Portfolio Construction using Black-Litterman Model and Factors

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Abstract

This report primarily explores the construction of asset portfolios through the Black-Litterman model, segmented into three pivotal sections.

Initially, the first segment embarks on an investigative journey into factors, engaging in both learning and back-testing processes. Specific factors are identified by sourcing corresponding Exchange-Traded Funds (ETFs) from Yahoo Finance. This involves the procurement and refinement of data for the chosen ETFs , the Standard & Poor's 500 Index (SPY) as a benchmark and the 13-week Treasury Bill rate (^IRX) as a proxy for the risk-free rate. Subsequently, a comparative analysis is conducted between the chosen ETFs and the benchmarks through rolling beta and changing alpha metrics.

The second section delves into the construction of the Black-Litterman model itself. It begins by establishing the prior distributions based on the market benchmark. Then the view part is argumented by integrating perspectives from macroeconomic analysts at major institutions, as well as incorporating my personal viewpoints to formulate the views matrix. The posterior distribution is then derived through Bayes's Formula and optimization constraints such as the Global Minimum Variance Portfolio (GMVP), mean-variance optimization, and the maximization of the Sharpe Ratio.

In the third section, the posterior outcomes are adjusted through alterations in risk-aversion coefficient and the scaling parameter Tau. This adjustment allows for an analysis and discussion of trends in the results, differences compared to the benchmark, and potential areas for optimization. For instance, formulating Black-Litterman views with the aid of machine learning classifiers and external data, as well as applying methods of denoising or shrinkage to approximate true estimates are considered as further improvement.

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Introduction

This report delves into portfolio construction using the Black-Litterman model, a sophisticated approach integrating traditional finance theory with contemporary artificial intelligence techniques. It focuses on creating an asset portfolio that aligns with market-consistent weights and leverages factors such as the Fama-French model, ETFs, and global market considerations. The report investigates various aspects of portfolio management, including factor analysis, risk aversion impacts, and dynamic view adjustments through Long Short-Term Memory (LSTM) networks.

I. Factor Data and Study (Back-testing)

1.1 Portfolio Choice and Data

The principal criterion for factor selection emphasizes the construction of a portfolio that promises optimal diversification. This portfolio undergoes rigorous computations and analyses, including the correlation matrix evaluation, in concert with rolling beta comparisons to market benchmarks. The construction of this portfolio will be modeled after the configuration delineated in Option B from Q&A paper.

Initially, to ensure the portfolio benefits from robust liquidity and a free market mechanism, the selection is centered on the largest financial market globally—the US market. Within this arena, ETFs that correspond to specific factors or market segments have been curated. For instance, ETFs aligned with the Fama-French 5 model, U.S. Treasuries-related ETFs, commodity ETFs, and sector-specific ETFs (notably in high-technology and healthcare) have been selected.Turning our attention to emerging markets, the China market is chosen for two primary reasons. First, its relatively low correlation with the US market enhances the diversification benefits. Second, my personal familiarity with the Chinese market provides a profound understanding of its current economic downturn and future expectations.

The Fama-French factors, serving as an extension to the original three-factor model, have been expanded by Fama and French to a more comprehensive five-factor model that refines the asset pricing theory. This model encapsulates the original triad of factors—market risk, size (SMB), and value (HMB)—and enriches it with two additional factors: the Profitability Factor (RMW - Robust Minus Weak) and the Investment Factor (CMA - Conservative Minus Aggressive). Consequently, a selection of corresponding ETFs has been made both in the US and Chinese markets. For bond assets, actively traded ETFs of US and Chinese bonds have been distinguished. In terms of industry factors, the healthcare and high-technology sectors, which are well-acquainted territories, form integral parts of the asset portfolio. Regarding the commodity category, a trinity of common commodities—oil, gold, and agriculture—have been included. Moreover, adhering to the recommendations from the Reading Q&A, a VIX factor will also be incorporated into the portfolio.

Therefore, we pick the following 26 factors, as the #0 to #24 in Table 1. There are two more factors, #25 and #26 that are treated as the risk-free rate and the benchmark, respectively.

Table 1: List of All Factors (27Time Series, including 25Factors 1 Benchmark and 1 Risk-free Rate

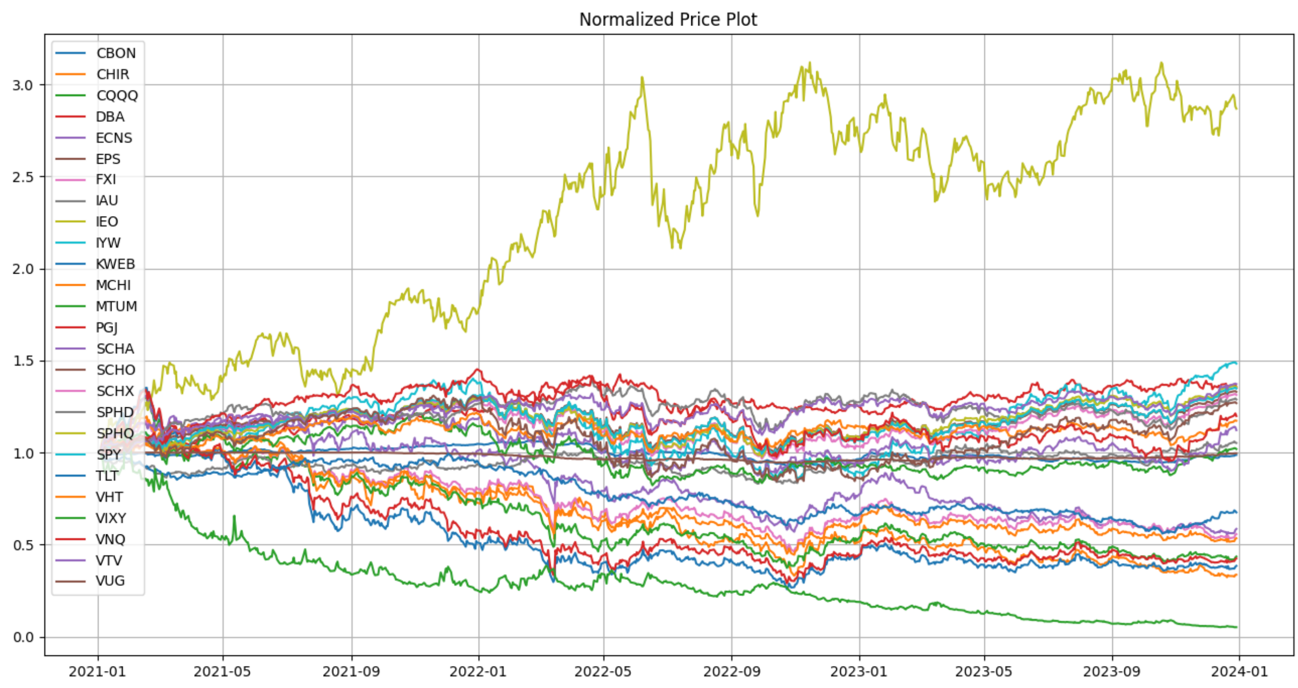
#	Type	yfinance Quote	Index Meaning	Variable Name	Factor
1	Bond China	CBON	VanEck China Bond ETF	china_bond	China Bond
2	Equity China	CHIR	Global X MSCI China Real Estate ETF	china_realestate	China Real Estate
3	Equity China	CQQQ	Invesco China Technology ETF	china_tech	China High Tech
4	Commodity	DBA	Invesco DB Agriculture Fund	commodity_agriculture	Agriculture
5	Equity China	ECNS	iShares MSCI China Small-Cap ETF	china_small	China Small Cap
6	Equity US	EPS	WisdomTree U.S. Earnings 500 Fund	us_earnings	US Earnings
7	Equity China	FXI	iShares China Large-Cap ETF	china_large	China Large Cap
8	Commodity	IAU	iShares Gold Trust	commodity_gold	Gold
9	Commodity	IEO	iShares U.S. Oil & Gas Exploration & Production ETF	commodity_oil	Oil and Gas
10	Equity US	IYW	iShares U.S. Technology ETF	us_tech	US High Tech
11	Equity China	KWEB	KraneShares CSI China Internet ETF	china_internet	China Internet
12	Equity China	MCHI	iShares MSCI China ETF	china_benchmark	China Benchmark Index
13	Equity US	MTUM	iShares MSCI USA Momentum Factor ETF	us_momentum	US Momentum
14	Equity China	PGJ	Invesco Golden Dragon China ETF	china_quality	China Quality
15	Equity US	SCHA	Schwab U.S. Small-Cap ETF	us_small	US Small Cap
16	Bond US	SCHO	Schwab Short-Term U.S. Treasury ETF	us_bond_shortterm	Short-term US Bond
17	Equity US	SCHX	Schwab U.S. Large-Cap ETF	us_large	US Large Cap
18	Equity US	SPHD	Invesco S&P 500 High Dividend Low Volatility ETF	us_dividend	US Dividend
19	Equity US	SPHQ	Invesco S&P 500 Quality ETF	us_quality	US Quality
20	Bond US	TLT	iShares 20+ Year Treasury Bond ETF	us_bond_longterm	Long-term US Bond
21	Equity US	VHT	Vanguard Health Care Index Fund	us_healthcare	US Health Care
22	Volatility	VIXY	ProShares VIX Short-Term Futures ETF (VIXY)	vix	VIX Short-term
23	Real Estate	VNQ	Vanguard Real Estate Index Fund	house_us	US Real Estate
24	Equity US	VTV	Vanguard Value Index Fund	us_value	US Value
25	Equity US	VUG	Vanguard Growth Index Fund	us_growth	US Growth
26	Equity US	SPY	SPDR S&P 500 ETF Trust	sp500	Benchmark
27	Rate	^IRX	13 Week Treasury Bill	tbill	3-Month Treasury Bill Rate

