Chapter 3

Problem Solving By Searching

Problem-Solving Agents

- Intelligent agents can solve problems by searching a state-space

- State-space Model
  - the agent’s model of the world
  - usually a set of discrete states
  - e.g., in driving, the states in the model could be towns/cities

- Goal State(s)
  - a goal is defined as a desirable state for an agent
  - there may be many states which satisfy the goal test
    - e.g., drive to a town with a ski-resort
  - or just one state which satisfies the goal
    - e.g., drive to Mammoth

- Operators (actions, successor function)
  - operators are legal actions which the agent can take to move from one state to another

Initial Simplifying Assumptions

- Environment is static
  - no changes in environment while problem is being solved
- Environment is observable
- Environment and actions are discrete
  - (typically assumed, but we will see some exceptions)
- Environment is deterministic

Example: Traveling in Romania

- On holiday in Romania; currently in Arad
- Flight leaves tomorrow from Bucharest

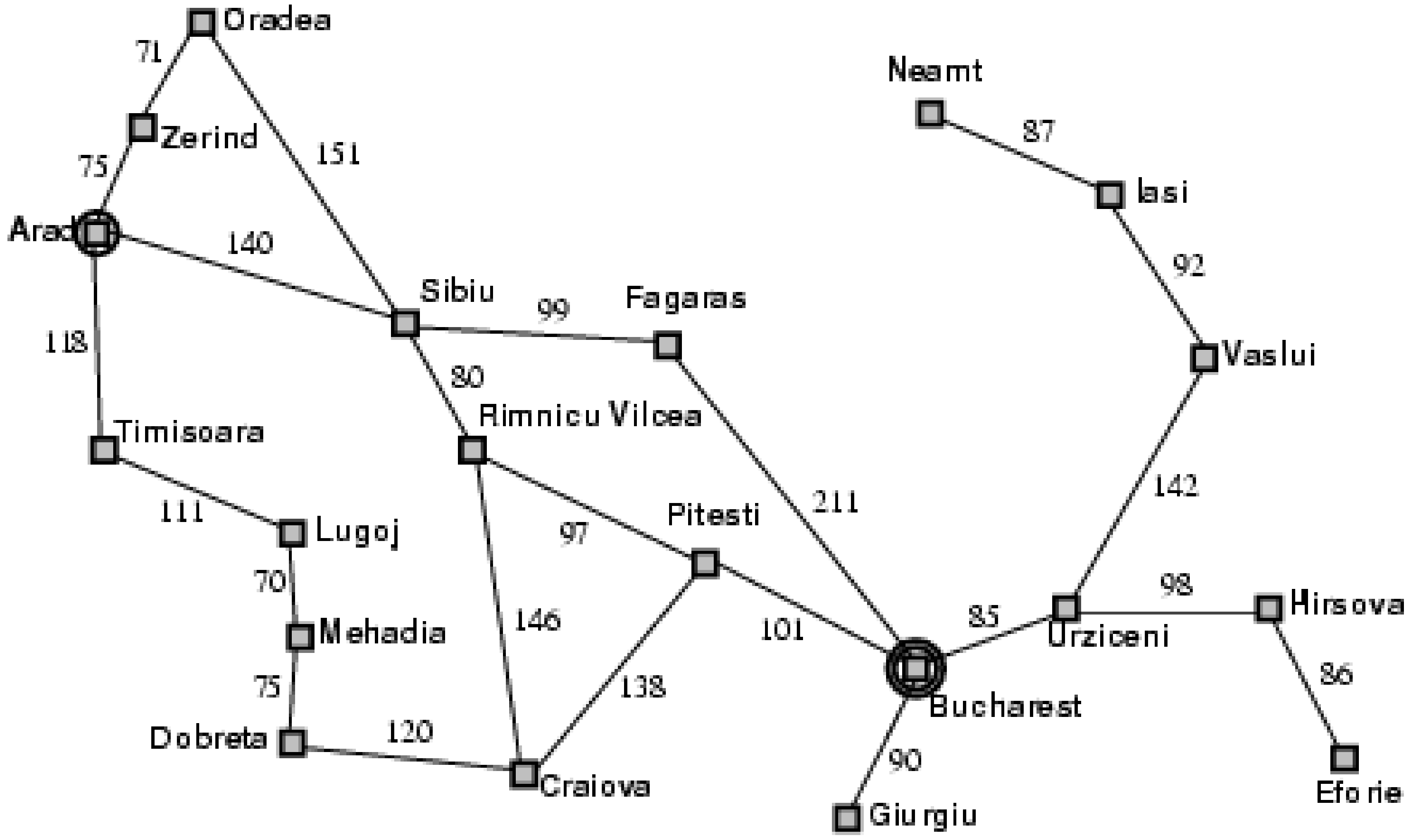
- Formulate goal:
  - be in Bucharest

- Formulate problem:
  - states: various cities
  - actions/operators: drive between cities

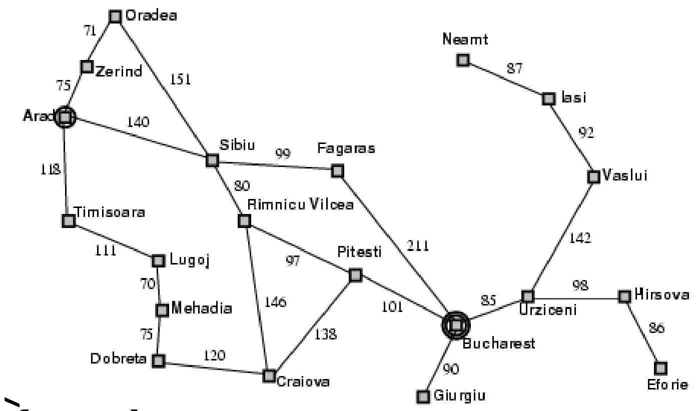
- Find solution
  - By searching through states to find a goal
  - sequence of cities, e.g., Arad, Sibiu, Fagaras, Bucharest

- Execute states that lead to a solution

Example: Traveling in Romania



State-Space Problem Formulation



A **problem** is defined by four items:

1. **initial state** e.g., "at Arad"

2. **actions** or successor function
   S(x) = set of action–state pairs
