# **HW17** Essemble

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Setup: Download the data, load it in your script, and omit any rows with missing values (NAs)

```
df <- read.csv('insurance.csv', header = T)
insurance <- na.omit(df)
str(insurance)</pre>
```

```
df <- read.csv('insurance.csv', header = T)
insurance <- na.omit(df)
str(insurance)

## 'data.frame': 1338 obs. of 7 variables:
## $ age : int 19 18 28 33 32 31 46 37 37 60 ...
## $ sex : chr "female" "male" "male" "male" ...
## $ bmi : num 27.9 33.8 33 22.7 28.9 ...
## $ children: int 0 1 3 0 0 0 1 3 2 0 ...
## $ smoker : chr "yes" "no" "no" "...
## $ region : chr "southwest" "southeast" "northwest" ...
## $ charges : num 16885 1726 4449 21984 3867 ...</pre>
```

```
k fold mse <- function(model, dataset, outcome, k=10) {
  shuffled_indicies <- sample(1:nrow(dataset))</pre>
  dataset <- dataset[shuffled indicies,]</pre>
  fold_pred_errors <- sapply(1:k, \(kth) {</pre>
    fold i pe(kth, k, model, dataset, outcome)
})
  pred errors <- unlist(fold pred errors)</pre>
  mse <- \(errs\) mean(errs^2)</pre>
  c(is = mse(residuals(model)), oos = mse(pred errors))
}
fold i pe <- function(i, k, model, dataset, outcome) {</pre>
  folds <- cut(1:nrow(dataset), breaks=k, labels=FALSE)</pre>
  test indices <- which(folds==i)</pre>
  test_set <- dataset[test_indices, ]</pre>
  train_set <- dataset[-test_indices, ]</pre>
  trained_model <- update(model, data = train_set)</pre>
  predictions <- predict(trained_model, test_set)</pre>
  dataset[test indices, outcome] - predictions
```

Question 1) Create some explanatory models to learn more about charges:

a. Create an OLS regression model and report which factors are significantly related to charges

```
ins_lm <- lm(charges ~ ., data=insurance)</pre>
summary(ins lm)
## Call:
## lm(formula = charges ~ ., data = insurance)
## Residuals:
                      Median
##
       Min
                 10
                                   3Q
                                          Max
## -11304.9 -2848.1
                      -982.1
                               1393.9 29992.8
##
## Coefficients:
                  Estimate Std. Error t value Pr(>|t|)
##
## (Intercept)
                  -11938.5
                                987.8 -12.086 < 2e-16 ***
                                11.9 21.587 < 2e-16 ***
## age
                     256.9
## sexmale
                    -131.3
                                332.9 -0.394 0.693348
## bmi
                                28.6 11.860 < 2e-16 ***
                     339.2
## children
                    475.5
                                137.8 3.451 0.000577 ***
## smokeryes
                   23848.5
                                413.1 57.723 < 2e-16 ***
## regionnorthwest -353.0
                                476.3 -0.741 0.458769
## regionsoutheast -1035.0
                                478.7 -2.162 0.030782 *
## regionsouthwest -960.0
                                477.9 -2.009 0.044765 *
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.01 '* 0.05 '.' 0.1 ' ' 1
## Residual standard error: 6062 on 1329 degrees of freedom
## Multiple R-squared: 0.7509, Adjusted R-squared: 0.7494
## F-statistic: 500.8 on 8 and 1329 DF, p-value: < 2.2e-16
```

#### Ans:

Age, childern, bmi, smokeyes, region southeast and southwest are significant. Meanwhile the rest are not which can be dropped for model improvement.

b. Create a decision (regression) tree with default parameters

```
tree model <- rpart(charges ~ ., data = insurance )</pre>
  tree model
## n= 1338
##
## node), split, n, deviance, yval
##
        * denotes terminal node
##
## 1) root 1338 196074200000 13270.420
##
    2) smoker=no 1064 38188720000 8434.268
      4) age< 42.5 596 13198540000 5398.850 *
##
      5) age>=42.5 468 12505450000 12299.890 *
##
##
    3) smoker=yes 274 36365600000 32050.230
##
      6) bmi< 30.01 130 3286655000 21369.220 *
      ##
```