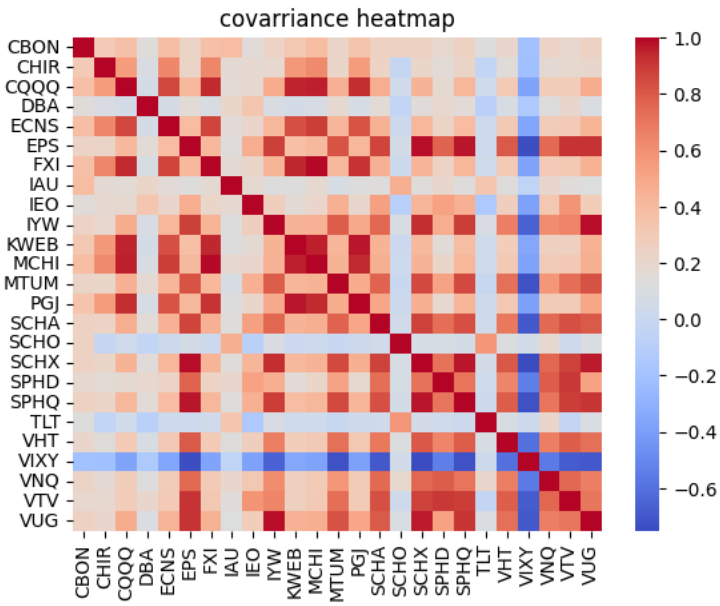
1.2 Back testing and analysis

1.2.1 Performance of the factors

Table 2 presents a comprehensive performance chart, encompassing 25 distinct factors alongside the benchmark index, SPY. Notably, a subset of these factors exhibits a significant degree of correlation. The chart reveals that the Oil Commodity ETF has demonstrated a pattern of high volatility and growth. This trend is likely attributable to the combined impacts of the recent pandemic and geopolitical issues over the past three years. Additionally, the Chinese Technology ETF is observed to have undergone a tumultuous decline. This downturn can be ascribed to previously inflated valuations, coupled with a stagnation in the growth of high-tech and internet companies in the post-pandemic era, necessitating a reassessment of their market values.

Table 2: Normalized Price Plot and Correlations





1.2.2 Rolling Beta and Changing Alpha

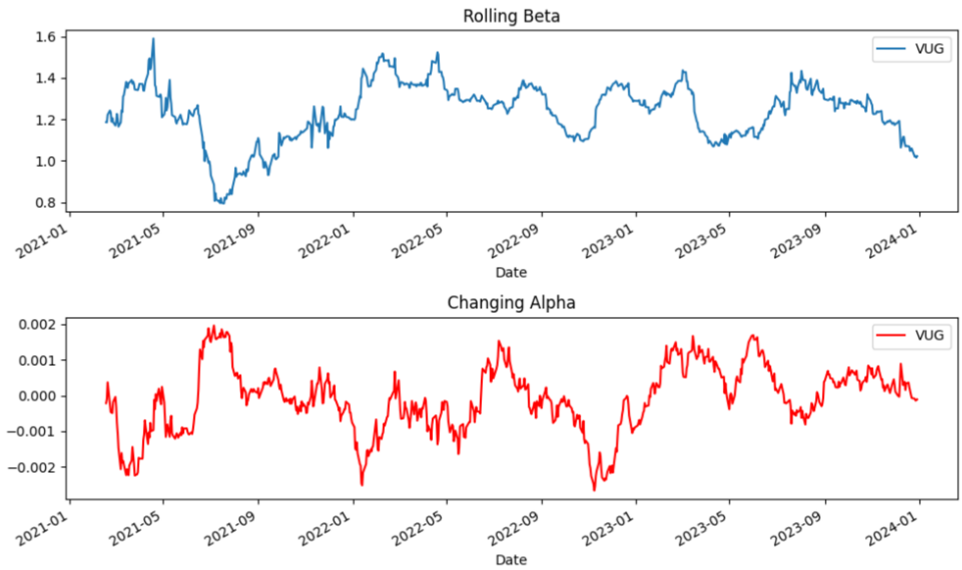
Rolling beta serves as a key indicator for measuring the systematic risk of an asset relative to the market, typically a benchmark index like the S&P 500. It quantitatively reflects the correlation between the returns of the asset and the market. The concept of Rolling Beta involves the calculation of the Beta value over a specified rolling window period. For instance, the Beta value might be calculated every 30 days based on the data from the preceding year.

