

定義をまとめておく．断りが無い限り X, Y は scheme であり， A, B, A_i, B_i は ring である．

1 About Scheme Property.

定義 1.1 (Locally Noetherian Scheme).

$$\begin{aligned} X &:: \text{locally noetherian} \\ \iff \exists \{A_i\}_{i \in I}, \quad &\left[X = \bigcup_{i \in I} \text{Spec } A_i \right] \wedge [A_i :: \text{noetherian.}] \\ \iff \forall \text{Spec } A \subseteq X, \quad &A :: \text{noetherian.} \end{aligned}$$

定義 1.2 (Noetherian Scheme).

$$\begin{aligned} X &:: \text{noetherian} \\ \iff \exists \{A_i\}_{i=1}^r, \quad &\left[X = \bigcup_{i=1}^r \text{Spec } A_i \right] \wedge [A_i :: \text{noetherian.}] \end{aligned}$$

定義 1.3 (Quasi-Compact Scheme).

$$\begin{aligned} X &:: \text{quasi-compact} \\ \iff \text{sp}(X) &:: \text{quasi-compact} \end{aligned}$$

以下の 3 つは教科書にないが，便宜上導入する．

定義 1.4 (Locally Finite Type Scheme over B).

$$\begin{aligned} X &:: \text{locally finite type scheme over } B \\ \iff \exists \{A_i\}_{i \in I}, \quad &\left[X = \bigcup_{i \in I} \text{Spec } A_i \right] \wedge [A_i :: \text{fin.gen. } B\text{-algebra.}] \end{aligned}$$

定義 1.5 (Finite Type Scheme over B).

$$\begin{aligned} X &:: B\text{-Fin.Gen. Scheme} \\ \iff \exists \{A_i\}_{i=1}^r, \quad &\left[X = \bigcup_{i=1}^r \text{Spec } A_i \right] \wedge [A_i :: \text{fin.gen. } B\text{-algebra.}] \end{aligned}$$

定義 1.6 (Finite Scheme over B).

$$\begin{aligned} X &:: B\text{-Fin.Gen. Scheme} \\ \iff \exists A, \quad &[X = \text{Spec } A] \wedge [A :: \text{fin.gen. } B\text{-algebra.}] \end{aligned}$$

2 About Scheme Morphism Property.

この section で $f : X \rightarrow Y$ は scheme morphism である．

定義 2.1 (Locally of Finite Type).

$$\begin{aligned}
& f :: \text{locally of finite type} \\
& \iff \exists \{B_i\}_{i \in I}, \left[Y = \bigcup_{i \in I} \text{Spec } B_i \right] \wedge [\forall i \in I, f^{-1}(\text{Spec } B_i) :: \text{locally finite type scheme over } B_i.] \\
& \stackrel{\text{Ex3.1}}{\iff} \forall \text{Spec } A \subseteq Y, f^{-1}(\text{Spec } A) :: \text{locally finite type scheme over } A.
\end{aligned}$$

定義 2.2 (Finite Type).

$$\begin{aligned}
& f :: \text{finite type} \\
& \iff \exists \{B_i\}_{i \in I}, \left[Y = \bigcup_{i \in I} \text{Spec } B_i \right] \wedge [\forall i \in I, f^{-1}(\text{Spec } B_i) :: \text{finite type scheme over } B_i.] \\
& \stackrel{\text{Ex3.3a}}{\iff} f :: \text{locally finite type+quasi-compact.} \\
& \stackrel{\text{Ex3.3b}}{\iff} \forall \text{Spec } A \subseteq Y, f^{-1}(\text{Spec } A) :: \text{finite type scheme over } A.
\end{aligned}$$

定義 2.3 (Finite).

$$\begin{aligned}
& f :: \text{finite} \\
& \iff \exists \{B_i\}_{i \in I}, \left[Y = \bigcup_{i \in I} \text{Spec } B_i \right] \wedge [\forall i \in I, f^{-1}(\text{Spec } B_i) :: \text{finite scheme over } B_i.] \\
& \stackrel{\text{Ex3.4}}{\iff} \forall \text{Spec } A \subseteq Y, f^{-1}(\text{Spec } A) :: \text{finite scheme over } A.
\end{aligned}$$

定義 2.4 (Quasi-Compact).

$$\begin{aligned}
& f :: \text{quasi-compact} \\
& \iff \exists \{B_i\}_{i \in I}, \left[Y = \bigcup_{i \in I} \text{Spec } B_i \right] \wedge [\forall i \in I, f^{-1}(\text{Spec } B_i) :: \text{quasi-compact.}] \\
& \stackrel{\text{Ex3.2}}{\iff} \forall \text{Spec } A \subseteq Y, f^{-1}(\text{Spec } A) :: \text{quasi-compact.}
\end{aligned}$$