

Module-3

INVENTORY CONTROL





Content

- *3.1. Basic concepts of inventory, Types of inventory, purpose of holding stock and influence of demand on inventory, Costs associated with Inventory management.*
- *3.2. Inventory Models: Deterministic models - instantaneous stock replenishment model, Production model, planned shortages and price discount model, Probabilistic models fixed quantity system(Q-system) and Fixed period system (p-system)*
- *3.3. Selective Inventory Control techniques - ABC analysis, HML analysis and VED analysis*





Basic concepts of inventory

Two Basic Concepts-

- The first of these concepts is that *much of a firm's inventory is really stored capacity*. That means that most inventory represents the use of the firm's capacity to create a product ahead of the actual demand of that product. That concept is one of the major issues that makes the planning and control for a pure service firm so different from a manufacturing firm. The average service firm does not have the luxury of planning and using the capacity ahead of the demand for its use, but instead must use the capacity only once the demand is created.
- The second concept is that *inventory is almost never a problem* in any company, in spite of the fact that it is often mentioned that “one of our problems is too much inventory.” *In most firms, inventory exists as a symptom of the way the business is run.*



Basic concepts of inventory

- The amount of material, a company has in stock at a specific time is known as inventory or in terms of money it can be defined as the total capital investment over all the materials stocked in the company at any specific time. Inventory may be in the form of,
 - raw material inventory
 - in process inventory
 - finished goods inventory
 - spare parts inventory
 - office stationary etc.
- As a lot of money is engaged in the inventories along with their high carrying costs, companies cannot afford to have any money tied in excess inventories. Any excessive investment in inventories may prove to be a serious drag on the successful working of an organization. Thus there is a need to manage our inventories more effectively to free the excessive amount of capital engaged in the materials







Inventory Decision

- How much to order? – This is decided by the manager as to how much quantity to order for optimal performance and effective utilization of resources.
- When to order? – This is the most important aspect the manager should emphasize on because this would decide when should the products be ordered.
- How much stock should be kept in safety? – This indicates how much quantity should be taken under consideration so that the stock can be used safely in the future without any hesitation.





Types of inventory

1. Source of demand –

- **Independent demand inventory**- The source of demand for this type of inventory is typically from sources outside the company itself, usually external customer. It is called independent since the demand for it is essentially independent of any internal actions of the firm. It is usually sellable product.
- **Dependent demand inventory**-The source of dependent demand inventory is directly dependent on internal decisions, primarily on the decision of how many of which product to produce at what time. It is usually components of sellable product
- Example – Sellable product is chair , required by customers – Independent Demand inventory
 - Components can be seats, legs, back support, nuts & bolts, etc.. – Dependent demand inventory for number of chairs to be produced





Types of inventory

2. Position of the inventory in the process

- **Raw materials** represent inventory that has been purchased for use in the production process, but have had no value added by the company's production process.
- **Work in process (WIP)** represent inventory that has had some value added, but still has additional processing to be completed before it can be used to meet customer demand.
- **Finished goods** represent inventory that has completed all the processing from the firm. It is generally ready to be used to meet customer demand, with the possible exception of packaging.
- **Maintenance, repair, and operations (MRO) inventory** is material used to support the company's business and production processes, but typically will never be directly sold to a customer. It is made up of spare parts, machine oil, cleaning supplies, office supplies, and so forth.





Types of inventory

3. Function or use of inventory in the process

- **Transit inventory** is inventory in motion from one activity to another. Inventory in the transportation system is the most common form.
- **Cycle inventory** is inventory that exists because for any time period the rate of replenishment exceeds the demand-a situation often caused because of order costs, setup costs, or packaging considerations.
- **Buffer inventory** is also called **safety stock**, and exists "just in case." Inventory that is maintained explicitly to protect the organization just in case one or more of these problems occur is called buffer inventory or safety stock.
- **Anticipation inventory** is inventory built up on purpose in anticipation of some demand in excess of the usual production output.
- **Decoupling inventory** is inventory that is purposely placed between operations to allow them to operate independently of one another.

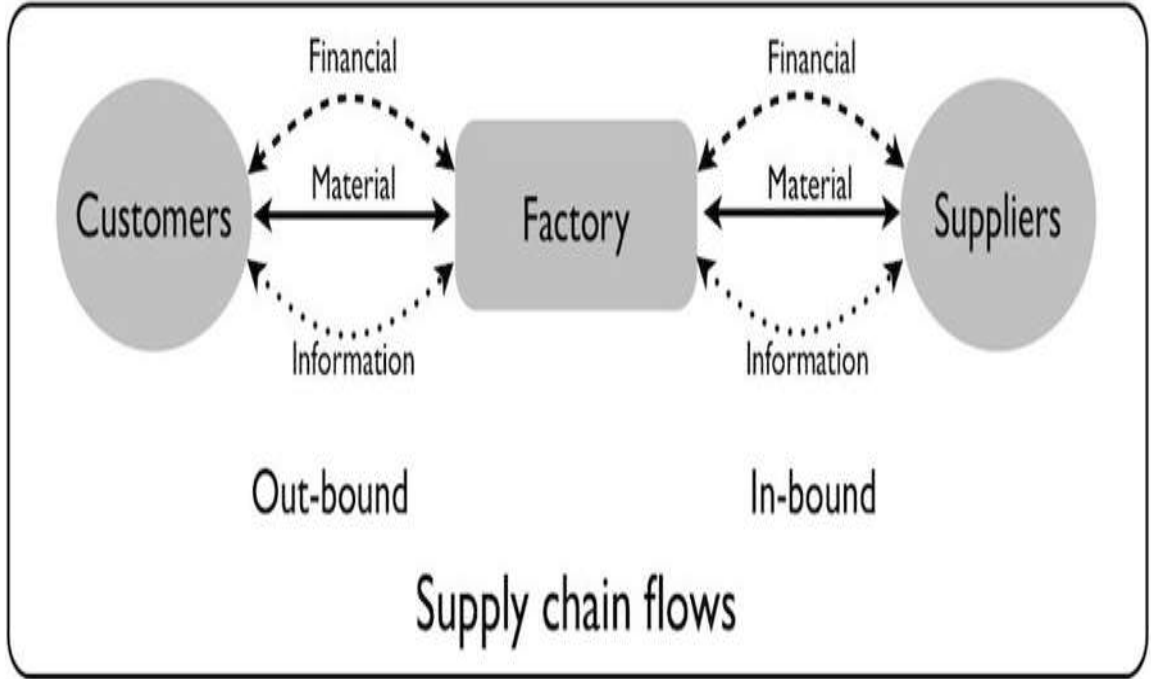




Purpose of holding stock

- Inventories are needed because demand and supply can not be matched for physical and economical reasons. There are several other reasons for carrying inventories in any organization.
- To safe guard against the uncertainties in price fluctuations, supply conditions, demand conditions, lead times, transport contingencies etc.
- To reduce machine idle times by providing enough in-process inventories at appropriate locations.
- To take advantages of quantity discounts, economy of scale in transportation etc.
- To decouple operations i.e. to make one operation's supply independent of another's supply. This helps in minimizing the impact of break downs, shortages etc. on the performance of the downstream operations. Moreover operations can be scheduled independent of each other if operations are decoupled.
- To reduce the material handling cost of semi-finished products by moving them in large quantities between operations.
- To reduce clerical cost associated with order preparation, order procurement etc.

Influence of demand on inventory





Inventory control - Terminology

- *Demand* : Number of items required per unit of time . This may be deterministic or probabilistic in nature.
- *Order cycle* : The time period between two successive orders.
- *Lead time*: time interval between ordering and receiving the order
- *ROL (Re Order Level)* : The point at which the replenishment action is initiated.
- *Reorder Quantity* : The quantity of material to be ordered at ROL. Normally it is EOQ (Economic Order Quantity)

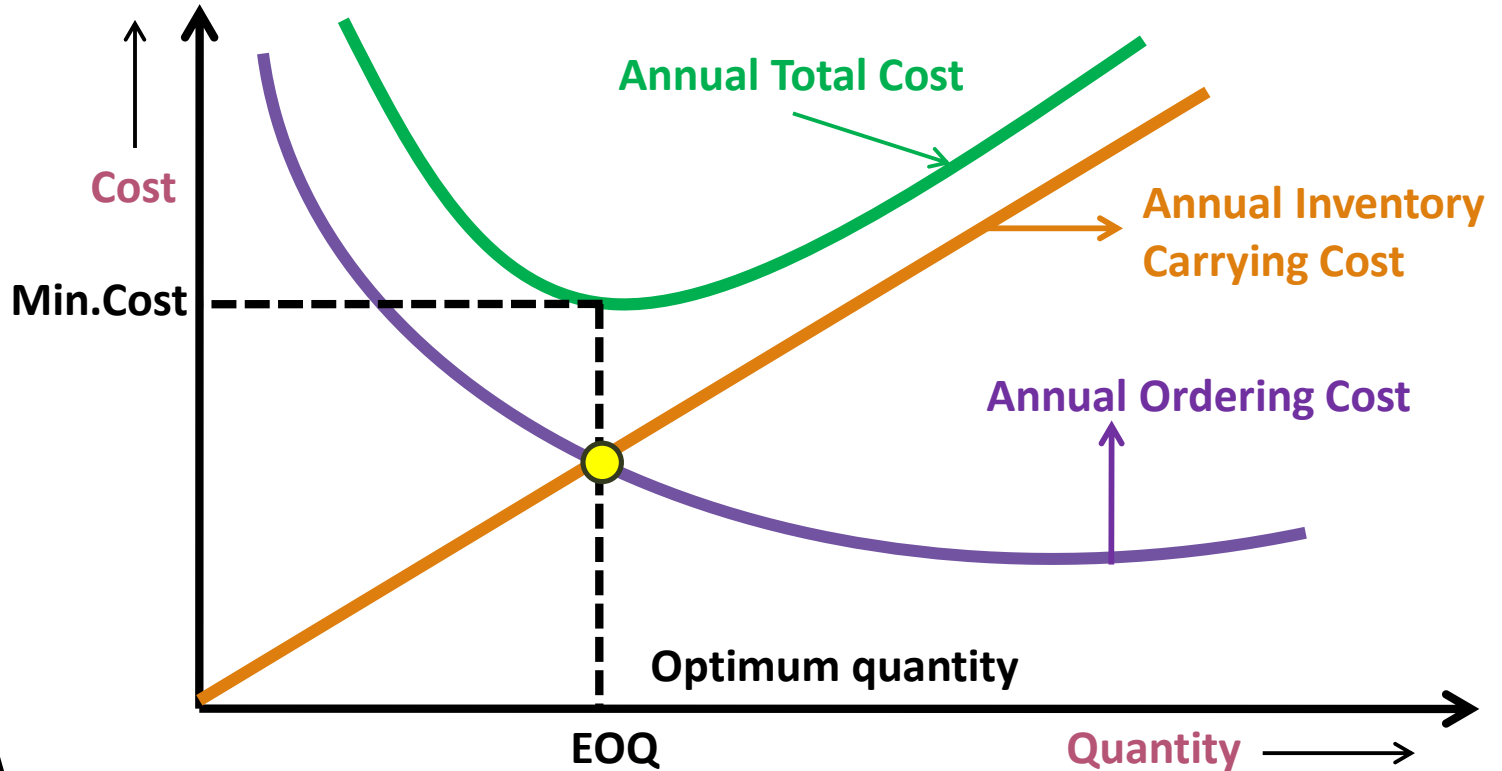




Costs associated with Inventory management

- **Unit cost:** it is usually the purchase price of the item under consideration. If unit cost is related with the purchase quantity, it is called as discount price.
- **Procurement/Ordering/Replenishment/Acquisition costs:** This includes the cost of order preparation, tender placement, cost of postages, telephone costs, receiving costs, set up cost, inspection costs, Accounting costs, Transportation costs etc.
- **Inventory Carrying/Holding costs:** This represents the cost of maintaining inventories in the plant. It includes the cost of insurance, security, warehouse rent, taxes, interest on capital engaged, spoilage, breakage, deterioration, obsolescence, depreciation, pilferage, Loss due to perishable nature, storage (rent, heating, lighting, cooling etc..) etc.
- **Stock out/shortage costs:** This represents the cost of loss of demand due to shortage in supplies. This includes cost of loss of profit, loss of customer, loss of goodwill, penalty, breakdown costs, loss of future costs, extra cost for urgent delivery etc.