This methodology enables the observation of the temporal variations in Beta, thereby providing insights into the changing risk exposure of the asset.

Changing alpha represents the excess return of an asset, that is, the actual return of the asset relative to its expected return, based on its Beta. Changing Alpha refers to the Alpha value calculated at different time points, which is instrumental in analyzing whether the performance of the asset manager has improved or deteriorated over time.

Given the complexity of the calculations, a step-by-step approach will be adopted. Initially, the daily return rates for each asset will be computed. Subsequently, these rates will be employed to calculate both the Rolling Beta and Changing Alpha. This process typically involves selecting a rolling window period, such as 30 or 60 days.

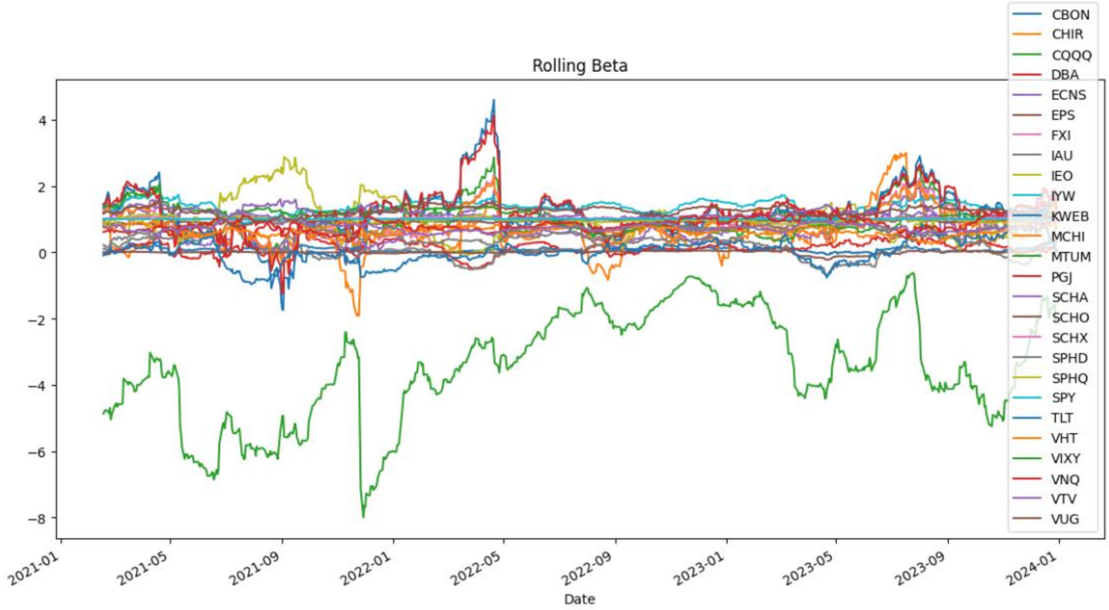
Table 3: Vanguard Growth Index Fund Rolling Beta and Changing Alpha

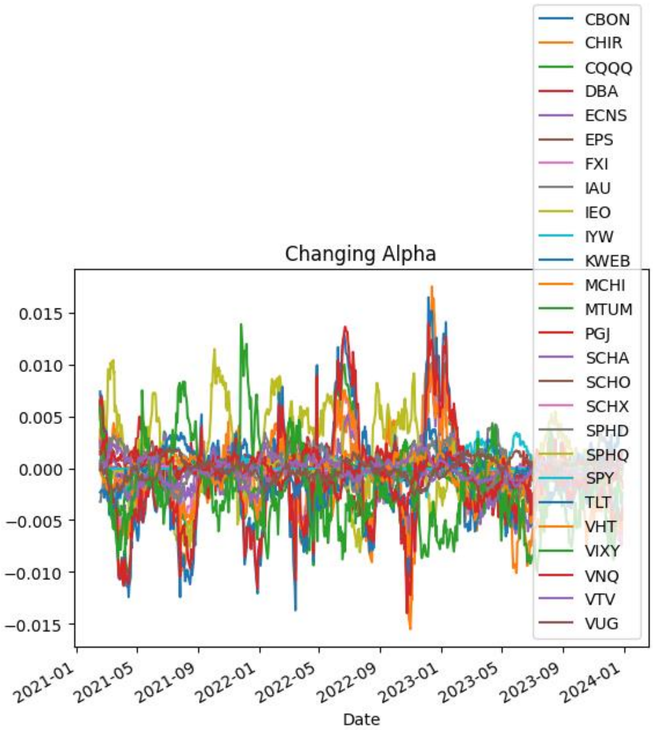


In Table 3, we present the relationship between individual factors and the market, as represented by SPY (SPDR S&P 500 ETF Trust), in terms of Rolling Beta and Changing Alpha. This table elucidates the systematic risk (Rolling Beta) and performance differential (Changing Alpha) of each factor in relation to the market benchmark.

Building upon this, Table 4 exhibits the Rolling Beta and Changing Alpha for a collective of factors in relation to the market (SPY). A salient feature of this analysis is the identification of distinct correlation levels each asset holds with the market. Notably, there is a pronounced disparity in the excess returns among these assets. For instance, the VIX Short-Term Futures ETF (VIXY) is characterized by a substantial negative correlation with the market, distinguishing itself in terms of its risk-return profile.

Table 4: All Factors Rolling Beta and Changing Alpha





This comparative analysis underscores the heterogeneous nature of asset-market correlations and the variability in generating excess returns. The distinctive negative correlation exhibited by VIXY exemplifies the divergent behavior of certain assets in relation to market trends. Such findings are crucial for understanding the nuanced dynamics of market correlation and its implications for portfolio management and risk assessment.

II. Black-Litterman Model Explained

2.1 Black-litterman Basic Procedure

The Black-Litterman model represents a significant advancement in portfolio allocation, offering a mathematical structure that synergizes individual expected returns with market equilibrium returns, as inferred from the Capital Asset Pricing Model (CAPM). This model effectively circumvents the typical pitfalls of mean-variance optimization, which is notably prone to the vagaries of expected return inputs, often leading to disproportionate and unrealistic asset weightings. By contrast, the Black-Litterman model ingeniously assimilates investors' perspectives with the market's equilibrium state, thus ensuring a more rational and balanced portfolio allocation strategy. Here’s a simplified explanation of the procedure:

1) **Determine the Market Equilibrium Returns**: This uses the Capital Asset Pricing Model (CAPM) to find the equilibrium expected returns for the assets based on the market portfolio.
2) **Formulate Investor Views**: Investors can express their views on the expected performance of various assets or asset classes. These views can be absolute (e.g., Asset X will have a return of Y%) or relative (e.g., Asset X will outperform Asset Y by Z%).
3) **Blend the Views with the Market Equilibrium**: The Black-Litterman model uses a Bayesian approach to combine the investor views with the market equilibrium returns. The result is a set of adjusted expected returns that reflect both the market equilibrium and the investor's views.
4) **Optimize the Portfolio**: With the adjusted expected returns and the covariance matrix, the model then uses mean-variance optimization to suggest the optimal asset weights in the portfolio.
5) **Assess the Results**: The output is a set of portfolio weights that aim to maximize return for a given level of risk, considering both market expectations and the investor's unique views.

