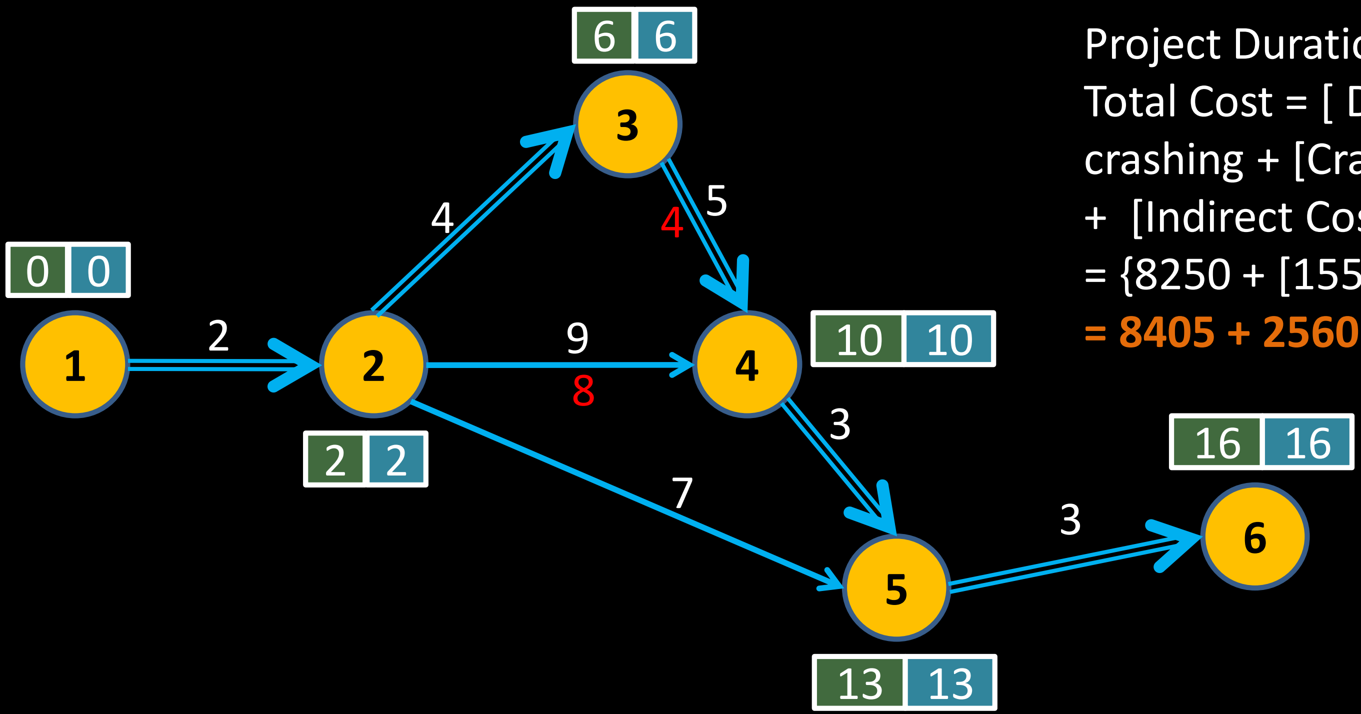
Total Cost = [Direct Cost of previous crashing + [Crash Cost X Crash time]} + [Indirect Cost X Duration]

= {7950 + [100 X 3]} + [160 X 17]

= 8250 + 2720 = 10,970/-

Crashing- Solution

5. Crashing Activity 2-4 & 3-4: If we crash 2-3 & 3-4 simultaneously by 1 day then total cost (100+55=155) which is less than indirect cost 160. so we can crash both together



Project Duration-16

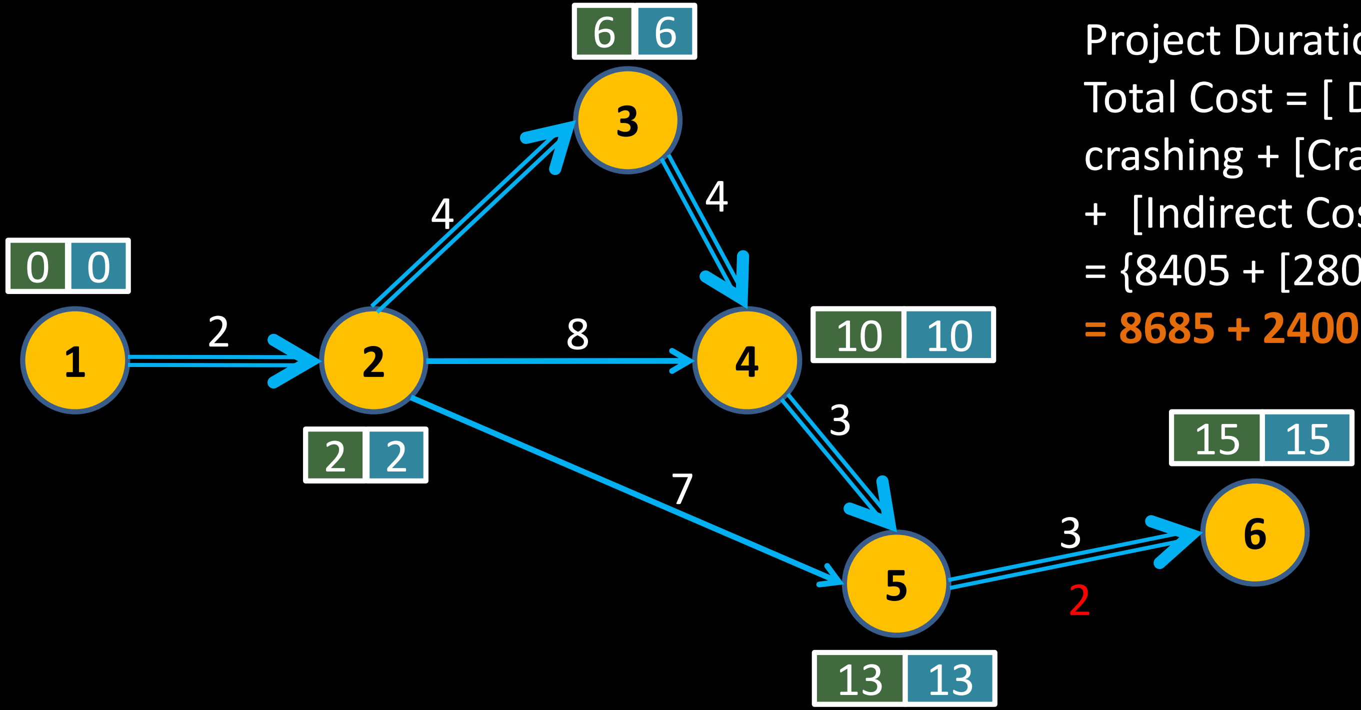
Total Cost = [Direct Cost of previous crashing + [Crash Cost X Crash time]} + [Indirect Cost X Duration]

= {8250 + [155 X 1]} + [160 X 16]

= 8405 + 2560 = 10,965/-

Crashing- Solution

6. Crashing Activity 5-6: As next min. slop is 280 for activity 5-6, we will Crash it now.



Project Duration-15

Total Cost = [Direct Cost of previous crashing + [Crash Cost X Crash time]} + [Indirect Cost X Duration]