INVENTORY COST RELATIONSHIP





Economic Order Quantity (EOQ)

- Annual carrying cost = Avg. inventory X Carrying Cost per unit
- Annual carrying cost varies with Quantity of Inventory
- Annual Ordering cost varies with number of orders i.e. this cost is high if the items are frequently ordered in small quantity
- Total Cost= Annual Ordering Cost + Annual Carrying Costs
- Min. Total cost is achieved when
$$\text{Annual ordering cost} = \text{Annual Carrying Cost}$$
- The optimum quantity for which Total cost is minimum , is called Economic Order Quantity (EOQ)

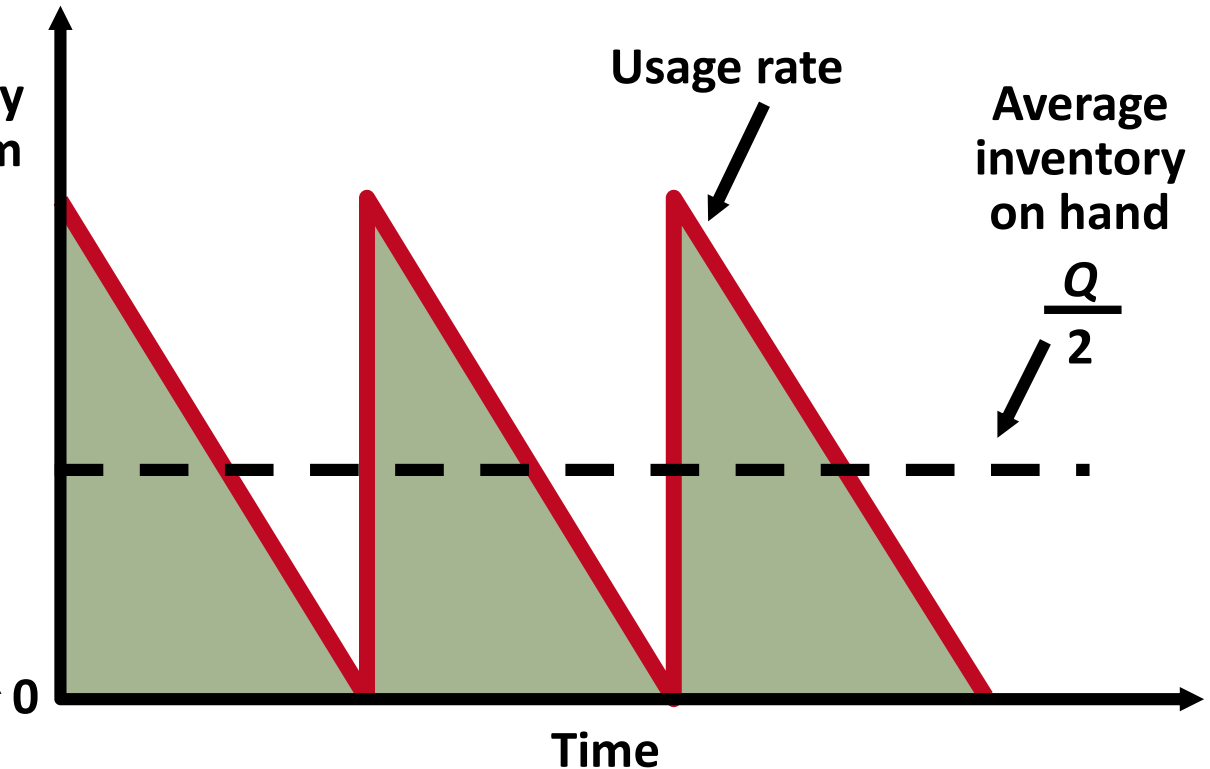


Inventory Usage Cycle

Inventory level

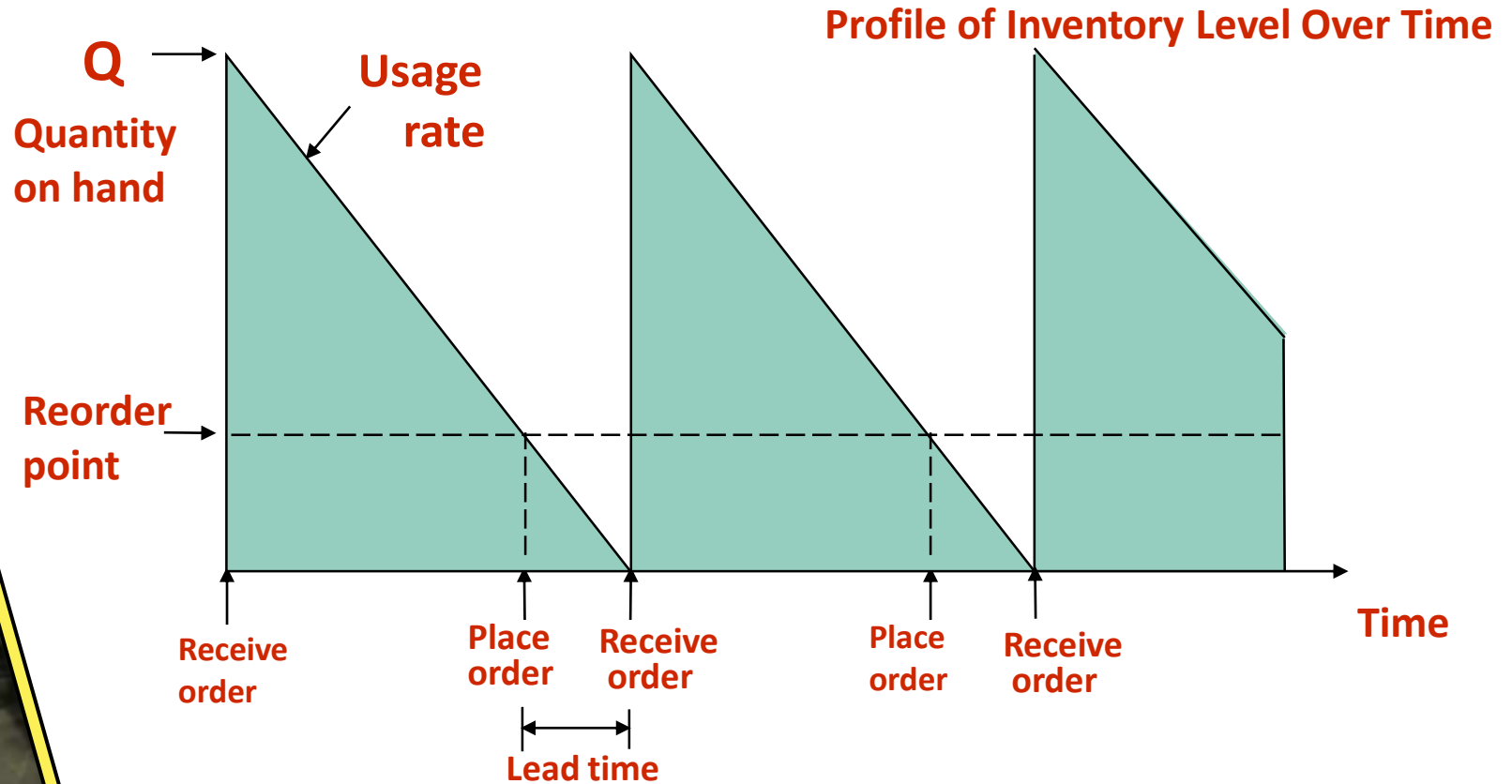
Order quantity
= Q (maximum
inventory
level)

Minimum
inventory





Inventory Cycle





Inventory Models

An inventory system can be modelled quantitatively based on **demand patterns**. They are

- **Deterministic inventory models** in which demand rate of an item is assumed to be constant.
- **Probabilistic inventory models** where the demand for an item fluctuates and is specified in probabilistic terms.

Based on **the frequency** at which **orders** are placed for procuring inventory, there are two models.

- **Single Period models:** Typically orders are made only once and are used for one time ordering for seasonal products or spare parts purchases.
- **Multi Period models:** Orders are placed multiple times over the entire production cycle.





Inventory Models

In **multi-period model**, based on the pattern of reviewing current inventory, they are further classified.

- **Continuous Review (also called Fixed Quantity or Q system):** Inventory is reviewed continuously and when inventory drops to a certain (prefixed) reorder level, a fixed quantity is ordered. This model is generally used for high volume, valuable, or important items.
- **Periodic Review (also called Fixed Period or P system):** Inventory is reviewed at (prefixed) periodic intervals irrespective of the levels to which inventory drops; an order is placed to bring up the inventory to the maximum level. This is used for moderate volume items.





Inventory Models

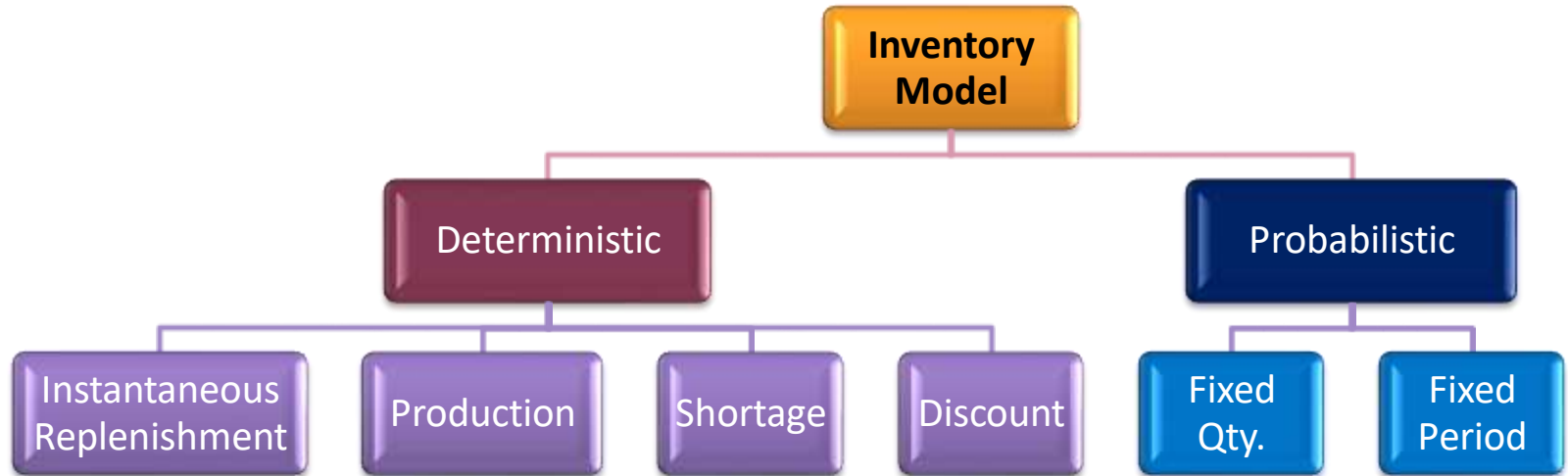
Several other systems use a combination of traditional approaches.