   e.g., *S(Arad) = {<Arad → Zerind, Zerind>, … }*

3. **goal test** (or set of goal states)
   e.g., *x* = "at Bucharest", *Checkmate(x)*

4. **path cost** (additive)
   e.g., sum of distances, number of actions executed, etc.
   c(x,a,y) is the step cost, assumed to be ≥ 0

A **solution** is a sequence of actions leading from the initial state to a goal state

Example: Formulating the Navigation Problem

- Set of States
  - individual cities
  - e.g., Irvine, SF, Las Vegas, Reno, Boise, Phoenix, Denver
- Operators
  - freeway routes from one city to another
  - e.g., Irvine to SF via 5, SF to Seattle, etc
- Start State
  - current city where we are, Irvine
- Goal States
  - set of cities we would like to be in
  - e.g., cities which are closer than Irvine
- Solution
  - a specific goal city, e.g., Boise
  - a sequence of operators which get us there,
    - e.g., Irvine to SF via 5, SF to Reno via 80, etc

Abstraction

- Definition of Abstraction:
  Process of removing irrelevant detail to create an abstract representation: ``high-level”, ignores irrelevant details

- Navigation Example: how do we define states and operators?
  - First step is to abstract “the big picture”
    - i.e., solve a map problem
    - nodes = cities, links = freeways/roads (a high-level description)
    - this description is an abstraction of the real problem
  - Can later worry about details like freeway onramps, refueling, etc

- Abstraction is critical for automated problem solving
  - must create an approximate, simplified, model of the world for the computer to deal with: real-world is too detailed to model exactly
  - good abstractions retain all important details