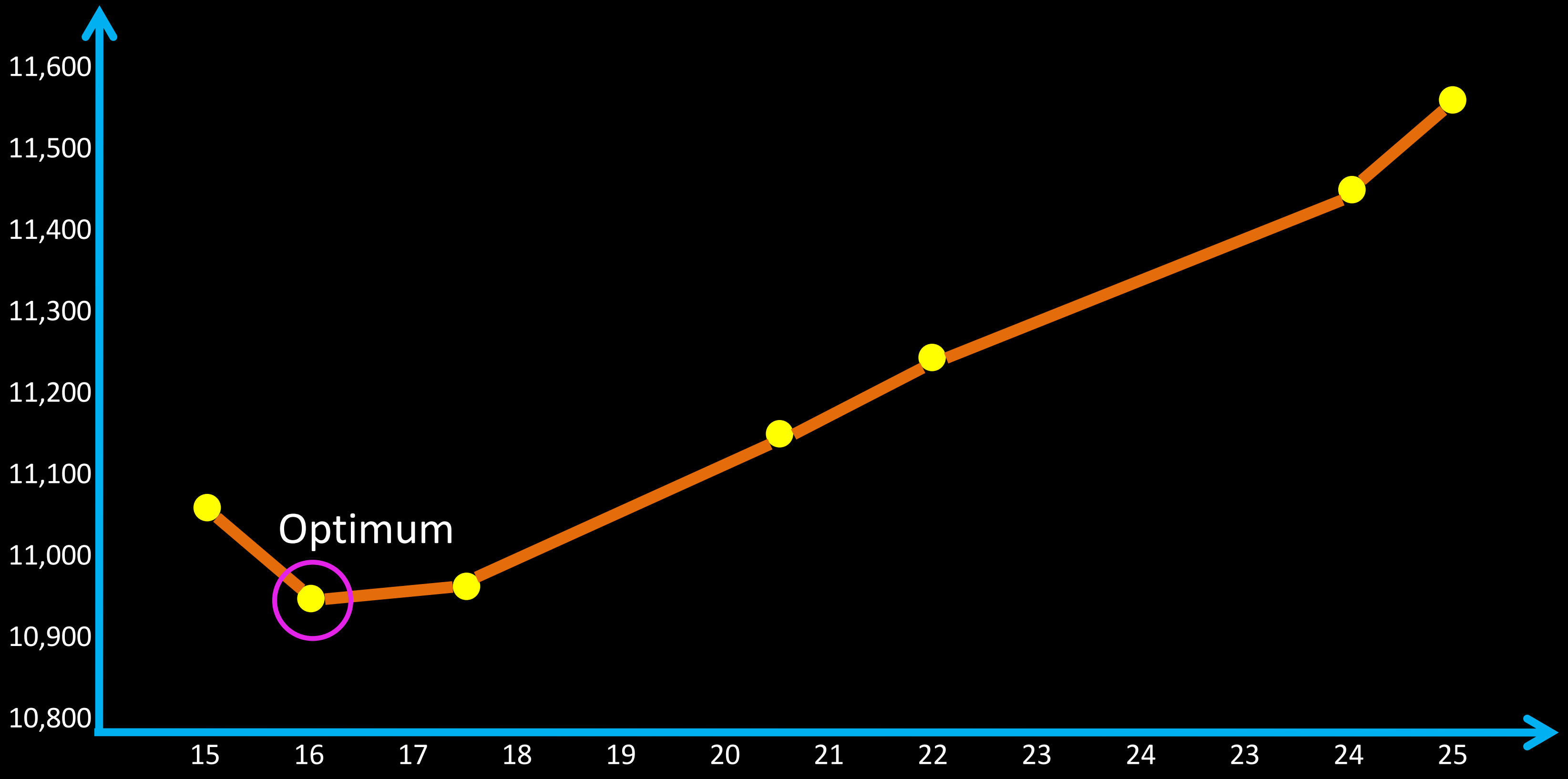
= {8405 + [280 X 1]} + [160 X 15]

= 8685 + 2400 = 11,085/-

Crashing- Solution

Crashed Activity	Days saved	Project duration	Direct project cost	Over heads 160Xduration	Total project cost
None	-	25	7560	4000	11650
1-2	1	24	7600	3840	11440
4-5	2	22	7770	3520	11250
2-3	2	20	7950	3200	11150
3-4	3	17	8250	2720	10970
3-4, 2-4	1	16	8405	2560	10965
5-6	1	15	8685	2400	11085

Crashing- Solution



Resource Leveling

- **Resource leveling** is a technique used to examine unbalanced use of resources (usually people or equipment) over time, and for resolving over-allocations or **conflicts** resulting from scheduling certain tasks **simultaneously**. Such conflicts are:
  – more resources such as **machines or people** are needed are available, **or**
  – a specific person is needed in both tasks, the tasks will have be rescheduled concurrently or even sequentially to manage the constraint.
- It is used to balance the workload of primary resources over course of the project[s], usually at the expense of one of traditional triple constraints **(time, cost, scope)**

Resource Leveling

- Resource leveling helps an organization to make use of the available resources to the maximum. The idea behind resource leveling is to reduce wastage of resources i.e., to stop over-allocation of resources. Resource leveling techniques provide a means of distributing resource usage over time **to minimize the period-by-period variations** in manpower, equipment, or money expended.
- The essential idea of resource leveling centers about the **rescheduling of activities within the limits of available float** to achieve better distribution of resource usage.

Resource Leveling -Example

A subcontractor needs to install flooring in two areas:

Area 1: This area has old vinyl tile that must be removed and replaced with new vinyl tile.

Area 2: This area has a concrete slab that needs to be topped with ceramic tile. This simple project is broken into the activities shown in the following table,

Activity	Description	Predecessor	Duration	Worker
A	Purchase and deliver material	-	5	2
B	Remove old vinyl tile	-	7	4
C	Install ceramic tile	A	3	3
D	Install new vinyl tile	A,B	5	3
E	Clean-up inspect	C,D	2	2

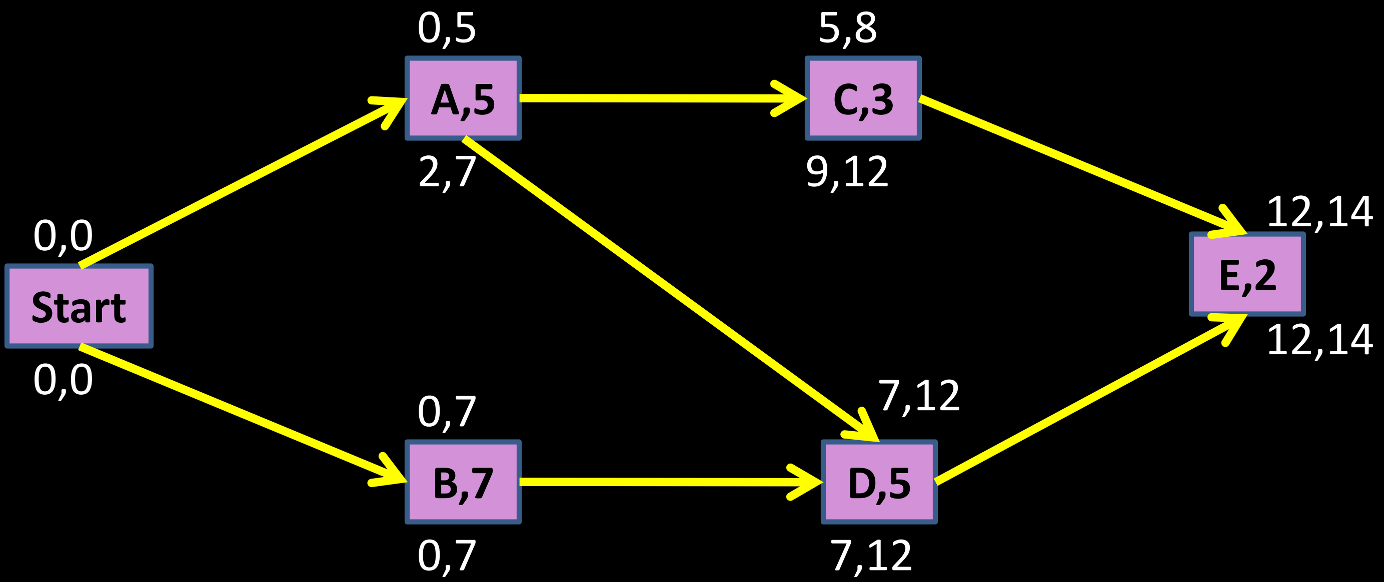
Resource Leveling - Solution

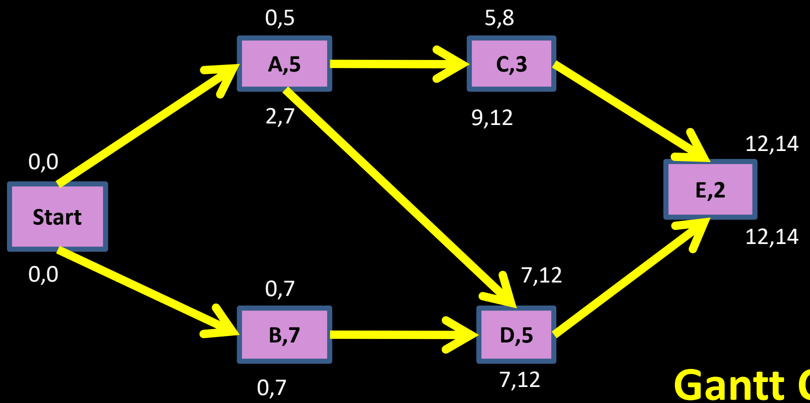
1. Draw the precedence network and perform the CPM calculations.
2. Allocate the required resources, then level them so that the subcontractor does not use more than six laborers at any time.
3. Find ways to improve the labor usage profile.

For the sake of simplicity, assume that any laborer can perform any task.