2.2 Determine the Market Equilibrium Returns (the prior)

2.2.1 Define the prior Distribution

The prior distribution of excess return $f_{\mu}(\mu)$

$$\mu \sim N(\pi, \tau\Sigma)$$

- $\tau = 1/252 \approx 0.004$ gives low uncertainty on estimates $\tau\Sigma$, and so we choose up to $\tau = 0.4$ = 0.4 to impose more uncertainty.
- π is the expected excess return.
- Σ is the variance of the estimate.

2.2.2 The Risk Aversion of the Market

Risk aversion (often denoted as λ or "market lambda" in financial models) is a measure of an investor's reluctance to accept risk. It can be estimated from the market portfolio based on the Capital Asset Pricing Model (CAPM). The formula to estimate:

$$\lambda_m = \frac{E(R_m) - R_f}{\sigma_m^2}$$

, where $E(R_m)$ is the expected return of the market portfolio which is the market benchmark by the S&P500 index, R_f is the risk-free rate by using by the 3-month Treasury Bond rate, and σ_m^2 is the variance of the market portfolio's return. Therefore, the risk aversion of market is $\lambda_m/2 \approx 1.4833$.

2.2.3 Weight Allocation

In the Black-Litterman model's framework, the equilibrium market weights are ideally informed by the assets' market capitalizations, which are posited to reflect the market portfolio's aggregate judgment of value. Yet, this approach encounters practical hurdles with ETFs, where direct market capitalization data may be obscure or imprecise, notably when using a data source such as Yahoo Finance, interfaced via the 'yfinance' library in Python. To address this issue, I tried to resort to an equal-weighting strategy as a pragmatic approximation. This stratagem operates on the premise that every ETF has equal importance from the market's standpoint, a robust assumption that strays from the original market equilibrium concept intrinsic to the Black-Litterman model. Additionally, the methodology employed here includes the computation of a benchmark allocation. This refers to the process of assigning weights to each ETF in the portfolio based on its proportionate market capitalization relative to the aggregate market capitalization of all ETFs within the portfolio. In other words, the weight of each ETF is determined by its market capitalization as a percentage of the total market capitalization of all ETFs in the portfolio. Therefore, each factor weight : ETF's total asset/ sum of all ETFs total assets.

Table 5_1: The Weight Allocation(Market Capacity)

Assets Class(ETFs)	Weight_market
CBON	31.6094%
CHIR	24.5705%
CQQQ	1.0649%
DBA	0.4600%
ECNS	3.0686%
EPS	2.2250%
FXI	5.7970%
IAU	2.4934%
IEO	1.3244%
IYW	0.1180%
KWEB	0.9352%
MCHI	0.0089%
MTUM	0.6904%
PGJ	0.8473%
SCHA	0.0010%
SCHO	0.1031%
SCHX	0.0249%
SPHD	1.8745%
SPHQ	8.1402%
TLT	0.0051%
VHT	0.1153%
VIXY	4.1650%
VNQ	0.1107%
VTV	10.2223%
VUG	0.0248%

2.2.4 Solving the Equilibrium Return by Reverse Optimization

Solve reverse optimisation problem to obtain equilibrium returns π from market index allocations w.

$$arg\ \underbrace{max}_{w}\{w^T\pi - \lambda_{mkt}w^T\Sigma w\}$$

By F.O.C

$$\frac{\partial}{\partial w} \equiv \pi - 2\lambda_{mkt}\Sigma w \equiv 0$$

$$\pi = 2\lambda_{mkt}\Sigma w$$

, where π is a vector of the equilibrium excess return and λ_{mkt} is the empirical risk aversion derived from the benchmark (in our case is SPY).

Table 5_2: The Equilibrium Returns

Assets Class(ETFs)	Equilibrium Return
CBON	1.1450%
CHIR	18.5740%
CQQQ	12.8829%
DBA	0.8770%
ECNS	10.2931%
EPS	3.0883%
FXI	11.9643%
IAU	1.7054%
IEO	4.6107%
IYW	4.3899%
KWEB	17.9020%
MCHI	11.5477%
MTUM	3.4674%
PGJ	16.4135%
SCHA	4.3564%
SCHO	0.0471%
SCHX	3.2880%
SPHD	2.1847%
SPHQ	3.2937%
TLT	0.0659%
VHT	2.0893%
VIXY	-8.5330%
VNQ	2.9879%
VTV	2.4554%
VUG	4.0285%

2.3 View

In the Black-Litterman model, "views" refer to the specific opinions or insights that an investor has regarding the expected returns of the assets in their portfolio. These views are subjective and are used to adjust the market equilibrium returns (derived from the CAPM) to better reflect the investor's expectations.

There are generally two types of views that can be incorporated into the Black-Litterman model:

1) **Absolute Views**: These express an opinion about the expected return of a single asset or a group of assets. For example, an investor might have the view that "Stock A will return 8% next year." Absolute views set a specific expected return for the asset(s) in question.
2) **Relative Views**: These express an opinion about the expected performance of one asset relative to another. For example, "Stock A will outperform Stock B by 3% next year." Relative views do not specify an expected return but rather express a differential expectation between assets.

Each view is quantified in the Black-Litterman model using two elements:

Q: A vector of the investor's views on the expected returns of the assets. ($K \times 1$ vector of expected returns of those K views)

P: A matrix that maps the views to the assets. This matrix identifies which assets the views apply to and the nature of the views (absolute or relative). ($K \times N$ matrix that has K views on N assets)

Here is three views in this report：

- The first view states only the ‘US Value’ would be expected to generate a 5% excess return.
- The second view states the return of holding ‘US Healthcare‘ would outperform using the ‘US Tech Stocks‘ index by 10%.
- The third view states the return of holding all China stocks and bonds would outperform using the ‘Oil and Gas’ index by 15%.

For confidence in views, each view is also associated with a confidence level, which is represented by a scalar or matrix (often denoted by Ω). This reflects how confident the investor is in each of the views being correct. A higher confidence level means that the investor's views will have a greater impact on the final adjusted returns. Here is the Ω :

$$\Omega = diag(P(\tau\Sigma)P^T$$

2.4 Posterior - Blend the Views with the Market Equilibrium

2.4.1 Black-litterman solution for computation

In the context of the Black-Litterman model, posterior expected returns emerge from the Bayesian update as a new, refined set of predictions that represent a compromise between market data and the investor's views. These updated returns inform the portfolio construction process, potentially leading to different asset allocations than those suggested by market equilibrium alone. The posterior results are tailored to the investor's perspective, offering a customized investment approach that leverages both market trends and individual analysis.