The State-Space Graph

- Graphs:
  - nodes, arcs, directed arcs, paths

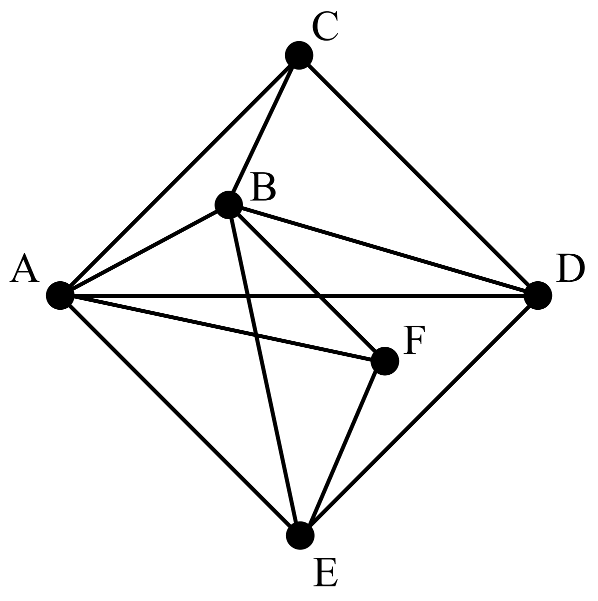
- Search graphs:
  - States are nodes
  - operators are directed arcs
  - solution is a path from start S to goal G

- Problem formulation:
  - Give an abstract description of states, operators, initial state and goal state.

- Problem solving:
  - Generate a part of the search space that contains a solution

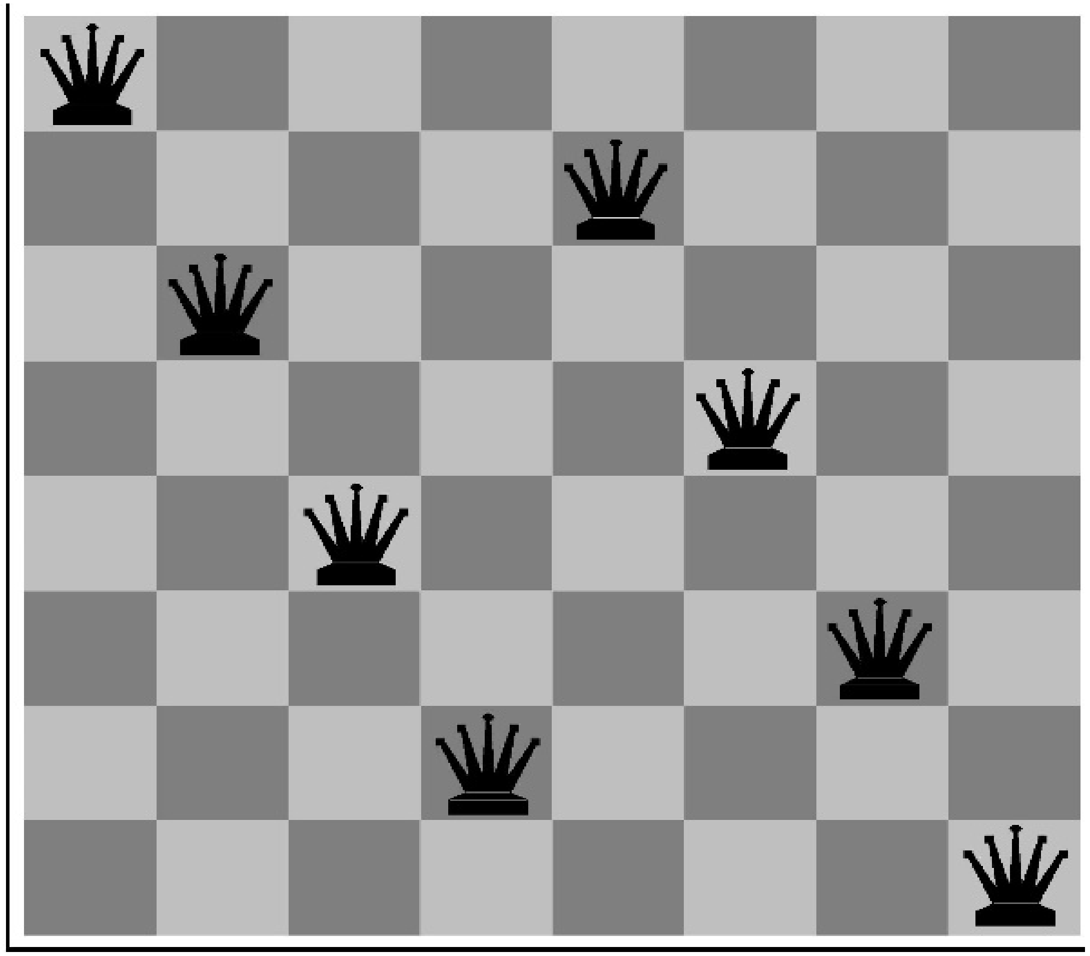
The Traveling Salesperson Problem

- Find the shortest tour that visits all cities without visiting any city twice and return to starting point.
- State: sequence of cities visited
- S_0 = A