Resource Leveling - Solution

Network diagram





Float for A is 2 Days
Float for C is 4 Days

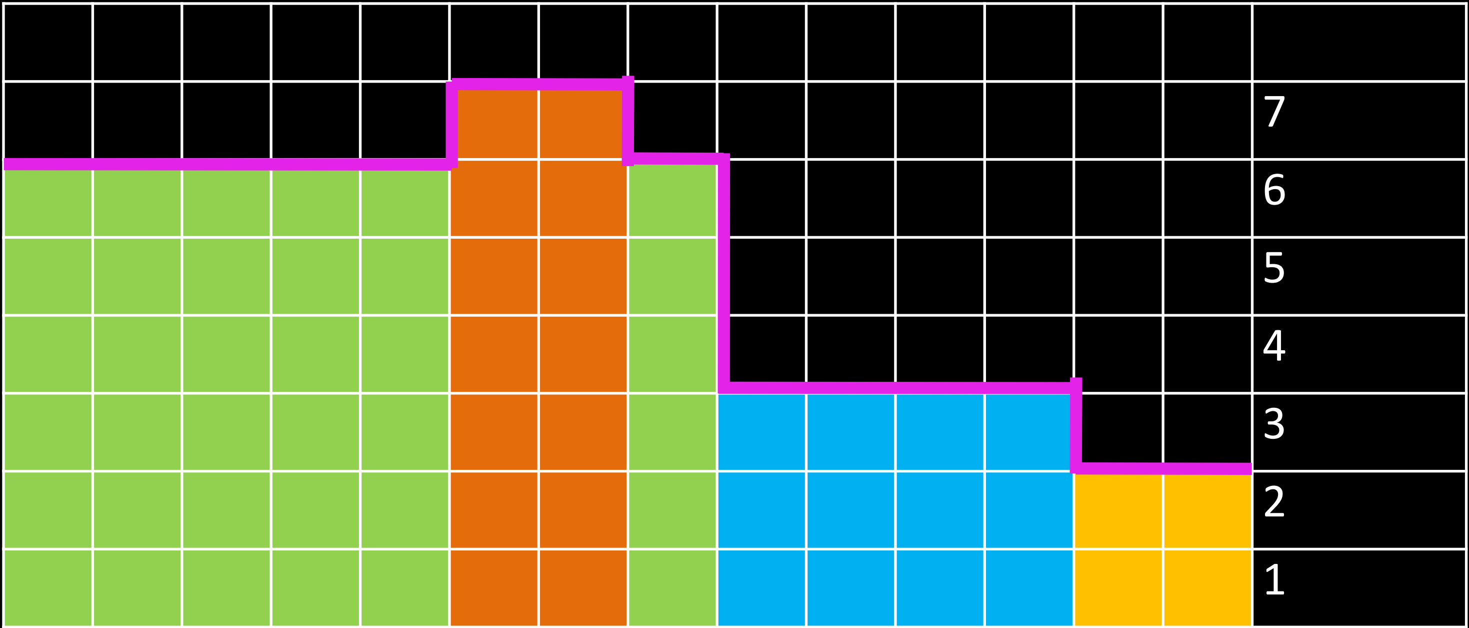
Gantt Chart

A	2	2	2	2	2	- - -	- - -								
B	4	4	4	4	4	4	4								
C						3	3	3	- - -	- - -	- - -	- - -			
D								3	3	3	3	3			
E													2	2	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Days
	6	6	6	6	6	7	7	6	3	3	3	3	2	2	Labour

Critical

- - - - - Float

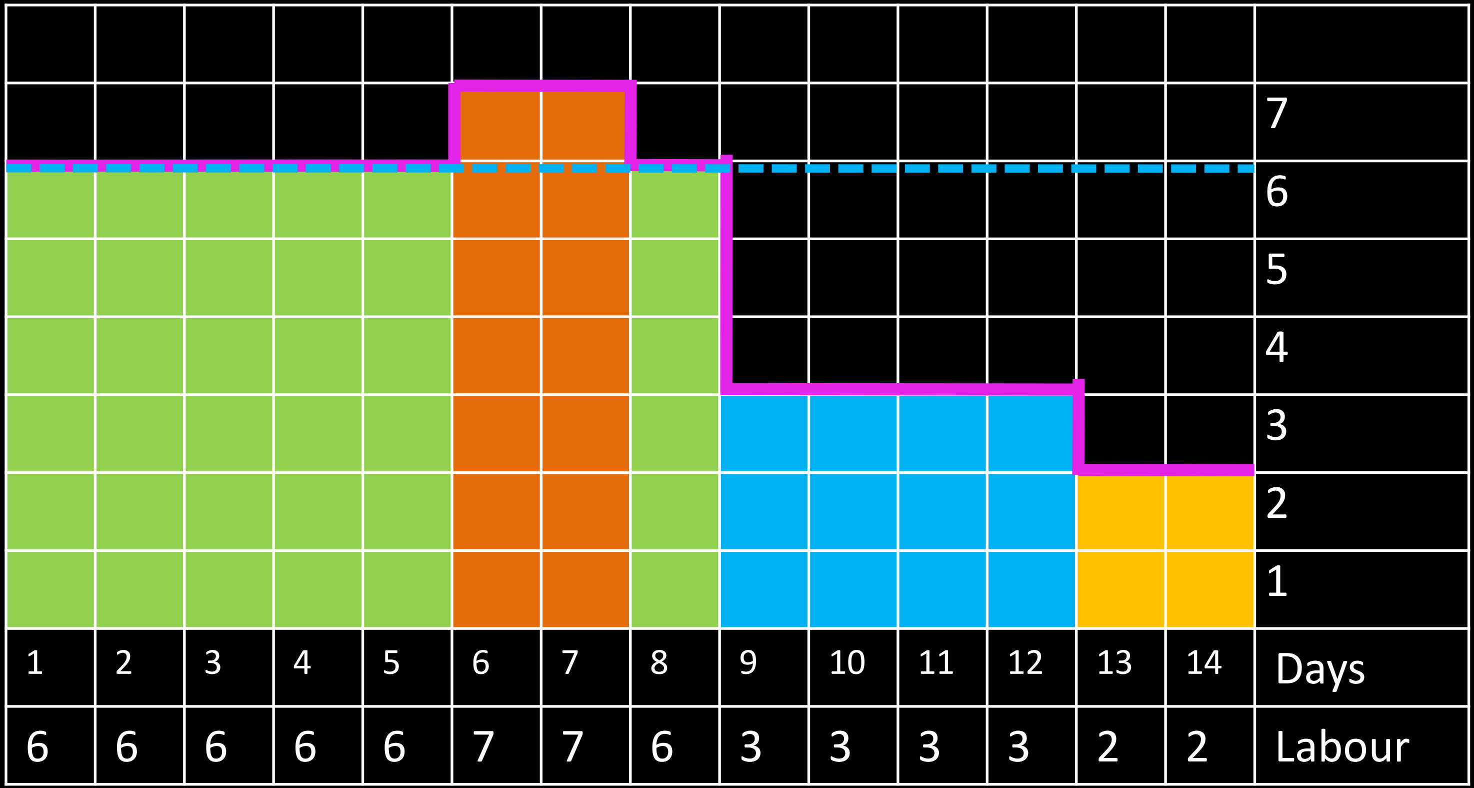
Resource Profile of Example

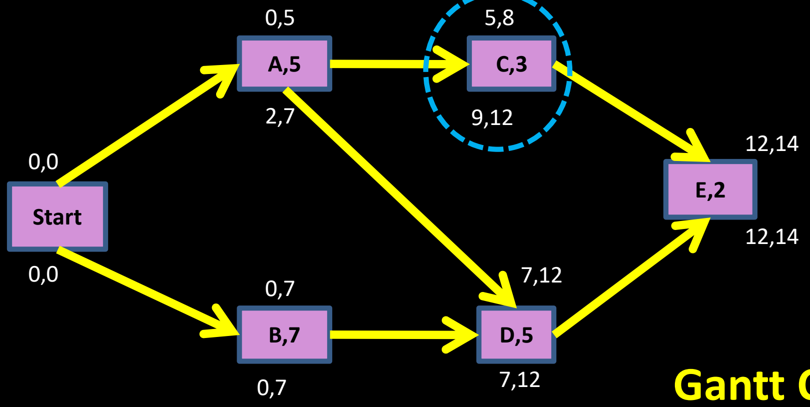


1	2	3	4	5	6	7	8	9	10	11	12	13	14	Days
6	6	6	6	6	7	7	6	3	3	3	3	2	2	Labour

Resource Profile of Example

Does not use more than six laborers





Float for A is 2weeks
Float for C is 4weeks

Does not use more than six laborers

Gantt Chart

A	2	2	2	2	2										
B	4	4	4	4	4	4	4								
C								3	3	3					
D								3	3	3	3	3			
E													2	2	
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	Days
	6	6	6	6	6	4	4	6	6	6	3	3	2	2	Labour