Attillio Meucci (2010) suggested computationally stable formulation (less matrix inversions) than original BL.

$$\mu_{BL} = \pi + \tau\Sigma P^T(\tau P\Sigma P^T + \Omega)^T(V - P\pi)$$

,where the right side of the equation is already known, so we can obtain the Posterior results.

2.5 Optimize the Portfolio

2.5.1 the Posterior weight with no constraints

In an unconstrained optimization, the portfolio weights are determined purely based on the maximization of expected utility, subject to the updated (posterior) expected returns and the covariances of the assets. The analytical solution, no constraints is:

$$w_{BL} = \frac{1}{2\lambda}\Sigma^{-1}\mu_{BL}$$

2.5.2 Global Minimum Variance Portfolio

The global minimum variance portfolio has the objective function of minimizing the portfolio variance.

$$w_{GMV} = arg\underbrace{min}_{w} w^T\Sigma w$$

$$s.t. w^T\vec{1} = 1$$

$$s.t. 0 \leq \forall w_i \leq 1$$

$$w_{GMV} = \frac{\Sigma^{-1}\vec{1}}{\vec{1}^T\Sigma^{-1}\vec{1}}$$

2.5.3 Markowitz Mean-Variance Portfolio

$$w_{mv} = arg\underbrace{max}_{w}\{w^T\mu_{BL} - \lambda w^T\Sigma w\}$$

$$s.t. w^T\vec{1} = 1$$

$$s.t. 0 \leq \forall w_i \leq 1$$

2.5.4 Maximum Sharpe Ratio Portfolio

$$w_{mv} = arg\underbrace{max}_{w}\frac{w^T(\mu_{BL} - rf)}{\sqrt{w^T\Sigma w}}$$

$$s.t. w^T\vec{1} = 1$$

$$s.t. 0 \leq \forall w_i \leq 1$$

An initial attempt was made to address the problem using CVXPY, a Python library for convex optimization. However, CVXPY operates under the strict guidelines of Disciplined Convex Programming (DCP) which necessitates that all optimization problems conform to the rules of convexity. The maximization of the Sharpe ratio presented a challenge in this context, as the Sharpe ratio is a ratio with the standard deviation (or volatility) in the denominator, resulting in a non-convex optimization problem due to the non-convex nature of the reciprocal of volatility. To circumvent this issue, the maximization of the square of the Sharpe ratio was proposed, effectively removing the square root from the denominator and rendering the problem convex. This approach is theoretically equivalent to maximizing the Sharpe ratio itself, as the optimal asset weights remain unaffected in the absence of leverage constraints. Despite this reformulation, the DCP constraints persisted, eluding resolution after multiple attempts. Consequently, the gradient ascent method was employed to calculate the solution, albeit with an acknowledgment of its susceptibility to converging to local maxima.

III. Comparative Analysis of Black-Litterman Outputs

3.1 Optimal allocations (your) vs benchmark for active risk & Expected returns vs implied equilibrium returns.

In the Black-Litterman model, the prior returns are typically derived from the market equilibrium, reflecting the collective expectations of market participants. These are often calculated using the reverse optimization method based on the Capital Asset Pricing Model (CAPM), yielding the implied equilibrium returns.

The posterior returns, on the other hand, are the result of blending the prior returns with the investor's specific views on the expected performance of the assets. The difference between the prior and posterior returns arises due to this incorporation of subjective views, which adjust the market equilibrium based on additional information or beliefs that the investor holds.

The primary reasons influencing the difference between the prior weights and the posterior weights are:

1) **Investor Views:** The investor's views on certain assets, whether they are expected to perform better or worse than the market consensus, can significantly shift the weights from their market-cap-weighted equilibrium positions.
2) **Confidence in Views:** The degree of confidence the investor has in their views also affects the posterior weights. Higher confidence leads to greater deviations from the equilibrium weights.
3) **Tau (τ):** This parameter scales the uncertainty of the prior. A higher τ indicates greater uncertainty in the prior returns, allowing the investor's views to have a more pronounced effect on the posterior returns.
4) **Risk Aversion:** The investor's risk aversion level influences the sensitivity of the portfolio weights to the expected returns. A more risk-averse investor will have a portfolio that less aggressively adjusts weights in response to the views.

In sum, in determining the implied equilibrium returns, two distinct approaches were employed to establish the prior weights: equal weighting and market capitalization weighting. Despite the challenges in accurately ascertaining the market capitalizations of each ETF, the latter methodology was utilized in subsequent analyses. It was observed that the selection of prior weights critically influences the resultant portfolio outcomes.

Table6: Return Vectors and Resulting Portfolio Weights

Assets Class(ETFs)	The Posterior Return E[R]	The Prior Return π	Difference E[R]-π	The Posterior Weight w_bl	Equal Weight w_eq	Difference w_bl-w_eq
CBON	0.8346%	1.0113%	-0.1767%	5.5995%	4.0000%	1.5995%
CHIR	10.5791%	11.7403%	-1.1612%	5.5995%	4.0000%	1.5995%
CQQQ	11.9319%	15.1207%	-3.1888%	5.5995%	4.0000%	1.5995%
DBA	0.8957%	1.1455%	-0.2498%	5.2975%	4.0000%	1.2975%
ECNS	8.8540%	10.7773%	-1.9233%	5.5995%	4.0000%	1.5995%
EPS	2.9903%	5.2417%	-2.2515%	5.2975%	4.0000%	1.2975%
FXI	10.6194%	12.9105%	-2.2911%	5.5995%	4.0000%	1.5995%
IAU	1.4946%	1.7176%	-0.2230%	5.2975%	4.0000%	1.2975%
IEO	4.6608%	6.3947%	-1.7339%	2.8817%	4.0000%	-1.1183%
IYW	3.0554%	8.1154%	-5.0600%	-47.7994%	4.0000%	-51.7994%
KWEB	16.8397%	20.8861%	-4.0464%	5.5995%	4.0000%	1.5995%
MCHI	10.3918%	12.7137%	-2.3219%	5.5995%	4.0000%	1.5995%
MTUM	3.6598%	6.1007%	-2.4409%	5.2975%	4.0000%	1.2975%
PGJ	15.4015%	19.6441%	-4.2426%	5.5995%	4.0000%	1.5995%
SCHA	4.7883%	7.4049%	-2.6166%	5.2975%	4.0000%	1.2975%
SCHO	0.0893%	0.1023%	-0.0130%	5.2975%	4.0000%	1.2975%
SCHX	3.1758%	5.8041%	-2.6283%	5.2975%	4.0000%	1.2975%
SPHD	2.8348%	3.3945%	-0.5597%	5.2975%	4.0000%	1.2975%
SPHQ	3.1230%	5.5068%	-2.3838%	5.2975%	4.0000%	1.2975%
TLT	0.6515%	0.8312%	-0.1797%	5.2975%	4.0000%	1.2975%
VHT	3.3584%	3.8041%	-0.4457%	58.3944%	4.0000%	54.3944%
VIXY	-8.4107%	-15.2743%	6.8636%	5.2975%	4.0000%	1.2975%
VNQ	3.6601%	5.1095%	-1.4494%	5.2975%	4.0000%	1.2975%
VTV	2.9790%	3.9840%	-1.0049%	5.7305%	4.0000%	1.7305%
VUG	3.2883%	7.3885%	-4.1002%	5.2975%	4.0000%	1.2975%