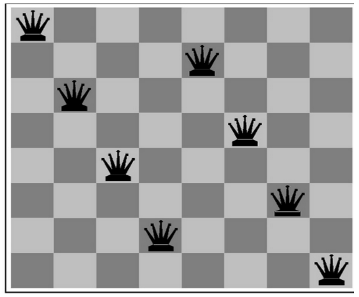


- G = a complete tour

Example: 8-queens problem



State-Space problem formulation



- states? -any arrangement of n<=8 queens
  -*or* arrangements of n<=8 queens in leftmost n columns, 1 per column, such that no queen attacks any other.

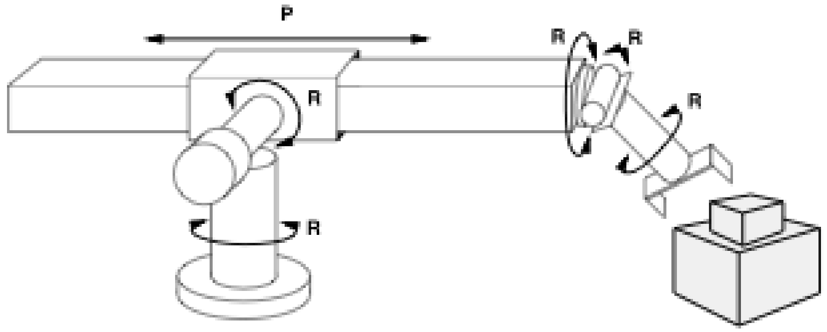
- initial state? no queens on the board

- actions? -add queen to any empty square
  -*or* add queen to leftmost empty square such that it is not attacked by other queens.

- goal test? 8 queens on the board, none attacked.

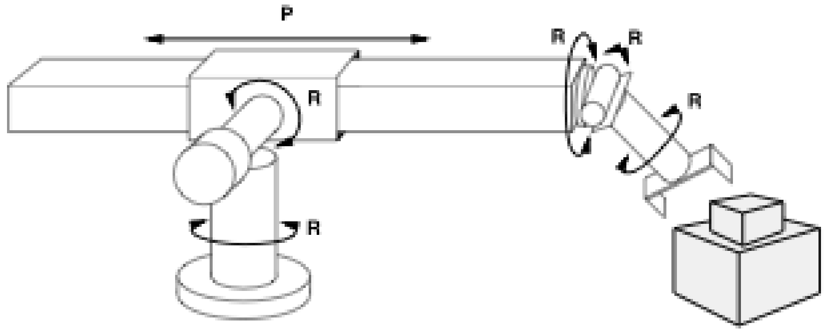
- path cost? 1 per move

Example: Robot Assembly



- States
- Initial state
- Actions
- Goal test
- Path Cost

Example: Robot Assembly



- States: configuration of robot (angles, positions) and object parts
- Initial state: any configuration of robot and object parts
- Actions: continuous motion of robot joints
- Goal test: object assembled?
- Path Cost: time-taken or number of actions

Learning a spam email classifier

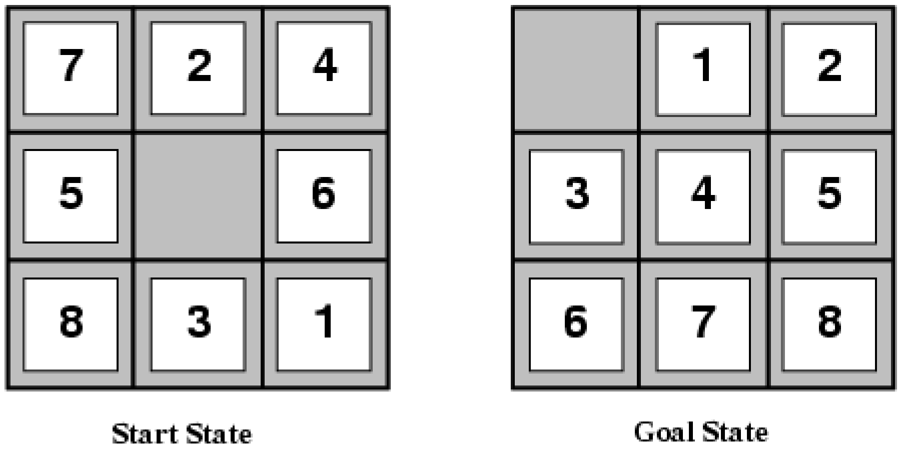
- States
- Initial state
- Actions
- Goal test
- Path Cost