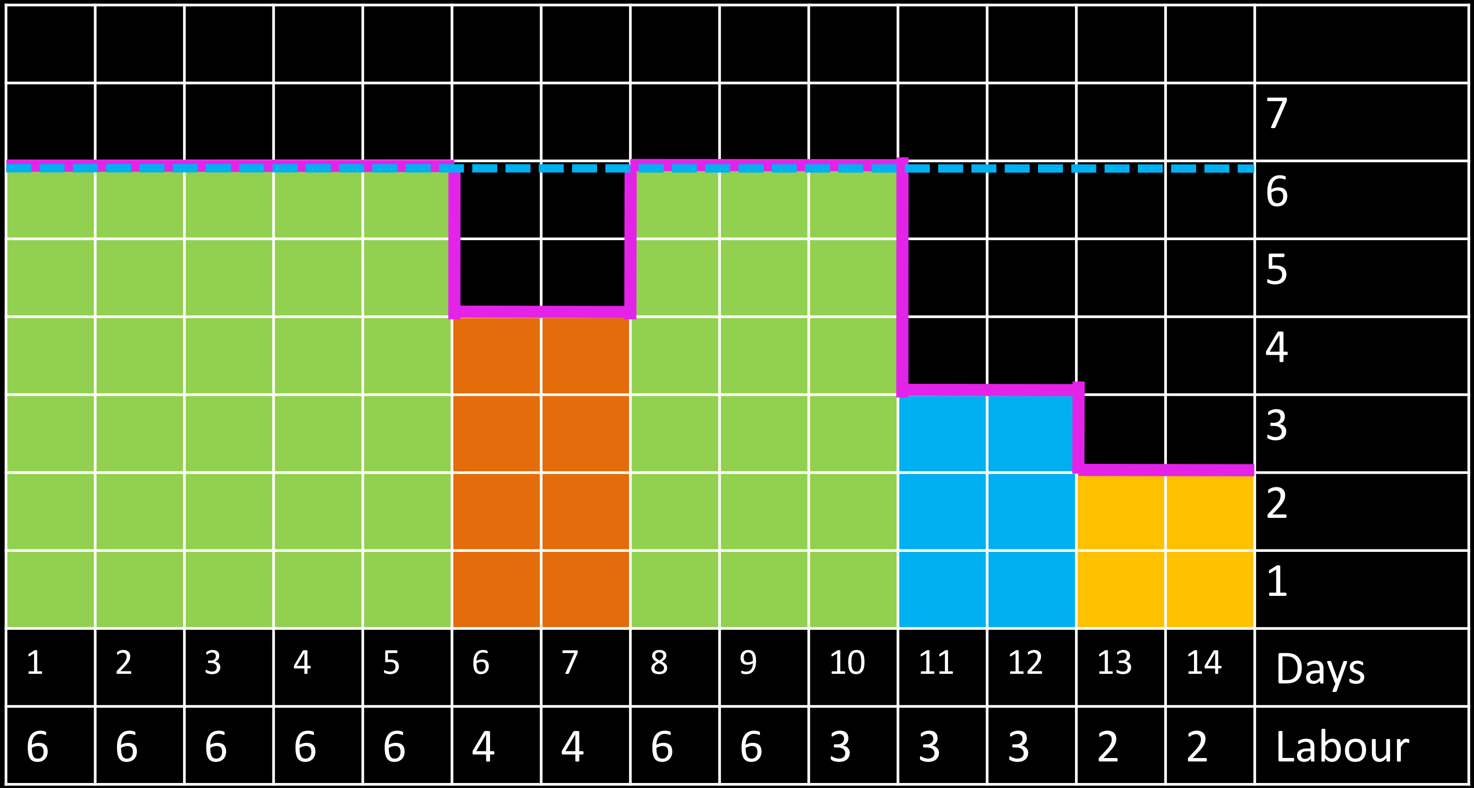
Critical

Float

≤ 6 laborers

Resource Profile of Example

Does not use more than six laborers



Sequencing

- Product sequencing
- Dispatching
- Progress report & expediting and control.
- Johnson's Rule for optimal sequence of
  - N jobs on 2 machine
  - Process n Jobs on 3 Machines (n/3 problem) and
  - Jackson Algorithm
  - Processing of 2 Jobs on m Machine (2/m) problem

Product sequencing

- **Sequencing**: the process of determining the job order on machines or work centers
  - Also known as *priority sequencing*
- **Priority rules**: the rules used in obtaining a job sequence
  - Can be simple or complex
  - Can use one or more pieces of information
  - Common rules shown on next slide

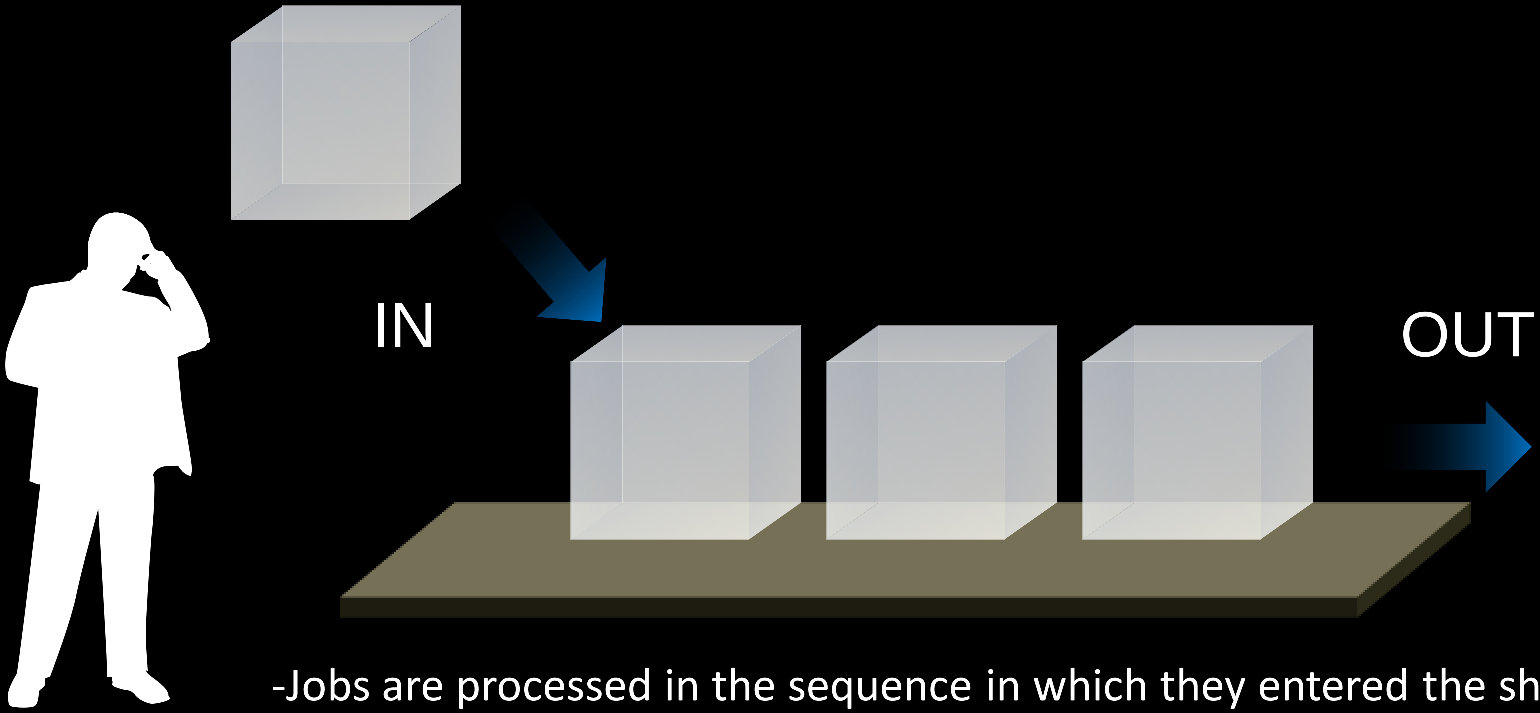
Priority Rules for Job Sequencing

- Prioritize jobs assigned to a resource
- If no order specified use first-come first-served (FCFS)
- Many other sequencing rules exist
- Each attempts to achieve to an objective

Sequencing Rules

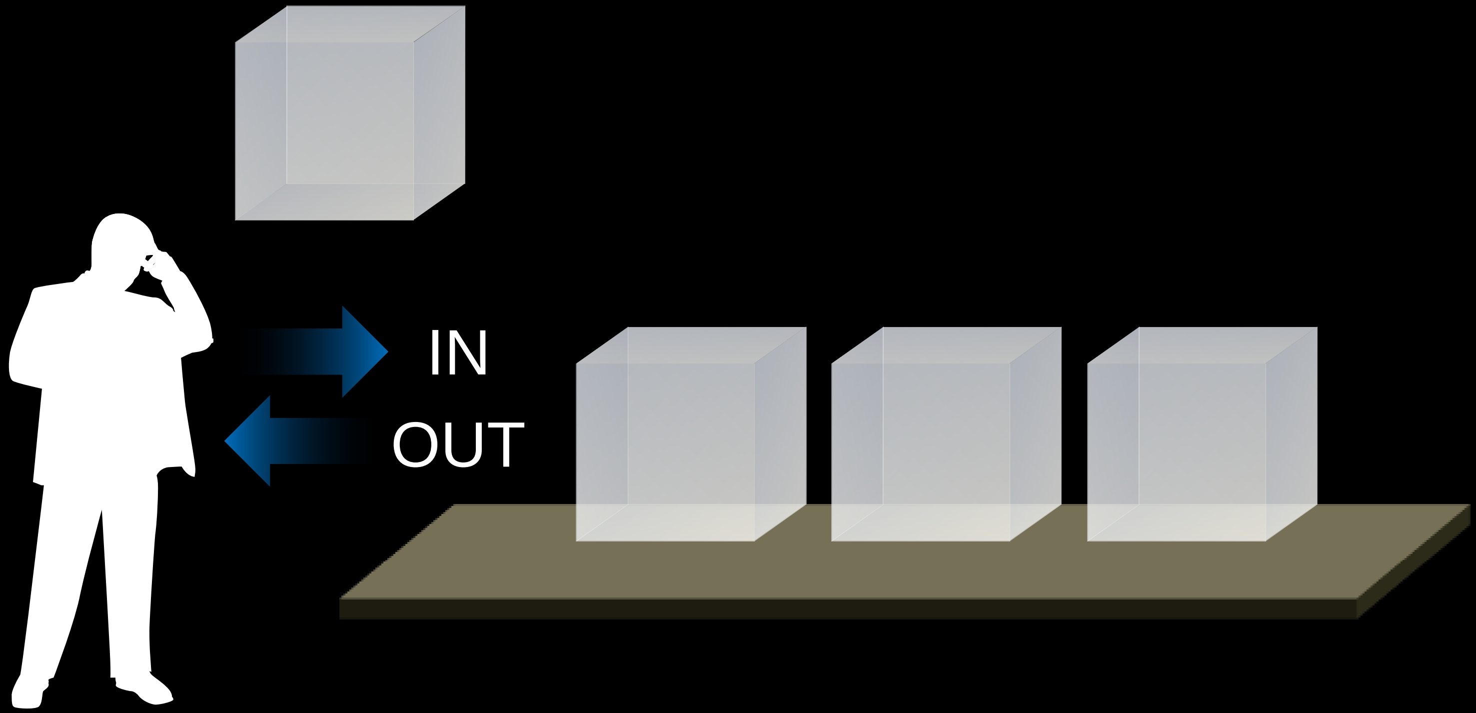
1. FCFS - first-come, first-served
2. LCFS - last come, first served
3. SPT - shortest processing time
4. EDD - earliest due date
5. CR - critical ratio
6. LWR - Least Work remaining
7. FOR – Fewest Operation Remaining
8. SL – Slack Time
9. ST/O - Slack time per operation
10. NQ – Next Queue
11. LSU - Least set –up