- **Optional replenishment system:** Inventory is reviewed on a fixed frequency and a specific quantity is ordered, if inventory is below a certain level. This is a mix of the P and Q systems.
- **Two-bin system:** An inventory amount equal to R is kept in reserve in a second bin. When the first bin is emptied, the second bin is emptied into the first and an order of size Q is placed.
- **One-bin system:** This is the P-system where one bin is reviewed at a fixed interval and inventory is brought up to a certain level.
- **ABC Inventory planning:** Inventory items are classified into three groups on the basis of annual dollar volume. Inventory items with a high dollar volume are more frequently reviewed compared to a low dollar volume.
 - A: high dollar volume items, say 15% of the total number of items
 - B: moderate dollar volume items, say the next 35%
 - C: low dollar volume items, the last 50%





Inventory Models





Deterministic models

- *Instantaneous stock replenishment model*
- *Production model*
- *Planned shortages and*
- *Price discount model*





Instantaneous stock replenishment model (Basic Inventory Model)

Let

D = Annual Demand (Units/year)

C_o = Ordering cost (Rs./Order)

C_p = Price per unit (Rs./Unit)

C_h = Inventory carrying cost (Rs./unit/time)

Q = Quantity Ordered

N = No. of order /year

T_c = Total cost/year

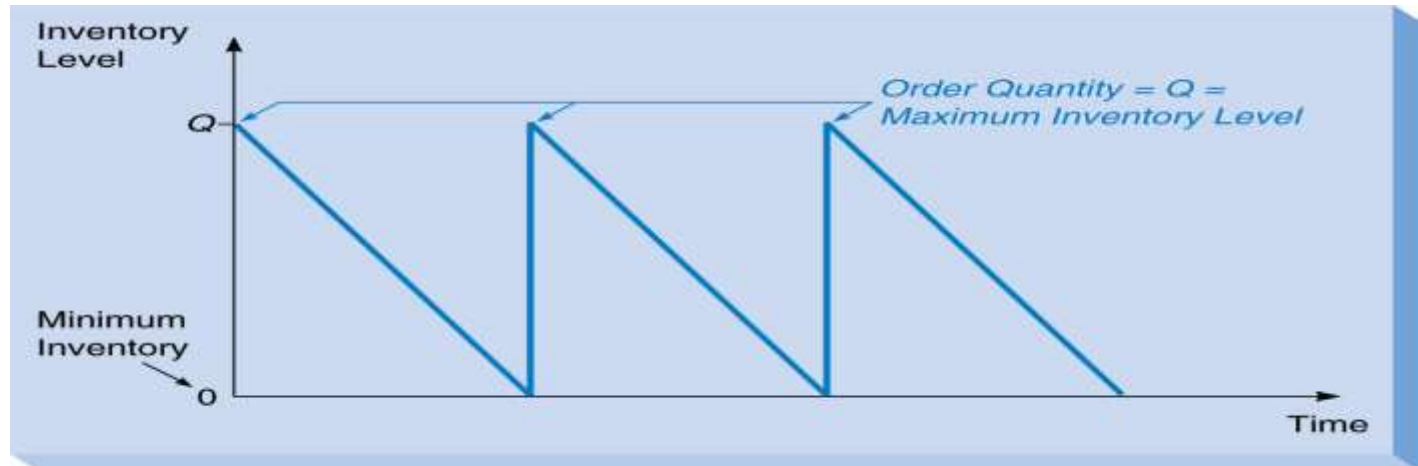
T_c^* = Total cost at Economic Order Quantity

Q^* = Economic order quantity

Instantaneous stock replenishment model (Basic Inventory Model)

Assumptions-

- Demand is deterministic, Constant & known
- Stock replenishment is instantaneous (Lead time is zero)
- Price of material is Fixed.
- Ordering Cost does not change with quantity





Two Methods for Carrying Cost

Carry cost (C_h) can be expressed either:

1. As a fixed cost, such as

$$C_h = \text{Rs. } 0.50 \text{ per unit per year}$$

2. As a percentage of the item's purchase cost (P)

$$C_h = C_p \times I$$

I = a percentage of the Unit cost



EOQ Total Cost

Total ordering cost = $(D/Q) \times C_o$

Total carrying cost = $(Q/2) \times Ch$

Total Unit cost = $C_p \times D$

Total Cost T_c = $(D/Q) \times C_o + (Q/2) \times Ch + C_p \times D$

To determine EOQ, differentiate T_c w.r.t Q and set 1st derivative equal to zero

Note:

- $(Q/2)$ is the average inventory level
- Purchase cost does not depend on Q



Finding EOQ (Q^)*

Recall that at the optimal order quantity EOQ (Q^*):

Carry cost = Ordering cost

$$(D/Q^*) \times C_o = (Q^*/2) \times C_h$$

Rearranging to solve for Q^* :

$$Q^* = \sqrt{(2DC_o / Ch)} \quad \text{or} \quad Q^* = \sqrt{(2DC_o / C_p \times I)}$$

$$T_c^* = \sqrt{(2DC_o Ch)}$$



Example-1

Usha corporation currently practices the following system for the procurement of items.

- Unit cost = Rs. 40/unit
- No. of order placed in a year= 8
- Ordering cost = Rs. 750/order
- Each time order quantity= 250
- Carrying cost= 40%

Comment on the ordering policy of the company and estimate the loss/profit to the company in not practicing scientific inventory policy.





Solution-1

Given

- Unit cost(C_p) = Rs. 40/unit
- No. of order placed in a year(N)= 8
- Ordering cost(C_o) = Rs. 750/order
- Each time order quantity(Q)= 250
- Carrying cost(I)= 40%

Now

- Annual Demand(D)= $N \times Q = 8 \times 250$





Solution-1

- Economic Order Quantity ($Q^* = \sqrt{2DC_o / Ch}$)

$$Q^* = \sqrt{[(2 \times 250 \times 8 \times 750) / (40 \times 0.4)]}$$

$$Q^* = 433 \text{ units}$$

- Min. Total cost ($T_c^* = \sqrt{2DC_o(C_p \times I)}$)

$$T_c^* = \sqrt{2 \times (250 \times 8) \times 750 \times (40 \times 0.4)}$$

$$T_c^* = \text{Rs. } 6928.28$$

Total cost under present system $T_c = \text{Annual ordering cost} + \text{Annual carrying cost}$

$$T_c = 8 \times 750 \times \frac{250}{2} \times 40 \times 0.4 = \text{Rs. } 8000/-$$

Present total cost - total cost for EOQ = $T_c - T_c^* = 8000 - 6928.28 = \text{Rs. } 1071.72 \text{ (Loss)}$



Solution-1

Given

- Unit cost(C_p) = Rs. 40/unit
- No. of order placed in a year(N)= 8
- Ordering cost(C_o) = Rs. 750/order
- Each time order quantity(Q)= 250
- Carrying cost(I)= 40%

Now

- Annual Demand(D)= $N \times Q = 8 \times 250$





Example-2

Indian telecoms entered into contract with precision instruments for the purchase of 12,500 instruments at the rate of Rs. 250/- per instrument during the year. The deliveries of the instrument will be made each time half a month after the order placed. Indian telecoms estimates its carrying cost at Rs. 48 per instrument per year. The cost of paper work, follow-up, transport and receipt work out to be Rs. 2000. how frequently should Indian telecom place orders with precision instruments? What is the re-order point?





Solution-2

Given- Annual Demand (D)= 12,500/-, Ordering cost(Co)= Rs.2000/order,
Inventory carrying cost(Ch)=Rs.48/unit/annum

$$\text{Economic order quantity } Q^* = \sqrt{(2DC_o / Ch)} = 1020.62 \approx 1021$$

$$\text{No. of orders to be placed in a year } N = \frac{D}{Q^*} = \frac{12500}{1021} = 12.24 \approx 12$$

The Re-order point = Lead time consumption

Lead time is given as ½ month (twice in a month= 2x12)

$$\text{Reorder point (RO)} = \frac{12500}{2 \times 12} = 520.83 \approx 521 \text{ units}$$



Production model

Assumption-

- The item is sold or consumed at constant demand rate which is known
- Set up cost is fixed and it does not change with lot size
- The increase in inventory is not instantaneous but it is gradual



Production model

Parameters

Q^* = Optimal production or batch quantity (EPQ/EBQ)