Table7: Return Vectors and Resulting Portfolio Weights(Market_cap)

Assets Class(ETFs)	The Posterior Return E[R]	The Prior Return π	Difference E[R]-π	The Posterior Weight w_bl	Market_Cap Weight w_mrk	Difference w_bl-w_mrk
CBON	1.0408%	1.1450%	-0.1043%	41.9899%	31.6094%	10.3804%
CHIR	17.8557%	18.5740%	-0.7183%	32.6677%	24.5705%	8.0972%
CQQQ	10.6070%	12.8829%	-2.2759%	1.5374%	1.0649%	0.4725%
DBA	0.8243%	0.8770%	-0.0528%	0.6093%	0.4600%	0.1492%
ECNS	9.0049%	10.2931%	-1.2881%	4.1911%	3.0686%	1.1225%
EPS	1.9627%	3.0883%	-1.1255%	2.9468%	2.2250%	0.7217%
FXI	10.4226%	11.9643%	-1.5417%	7.8045%	5.7970%	2.0075%
IAU	1.6207%	1.7054%	-0.0847%	3.3022%	2.4934%	0.8088%
IEO	4.3190%	4.6107%	-0.2917%	0.7372%	1.3244%	-0.5871%
IYW	1.0426%	4.3899%	-3.3474%	-42.6751%	0.1180%	-42.7931%
KWEB	14.9627%	17.9020%	-2.9393%	1.3656%	0.9352%	0.4304%
MCHI	9.9530%	11.5477%	-1.5947%	0.1388%	0.0089%	0.1300%
MTUM	2.1641%	3.4674%	-1.3033%	0.9144%	0.6904%	0.2240%
PGJ	13.4077%	16.4135%	-3.0058%	1.2493%	0.8473%	0.4019%
SCHA	3.1087%	4.3564%	-1.2477%	0.0013%	0.0010%	0.0003%
SCHO	0.0379%	0.0471%	-0.0093%	0.1365%	0.1031%	0.0334%
SCHX	1.8649%	3.2880%	-1.4231%	0.0330%	0.0249%	0.0081%
SPHD	2.3753%	2.1847%	0.1906%	2.4826%	1.8745%	0.6081%
SPHQ	2.0663%	3.2937%	-1.2274%	10.7807%	8.1402%	2.6405%
TLT	-0.1260%	0.0659%	-0.1918%	0.0068%	0.0051%	0.0017%
VHT	2.2536%	2.0893%	0.1643%	42.9840%	0.1153%	42.8688%
VIXY	-5.0748%	-8.5330%	3.4582%	5.5160%	4.1650%	1.3510%
VNQ	2.5042%	2.9879%	-0.4837%	0.1466%	0.1107%	0.0359%
VTV	2.3299%	2.4554%	-0.1256%	27.3660%	10.2223%	17.1436%
VUG	1.4253%	4.0285%	-2.6032%	0.0328%	0.0248%	0.0080%

3.2 Different type of optimization method comparison

Global Minimum Variance Portfolio (GMVP):

- The GMVP aims to minimize the portfolio's total variance (or standard deviation), regardless of the expected returns. The result is the portfolio with the lowest possible risk.
- This approach is purely risk-focused and does not consider the expected returns of the assets.

Markowitz Mean-Variance Optimization:

- Proposed by Harry Markowitz, this framework seeks to construct a portfolio that achieves the best possible balance between expected return and risk (as measured by variance).
- Investors can set a desired return level, and the optimization will find the portfolio with the minimum risk for that level of return (or vice versa).
- It is based on the efficient frontier concept, where portfolios are selected that offer the highest expected return for a defined level of risk.

Maximum Sharpe Ratio:

- This optimization seeks to maximize the Sharpe ratio of the portfolio, which is the expected return more than the risk-free rate per unit of volatility.

- The outcome is the portfolio on the efficient frontier with the highest risk-adjusted return, which theoretically offers the best reward for the risk taken.
- This approach is particularly popular as it considers both risk and return in its assessment, aiming to provide the most efficient use of risk.

From the results, the outcomes derived from maximum Sharpe ratio appear to be more balanced, potentially attributable to the incorporation of the view matrix. Conversely, the other two methods yield more extreme results, particularly the Global Minimum Variance Portfolio, which is most susceptible to the influence of the covariance matrix.

Table8: Three Type of Optimisation Weights(Market_cap)

Assets Class(ETFs)	Market_Cap Weight w_mrk	The BL Weight (with no contraints)	GMVP Weight	Mean-Varriance Weight	Maximum_SR Weight
CBON	31.6094%	41.9899%	3.6433%	0.0000%	6.4286%
CHIR	24.5705%	32.6677%	0.0547%	35.1553%	2.7810%
CQQQ	1.0649%	1.5374%	0.0000%	0.0000%	2.8486%
DBA	0.4600%	0.6093%	1.9709%	0.0000%	5.3992%
ECNS	3.0686%	4.1911%	0.0000%	1.8244%	3.5658%
EPS	2.2250%	2.9468%	0.0000%	0.0000%	4.8399%
FXI	5.7970%	7.8045%	0.0000%	8.3417%	2.9102%
IAU	2.4934%	3.3022%	0.0000%	0.3136%	3.4697%
IEO	1.3244%	0.7372%	0.3959%	3.6526%	2.3814%
IYW	0.1180%	-42.6751%	0.0000%	0.0000%	4.1330%
KWEB	0.9352%	1.3656%	0.0000%	0.0000%	2.9647%
MCHI	0.0089%	0.1388%	0.0000%	1.1583%	3.1716%
MTUM	0.6904%	0.9144%	0.9917%	0.0000%	5.9882%
PGJ	0.8473%	1.2493%	0.0000%	0.0000%	2.9270%
SCHA	0.0010%	0.0013%	0.0000%	0.0000%	3.9742%
SCHO	0.1031%	0.1365%	91.6260%	0.0000%	6.7589%
SCHX	0.0249%	0.0330%	0.0000%	0.0000%	4.7497%
SPHD	1.8745%	2.4826%	0.0000%	17.9322%	4.3033%
SPHQ	8.1402%	10.7807%	0.0000%	0.0000%	4.7127%
TLT	0.0051%	0.0068%	0.0000%	0.0000%	3.8102%
VHT	0.1153%	42.9840%	0.0000%	23.8338%	5.0552%
VIXY	4.1650%	5.5160%	0.4924%	7.7880%	0.0000%
VNQ	0.1107%	0.1466%	0.0000%	0.0000%	3.3539%
VTV	10.2223%	27.3660%	0.8251%	0.0000%	5.0976%
VUG	0.0248%	0.0328%	0.0000%	0.0000%	4.3756%

3.3 Different types of risk aversion

The exam paper asks to calculate the Black-Litterman weights under three different scenarios. Also we can get market risk aversion from bench mark (SPY).