1. FCFS - first come, first served



-Jobs are processed in the sequence in which they entered the shop
- The simplest and nature way of sequencing as in queuing of a bank

2. LCFS - last come, first served



Sequencing methods

3. SPT (shortest processing time)
   – Jobs are sequenced in increasing order of their processing time
   – The job with shortest processing time is first, the one with the next shortest processing time is second, and so on
4. EDD (earliest due date) - **Jackson Algorithm**
   – Jobs are sequenced in increasing order of their due dates
   – The job with earliest due date is first, the one with the next earliest due date is second, and so on

Sequencing methods

5. CR (Critical ratio)

   - Critical ratio is the remaining time until due date divided by processing time;

   - Scheduling the job with the smallest CR next;

   -CR provides the balance between SPT and EDD, such that the task with shorter remaining time and longer processing time takes higher priority;

   - CR will become smaller as the current time approaches due date, and more priority will be given to one with longer processing time;

   - For a job, if the numerator of its CR is negative (the job has been already later), it is naturally scheduled next; 6. If more than one jobs are later, higher priority is given to one that has shorter processing time (SPT).

6. Least work remaining (LWR) It is an extension of SPT. It indicates that work be scheduled according to the processing time remaining before the job is considered to be complete.

Sequencing methods

7. Fewest operations remaining (FOR) It is another form of SPT. It sequences jobs based on the number of successive operations remaining until the job is considered.
8. Slack time (ST) Slack is determined by subtracting the sum of set-up and processing times from the time remaining until the job due date. Jobs are processed in order of the smallest amount of slack.
9. Slack time per operation (ST/O) The slack time is divided by the number of operations remaining until the job is complete with the smallest values being scheduled first.

Sequencing methods

10. Next queue (NQ) It depends on the machine utilization. It considers queues at each of the succeeding work centers at which the jobs will go.
11. Least set –up (LSU) It maximizes utilization. Least set-up selects first the job minimizes changeover time on given machine.

Sequencing – Example 1

The processing time for five jobs and their due dates are given for a single machine scheduling is given below in table. Determine:

1. The sequence
2. Total completion time
3. Average completion time
4. Average number of jobs in the system and
5. Average job lateness using the following priority sequencing rules
   I. Shortest Processing time (SPT)
   II. Earliest due date (EDD)
   III. Longest processing time (LPT)
   IV. Compare the above characteristics for the three sequencing rules

Job (j)	**1**	**2**	**3**	**4**	**5**
Processing Time (tj) hrs	9	7	5	11	6
Due date (in days) (dj)	16	20	25	15	40

Sequencing – Solution 1

Shortest Processing time (SPT) – As per the rule, the job with shortest processing time is scheduled first immediately followed by the next lowest processing time and so on.

Job Sequence	Processing time (tj) days	Flow time (Ft) days	Due Date (dj) days	Job lateness (days)
3	5	5	25	0
5	6	11	40	0
2	7	18	20	0
1	9	27	16	11
4	11	38	15	23
Total	38	99		34

Sequencing – Solution 1

Various Characteristics are:

I. Total completion time (flow time)= 38 days

II. Average completion time
= Total flow time/No. of jobs = 99/5 = 19.8 days

III. Average number of jobs in the system
= Total flow time/ Total Process time (completion)
= 99/38 = 2.61 jobs

IV. Average job lateness
= Total Job lateness/ No. of jobs
= 34/5 = 6.8 days

Sequencing – Solution 1

Earliest Due Date (EDD) – As per the rule, priority is given to t he job with earliest due date. Arranging the jobs as per the EDD rules gives following table.

Job Sequence	Processing time (tj) days	Flow time (Ft) days	Due Date (dj) days	Job lateness (days)
4	11	11	15	0
1	9	20	16	4
2	7	27	20	7
3	5	32	25	7
5	6	38	40	0
Total	38	128		18

Sequencing – Solution 1

Various Characteristics are:

I. Total completion time (flow time)= 38 days

II. Average completion time
= Total flow time/No. of jobs = 128/5 = 25.6 days

III. Average number of jobs in the system
= Total flow time/ Total Process time (completion)
= 128/38 = 3.37 jobs

IV. Average job lateness
= Total Job lateness/ No. of jobs
= 18/5 = 3.6 days

Sequencing – Solution 1

Longest Processing Time (LPT) – As per the rule, priority is given to t he job with longest processing time. Arranging the jobs as per the LPT rules gives following table.

Job Sequence	Processing time (tj) days	Flow time (Ft) days	Due Date (dj) days	Job lateness (days)
4	11	11	15	0
1	9	20	16	4
2	7	27	20	7
5	6	33	40	0
3	5	38	25	13
Total	38	129		24

Sequencing – Solution 1

Various Characteristics are:

I. Total completion time (flow time)= 38 days

II. Average completion time
= Total flow time/No. of jobs = 129/5 = 25.8 days

III. Average number of jobs in the system
= Total flow time/ Total Process time (completion)
= 129/38 = 3.39 jobs

IV. Average job lateness
= Total Job lateness/ No. of jobs
= 25/5 = 4.8 days

Sequencing – Solution 1

Comparison of Priority rules

Priority rule	Total Completion time (days)	Average Completion time (days)	Average no. of jobs in the system	Average Job Lateness
SPT	38	19.8	2.61	6.8
EDD	38	25.6	3.37	3.6
LPT	38	25.8	3.39	4.8

Sequencing – Example 2

The following jobs are waiting to be processed in a turning shop on 23rd July 2020. The estimates of the time needed to complete the jobs are as follows in the table. Sequence the job based on minimum critical ratio.

Jobs (j)	Due Date	Processing time (days)
1	31st July	9
2	2nd August	6
3	16th August	24
4	29th July	5
5	30th August	30

Sequencing – Solution 2

- The Critical ratio is computed as
- Critical Ratio (CR) = $\dfrac{\text{Time remaining for due date of the job}}{\text{Time needed to complete the job}}$

  $= \dfrac{\text{Time remaining}}{\text{Work remaining}}$
- As per the critical ration rule, a job with the minimum critical ratio is given the first preference, i.e., the lower is the critical ratio. Higher is it's priority.
- The denominator of CR, i.e., the time needed to complete the job includes the processing time remaining plus the transfer time plus the estimated waiting time remaining for the job to go through before it is completely processed .

Sequencing – Solution 2

Job Sequence	Due Date	Processing time (Tr) days	Time remaining for due date of the job in days (Tn)	Critical Ratio CR=Tn/Tr
1	31st July	9	8	8/9=0.89
2	2nd August	6	10	10/6=1.167
3	16th August	24	24	24/24=1.00
4	29th July	5	6	6/5=1.20
5	30th August	30	38	38/30=1.27

Sequencing – Solution 2

- CR<1 means that job is already late
- CR=1 indicates that the job is on schedule
- CR>1 indicates that the job has some slack available to it.
- From the table, job 1 has lowest CR and has to be processed first and job 2 has highest CR and it is scheduled last
- The sequence is

1	3	4	5	2

Expediting and Control

- Control phase is effected by dispatching, inspection and expediting materials control, analysis of work-in-process. Finally, evaluation makes the PPC cycle complete and corrective actions are taken through a feedback from analysis. A good communication, and feedback system is essential to enhance and ensure effectiveness of PPC.
- Expediting: This is the control tool that keeps a close observation on the progress of the work. It is logical step after dispatching which is called 'follow-up'. It coordinates extensively to execute the production plan.
- Expediting is a process of tracking a job's progress and taking special actions to move it through the facility. In tracking a job's progress, special action may be needed to keep the job moving through the facility on time. Manufacturing or service operations disruptions-equipments breakdowns, unavailable materials, last-minute priority changes, require managers to deviate from plans and schedules and expedite an important job on a special handling basis.