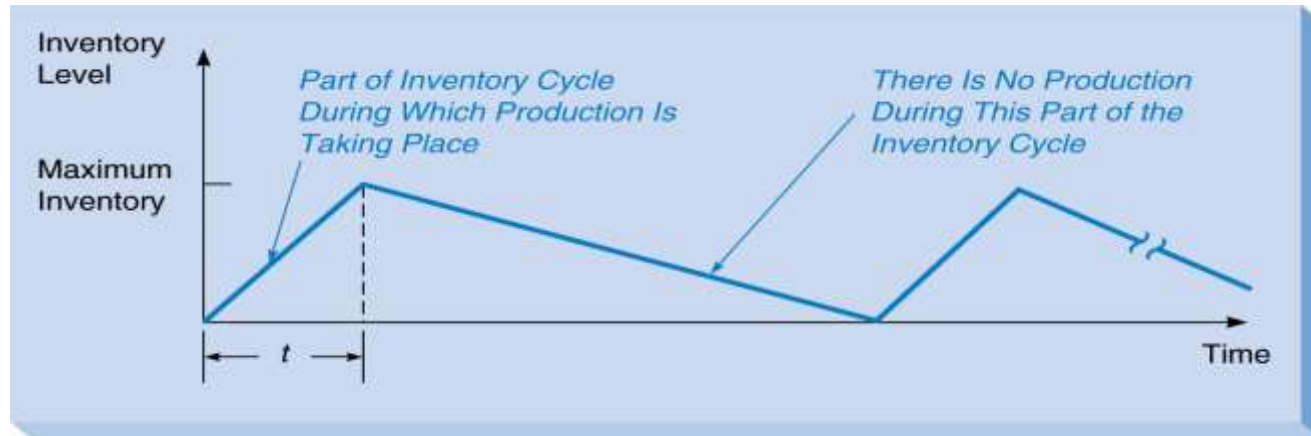
C_o = Setup cost

D = annual demand

d = daily demand rate

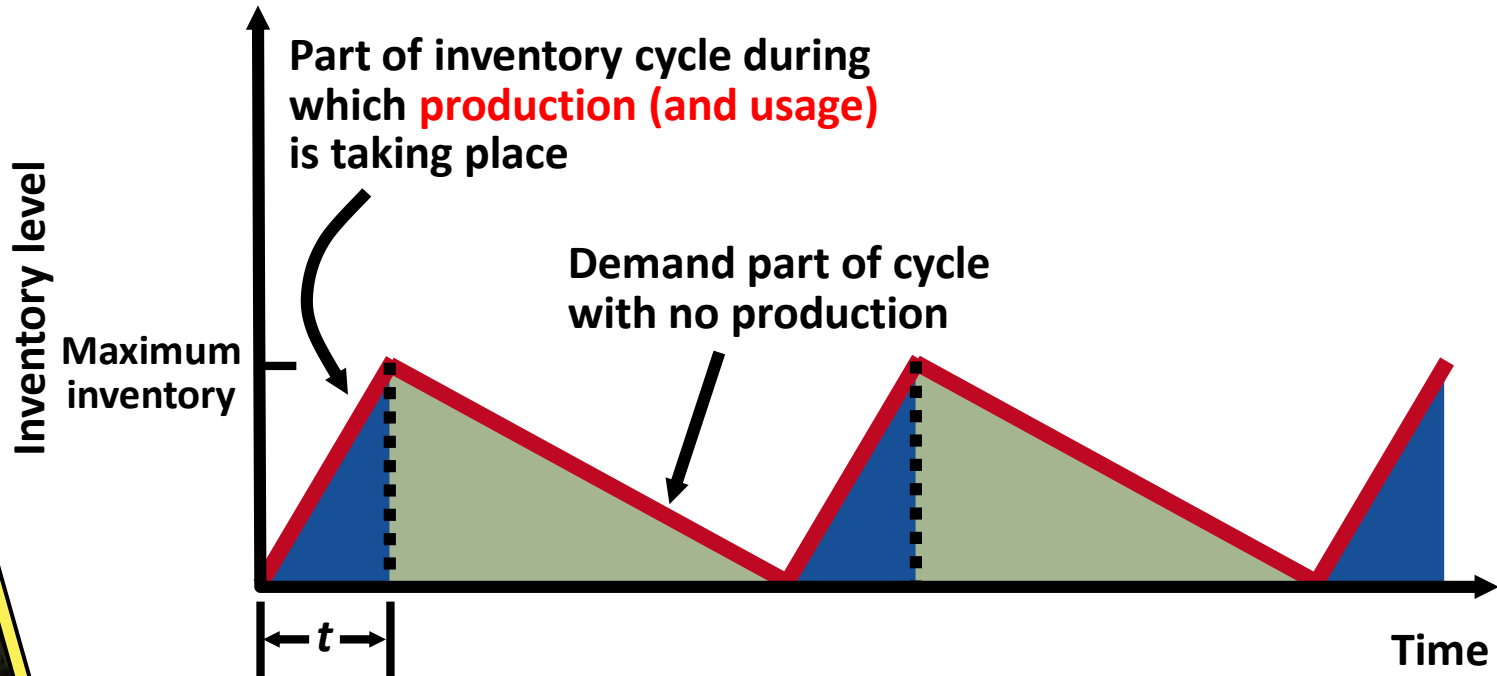
p = daily production rate

t = Length of the production run in days





Production model





Production model

We will need the average inventory level for finding carrying cost

Average inventory level is $\frac{1}{2}$ the maximum

$$\text{Max inventory} = Q \times (1 - d/p)$$

$$\text{Ave inventory} = \frac{1}{2} Q \times (1 - d/p)$$

$$\text{Setup cost} = (D/Q) \times C_o$$

$$\text{Carrying cost} = [\frac{1}{2} Q \times (1 - d/p)] \times C_h$$

$$\text{Production cost} = P \times D$$

$$\text{Total cost} = (D/Q) \times C_o + [\frac{1}{2} Q \times (1 - d/p)] \times C_h + P \times D$$

As in the EBQ model, at the optimal quantity Q^* we should have:

$$\text{Setup cost} = \text{Carrying cost}$$

$$(D/Q^*) \times C_o = [\frac{1}{2} Q^* \times (1 - d/p)] \times C_h$$

$$\text{Rearranging to solve for } Q^*: \quad Q^* = \sqrt{(2DC_o / [C_h(1 - d/p)])}$$



Production model

$$Q^* = \sqrt{\frac{2DCo}{Ch \times I [1 - (d/p)]}}$$

Where I = % of Holding cost

$$Q^* = \sqrt{\frac{2DCo}{Ch \left[1 - \frac{\text{annual demand rate}}{\text{annual production rate}} \right]}}$$

$$Tc^* = \sqrt{2DCo \times Ch \times I [1 - (d/p)]}$$

Production Time $t = Q^*/p$

Optimum number of production run

$$N^* = \frac{\text{Annual Demand}}{\text{Annual Batch Quantity (EBQ)}} = \frac{D}{Q^*}$$



Example - 3

A contractor undertakes to supply diesel engines to a truck manufacturer at the rate of 25 per day. He finds that the cost of holding a completed engine in stock is Rs. 1.6 per month. Production of engine is in batches and each time a new batch is started, there are set-up cost of Rs. 10,000/-. How frequently should the batches be started and what will be minimum average inventory cost and production time if the production rate is 40 engines/day? Assume 300 working days in a year.





Solution -3

- Production rate (p)= 40/day
- Demand rate (d)= 25/day
- Annual demand (D) = 25x300=7500 per annum (300 working days/year)
- Set-up cost (Co) = 10,000/-
- Inventory carrying cost (Ch) =Rs. 1.6 x 12 per year
- Economic manufacturing quantity= EBQ= $Q^* = \sqrt{\frac{2DCo}{Ch \times [1 - (d/p)]}}$

$$Q^* = \sqrt{\frac{2 \times 7500 \times 10,000}{19.2 [1 - (25/40)]}} = 4564.35 \approx 4565$$

No. of Production runs per annum $N = D/Q^* = 7500/4565 = 1.642 \approx 2$ orders

Frequency of production run (Time between two runs) = $1/N = \frac{1}{2} = 0.5$ years

$$\text{Total annual cost } Tc^* = \sqrt{2DCo Ch [1 - (d/p)]} = \text{Rs.32863.35/year}$$



Example - 4

A Pharma company has a demand for 10,00,000 bottles. Each empty bottle costs the company Rs. 1. Empty bottles are supplied by M/s Rupa Glass Ltd. The R.O.L system of stock replenishment is followed. Ordering cost is Rs. 12.5/order and inventory carrying cost is 25 per cent of cost per bottle. The demand is constant throughout the year. The lead time is 15 days.

Determine-

1. Economic order quantity
2. Lead time consumption
3. Re-order level
4. Average inventory





Solution - 4

1. Economic order quantity $Q^* = \sqrt{(2DC_o / Ch)} = 10,000$ bottles
2. Lead time consumption = lead time(in months) x Monthly consumption
$$= \frac{15}{30} \times \frac{10,00,000}{12}$$
$$= 41667 \text{ units}$$
3. Safety stock is assumed equal to lead time consumption.
Re-order level = Safety stock + Lead time consumption
$$= 41667 + 41667$$
$$= 83334 \text{ units}$$
4. Average inventory =
$$\frac{\text{Max. Inventory} + \text{Min. Inventory}}{2}$$
$$= \frac{(41667 + 10,000) + (41667)}{2}$$
$$= 46667 \text{ bottles}$$





Planned shortages

In many practical situations stock-outs or shortages are permitted and are economically justifiable also. This situation is observed usually when cost per unit is very high.

Let,

C_s = Shortage cost per unit period

S = Balance units after back orders are satisfied

$Q-S$ = Number of shortages per order

t_1 = Time period for which inventory is positive

t_2 = Time period for which shortage exists

D = Annual Demand (Units/year)

C_o = Ordering cost (Rs./Order)

C_p = Price per unit (Rs./Unit)

C_h = Inventory carrying cost (Rs./unit/time)

Q = Quantity Ordered

N = No. of order /year

T_c = Total cost/year

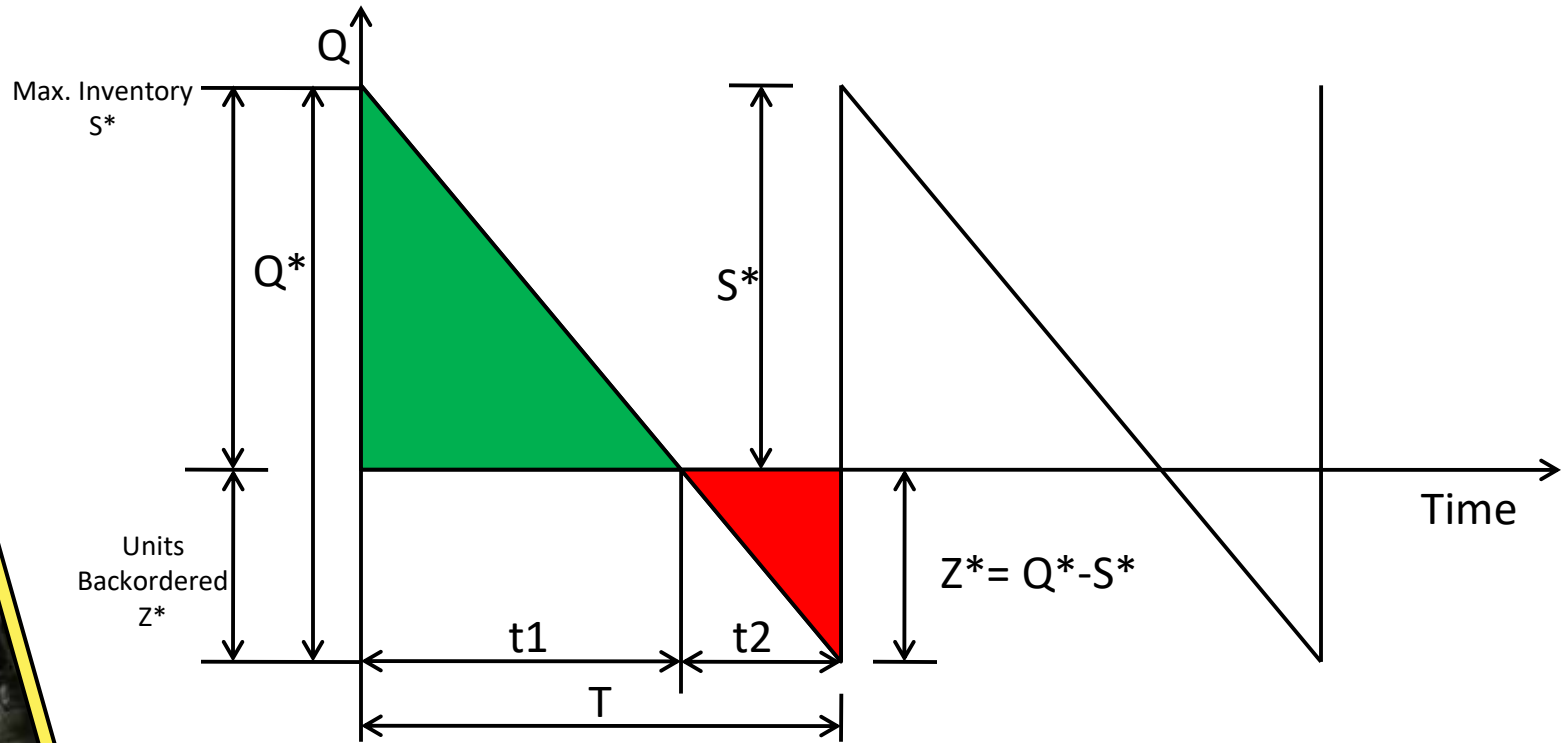
T_c^* = Total cost at Economic Order Quantity

Q^* = Economic order quantity





Planned shortages



Planned shortages

Assumption- No loss of sale due to stock out or shortage

1. Economic order Quantity :

$$Q^* = \sqrt{\left[\frac{2DCo}{Ch} \right] \left[\frac{Cs+Ch}{Cs} \right]}$$

2. Optimal Remaining units after back ordering :

$$S^* = \sqrt{\left[\frac{2DCo}{Ch} \right] \left[\frac{Cs}{Cs+Ch} \right]}$$

3. Optimal units back ordered :

$$Z^* = Q^* - S^* = Q^* \left[1 - \frac{Cs}{Ch+Cs} \right]$$

4. Total Optimal Inventory Cost :

$$Tc^* = \sqrt{2DCoCh \left[\frac{Cs}{Cs+Ch} \right]}$$



Example-5

The demand for an item is deterministic and constant over time and is equal to 600 units per year. The unit cost of the item is Rs.50 while the cost of placing an order is Rs.5. The inventory carrying cost is 20% of the cost of inventory per annum and the cost of shortage is Rs.1 per month. Find the optimal ordering quantity when stock outs are permitted. If the stock outs are not permitted what would be the loss to company. (Dec.17)

Given- $D=600$ units, $C_p=Rs.50/-$, $C_o= Rs5/-$, $I= 20\%$

$Ch= C_p \times I= 50 \times 0.2=Rs.10/-$, $C_s=Rs.1/month=Rs.12/year$



Solution-5

When stock outs are permitted,

1. Optimal order quantity $Q^* = 33$ units
2. Max. no. of back orders $Z^* = 15$
3. Total expected annual cost $Tc^* = \text{Rs. } 181/-$

When stock outs are not permitted,

1. Optimal order quantity $Q^* = 24.5$ units
2. Total relevant annual cost $Tc^* = \text{Rs. } 245/-$

Thus stock outs are not permitted





Example-6

The annual demand for a machine component is 24,000 units. The carrying cost is Rs.0.40/unit/year, ordering cost is Rs.20/order and the shortage cost is Rs.1000/unit/year. Find the values of followings:

1. Economic order quantity
2. Maximum inventory
3. Maximum shortage quantity
4. Cycle time
5. Inventory period (t_1)
6. Shortage period (t_2)





Solution-6

Given- $D = 24,000$ units, $Ch = \text{Rs.}0.40/\text{unit}/\text{year}$, $Co = \text{Rs.}20/\text{order}$,
 $Cs = \text{Rs.}1000/\text{unit}/\text{year}$.

