Mutation-এ, অল্প change হয়,

12/7/23

CSE-301

Probability models

Sample Space:

$S = \{H, T\}$; $S = \{1, 2, 3, 4, 5, 6\}$

~~Each value is called an event~~

Event: Any subset E of sample space S

E, F are mutually exclusive if $E \cap F = \emptyset$

$P(D \mid T)\, P(T)$

a function on event

Probability : $P(E)$

i) $0 \le P(E) \le 1$

ii) $P(S) = 1$

iii) $E_1, E_2, \ldots$ mutually exclusive events, $P\left(\bigcup_{i=1}^{\infty} E_i\right) = \sum_{i=1}^{\infty} P(E_i)$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$

Not mutually Exclusive

Ex: $S = \{HH, HT, TH, TT\}$

First head
$E = \{HH, HT\}$

Next head
$F = \{TH, HH\}$

$P(E) = \frac{2}{4} = \frac{1}{2}$

$P(F) = \frac{1}{2}$

$$P(E \cup F) = P(E) + P(F) - P(E \cap F)$$
$$= \frac{1}{2} + \frac{1}{2} - \frac{1}{4} = \frac{3}{4}$$

$$P\left(\bigcup_{i=1}^{n} E_i\right) = \sum_{i=1}^{n} P(E_i) - \sum_{i<j} P(E_iE_j) + \sum_{i<j<k} P(E_iE_jE_k) - \cdots + (-1)^{n+1} P(E_1E_2 \ldots E_n)$$

Not mutually exclusive

$\boxed{E_i \cap E_j \rightarrow E_iE_j}$

Conditional Probability:

$$P(E|F) = \frac{P(EF)}{P(F)}$$

Ex: Number 1 – 10. E be the event that number is 10. F be the event that the number is at least 5.

$$P(E|F) = \frac{P(EF)}{P(F)} = \frac{\frac{1}{10}}{\frac{6}{10}} = \frac{1}{6}$$

Ex: $S = \{bb, bg, gb, gg\}$

E = both boys , F = at least one boy.

$$P(E|F) = \frac{P(EF)}{P(F)} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$$

Ex: 3 player, কেউই নিজের cap পাবে না,

E_i = Event that ith man gets his own hat.

P(non gets his own hat) = ?

Calculation on next page

$$1 - P(E_1 \cup E_2 \cup \cdots \cup E_n) = 1 - \frac{2}{3} = \frac{1}{3}$$

$.95 \times$

$.01 \times$ 99.5%

HTHH,

$P(E_i) = \frac{1}{3}$, $P(E_iE_j) = P(E_j|E_i)\,P(E_i)$

$= \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$P(E_iE_jE_k) = P(E_k|E_iE_j)\,P(E_iE_j)$

$= \frac{1}{1} \times \frac{1}{6} = \frac{1}{6}$

$P(E_1 \cup E_2 \cup E_3) = P(E_1) + P(E_2) + P(E_3) - P(E_1E_2) - P(E_2E_3) - P(E_1E_3) + P(E_1E_2E_3)$

$= \frac{1}{3} + \frac{1}{3} + \frac{1}{3} - \frac{1}{6} - \frac{1}{6} - \frac{1}{6} + \frac{1}{6} = \frac{2}{3}$

$P(E|F) = \frac{P(EF)}{P(F)}$; $P(EF) = P(E|F)\,P(F)$

if E & F independent, $P(E|F) = P(E)$

$P(EF) = P(E) \cdot P(F)$

$F \cup F^c = S$

Bayes Theorem:

$$P(F|E) = \frac{P(E|F)\,P(F)}{P(E|F)\,P(F) + P(E|F^c)\,P(F^c)}$$

$F_1, F_2, \ldots F_n$ mutually exclusive $\bigcup_{i=1}^{n} F_i = S$

$$P(F_j|E) = \frac{P(E|F_j)\,P(F_j)}{\sum_{k=1}^{n} P(E|F_k)\,P(F_k)}$$

Ex:

l — word ; $f_1, f_2, f_3 \rightarrow$ files

F_i = Event that l is in file i.

$P(F_1) = P(F_2) = P(F_3) = \frac{1}{3}$

α_i (Probability of finding l in file i by Q search)

E = Q Searched file $\underline{1}$ & did not find l

$$P(F_1 \mid E) = ? = \frac{P(E \mid F_1)\,P(F_1)}{P(E \mid F_1)P(F_1) + P(E \mid F_2)P(F_2) + P(E \mid F_3)P(F_3)}$$

searching in file 1 but l is in 2

$$= \frac{(1-\alpha_1)\cdot\frac{1}{3}}{(1-\alpha_1)\frac{1}{3} + 1 \times \frac{1}{3} + 1 \times \frac{1}{3}}$$

$$= \frac{1-\alpha_1}{3-\alpha_1}$$