Progress report

- Progressing function can be divided into three parts, i.e., follow up of materials, follow up of work-in-process and follow up of assembly.
- The duties include: (a) Identification of bottlenecks and delays and interruptions because of which the production schedule may be disrupted. (b) To devise action plans (remedies) for correcting the errors. (c) To see that production rate is in line with schedule.

Dispatching

- It is concerned with starting the processes.
- It gives necessary authority to start a particular work, which has already being planned under Routing and scheduling.
- For starting the work, essential orders and instructions are given. Therefore, the complete definition of dispatching → "Released of order and instructions for the starting of production for any item in accordance with the route sheet and scheduled chart.

Function of Dispatching

1. After dispatching is done, required materials are moved from stores to m/c(s) and from operation to operation.
2. Authorizes to take work in hand as per schedule.
3. To distribute m/c loading and schedule charts route sheets and other necessary instructions and forms.
4. To issue inspection orders, clearly stating the type of inspection required at various stages.
5. To order too section for issuing proper tools, jigs, fixtures and other essential articles.

Forms used in Dispatching

Following are some of the more common forms used in dispatching.

1. Work orders: while starting the production, work orders are issued to departments to commence the desired lot of product.
2. Time cards: Each operator is supplied with this card in which he mentions the time taken by each operation and other necessary information's. there are helpful for wage payment.
3. Inspection Tickets: These tickets are sent to the inspection department which shows the quality of work required and stages at which inspection is to be carried out.
4. Move Tickets: These tickets are used for authorizing over the movement of material from store to shop and from operation to operation.
5. Tool & Equipment Tickets: It authorizes the tool department that new tools, gauges, jigs, fixtures and other required equipment may be issued to shop.

Johnson's Algorithm

- The general sequencing problem is stated as 'N' jobs (1,2,3….n) to be processed through 'M' machines (m_1,m_2,m_3….m_n) at a time by following , order of processing (precedence) and a given processing time, with minimizing elapsed time.
- Johnson's algorithm is the technique for minimizing completion time for a group of jobs to be processed in optimal sequence.

Johnson's Algorithm

Johnson's algorithm is used for:

1. N jobs and 2 machines
2. N jobs and 3 machines

Assumptions -

1. No machines can process more than one job at a time
2. Processing times are independent of processing job
3. Each job once started once one machine is continued till completion on it
4. Time involved in moving a job from one machine to another is negligibly small

Johnson's Algorithm

Steps:

1. List time required to process each job at each machine. Set up a one-dimensional matrix to represent desired sequence with # of slots equal to # of jobs.
2. Select smallest processing time at either machine.
   - If that time is on machine 1, put the job as near to beginning of sequence as possible.
   - If smallest time occurs on machine 2, put the job as near to the end of the sequence as possible. Remove the job from the list.
3. Remove the job from the list.
4. Repeat steps 2-3 until all slots in matrix are filled & all jobs are sequenced.

N Jobs on 2 machines (N/2)Example

Five jobs are to be processed on two machines M1 and M2 in the order M1M2. Processing time in hours are given below in the table .

Job	Machine Centre-1	Machine Centre-2
A	5	6
B	16	5
C	8	2
D	9	17
E	4	6

Determine the sequence that minimizes total elapsed time. Find out the total elapsed time and idle time, if any , on M2.

N Jobs on 2 machines (N/2) Solution

Job	Machine Centre-1	Machine Centre-2
A	5	6
B	16	5
C	8	2
D	9	17
E	4	6

N Jobs on 2 machines (N/2) Solution

The minimum processing time, 2, is given by Job C on Machine 2.
So, Schedule Job C in the end.

Job	Machine Centre-1	Machine Centre-2
A	5	6
B	16	5
C	8	2
D	9	17
E	4	6

				C

N Jobs on 2 machines (N/2) Solution

The minimum processing time, 2, is given by Job C on Machine 2.
So, Schedule Job C in the end.

Job	Machine Centre-1	Machine Centre-2
A	5	6
B	16	5
C	8	2
D	9	17
E	4	6

				C

N Jobs on 2 machines (N/2) Solution

After removing job C, the minimum processing time, 4, is given by Job E on Machine 1. So, Schedule Job E in the beginning.

Job	Machine Centre-1	Machine Centre-2
A	5	6
B	16	5
D	9	17
E	4	6

E				C

N Jobs on 2 machines (N/2) Solution

Job E is removed. Now, there is a tie. Minimum processing time is given by Jobs A and B. Break ties arbitrarily. Schedule one of Job A or Job B.

Job	Machine Centre-1	Machine Centre-2
A	5	6
B	16	5
D	9	17

E	A			C

N Jobs on 2 machines (N/2) Solution

Job A is chosen arbitrarily. Job A is scheduled in the beginning, because its minimum time is given on Machine 1. Beginning means position 2 because position 1 is taken

Job	Machine Centre-1	Machine Centre-2
B	16	5
D	9	17

E	A		B	C

N Jobs on 2 machines (N/2) Solution

Next, Job B is scheduled. It's scheduled in the end, because its minimum processing time is given on Machine 2.

Job	Machine Centre-1	Machine Centre-2
D	9	17

E	A	D	B	C

N Jobs on 2 machines (N/2) Solution

The sequencing is complete after assigning the remaining Job D to the remaining position. Next, the make span is computed.

E	A	D	B	C

N Jobs on 2 machines (N/2) Solution

To compute the elapsed time:

1. The starting time of machine M_1 is assumed as 0.00
2. The machine M2 starts processing job 2 only when it comes out of machine M1 after completion. So it is idle for 1 hour in the start till job comes to it from machine 1.
3. The minimum total elapsed time is 40 hrs to process all the 5 jobs through two machines M1 and M2

The idle time on machine M1 = 4+1+2 = 7 hrs.

N Jobs on 2 machines (N/2) Solution

Job	Machine Centre-1	Machine Centre-2
A	5	6
B	16	5
C	8	2
D	9	17
E	4	6

Optimal sequence	Machine M1			Machine M2			Idle time M2
	In	Processing time	Out	In	Processing time	Out	
E	0	4	4	4	6	10	4
A	4	5	9	10	6	16	-
D	9	9	18	18	17	35	2
B	18	16	34	35	5	40	-
C	34	8	42	42	2	44	2

The minimum total elapsed time is **44** hrs to process all the 5 jobs through two machines M1 and M2. The idle time on machine M2 = 4+2+2 = 8 hrs.

N Jobs on 3 machines (N/3)

Conditions to be satisfied to solve N/3 problems by Johnson's Methods:

1. Each job has to go through all the three machines M1,M2,M3.
2. The smallest processing time on machine M1 ≥ Largest processing time on machine M2
3. The smallest processing time on machine M3 ≥ Largest processing time on machine M2

If either or both of these conditions are satisfied then the given problem can be solved by Johnson's algorithm.

N Jobs on 3 machines (N/3)

Steps:

1. Convert the 3 machine problem in to 2 machine problem by introducing two fictitious machines G and H.
   – Gi =M1i=M2i
   – Hi = M2i=M3i (i=1,2,3…n)
2. Once the problem is converted in to N/2 problem, the sequence is determined by Johnson’s algorithm for N/2 problem
3. For the optimal sequence determined, find out the minimum total elapsed time associated with machines.

N Jobs on 3 machines (N/3)

Breaking Rules:

1. If there are equal smallest processing time one for each machine, place the job on machine 1, first in the sequence and one in machine 2 last in the sequence.
2. If the equal smallest time are both for machine 1 , select the job with lower processing time in machine 2 for placing first in the sequence.
3. If equal smallest times are both for machine 2, select the one with lower processing time in machine 1, for placing last in the sequence.

N Jobs on 3 machines (N/3)

Seven jobs are to be processed through 3 machines M1, M2 and M3 in the order M1,M2,M3. The processing times are given in hrs. to process each one of the 3 jobs through all the machines find the optimal sequence of the jobs. Also find the minimum total elapsed time and idle times on M2 and M3.

Jobs	M1	M2	M3
A	3	4	6
B	8	3	7
C	7	2	5
D	4	5	11
E	9	1	5
F	8	4	6
G	7	3	12

N Jobs on 3 machines (N/3)

Step-1: Check for conditions to be satisfied

- Minimum processing time on M1 = 3
- Minimum processing time on M3 = 5
- Maximum processing time on M2 = 5

Condition 1 :

- Min. time on M1 ≥ Max. time on M2
- Here 3 is not ≥ 5, so condition 1 is not satisfied

Condition 2:

- Min. time on M3 ≥ Max. time on M2
- Here 5 = 5, so condition 2 is satisfied
- Hence this problem can be solved by Johnson's algorithm

Jobs	M1	M2	M3
A	3	4	6
B	8	3	7
C	7	2	5
D	4	5	11
E	9	1	5
F	8	4	6
G	7	3	12

N Jobs on 3 machines (N/3)

Step-2: Convert N/3 problem in to N/2 problem by assuming fictitious machines G and H.

$$G = M1i + M2i \text{ and } H = M2i + M3i$$

Jobs	M1	M2	M3
A	3	4	6
B	8	3	7
C	7	2	5
D	4	5	11
E	9	1	5
F	8	4	6
G	7	3	12

Jobs	Gi	Hi
A	7	10
B	11	10
C	9	7
D	9	16
E	10	6
F	12	10
G	10	15

N Jobs on 3 machines (N/3)

Step-3: Determine the sequence using N/2 procedure.