Find : 1. Q^* , 2. S^* , 3. Z^* , 4. T , 5. (t_1) , 6. (t_2)

1. $Q^* = 1580$ units
2. $S^* = 1520$
3. $Z^* = 1580 - 1520 = 60$ units
4. $T = (Q^*/D) \times 365 = 24$ days cycle time
5. $t_1 = (S^*/D) \times 365 = 23$ days (inventory period)
6. $t_2 = T - t_1 = 24 - 23 = 1$ day





Price discount model

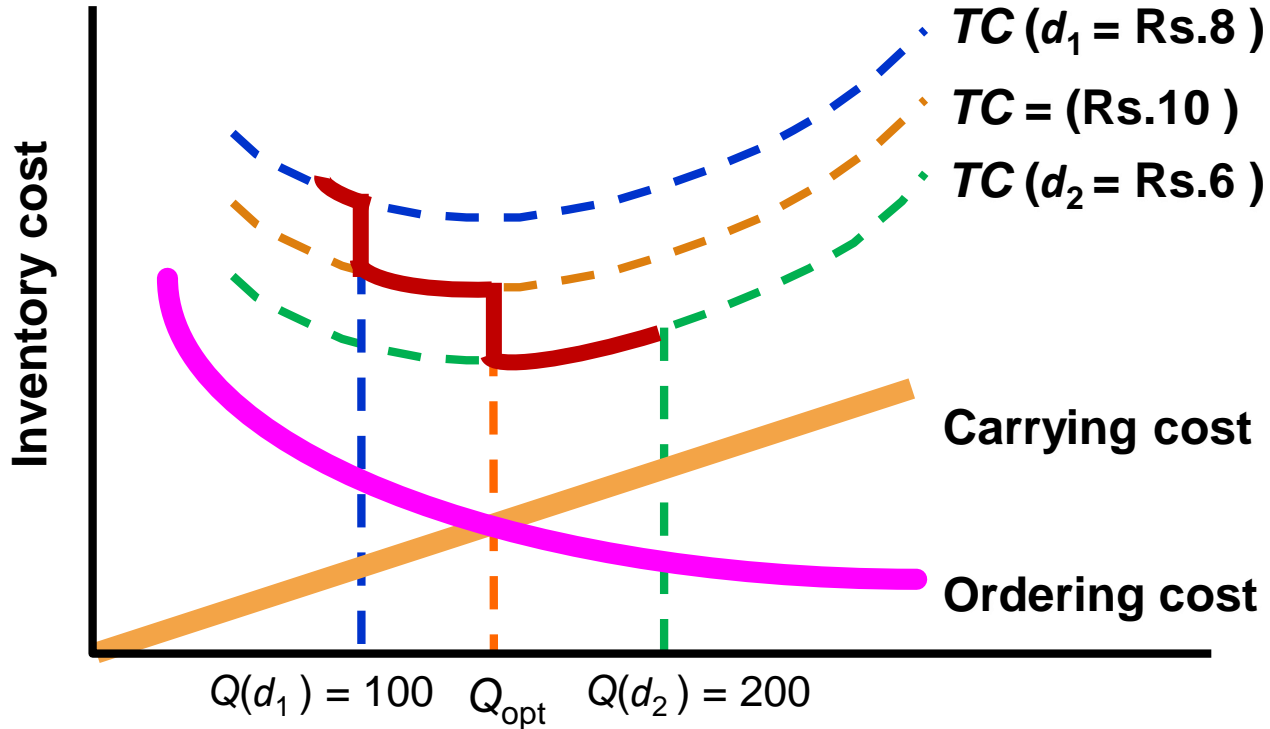
- Quantity discount model is used when the vendor (supplier) offers a discount for buying in large quantities.
- The classical EOQ model is based on the assumption that the cost of an item under consideration is uniform.
- But in real life, it is very common to find cost discounts on quantities for which the order is placed.
- Lower rates are highlighted if the quantity of goods is high.
- So, the quantity ordered should be carefully examined taking into consideration the price levels of different quantity ranges.
- When the unit cost price is uniform, the purchasing cost is inadequate to determine the order size.
- But under the conditions of price break, the item cost, being a function of order quantity, is the incremental cost and must be included in the cost model.





Price discount model

When carrying costs are specified as a percentage of unit price, the total cost curve is broken into different total cost curves for each discount range





Example - 7

A materials manager adopts the policy to place an order for a min. quantity of 500 of a particular item in order to avail a discount of 10%. It was found from the company records that for last 8 orders were placed each of size 200 units, ordering cost is Rs.500/order. Inventory carrying cost at 40% and unit cost is Rs. 400/-. Is the purchase manager justified in his decision. What is the effect of this decision on the company?

Given – $D = 200 \times 12 = 1600$ units, $Ch = 0.4 \times 400$, $Co = \text{Rs. } 500/\text{order}$



Solution - 7

Given – $D = 200 \times 12 = 1600$ units, $C_h = 0.4 \times 400$, $C_o = \text{Rs. } 500/\text{order}$

$$Q^* = \sqrt{(2DC_o / C_h)} = \sqrt{2 \times 1600 \times 500 / 0.4 \times 400} = 100 \text{ units}$$

(i) Annual Total inventory cost for $Q^*=100$ units

$$\begin{aligned} Tc^* &= D \times C_p + (D/Q^*) \times C_o + (Q^*/2) \times C_h \\ &= 1600 \times 400 + (1600/100) \times 500 + (100/2) \times 0.4 \times 400 = 6,56,000/- \end{aligned}$$

(ii) Annual Total cost of present policy ($Q=200$)

$$\begin{aligned} Tc &= D \times C_p + (D/Q) \times C_o + (Q/2) \times C_h \\ &= 1600 \times 400 + (1600/200) \times 500 + (200/2) \times 0.4 \times 400 = 6,60,000/- \end{aligned}$$

(iii) Proposed inventory cost for discount policy (10%)

Discounted Price = 10% discount on Rs.400 = Rs. **360**/- on $Q_1 = 500$ units

$$\begin{aligned} Tc_1 &= D \times C_{p1} + (D/Q_1) \times C_o + (Q_1/2) \times C_h \\ &= 1600 \times 360 + (1600/500) \times 500 + (500/2) \times 0.4 \times 360 = 6,13,600/- \end{aligned}$$

Manager's decision is justified as it is going to save 46,400/- per year over present system



Probabilistic models

- The Probabilistic inventory model is closely aligned to the manufacturing and retail reality that from time to time, demand will vary.
- Demand variations cause shortages, particularly during lead time if a retailer only has a limited amount of inventory stock to cover the demand during the lead time when replenishment stock has not arrived.

The probabilistic inventory model incorporates demand variation and lead time uncertainty based on three possibilities.

- The first is when lead time demand is constant but the lead time itself varies
- The second is when lead time is constant but demand fluctuates during lead time
- The third possibility is when both lead time and demand during lead time vary.





Probabilistic models

- *Fixed quantity system(Q-system) and*
- *Fixed period system (P-system)*





- Probabilistic inventory models consisting of probabilistic supply and demand are more suitable in most circumstances. Two methods are used based on the frequency of order placement for procuring inventory stock, these are **single period** and **multi-period** inventory systems.
- The term single period term refers to the situation where the inventory stock is perishable, and orders are typically only made once. Generally, for one time ordering of seasonal products or where demand exists only for the period in which it is ordered. For example, a newspaper sold today will not be sold at the same price tomorrow nor will summer clothing items be likely to sell during the winter season
- In single period models, only demand is the major variable factor and lead time does not play any role in the decision process.
- But, in multi-period models, both demand and lead time play major role in the decision process.
- The variation in demand and/or in lead time imposes risks.
- We cushion the effects of demand and lead time variation by absorbing risks in carrying larger inventories, called buffer stocks or safety stocks.
- The larger we make these safety stocks, the greater our risk, in terms of the funds tied up in inventories, the possibility of obsolescence and so on.



LEVELS OF STOCKS

- **Minimum Stock Level** : This is also known as safety or buffer stock. This is the level below which stock is not allowed to fall. When this level is reached, it triggers of urgent action to bring forward delivery of the next order, it is sometimes called the danger level. In fixing their level, the main factor to be taken into account is the effect which a run-out of stock would have upon the flow of work or operations.
- **Re-Ordering Stock Level** : This is known as Re-order level. This is the level at which ordering action is taken for the material to be delivered before stock falls below the minimum. Two main factors are involved in deciding the re-order level, (i) the anticipated rate of consumption, and (ii) the estimated time which will elapse between the raising of a provision demand and the actual availability of goods in store after receipt and inspection, i.e. the lead time.
- **The Hastening Stock Level** : This is the level at which it is estimated that hastening action is necessary to request suppliers to make early delivery. It is fixed between the minimum level and the re-order level. This level is subjectively based on experience.
- **The Maximum Stock Level** : This is the level of stock above which the stock should not be allowed to rise. The purpose of this level is to curb excess investment. In fixing the maximum, the main consideration is usually financial, and the figure is arranged so that the value of stock will not become excessive at any time. Other points affecting this level are the possibility of items becoming obsolete, and the danger of deterioration in perishable commodities.





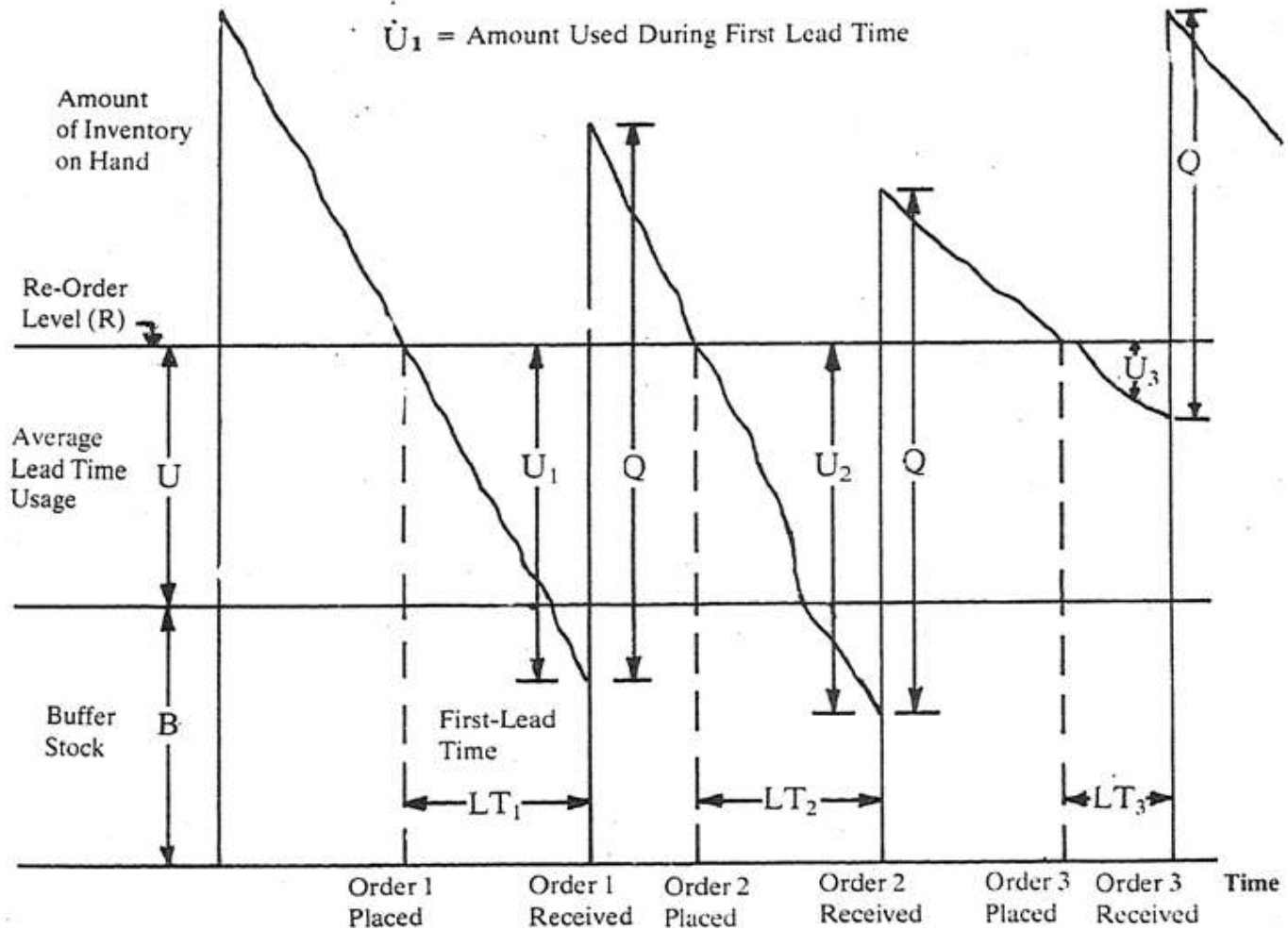
Fixed quantity system(Q-system)