Learning a spam email classifier

- States: settings of the parameters in our model
- Initial state: random parameter settings
- Actions: moving in parameter space
- Goal test: optimal accuracy on the training data
- Path Cost: time taken to find optimal parameters

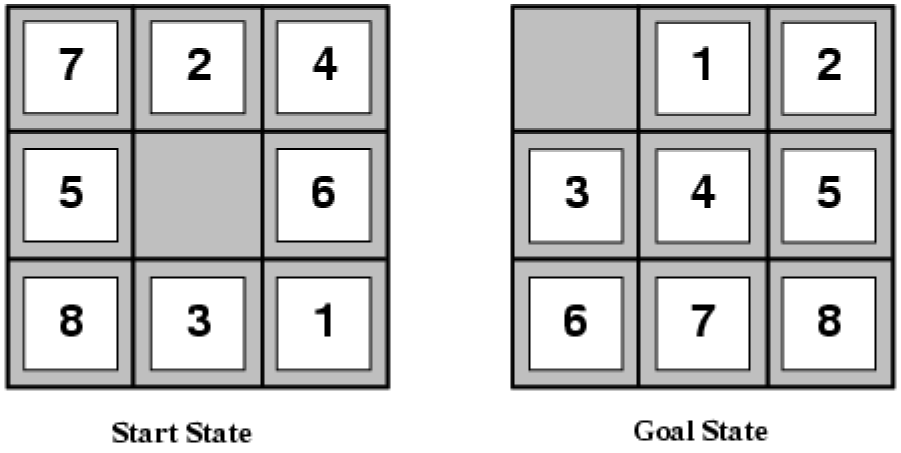
(Note: this is an optimization problem – many machine learning problems can be cast as optimization)

Example: 8-puzzle



- states?
- initial state?
- actions?
- goal test?
- path cost?

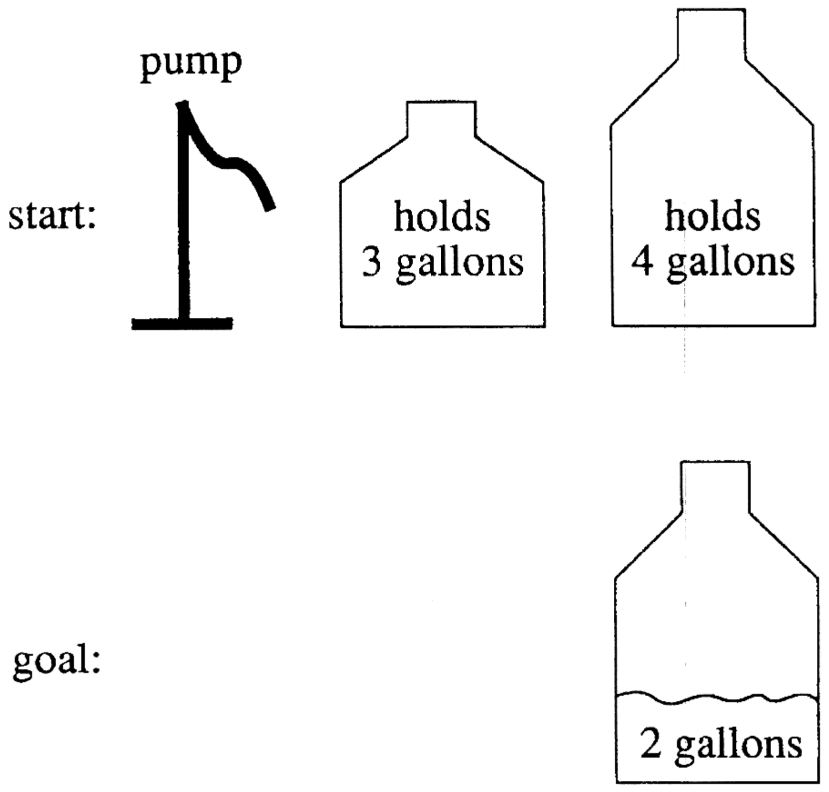
Example: 8-puzzle



- states? locations of tiles
- initial state? given
- actions? move blank left, right, up, down
- goal test? goal state (given)
- path cost? 1 per move