- Minimum processing time is 6 for job E and is associated with machine H. So process job E last in the sequence.

Jobs	Gi	Hi
A	7	10
B	11	10
C	9	7
D	9	16
E	10	6
F	12	10
G	10	15

						E

N Jobs on 3 machines (N/3)

Jobs	Gi	Hi
A	7	10
B	11	10
C	9	7
D	9	16
F	12	10
G	10	15

A					C	E

N Jobs on 3 machines (N/3)

Jobs	Gi	Hi
B	11	10
D	9	16
F	12	10
G	10	15

A	D				C	E

N Jobs on 3 machines (N/3)

Minimum time is 10 associated with

1. Machine Gi for Job G. so do the job g next in the sequence
2. Tie between job F and job B for machine Hi . As per the rule job B should be processed last

Jobs	Gi	Hi
B	11	10
F	12	10
G	10	15

A	D	G	F	B	C	E

N Jobs on 3 machines (N/3)

Optimal sequence	Machine M1			Machine M2			Idle time M2	Machine M3			Idle time M2
	In	Processing time	Out	In	Processing time	Out		In	Processing time	Out	
A	0	3	3	3	4	7	3	7	6	13	7
D	3	4	7	7	5	12	-	13	11	24	-
G	7	7	14	14	3	17	2	24	12	36	-
F	14	8	22	22	4	26	5	36	6	42	-
B	22	8	30	30	3	33	4	42	7	49	-
C	30	7	37	37	2	39	4	49	5	54	-
E	37	9	46	46	1	47	7	54	5	59	-
							25				7

Min. total elapsed time=59 hrs., Idle time of M2=25 hrs, idle time of M3=7 hrs

2 Jobs on M machines (2/m)

- For these types of problems, there are two jobs J1, J2 each of which has to be processed on m machines M1, M2, …, Mm with different sequences of machines for processing each of the jobs.
- Each machine can perform only one job at a time. The exact or expected processing times on all the given machines are known.
- The objective is to find a sequence of processing the jobs, which minimizes the total elapsed time from the start of the first job to the completion of the last job.

2 Jobs on M machines (2/m)

- A problem of this type can be solved with the help of **graphical method**.
- We have to use the following steps in this method:
- Step 1:Represent the processing times of job 1 on different machines along the x-axis and processing times of job 2 on different machines along the y-axis.
- Step 2: Draw the vertical/horizontal lines through processing times as given in Step 1 and shade the common area for processing of two jobs on the same machine at different times since each machine can perform only one job at a time.

2 Jobs on M machines (2/m)

- Step 3: Starting from the origin, we move diagonally, horizontally or vertically through various stages of completion of the processing of both the jobs through different machines at the same time until all the processing times are finished. If both the jobs are being processed simultaneously at different machines at the same time, we move diagonally. If Job 2 is under process at any machine and Job 1 is idle, i.e., Job 1 is waiting to be processed at the same machine through which Job 2 is being processed, we move vertically; and we move horizontally when Job 1 is being processed and Job 2 is idle.

2 Jobs on M machines (2/m)

- Step 4: An optimal sequence for the jobs is the shortest line consisting of combinations of horizontal, vertical and diagonal (45) lines from the origin. Since both the jobs are being processed simultaneously on the diagonal line, effort should be made to select a path on which diagonal movement is maximum.
- So, we first try to move diagonally but keep in mind that diagonal movement through the blocked out (shaded) areas is not allowed since both the jobs cannot be processed simultaneously on a machine. Then horizontal or vertical movement is chosen keeping in view that total elapsed time should be minimum. The elapsed time can be obtained by adding the idle time for either job to the processing time for that job.

2 Jobs on M machines -Example

- A machine shop has five machines A, B, C, D and E. Two jobs must be processed through each of these machines. The time (in hours) taken on each of these machines and the necessary sequence of jobs through the shops are given in table below.
- Use the graphical method to obtain the total minimum elapsed time.

Job 1	**Sequence**	**A**	**B**	**C**	**D**	**E**
	Time	2	4	3	6	6
Job 2	Sequence	C	A	D	E	B
	Time	4	6	3	3	6

2 Jobs on M machines -Solution

- We are given the job sequences and processing time of 2 jobs at 5 machines. We follow the graphical method to find the minimum total elapsed time from starting the first job at the first machine to completion of the second job at the last machine.
- We first represent the processing time of job 1 on different machines, i.e, 2, 4, 3, 6, 6 along the x-axis and the processing time of job 2, i.e., 4, 6, 3, 3, 6 along the y-axis.

2 Jobs on M machines -Solution

- Then we draw the vertical lines through processing times of Job 1. We draw the first vertical line at 2 hrs, the second at 2 + 4 = 6 hrs, the third at 6 + 3 = 9 hrs, and so on.
- Similarly, we draw the horizontal lines through processing times of Job 2. We draw the horizontal lines at 4, 10, 13, 16 and 22 hrs in this case. We now shade the common area for processing of jobs 1 and 2 on one machine at the same time as each machine can perform only one job at a time.

In this case we find the common areas to be:

i. between 0 to 2 hours on the x-axis and 4 to 10 hours on the y-axis for machine A,

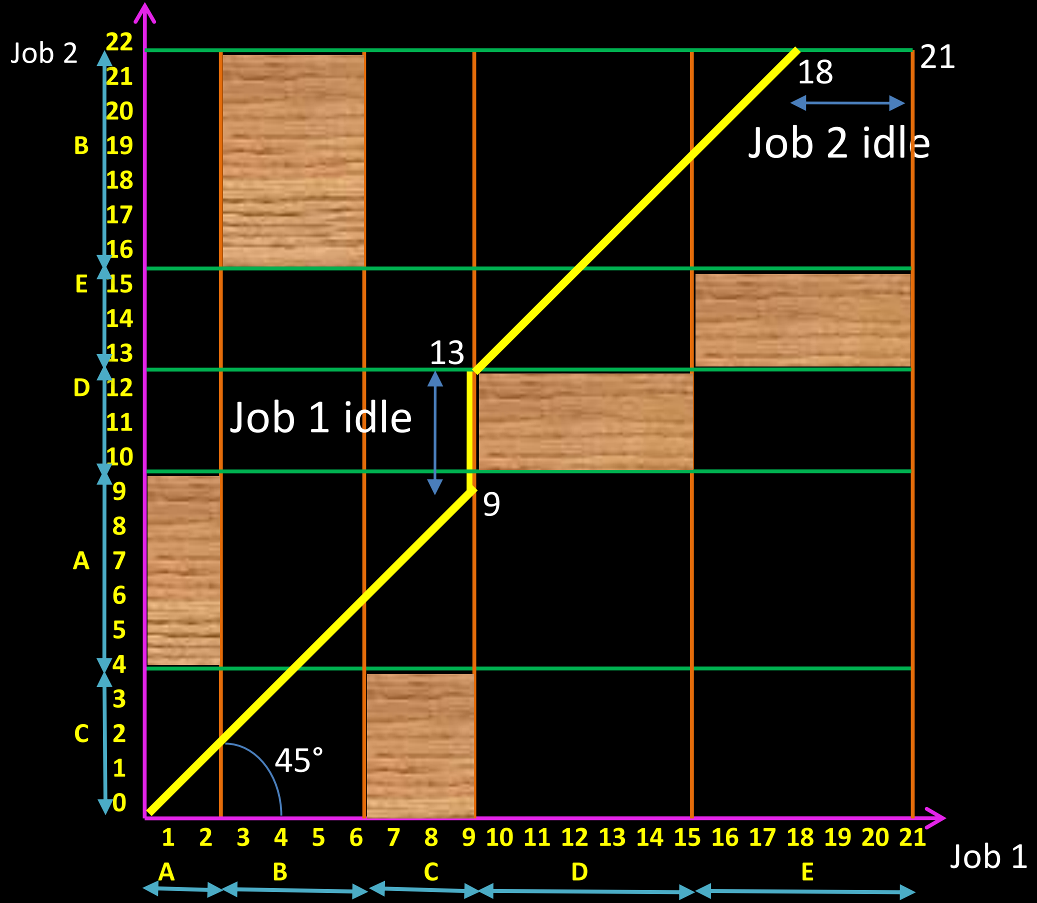
ii. between 2 to 6 hours on the x-axis and 16 to 22 hours on the y-axis for machine B,

iii. between 6 to 9 hours on the x-axis and 0 to 4 hours on the y-axis for machine C,

iv. between 9 to 15 hours on the x-axis and 10 to 13 hours on the y-axis for machine D,

v. between 15 to 21 hours in the x-axis and 13 to 16 hours on the y-axis for machine E.