Also known as Perpetual Inventory System; Re-order Inventory System or Q- system.

Characteristics:

- 1. Re-order quantity is fixed and normally it is equals EOQ**
- 2. Depending upon demand, the time interval of ordering varies**
- 3. Replenishment action is initiated when stock level falls Re-order level(ROL)**
- 4. Safety stock is maintained to account for increase in demand during lead time**







Fixed quantity system(Q-system)

- With the multi-period method orders are placed multiple times over an entire production cycle and are further classified as continuous review or periodic review inventory.
- Continuous review inventory is reviewed constantly and when inventory stock drops to a certain predetermined par or reorder level, a fixed quantity is ordered. Continuous review is commonly used for high volume, valuable or important stock items
- Periodic review inventory is examined at periodic intervals in predetermined timeframes, irrespective of the levels to which inventory levels drop. At this time an order is then placed to bring inventory up to the maximum level, the method is largely used for moderate volume items





Fixed quantity system(Q-system)

Setting of various levels for Q system

- For fixed order quantity system, the various levels can be fixed for the following conditions:
 - Variable Demand and Constant Lead Time
 - Constant Demand and Variable Lead Time
 - Variable Demand and Variable Lead Time





Variable Demand and Constant Lead Time

U = random variable representing demand during lead time

s = standard deviation of demand during lead time

\bar{U} = Expected lead time demand t_L = lead time

\bar{d} = average daily demand

σ_d = standard deviation of daily demand

\bar{D} = expected annual demand B = buffer stock or safety stock

Z = number of standard deviations needed for a specified confidence level R = Re-order level

Thus, re-order level is given as: $R^* = \bar{U} + B$ = Average lead time Demand + Buffer Stock

where buffer stock, $B = Z\sigma_u$

and lead time consumption, $\bar{U} = \bar{d} t_L$

and σ_u = standard deviation for lead time consumption = $\sqrt{t_L \sigma_d^2}$

There order size is, $Q^* = \sqrt{\frac{2C_o D}{C_h}}$

Maximum Level = $Q^* + B$

Average Inventory Level = $\frac{Q^*}{2} + B$



Example - 8

The daily demand of an item is normally distributed with a mean of 50 units and standard deviation of 5 units. Lead time is 6 days. The cost of placing an order is Rs. 8, and annual holding costs are 20% of the unit price of Rs. 1.20. A 95% service level is desired. Back-orders are allowed but there is no stock-out cost. Find the various levels.





Solution - 8

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2 \times (50)(365)(8)}{(0.2)(1.20)}} = 1103 \text{ units}$$

From the normal distribution, a 95% confidence level gives $Z = 1.645$. Thus

$$R^* = \bar{U} + B = \bar{d}t_L + Z \sigma_u = 50(6) + 1.645 \sigma_u$$

$$\text{where } \sigma_u = \sqrt{t_L \sigma_d^2} = \sqrt{6(5)^2} = \sqrt{150} = 12.2$$

Therefore, safety stock = $1.645 (12.2)$ and

$$R^* = 50(6) + 1.645 (12.2) = 300 + 20 = 320 \text{ units.}$$

Operating doctrine is to order 1103 units when we reach an order point of 320 units.

$$\text{Average inventory level} = \frac{1103}{2} + 20 = 551.5 + 20 = 571.5 \text{ units}$$

$$\text{Maximum inventory level} = 1123 \text{ units.}$$





Example - 9

Consider previous example with following observations for 7 days.

Actual daily demand (in units)	1-200	201-400	401-600	601-800
No. of occurrences(days)	3	2	1	1
Relative frequency occurrences	42.8%	26.8%	14.3%	14.3%





Solution - 9

We find the expected daily demand and the standard deviation of daily demand

$$\begin{aligned}\text{Expected demand} = \bar{d} &= \frac{100(3) + 300(2) + 500(1) + 700(1)}{7} \\ &= 300 \text{ units per day}\end{aligned}$$

$$\begin{aligned}\sigma_d &= \sqrt{\sum_{i=1}^n \frac{(\text{Demand} - \text{Expected Demand})^2}{n}} \\ &= \sqrt{\frac{3(100-300)^2 + 2(300-300)^2 + 1(500-300)^2 + 1(700-300)^2}{7}} = 214 \text{ units.}\end{aligned}$$

Therefore

$$\bar{U} = \bar{d}t_L = 300 \times 6 = 1800 \text{ units.}$$

$$\sigma_u = \sqrt{\frac{6(320000)}{7}} = 524$$

$$\text{Therefore, } R^* = 1800 + 1.645(524) = 1800 + 862 = 2662 \text{ units}$$

$$\text{Safety Stock} = 862 \text{ units}$$

$$Q^* = \sqrt{\frac{2 \times (300)(365)(8)}{(0.2)(1.2)}} = 2700 \text{ units}$$

Company is to order 2700 units when inventory level reaches to the re-order point of 2662 units.

$$\text{Maximum Level} = 2700 + 862 = 3562 \text{ units}$$



Constant Demand and Variable Lead Time

Re-order level = $R^* = U + B$

where $U = dt L = (\text{Daily demand} \times \text{expected lead time})$

Buffer stock, $B = z\sigma U$

$= Z (\text{demand}) (\text{Standard deviation of lead time})$

Another, simple way of determining the safety stock is :

$B = (\text{Maximum Lead Time} - \text{Normal Lead Time}) (\text{Demand during Lead Time})$

Further more, the levels of safety stock depend upon what extent an organization is prepared to accept stock-out risk (SOR). Since it is difficult to obtain an accurate estimate for the shortage cost, the management must specify reasonable service level (SL) so as to determine safety stock necessary to keep the stock-out risk within the prescribed limits.





Constant Demand and Variable Lead Time

The service level is the probability of not running out of stock on any stock cycle, i.e. per cent of order cycle in which all the demand can be supplied from the stock: Service level (SL) = 100% - Stock-out Risk (SOR)

Optimal stock-out risk (SOR) = C_1/C_2

where SOR is obtained by marginal analysis of evaluating the cost of shortage for one unit i.e.

(1 unit short) (average stock-out per year) (Cost per unit short) =
= C_1 or $(\text{SOR}/\text{yr}) C_2 = C_1$





Example - 10

A company is ordering an item 4 times a year and has specified a service level of one stock-out per 3 years. The history of re-order lead times is shown below. Daily demand of the item is 40 units. Find the re-order level.

Order placed (date/month)	7/1	3/2	16/3	6/4	2/5	2/6
Order Received (lead Time)	18/1	21/2	20/4	28/4	20/5	23/6
Calendar days	11	18	35	22	18	21
Working days	7	12	25	16	14	15



Solution - 10

$$\text{Average lead time} = \bar{t}_L = \frac{7+12+25+16+14+15}{6} = 14.83 \text{ days}$$

$$\text{Variance of lead time} = \frac{(7-14.83)^2 + (12-14.83)^2}{6-1} + \dots = 34.97 (\text{days})^2$$

$$U = d\bar{t}_L = 40(14.83) = 593.3 \text{ units demanded per lead time}$$
$$\text{variance } U = (34.97)(40)^2$$
$$\text{or } \sigma_U = 40\sqrt{34.97} = 40 \times 5.91 = 236.5$$

$$\text{Desired SOR yr} = \frac{\text{No. of stock-outs per year}}{\text{No. of cycles per year}} = \frac{1/3}{4} = \frac{1}{12} = 0.083$$

= Probability of stock-out per order cycle

Thus, from normal table, for service level of $100\% - (8.3)\% = 91.7\%$ the value of z from the normal distribution table is 1.39

$$\text{Therefore, safety stock (Buffer stock)} = (1.39)(236.5)$$
$$= 328.73 \text{ units}$$

and

$$\text{Re-order level} = R^* = \bar{U} + Z\sigma_U$$
$$593.3 + 328.73 = 922 \text{ units.}$$





Variable Demand and Variable Lead Time

- The various levels are as follows
- Reorder level: $R = \bar{U} + Z\sigma_U$
- Buffer stock: $B = Z\sigma_U$
- $EOQ = \sqrt{(2DC_o / Ch)}$
- Where Z is a constant of proportionality determined from the Normal distribution table corresponding to the expected service level.
- \bar{U} = Expected consumption during lead time
= (\bar{d} avg daily demand) ($\bar{\tau}_L$ avg lead time)
- σ_U = std deviation of U (or $\text{Var } U$)
- $\text{Var } U = \sigma_d^2 = \text{Variance of lead time}$
- $\bar{\tau}_L$ = average lead time
- \bar{d} = Average daily demand



Example - 11

Consider Example No. 10 with the following change in demand. The average daily demand of the item is 40 units per day and variance of daily demand is $(30 \text{ units/day})^2$