(See Next Page)

i. Plot a visual representation of the tree
 Here, I only show the plot of Method 2.
 However, welcome to apply my code if you're interested.

```
# Method 1
prp(tree_model, # Model
                # No abbreviation
                 faclen=0,
                 # vertical branches
                fallen.leaves=TRUE,
                                                                                                                                                                                                                                                        yes smoker = no no
                # Leaf's shadow
                 shadow.col="gray",
                                                                                                                                                                                                                               age < 43
                                                                                                                                                                                                                                                                                                                       bmi < 30 .
# number of correct classificatio
ns / number of observations in th
at node
                                                                                                                                                                                                             5399
                                                                                                                                                                                                                                                        12e+3
                                                                                                                                                                                                                                                                                                    21e+3
                                                                                                                                                                                                                                                                                                                                                 42e+3
  extra = 1)
                                                                                                                                                                                                            n=596
                                                                                                                                                                                                                                                         n=468
                                                                                                                                                                                                                                                                                                     n=130
                                                                                                                                                                                                                                                                                                                                                 n=144
# Method 2
rpart.plot(tree_model,
                                                                                                                                                                                                                                                                                  1
                                                                                                                                                                                                                                                                            13e+3
                                             # color scheme
                                                                                                                                                                                                                                                                             100%
                                                 box.palette = "GnBu",
                                                                                                                                                                                                                                           yes smoker = no no
                                             # dotted branch lines
                                                                                                                                                                                                                       2
                                                branch.lty = 3,
                                                                                                                                                                                                                  8434
                                                                                                                                                                                                                                                                                                                                        32e+3
                                             # node boxes shadows
                                                                                                                                                                                                                   80%
                                                                                                                                                                                                                                                                                                                                          20%
                                                 shadow.col = "gray",
                                                                                                                                                                                                           age < 43
                                                                                                                                                                                                                                                                                                                                  bmi < 30
                                              # show the node number
S
                                             nn = TRUE)
                                                                                                                                                                                                                                                     5
                                                                                                                                                                                                                                              12e+3
                                                                                                                                                                                                                                                                                                        21e+3
                                                                                                                                                                                                                                                                                                            10%
                                                                                                                                                                                                                                                35%
# Method 3
                                                                                                                                                                                                                                                                                1
# plot(as.party(tree model))
                                                                                                                                                                                                                                                                           smoker
                                                                                                                                                                                                                           2
                                                                                                                                                                                                                                                                                                                                      bmi
                                                                                                                                                                                                                         age
                                                                                                                                                                                                                                                                                                                       < 30.01
                                                                                                                                                                                                                                                                                                                                                 ≥ 30.01
                                                                                                                                                                                                           < 42.5
                                                                                                                                                                    70000 \frac{\text{Node 3 (n = 596)}}{10000} \frac{\text{Node 4 (n = 468)}}{10000} \frac{\text{Node 6 (n = 130)}}{10000} \frac{\text{Node 7 (n = 144)}}{10000} \frac{\text{Node 7 (n = 144)}}{10000
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                                                                                                                                                                                                                                                                                               0
```

- ii. How deep is the tree (see nodes with "decisions" ignore the leaves at the bottom)
- Ans: 2 level after ingore the leafs
- iii. How many leaf groups does it suggest to bin the data into?
- Ans: 4 groups of leaf
- iv. What is the average charges of each leaf group?

Plz refer to the plot of ,ethod 2, and scroll down til the part of Node number 4-7

```
summary(tree_model)
## Node number 4: 596 observations
##
     mean=5398.85, MSE=2.214521e+07
##
## Node number 5: 468 observations
     mean=12299.89, MSE=2.672104e+07
##
##
## Node number 6: 130 observations
##
     mean=21369.22, MSE=2.528196e+07
##
## Node number 7: 144 observations
##
     mean=41692.81, MSE=3.374313e+07
```

v. What conditions (decisions) describe each group?