2 Jobs on M machines -Solution



Total Idle time for Job 1 is 13-9 = 4 hrs.
Total Idle time for Job 2 is 21-18 = 3 hrs.

The total elapsed time for Job 1 is 21 + 4 = 25 hours.
The total elapsed time for job 2 is 22 + 3 = 25 hours

2 Jobs on M machines -Solution

- We now have to find an optimal sequence for processing of the jobs 1 and 2, i.e., the shortest line consisting of combinations of horizontal, vertical and diagonal (45°) lines from the origin. For this, we start from the origin and move diagonally first up to the point (9, 9) since both the jobs 1 and 2 are being processed simultaneously.
- After being processed through machines A, B and C, job 1 becomes idle since job 2 is being processed in machine A and it is then being processed in machine D. Therefore, we move vertically which means Job 2 is under process and Job 1 is idle. The idle time for job 1 is 4 hours in this case.
- Again we move diagonally up to the point (18, 22) since both the jobs 1 and 2 are being processed simultaneously. At this stage, job 2 is completed but job 1 is being processed through machine E. Therefore, we move horizontally, which means Job 1 is being processed and Job 2 is idle. The idle time for job 2 is 3 hours since job 1 is processed through machine E between 15 to 21 hours.
- In this case, the idle time for job 1 is found to be 4 hours. Therefore, the total elapsed time for Job 1 is 21 + 4 = 25 hours. Similarly, the idle time for job 2 is seen to be 3 hours when job 1 was under processing between 18 to 21 hrs and the processing for job 2 had been completed. Therefore, the total elapsed time for job 2 is 22 + 3 = 25

2 Jobs on M machines -Practice

1. Using the graphical method, calculate the minimum time needed to process job 1 and job 2 on five machines A, B, C, D and E, that is, for each machine find the job which should be done first. Also calculate the total time needed to complete both jobs.
    1. Job 1 Sequence : A B C D E Time (in hrs) : 1 2 3 5 1
    2. Job 2 Sequence : C A D E B Time (in hrs) : 3 4 2 1 5
2. Use the graphical method to minimize the time required to process the following jobs on the machines, that is, for each machine specify the job that should be done first. Also calculate the total elapsed time for completing both jobs.
    1. Job 1 Sequence : A B C D E Time (in hrs) : 6 8 4 12 3
    2. Job 2 Sequence : B C A D E Time (in hrs) : 10 8 6 4 12

Jackson Algorithm (n/m)

- We often come across with the *n* jobs to sequence on m machines or men or
  work centers in practice.
- However it is not very difficult to sequence in such case.
- The method we apply here is based on the Jackson's conditions as explained here.
- Let there be *n* jobs, of which each is to be processed through m work centers/machines Say

$$W1, W2, W3, \ldots Wm$$

Steps- Jackson Algorithm (n/m)

- **Step 1-** Find minimum process time of $W1$ and Wm series, and maximum process time of all the middle series.
- **Step 2 -** Check whether
  Minimum of $W1$ series ≥ Maximum of middle series.
  Minimum of Wm series ≥ Maximum of middle series.
- **Step 3 -** If the equations of step 2 are not satisfied, the method fails. Otherwise move to next step.
- **Step 4 -** Convert *m* Work center problem to two-work center problem by two fictitious machines say *A* and *B*.
  A = The algebraic sum of process time of $W1$ and all corresponding timings of the mediocre series.
  B = The algebraic sum of process time of Wm and all corresponding timings of the mediocre series.
  (However in general practice we can find the sequence by taking the timings of $W1$ and Wm if the sums of the mediocre series are fixed positive constants).

Numerical- Jackson Algorithm

Five machine parts are to be reassembled after yearly maintenance in the order *P*, *Q*, *R*, *S*, and *T* on four machines *A*, *B*, *C* and *D*. Find the optimal sequence for the machines when passing is not allowed of which the repair time (in hours) is given below. Also find the total elapsed time, and individual idle times for the repairmen on each machine and machine part. (Consider that the machine will be handed over to the production department immediately after reassembling all the parts).

Machine\ Part	P	Q	R	S	T
A	7	5	2	3	9
B	6	6	4	5	10
C	5	4	5	6	8
D	8	3	3	2	6

Solution-Jackson Algorithm (n/m)

Checking Sequencing algorithm rules :
Minimum repair time of P series [Minimum $(t_{P,j})$] = $(t_{P,C})$ = 5
Minimum repair time of T series [Minimum $(t_{T,j})$] = $(t_{T,D})$ = 6
And maximum repair time of Q, R, S series [Maximum $(t_{Q,j}\ t_{R,j}\ t_{S,j})$] = {6, 5, 6}, respectively.
Since the condition Minimum $(t_{T,j}) >= $ Maximum $(t_{Q,j}\ t_{R,j}\ t_{S,j})$ is satisfied, the given problem can be converted into a four jobs and two machines (say X and Y) problem as

– $t_{X,j} = (t_{P,j} + t_{Q,j} + t_{R,j} + T_{S,j})$

– $t_{Y,j} = (t_{Q,j} + t_{R,j} + t_{S,j} + T_{T,j})$

Machine\Job	A	B	C	D
Machine X	17	21	20	16
Machine Y	19	25	23	14

Solution- Jackson Algorithm (n/m)

- Now, using the optimal sequence algorithm, the following optimal sequence can be obtained.

A	C	B	D

- The total elapsed time corresponding to the above sequence can be computed as shown in the following table, using the individual processing times as given in the original problem :

Machine\Part	P	Q	R	S	T
A	0-7	7-12	12-14	14-17	17-26
C	7-12	12-16	16-21	21-27	27-35
B	12-18	18-24	24-28	28-33	35-45
D	18-26	26-29	29-32	33-35	45-51

Solution- Jackson Algorithm (n/m)

- From the above table, it can be understood that the minimum total elapsed time is 51 hours.
- The idle times for the man who reassembles the machine parts are as follows :
  For P = 51 – 26 = 25 hrs.
  For Q = 7 + (18 – 16) + (26 – 24) + (51 – 29) = 33 hrs.
  For R =12 + (16 – 14) + (24 – 21) + (29 – 28) + (51 – 32) = 37 hrs.
  For S = 14 + (21 – 17) + (28 – 27) + (51 – 35) = 34 hrs.
  For T = 17 +(27 – 26) = 18 hrs.

Solution- Jackson Algorithm (n/m)

- The idle times for the repairmen working on the machines are as follows :
- For *A* = No idle time and will be handed over after 26 hours.
- For *B* = 7 idle hrs and will be handed over after 35 hours.
- For *C* = 12 + (35 – 33) = 14 idle hrs and will be handed over after 45 hours.
- For *D* = 18 + (33 – 32) + (45 – 35) = 29 idle hrs and will be handed over after 51 hours.

Practice- Jackson Algorithm (n/m)

A mechanic has to assemble 5 machines, in which each assembly takes three stages, for which he employed three persons. The sequence of the stages is fixed as stage I, stage II and stage III. Determine the optimal sequence for the machines to be assembled to minimize the time elapsed from the start of first machine the end of last machine. The time required, in days for each machine at each stage is given in the following matrix. Also find the time that the mechanic could promise to deliver each machine. If the mechanic can de-link a person from the present process if his stage work is completed or not started, what will be the net idle time that the process demands?

Machine Code	Stage 1	Stage 2	Stage 3
M0235988	6	8	15
K0242115	16	10	18
V6566573	14	2	10
G3042721	10	4	12
S6567763	8	6	20

Practice- Jackson Algorithm (n/m)

There are 5 spare parts made, each of which must go through machines M1, M2, M3 in order M1, M3, M2. Processing times are given below.
(i) Determine the optimal sequence, total elapsed time and idle time of each machine. If it can be processed by the sub-contract services by three outside(ii) parties, P1, P2 and P3 to process on M1, M2 and M3 respectively, schedule the parties optimally, under the conditions that the parties may be called on any day but the contract once started should be continued till the last job of the respective party is completed and the payment should be made for the process delays also. (iii) What will be the amounts paid to each party if it costs Rs. 10/- for each working hour and Rs. 5/- for each waiting hour?

Spare Part	1	2	3	4	5
M1	8	5	4	6	5
M2	10	13	11	10	12
M3	6	2	9	7	4

Practice- Jackson Algorithm (n/m)

- Find the sequence that minimizes the total elapsed time in hrs to complete the following jobs on 3 machines. Prepare Gantt chart.

Spare Part	A	B	C	D	E
M1	3	4	7	5	2
M2	3	8	2	1	5
M3	5	8	10	7	6

- Find the sequence that minimizes total machining time. The data is :

Tasks	A	B	C	D	E	F
Time on Machine 1	4	9	8	5	10	9
Time on Machine 2	5	4	3	6	2	5
Time on Machine 3	7	8	6	12	6	7

Practice- Jackson Algorithm (n/m)

- Find the sequence that minimizes total elapsed time

Tasks	A	B	C	D	E	F	G
Time on Machine 1	3	8	7	4	9	8	7
Time on Machine 2	4	3	2	5	1	4	3
Time on Machine 3	6	7	5	11	5	6	12