Chap 5 : Exponential Distribution.

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$$

$$F(x) = \int_{-\infty}^{x} f(y)\, dy = 1 - e^{-\lambda x}$$

$$P\{X > x\} = 1 - F(x) = e^{-\lambda x}$$

$$E[X] = \int_{-\infty}^{\infty} x f(x)\, dx = \frac{1}{\lambda}$$

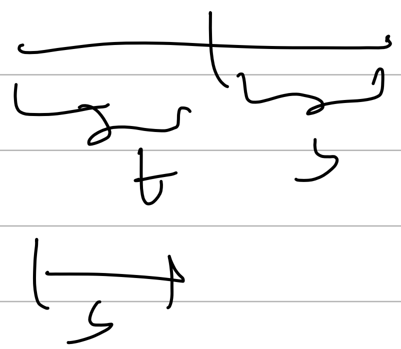
$$\phi(t) = \frac{\lambda}{\lambda - t}$$

$$Var[X] = \frac{1}{\lambda^2}$$

Properties of Exponential Distribution

(i) A random variable X is said to be without memory or memoryless if

$$P\{X > s + t \mid X > t\} = P\{X > s\}$$



For Exp. Distribu

$$\frac{P\{X > s + t,\ X > t\}}{P\{X > t\}} = P\{X > s\}$$

$$P\{X > s + t\} = P\{X > s\}\, P\{X > t\}$$

$$e^{-\lambda(s+t)} = e^{-\lambda s} e^{-\lambda t}$$

equal , memoryless

exponential is the only memoryless dist.

Ex:

X: Amount of ~~t~~ time a customer spends in a bank

$\lambda \rightarrow$ arrival rate.

$\frac{1}{10}$ cust/sec.

(i) $P\{X > 15\} = e^{-\lambda x} = e^{-\frac{1}{10} 15}$

$= e^{-3/2}$

(ii) $P\{X > 15 \mid X \geq 10\}$

$= P\{X > 5\} = e^{-\frac{1}{10} \times 5} = e^{-1/2}$

Exponential Distribution (Memoryless)

* Lifetime of a bulb is exp. distr. with mean 10 h.

$P\{\text{lifetime} > t+5 \mid \text{lifetime} > t\}$

$$= \frac{1 - F(t+5)}{1 - F(t)}$$

$$P\{\text{lifetime} > 5\} = 1 - F(5)$$

$$= 1 - (1 - e^{-\lambda 5})$$

$$= e^{-5\lambda} = e^{-5 \times \frac{1}{10}}$$

$$= e^{-\frac{1}{2}}$$

$$\begin{cases} // \lambda = \frac{1}{10} \\ // \lambda = \frac{1}{\text{mean}} \end{cases}$$

2 টি bulb. 1 টি 5 hour
2 টি 10 hour জ্বলে

$P\{X_1 < X_2\}$

X_1 and X_2 are independent RVs with

mean $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}$

$$\int_0^\infty P\{X_1 < X_2 \mid X_1 = x\} \lambda_1 e^{-\lambda_1 x} \, dx$$

$$= \int_0^\infty P\{X_2 > x\} \lambda_1 e^{-\lambda_1 x} \, dx$$

$$= \int_0^\infty e^{-\lambda_2 x} \lambda_1 e^{-\lambda_1 x} \, dx$$

$$= \lambda_1 \left[\frac{e^{-(\lambda_1 + \lambda_2) x}}{-(\lambda_1 + \lambda_2)} \right]_0^\infty = \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

$$\lambda_1 = \frac{1}{1000} \qquad \lambda_2 = \frac{1}{500}$$

$$P\{X_1 < X_2\} = \frac{\frac{1}{1000}}{\frac{1}{1000} + \frac{1}{500}} = \frac{1}{3}$$

Counting Process

It's a stochastic process (Collection of RV)

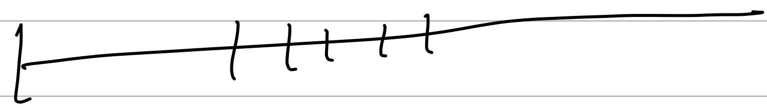
A stochastic process $\{N(t), t \geq 0\}$ is said to be a counting process if $N(t)$ represents the ~~total~~ number of events up to time t.