- Market (SPY): $\lambda_{mkt} = \frac{2.966}{2} = 1.4833$
- Near-Kelly Investor:$\lambda = \frac{0.01}{2} = 0.005$
- Average Investor (Market): $\lambda = \frac{2.24}{2} = 1.12$
- Risk-averse Investor (Trustee): $\lambda = \frac{6}{2} = 3$

Risk aversion is a crucial parameter that significantly influences the allocation results. The level of risk aversion dictates how much the expected excess returns are scaled when combining the investor's views with the market equilibrium. Here's how different levels of risk aversion can affect the outcomes:

Near-Kelly Investor (Higher Risk Aversion):

- Investors with higher risk aversion place a greater emphasis on the variance (risk) component of the portfolio.
- The optimization will lean towards less risky assets, potentially reducing the weights of higher-risk assets even if they have higher expected returns.
- The resulting portfolio is typically more conservative, focusing on stability rather than maximizing returns.

Average Investor (Lower Risk Aversion):

- Investors with lower risk aversion are more willing to accept risk in exchange for potentially higher returns.
- The optimization will favor assets with higher expected returns, despite their higher risk.
- The resulting portfolio is often more aggressive, seeking to maximize returns, which can increase the portfolio's overall risk.

Risk-averse Investor (Moderate Risk Aversion):

- A moderate level of risk aversion seeks a balance between risk and return.

- The optimization will aim to construct a more diversified portfolio that neither overly prioritizes risk nor return.
- This typically leads to a balanced portfolio that aligns with the classic risk-return trade-off.

The choice of risk aversion coefficient in the Black-Litterman model should reflect the investor's individual risk tolerance and investment objectives. It is a subjective input that can significantly affect the portfolio construction process, altering the final asset weights to align with the investor's risk-reward profile.

Table9: Black-Litterman Results under Different Risk Aversion Scenarios

Assets Class(ETFs)	weight_BL RealMarket	weight_BL Kelly	weight_BL AverageMarket	weight_BL Trustee
CBON	41.9899%	358.2752%	32.0637%	31.1470%
CHIR	32.6677%	351.2363%	25.0248%	24.1080%
CQQQ	1.5374%	327.7306%	1.5191%	0.6024%
DBA	0.6093%	0.4600%	0.4600%	0.4600%
ECNS	4.1911%	329.7344%	3.5229%	2.6062%
EPS	2.9468%	2.2250%	2.2250%	2.2250%
FXI	7.8045%	332.4628%	6.2512%	5.3345%
IAU	3.3022%	2.4934%	2.4934%	2.4934%
IEO	0.7372%	-2612.0017%	-2.3097%	5.0240%
IYW	-42.6751%	-7728.1542%	-40.6662%	-19.0631%
KWEB	1.3656%	327.6009%	1.3894%	0.4727%
MCHI	0.1388%	326.6746%	0.4631%	-0.4536%
MTUM	0.9144%	0.6904%	0.6904%	0.6904%
PGJ	1.2493%	327.5131%	1.3016%	0.3849%
SCHA	0.0013%	0.0010%	0.0010%	0.0010%
SCHO	0.1365%	0.1031%	0.1031%	0.1031%
SCHX	0.0330%	0.0249%	0.0249%	0.0249%
SPHD	2.4826%	1.8745%	1.8745%	1.8745%
SPHQ	10.7807%	8.1402%	8.1402%	8.1402%
TLT	0.0068%	0.0051%	0.0051%	0.0051%
VHT	42.9840%	7728.3875%	40.8995%	19.2964%
VIXY	5.5160%	4.1650%	4.1650%	4.1650%
VNQ	0.1466%	0.1107%	0.1107%	0.1107%
VTV	27.3660%	9263.5357%	30.8039%	4.8584%
VUG	0.0328%	0.0248%	0.0248%	0.0248%