Average lead time consumption, \bar{U}

$$= \bar{d} \bar{t}_L$$

$$= 40 \times 14.83 = 593.3 \text{ units}$$

$$\text{Var}(U) = (\text{Var } d) \bar{t}_L + (\text{Var } t_D) (\bar{d})^2$$

$$= 30 \times 14.83 + (34.97) (40)^2$$

$$= 56,397$$

Therefore, $\sigma_U = \sqrt{\text{Var}(U)} = 237.5$ units per lead time

Desired service level is 91.7%, therefore

$$\text{Buffer stock} = Z\sigma_U = (1.39) (237.5)$$

$$= 330.1$$

and

$$\text{Recorder level} = R^* = 593.3 + 330.1$$

$$= 923.4 \text{ units.}$$



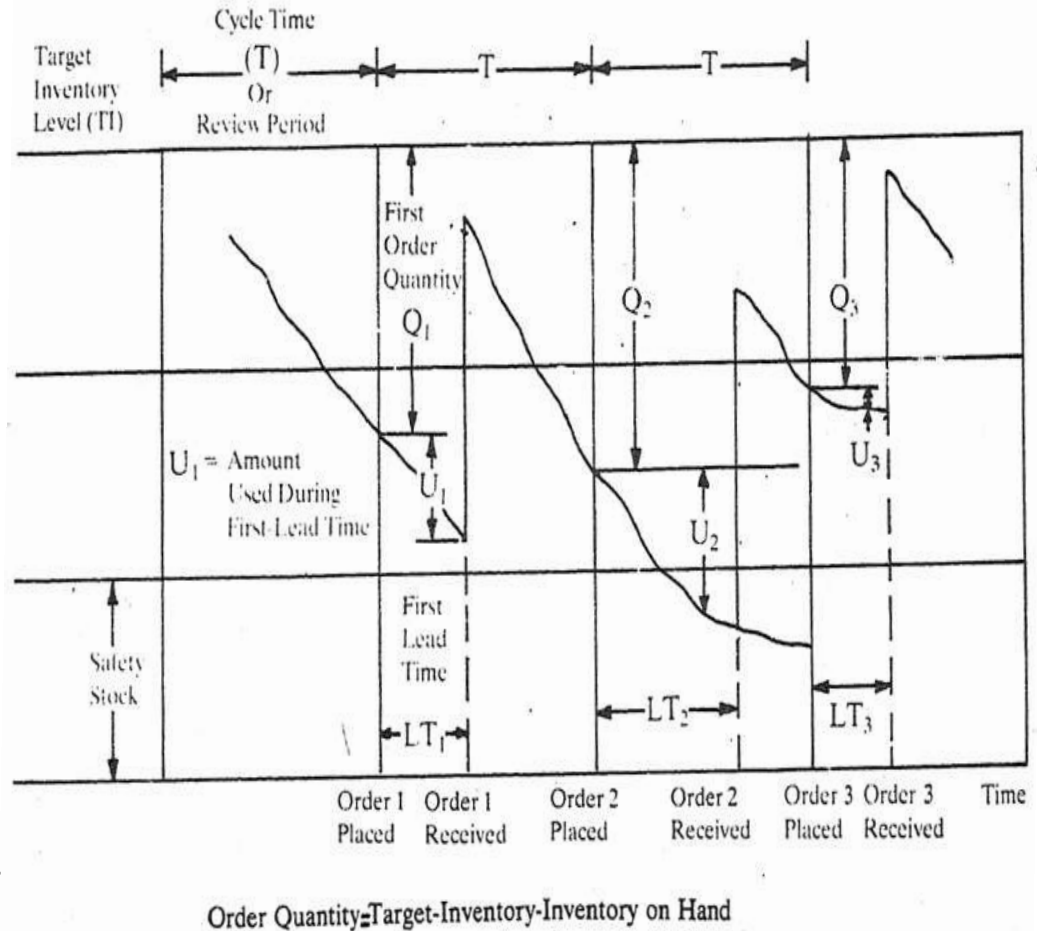


Fixed period system (p-system)

- This system involves the reviewing of stock levels at a **fixed interval of time** known as review period and placing replenishment orders at the end of each period.
- The replenishment **quantity is variable** and corresponds to the amount of stock required to bring the stock ordered and the stock on hand up to a target level.
- The key variable for system are T , the review period (fixed time between reviews of inventory records) and TI , the target inventory level on which orders are based.
- This system is also known as Fixed Period System or Replenishment Inventory System or P-system.

- Each review period is exactly T days long; at the end of this period, an order is placed for a quantity sufficient to replenish inventory to the target level Tl .
- After the re-order lead time t_L , the shipment arrives and goes into inventory.
- The order quantities are different each period, being calculated as $Q = Tl$ minus inventory on hand minus previous orders not yet received. The formula for target inventory for a fixed interval T is as follows :

$$Tl = \text{Av. Review period} + \text{Avg. Lead Time Demand} + \text{Buffer stock}$$





Fixed period system (p-system)

Variable Demand and Constant Lead Time

- The average demand during the lead time and review period is $\bar{d}(T+t_L)$. Buffer stock, to be hold during this period is, $B = Z\sigma_d \sqrt{T+t_L}$
- Where Z is the number of standard deviations to give the required service level. The target inventory level is
- $TI = \bar{d}(T+t_L) + Z\sigma_d \sqrt{T+t_L}$

Control Levels for Constant Demand and, Variable Lead Time

- Average review period demand = $T\bar{d}$
- Average lead time demand = $\bar{t}_L \bar{d}$
- Buffer stock = $B = Z\bar{d} \sigma_{t_L}$
- Thus, $TI = T\bar{d} + \bar{t}_L \bar{d} + Z\bar{d} \sigma_{t_L} = (T + \bar{t}_L) \bar{d} + Z\bar{d} \sigma_{t_L}$
- where t_L = standard deviation of lead time



Fixed period system (p-system)

Control Levels for Variable Demand and Variable Lead Time

- Average review period demand = $T\bar{d}$
- Average lead time demand = $\bar{t}_L \bar{d}$

$$\text{Buffer (Safety) stock} = Z\sqrt{T(\text{Var } d) + \text{Var } (U)}$$

where $(\text{Var } d) = \sigma_d^2 = \text{Variance of daily demand}$

$$\begin{aligned}(\text{Var } U) &= \sigma_U^2 = \text{Variance of lead time demand} \\ &= (\text{Var } d) (\bar{t}_L) + (\text{Var } t_L) (\bar{d})^2\end{aligned}$$

Thus

$$TI = (T + \bar{t}_L)\bar{d} + Z\sqrt{(T + \bar{t}_L)\sigma_d^2 + \bar{d}^2\sigma_{t_L}^2}$$

The review period T is usually fixed as

$$T = \sqrt{\frac{2C_o}{C_h\bar{D}}}$$



Example - 12

The daily demand of an item is normally distributed with a mean of 50 units and standard deviation of 5 units. Lead time is 6 days. The cost of placing an order is Rs. 8, and annual holding costs are 20% of the unit price of Rs. 1.20. A 95% service level is desired. Back-orders are allowed but there is no stock-out cost. Find the various levels.





Solution - 12

- This is a case of constant lead time and variable demand.
- Review period is fixed equal to

$$T = \sqrt{\frac{2 \times 8}{(0.2)(1.2)(50)(365)}}$$

- $T = 0.06 = 22$ days
- Lead time = 6 days
- Average demand during review period $T = 22 \times 50 = 1100$ units
- Average demand during lead time, $t_L = 6 \times 50 = 300$ units
- Safety stock = $Z\sigma_d \sqrt{(T+t_L)} = 1.645 \times 5 \times = 43.5$ or 44 units
- Therefore, target inventory level $TI = 1100 + 300 + 44 = 1444$ units.





Example - 13

A company is ordering an item 4 times a year and has specified a service level of one stock-out per 3 years. The history of re-order lead times is shown below. Daily demand of the item is 40 units. Find the re-order level.

Past record of Lead Times (1990) (Date/Month)						
Order placed (date) :	7/1	3/2	16/3	6/4	2/5	2/6
Order received (Lead Times) :	18/1	21/2	20/4	28/4	20/5	23/6
Calendar days ∴	11	18	35	22	18	21
Working days :	7	12	25	16	14	15



Solution - 13

This is a case of constant demand and variable lead time.

For this example, review period is fixed as, $T = \frac{1}{4}$ years = 90 days

Then, average demand during review period = $90 \times 40 = 3600$ units

Average lead time, $\bar{t}_L = 14.83$ days

Therefore, average demand during average lead time = $40 \times 14.83 = 593.3$ units

Variance of lead time = 34.97 (days)²

Safety stock for the service level of 91.7% confidence

$$= 1.39 \times 40 \times \sqrt{34.97} = 351.4 \text{ units}$$

Therefore, the target inventory level

$$TI = 3600 + 593.3 + 351.4 = 4544.7 \text{ or } 4545 \text{ units.}$$





Example - 14

- Consider Example No. 13 with the following change in demand.
- The average daily demand of the item is 40 units per day and variance of daily demand is $(30 \text{ units/day})^2$



Solution - 14

- review period is fixed as, $T = \frac{1}{4}$ years = 90 days
- Thus average consumption during review period = $90 \times 40 = 3600$ units
Average consumption during average lead time = $40 \times 14.83 = 593.3$ units

$$\begin{aligned}\text{Safety Stock} &= Z \sqrt{(T + \bar{t}_L) \sigma d^2 + d^2 \sigma t_L^2} \\ &= (1.39) \sqrt{(22 + 14.83) (30)^2 + (40)^2 (34.97)} \\ &= (1.39) \sqrt{(36.83) (900) + (1600) (34.97)} \\ &= (1.39) \sqrt{33147 + 55952} \\ &= (1.39) (298.49456) \\ &= 414.90 \text{ units}\end{aligned}$$

- Therefore, the Target Inventory level
- = $3600 + 593.3 + 414.9$
- = 4608.2 units



Areas Under the One-Tailed Standard Normal Curve

This table provides the area between the mean and some Z score.
For example, when Z score = 1.45 the area = 0.4265.



Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990
3.1	0.4990	0.4991	0.4991	0.4991	0.4992	0.4992	0.4992	0.4992	0.4993	0.4993
3.2	0.4993	0.4993	0.4994	0.4994	0.4994	0.4994	0.4994	0.4995	0.4995	0.4995
3.3	0.4995	0.4995	0.4995	0.4996	0.4996	0.4996	0.4996	0.4996	0.4996	0.4997
3.4	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4997	0.4998
3.5	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998	0.4998
3.6	0.4998	0.4998	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.7	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.8	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999	0.4999
3.9	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000	0.5000

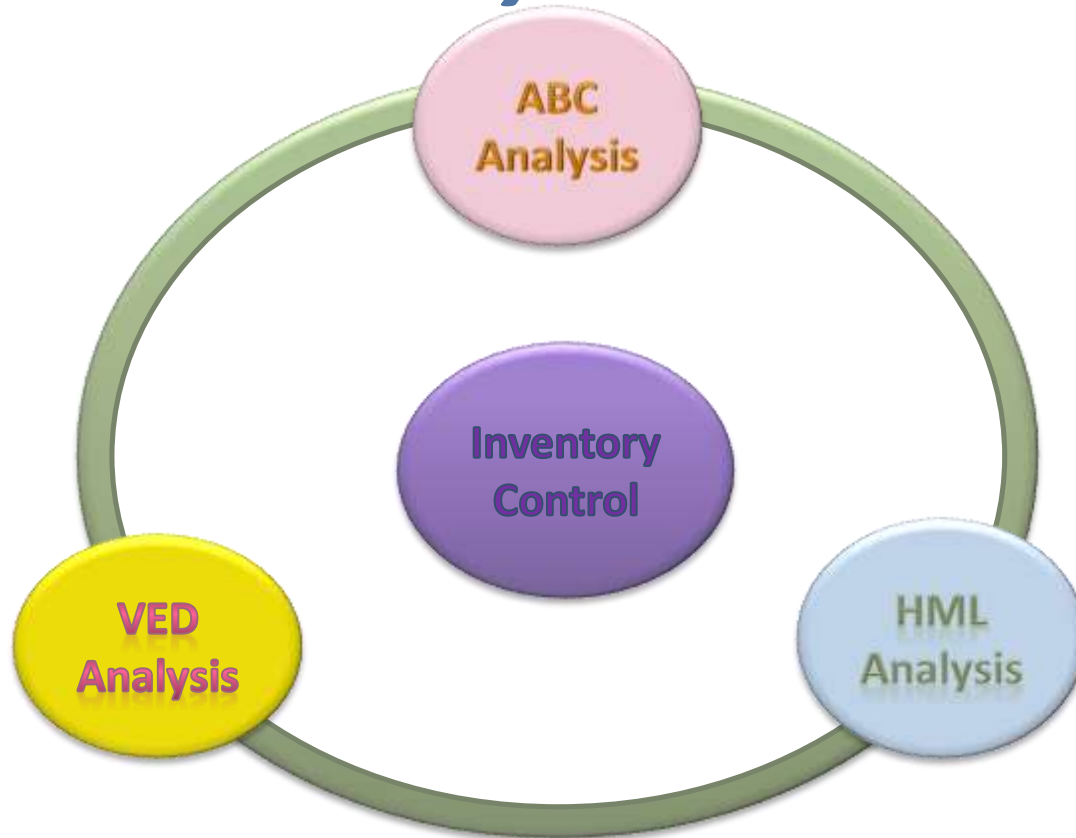
Tables of the Normal Distribution



Probability Content from $-\infty$ to Z

Z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Selective Inventory Control techniques





ABC analysis

- ABC plan is based upon segregation of materials for selection control.
- It measures the money value, i.e., cost significance of each material item in relation to total cost and material value.
- The study of each item of stock in terms of its usage, lead time, technical or other problems and its relative money value in the total investment in inventories.
- Critical, i.e., high value items deserve very close attention, and low value items need to be devoted minimum expense and effort in the task of controlling inventories.