(i) $N(t) \geq 0$

(ii) $N(t)$ is integer

(iii) if $s < t$, $N(s) \leq N(t)$

(iv) for $s < t$, $N(t) - N(s) = \#$ of events in $(s, t]$



① Independent Increment

A counting process is said to possess independent increment if the number of events in disjoint time intervals are independent.

② Stationary Increment

কোনো interval এ কতগুলো আছে তা interval এর length এর উপর নির্ভর করে।

A counting process is said to possess stationary increment if the distribution of # events in any interval depends only on the length of the interval.

Poisson Process:

A counting process $\{N(t), t \geq 0\}$ is said to be a Poisson process having rate λ, $(\lambda > 0)$ if

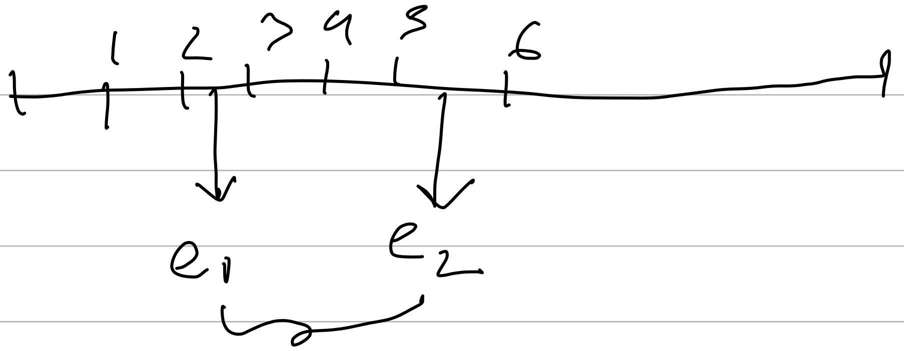
(i) $N(0) = 0$

(ii) The process has independent increment property.

(iii) The process has stationary inc. property

i.e., The # of events in an interval of length t is poisson distributed, with mean λt.

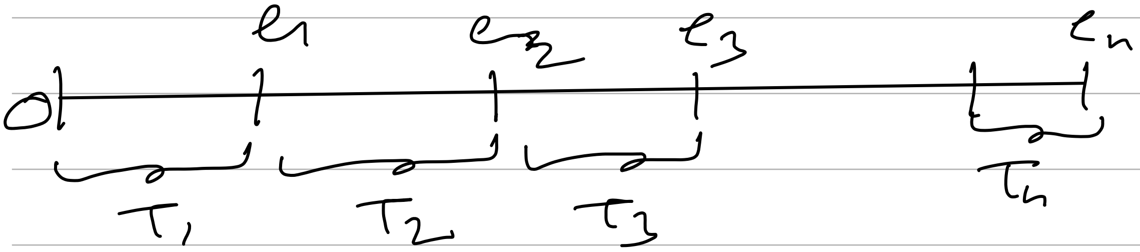
$$P\{N(t+s) - N(s) = n\} = e^{-\lambda t} \frac{(\lambda t)^n}{n!}$$



time টি exponentially distributed.

Distribution of Inter arrival time

2টি event এর মধ্যবর্তী সময়



$\{T_n, \ n = 1, 2 \ldots\}$, inter arrival time.

$$P\{T_1 > t\} = P\{\text{no events in } (0, t]\}$$
$$= P\{N(t) = 0\}$$

$$= \frac{e^{-\lambda t}(\lambda t)^{0}}{0!} = e^{-\lambda t}$$

$\therefore T_1$ is exponentially distributed with parameter λ, mean $\frac{1}{\lambda}$.

$$P\{T_2 > t \mid T_1 = s\}$$

$$= P\{\text{No events in } (s, s+t] \mid T_1 = s\}$$

$$= P\{\text{No events in } (s, s+t]\}$$ // independent increment // $T_1 = s$ ~~and~~

$$= P\{0 \text{ events in } (0, t]\}$$

[stationary inc]

$$= e^{-\lambda t}$$

$\therefore T_2$ is exp dist. ~~me~~ with mean $\frac{1}{\lambda}$

$\rightarrow T_n$ '' . '' '' '' $\frac{1}{\lambda}$