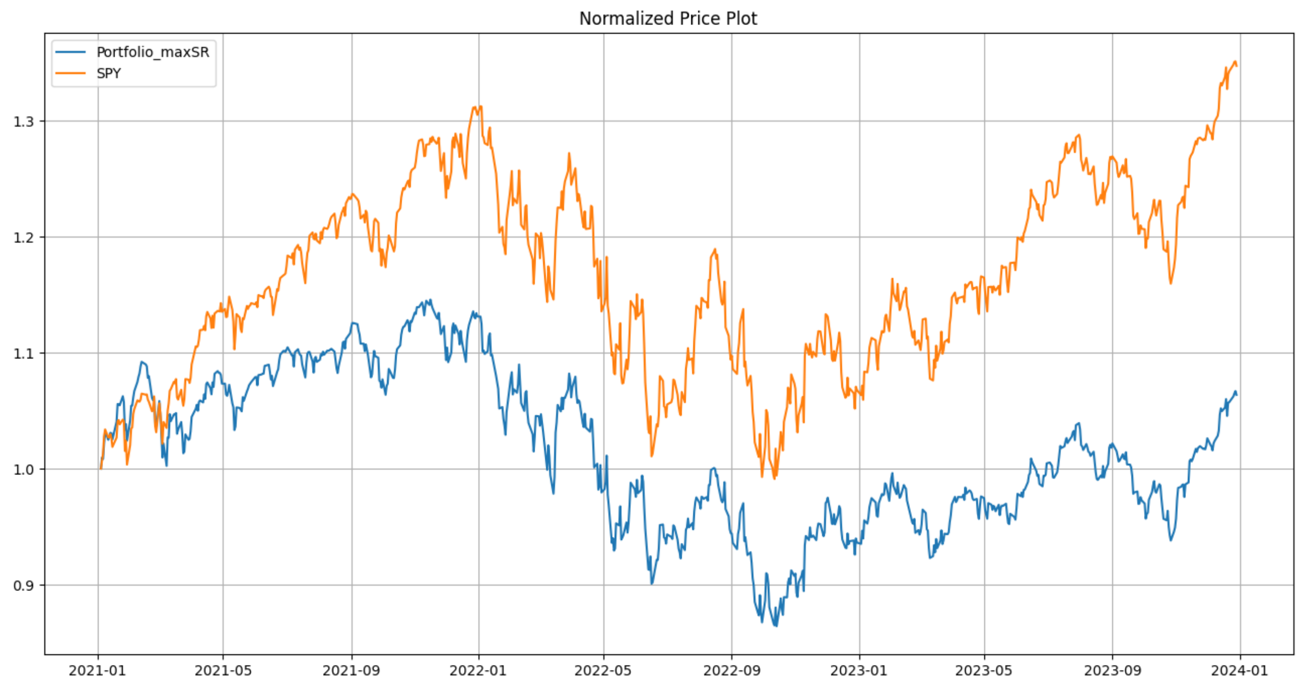
3.4 Different volatility of portfolio

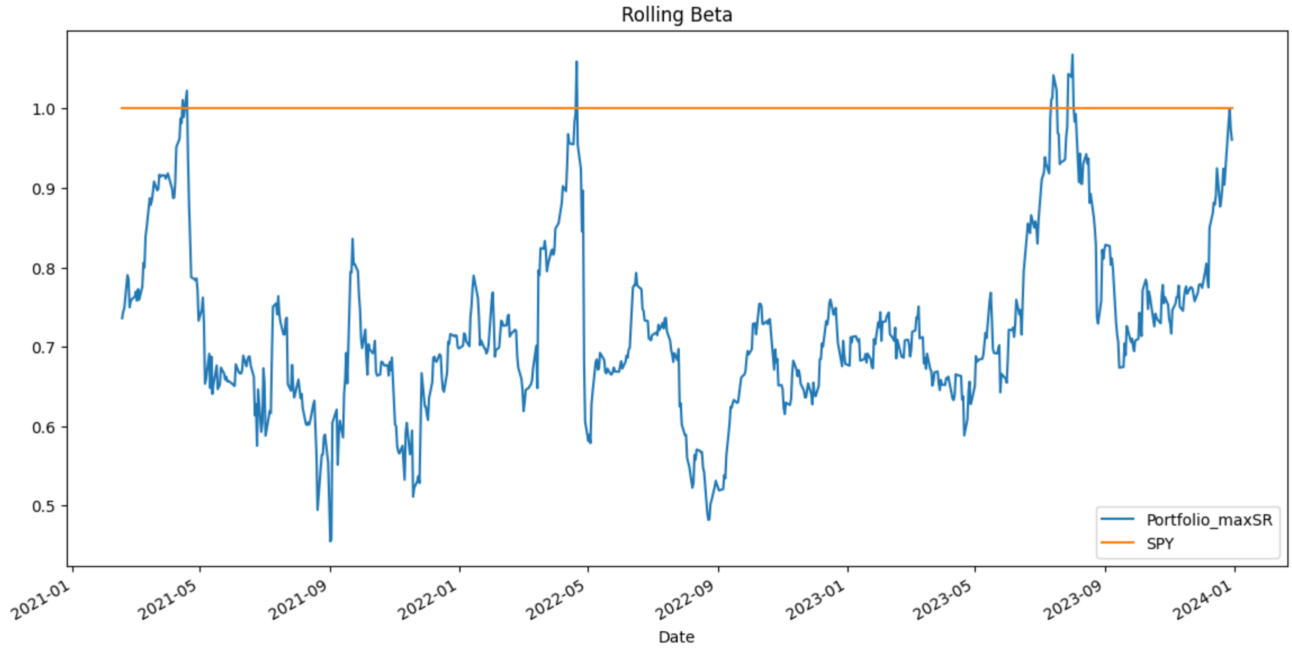
Varying the volatility assumptions within the Black-Litterman model impacts the posterior weights by altering the perceived risk of each asset and the overall risk of the portfolio. The model's inherent ability to blend market equilibrium with investor views allows it to adjust weights dynamically in response to changes in volatility, aiming to produce a portfolio that aligns with the desired risk level and investment outlook.

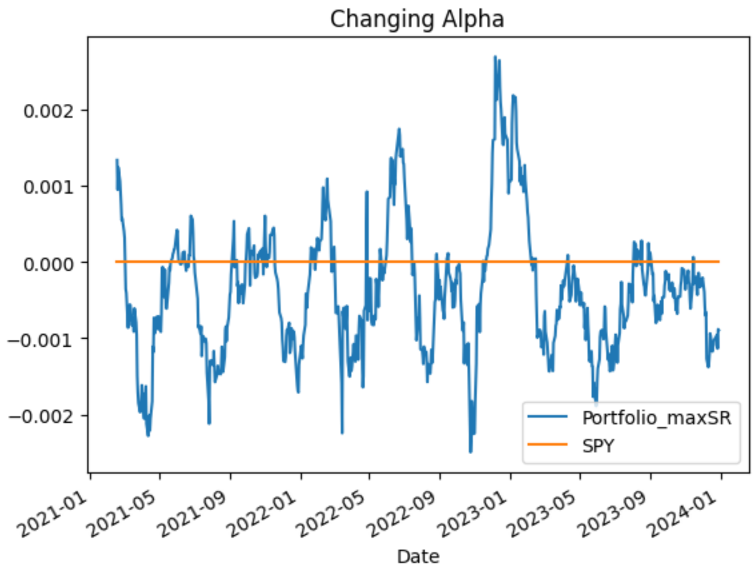
Owing to the complexities involved in generating a stacked area chart, the volatility and portfolio weight outcomes have been documented within a Jupyter notebook environment.

3.5 Compare performance of portfolio and benchmark

The Maximum Sharpe Ratio optimization was selected for comparing my portfolio against the benchmark. As evidenced by Tables 10 and 11, due to the unreliability of views and timing issues in their formulation, my portfolio did not outperform the benchmark. This is further corroborated by the lack of excess returns in the rolling beta and changing alpha analyses, along with an underperformance in market beta. A detailed discussion on the updates to the views will be provided in the 'Discussion' section.







Discussion and Improvement

Throughout the course of this project, a significant amount of time was dedicated to understanding concepts and handling data intricacies, wherein it was found that portfolio weights are greatly influenced by views and prior weights. Therefore, selecting assets in alignment with market-consistent weights becomes crucial. This necessitates filtering out ETFs corresponding to each factor and calculating their proportions. As for dynamic views, due to the substantial workload, implementing dynamic view adjustments may not be feasible in time. Here are some of my thoughts and improvements: using LSTM to update views and achieve a more automated portfolio.

To elaborate, integrating machine learning, particularly Long Short-Term Memory (LSTM) networks, into the Black-Litterman model for predicting views is a sophisticated approach that marries traditional finance theory with contemporary AI techniques. This amalgamation aids in transforming the model from a static to a dynamic framework. The process involves:

1) **Data Collection and Preprocessing:** Accumulating historical price data for portfolio assets, converting this data to log returns, normalizing it, and structuring it for LSTM applicability.
2) **LSTM Model Development:** Constructing an LSTM network to forecast future returns and training this model with historical data.
3) **Integration of Predicted Views:** Translating LSTM-predicted returns into the views matrix for the Black-Litterman model and defining the confidence level in these views.
4) **Dynamic Black-Litterman Model:** Regularly updating views with LSTM predictions and recalculating posterior returns and weights.
5) **Back-testing and Evaluation:** Assessing the dynamic model's performance and fine-tuning based on the results.

Key considerations include the complexity and potential overfitting of LSTM models, the reliance on data quality and availability, and the interpretability challenges of machine learning models. By leveraging LSTM for dynamic views, the traditional Black-Litterman model can be enhanced to be more adaptive to market shifts and new data, albeit contingent on data integrity, machine learning model robustness, and seamless integration of views into the framework.

Conclusion

The findings underscore the complexity of portfolio management in the Black-Litterman framework, highlighting the significant influence of views and prior weights on portfolio outcomes. The project reveals the importance of aligning asset selection with market weights and suggests enhancements like integrating LSTM for dynamic view adjustments. This approach may offer a more responsive and automated portfolio that adapts to market shifts, contingent on data integrity and model robustness. Future improvements could include machine learning classifiers and advanced statistical techniques to refine asset pricing and portfolio optimization strategies.

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