ABC analysis

Organization consists of thousands of items with varying prices and usage rate

Recognizes that some inventory items are more important than others

- A group items are considered critical (often about 70% of Total value and 10% of items)
- B group items are important but not critical (often about 20% of Total value and 20% of items)
- C group items are not as important (often about 10% of Total value and 70% of items)



ABC analysis

CATEGORY	NO. OF ITEMS(%)	ITEM VALUE(%)	MANAGEMENT CONTROL
A	10	70 (HIGHEST)	MAXIMUM
B	20	20(MODERATE)	MODERATE
C	70	10(LEAST)	MINIMUM
TOTAL	100	100	



ABC analysis steps

The important steps involved in segregating materials or inventory control are:

1. Find out future use of each item of stock in terms of physical quantities for the review forecast period.
2. Determine the price per unit for each item.
3. Determine the total project cost of each item by multiplying its expected units to be used by the price per unit of such item.
4. Beginning with the item with the highest total cost, arrange different items in order of their total cost as computed under step (3) above.
5. Express the units of each item as a percentage of total costs of all items.
6. Compute the total cost of each item as a percentage of total costs of all items,4,5,6.



Policy and control based on ABC

Class	Characteristics	Policy	Methods
A	Few items Most of turnover	Tight control Personal supervision Communication Balanced safety stock	Frequent monitoring- daily Accurate records Sophisticated forecasting High service policy
B	Important Items Significant turnover	Lean stock policy Typical stock control Fast appraisal methods Managed by exception	Less frequent monitoring-weekly Calculated safety stocks Limited order value Computerised Exception reporting
C	Many items Low turnover value Few movement of low-value items	Minimum supervision Supply of order where possible- keep stock centrally Large order quantities Zero or high safety stock policy	Simple system Avoid stock shortages or excess Infrequent ordering Automatic system



ABC analysis

Advantages

1. It ensures a closer and a more strict control over such items, which are having a sizable investment in there.
2. It releases working capital, which would otherwise have been locked up for a more profitable channel of investment.
3. It reduces inventory-carrying cost.
4. It enables the relaxation of control for the 'C' items and thus makes it possible for a sufficient buffer stock to be created.
5. It enables the maintenance of high inventory turn over rate.





ABC analysis


Disadvantages

1. Proper standardization & codification of inventory items needed.
2. Considers only money value of items & neglects the importance of items for the production process or assembly or functioning.
3. Periodic review becomes difficult if only ABC analysis is recalled.
4. When other important factors make it obligatory to concentrate on “C” items more, the purpose of ABC analysis is defeated.





ABC analysis Numerical



Mahalakshmi Book Bindery divides SKUs (Stock keeping units) into three classes, according to their money value usage. Calculate the usage values of the following SKUs and determine which is most likely to be classified as class A. Also draw the curve.

SKU No.	Description	Qty used annually	Unit value (Rs. '00)
1	Boxes	500	3.00
2	Cardboard	18,000	0.02
3	Cover stock	10,000	0.75
4	Glue (Gallons)	75	40.00
5	Inside cover	20,000	0.05
6	Reinforcing tap (meters)	3,000	015
7	Signatures	150,000	045



ABC analysis Numerical Solution

- The annual usage for each item is determined by multiplying the annual usage quantity by the value per unit.
- The SKUs are then sorted by annual dollar usage, in declining order.
- Finally, A–B and B–C class lines are drawn roughly, according to the guidelines presented in the text.
- Here, class A includes only one SKU (signatures), which represents only 1/7, or 14 percent, of the SKUs but accounts for 83 percent of annual usage.
- Class B includes the next two SKUs, which taken together represent 28 percent of the SKUs and account for 13 percent of annual usage.
- The final four SKUs, class C, represent over half the number of SKUs but only 4 percent of total annual usage.





ABC analysis Numerical Solution

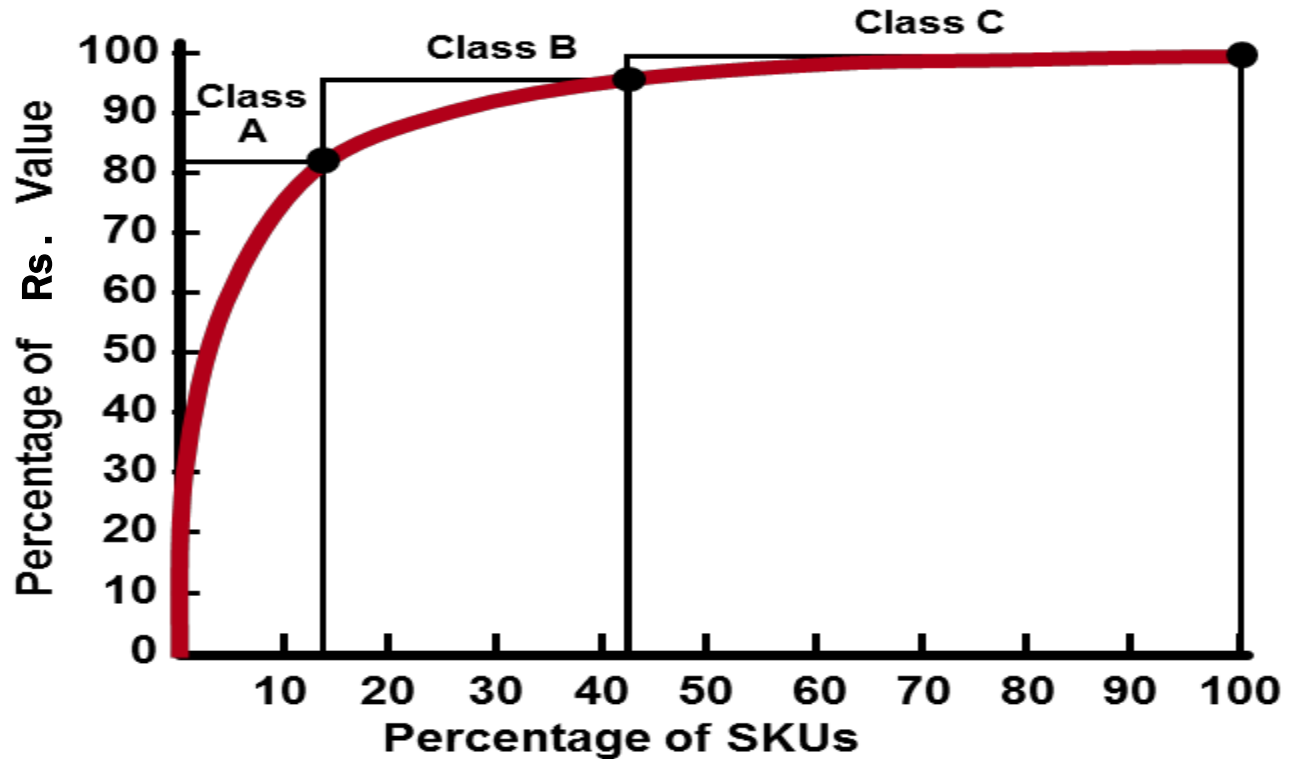
SKU No.	Description	Qty used annually		Unit value (Rs. '00)		Annual Usage (Rs.'00)
1	Boxes	500	X	3.00	=	1,500
2	Cardboard	18,000	X	0.02	=	360
3	Cover stock	10,000	X	0.75	=	7,500
4	Glue (Gallons)	75	X	40.00	=	3,000
5	Inside cover	20,000	X	0.05	=	1,000
6	Reinforcing tap (meters)	3,000	X	0.15	=	450
7	Signatures	150,000	X	0.45	=	67,500
TOTAL						81,310



ABC analysis Numerical Solution

SKU No.	Description	Qty used annually	Unit value (Rs. '00)	Annual Usage (Rs.'00)	% of total	Cumulative % of Rs. Value	Cumulative of SKUs	Class
7	Signatures	150,000	045	67,500	83.0	83.0 %	14.3 %	A
3	Cover stock	10,000	0.75	7,500	9.2	93.2 %	28.6 %	B
4	Glue (Gallons)	75	40.00	3,000	3.7	95.9 %	42.9 %	B
1	Boxes	500	3.00	1,500	1.8	97.8 %	57.1 5	C
5	Inside cover	20,000	0.05	1,000	1.2	99.0 %	71.4 %	C
6	Reinforcing tap (meters)	3,000	015	450	0.6	99.6 %	85.7 %	C
2	Cardboard	18,000	0.02	360	0.4	100 %	100 %	C
TOTAL				81,310				

ABC analysis Numerical Solution





HML analysis

- **H- HIGH VALUE**
- **M- MEDIUM VALUE**
- **L – LOW VALUE**





HML analysis

The cost per item (per piece) is considered for this analysis. High cost items (H), Medium Cost items (M) and Low Cost item (L) help in bringing controls over consumption at the departmental level.

Uses and application

- To assess storage & Security Requirements

- To keep control over consumption at the departmental head level

- To determine the frequency of stock verification

- To evolve buying policies to control purchases

- To delegate authorities to different buyers to make petty cash purchase

- High priced items in cupboards e.g. bearings, worm wheels

- Authority to indents of High & Medium priced items to departmental head after careful scrutiny

- checking frequency: more for high priced items and less for L category

- Excess supply: Not accepted in case of H & M category, Acceptable in case of L category

- H & M by senior & L by junior buyers





VED analysis

- V- part is VITAL(high stock level)
- E- part is ESSENTIAL (moderate stock level)
- D- part is DESIRABLE (minimum stock level)





VED analysis

In VED method (vital, essential, desirable) the degree of criticality can be stated as the material is vital to process of production, or essential to the process of production or desirable for process of production.

Vital:-

- Items without which treatment comes to standstill i.e. non-availability cannot be tolerated
- The vital items are stocked in abundance, essential and very strict control.

Essential:-

- Items whose non-availability can be tolerated for 2-3 days, because similar or alternative items are available.
- Essential items are stocked in medium amounts, purchase is based on rigid requirements and reasonably strict watch.

Desirable:-

- Items whose non-availability can be tolerated for a long period.
- Desirable items are stocked in small amounts and purchase is based on usage estimates.





VED analysis

- This method is based on how important a particular part is irrespective of its cost.
- In health care industry, irrespective of the cost how effectively the medicine saves the life is the basis for VED analysis.
- Their availability must be ensured at all items for smooth production, so need to be strictly controlled.
- Essential items follow vital items in hierarchy of importance.
- Desirable items are least importance in terms of functional considerations, which are loosely controlled at the lower level





VED analysis

CONTROL OF VED ITEMS:-

- ❖ *Category 1 : These items are the most important ones and require control by the administrator himself.*
- ❖ *Category 2 : These items are of intermediate importance and should be under control of the office in charge of the stores.*
- ❖ *Category 3 : These items are the least importance which can be left under the control of the store keeper.*





VED analysis

ADVANTAGES

- ✓ It is useful for monitoring and control of stores and spares inventory by classifying them into three categories.
- ✓ Determine the criticality of an item and its effect on production and other services.
- ✓ It is useful for controlling and maintain the stock of various types