A Water Jug Problem

- You have a 4-gallon and a 3-gallon water jug
- You have a faucet with an unlimited amount of water
- You need to get exactly 2 gallons in 4-gallon jug



Puzzle-solving as Search

- State representation**: (x, y)**
  - x: Contents of four gallon
  - y: Contents of three gallon
- Start state**: (0, 0)**
- Goal state **(2, n)**
- Operators
  - Fill 3-gallon from faucet, fill 4-gallon from faucet
  - Fill 3-gallon from 4-gallon , fill 4-gallon from 3-gallon
  - Empty 3-gallon into 4-gallon, empty 4-gallon into 3-gallon
  - Dump 3-gallon down drain, dump 4-gallon down drain

Production Rules for the Water Jug Problem

1 (x,y) ☾ $(4,y)$ if $x < 4$	Fill the 4-gallon jug
2 (x,y) ☾ $(x,3)$ if $y < 3$	Fill the 3-gallon jug
3 (x,y) ☾ $(x - d,y)$ if $x > 0$	Pour some water out of the 4-gallon jug
4 (x,y) ☾ $(x,y - d)$ if $x > 0$	Pour some water out of the 3-gallon jug
5 (x,y) ☾ $(0,y)$ if $x > 0$	Empty the 4-gallon jug on the ground
6 (x,y) ☾ $(x,0)$ if $y > 0$	Empty the 3-gallon jug on the ground
7 (x,y) ☾ $(4,y - (4 - x))$ if $x + y \geq 4$ and $y > 0$	Pour water from the 3-gallon jug into the 4-gallon jug until the 4-gallon jug is full

The Water Jug Problem (cont’d)

8 (x,y) ☾ $(x - (3 - y),3)$ if $x + y \geq 3$ and $x > 0$	Pour water from the 4-gallon jug into the 3-gallon jug until the 3-gallon jug is full
9 (x,y) ☾ $(x + y, 0)$ if $x + y \leq 4$ and $y > 0$	Pour all the water from the 3-gallon jug into the 4-gallon jug
10 (x,y) ☾ $(0, x + y)$ if $x + y \leq 3$ and $x > 0$	Pour all the water from the 4-gallon jug into the 3-gallon jug

One Solution to the Water Jug Problem

Gallons in the 4-Gallon Jug	Gallons in the 3-Gallon Jug	Rule Applied
0	0	2
0	3	9
3	0	2
3	3	7
4	2	5
0	2	9
2	0	

VLSI Layout Problem

- Require positioning millions of components and connections on a chip to **minimize area**, **minimize circuit delays**, **minimize stray capacitances**, and **maximize manufacturing yield**
- The layout problem comes after the logical design phase and is usually split into two parts: **cell layout** and **channel routing**
- In cell layout, the primitive components of the circuit are grouped into cells, each of which performs some recognized function

VLSI Layout Problem (cont’d)

- Each cell has a fixed footprint (size and shape) and requires a certain number of connections to each of the other cells
- The aim is to place the cells on the chip so that they do not overlap and so that there is room for the connecting wires to be placed between the cells
- Channel routing finds a specific route for each wire through the gaps between the cells. These search problems are extremely complex, but definitely worth solving.

Next Topics

- Uninformed search
  - Breadth-first, depth-first
  - Uniform cost
  - Iterative deepening

- Informed (heuristic) search
  - Greedy best-first
  - A*
  - Memory-bounded heuristic search
  - And more….

- Local search and optimization
  - Hill-climbing
  - Simulated annealing
  - Genetic algorithms

Summary

- Problem-solving agents where search consists of
  - state space
  - operators
  - start state
  - goal states

- Abstraction and problem formulation