#### Ans:

- 1. Whether is a smoker or not.
- 2. BMI < 30 or not
- 3. Age < 43 or not

Question 2) Let's use LOOCV to se how how our models perform predictively

Split-sample Testing

```
set.seed(22)
train.index <- sample(x=1:nrow(insurance), size=ceiling(0.8*nrow(insurance) ))
train <- insurance[train.index, ]
test <- insurance[-train.index, ]
charges_actual <- test$charges</pre>
```

a. What is the RMSEoos for the OLS regression model?

```
# Train
OLS_model <- lm(charges ~., data = insurance )
lm_trained <- update(OLS_model, data=train)
# Predict
preds <- predict(lm_trained, test)</pre>
```

```
# Test
RMSEoos <- function(actuals, preds) {
  (mean( (actuals - preds)^2 ))^0.5}
# one sample RMSE
RMSE OLS <- RMSEoos(charges actual, preds)</pre>
cat('one sample RMSE:',RMSE OLS, '\n\n')
## one sample RMSE: 6222.852
# K-fold (LOOCV) RMSE
MSE <- k_fold_mse(lm_trained, insurance, "charges", k = 1000)</pre>
RMSE <- MSE^0.5
cat('K-fold (LOOCV) RMSE:\n is
                                oos\n',RMSE, '\n')
## K-fold (LOOCV) RMSE:
## is
              005
## 6002.415 6086.806
```

# Question 3) How bagging helps our models

a. Write bagged\_learn(...) and bagged\_predict(...) functions using the hints in the class notes and help from your classmates on Teams. Feel free to share your code generously on Teams to get feedback, or ask others for help.

The function I designed can calculate the bagging model of the Random Forest, regression (OLS), and Decision Tree. (See Next Page)

```
bagged_learn <- function(model, dataset, model_type, b=100 , drop out = 0.4) {</pre>
    lapply(1:b, \(i) {
        Index <- sample(x=1:nrow(dataset), size=ceiling(0.8*nrow(dataset)), replace</pre>
= T)
        Train=dataset[Index,]
        if (model type == "OLS" ){
          Model <- update(model, data= Train)</pre>
        }
        else if (model_type == "RF" ){
          # Need rpart tree based model as input
          # it only works as dependent variable is continuous variable
          Index row=sample(nrow(Train),round(nrow(Train)* (1-drop out)))
          Train<-Train[Index row,]</pre>
          Model <- update(model, data= Train)</pre>
        }
        else if (model type == "DT" ){
          #Model<-rpart(model,data=Train ,method='anova')</pre>
          Model <- update(model, data= Train)</pre>
        }
        else{
          print("Warming: model type only allow OLS(Linear LM), RF (RandomForest), D
T (DecisionTree)")
          break
        }})}
bagged predict <- function(bagged models, new data) {</pre>
    predictions <- lapply(bagged_models, \(x) predict(x, new_data))</pre>
    as.data.frame(predictions) |> apply(1, mean)}
```

### b. What is the RMSEoos for the bagged OLS regression?

• Bagged learn for regression model

```
model_list <- bagged_learn(OLS_model,data = insurance, b =100, model_type = "OLS")

bagged_pred <- unlist(bagged_predict(model_list, test))

preds <- predict(tree_model, test)
RMSoos_OLS <- RMSEoos(charges_actual, bagged_pred)

RMSoos_OLS ## [1] 6175.428</pre>
```

Ans: The RMSEoos for the bagged OLS regression is 6175.428

# c. What is the RMSEoos for the bagged decision tree?

Bagged\_learn for Decision Tree Model

```
model_list <- bagged_learn(tree_model,data = insurance, b =100, model_type = 'DT')
bagged_pred <- unlist(bagged_predict(model_list, test))
RMSoos_Tree <- RMSEoos(charges_actual, bagged_pred)
RMSoos_Treem ## [1] 4815.196</pre>
```

Ans: The RMSEoos for the bagged decision tree is 4815.196

## Question 3) How boosting helps our models.

a. Write boosted\_learn(...) and boosted\_predict(...) functions using the hints in the class notes and help from your classmates on Teams. Feel free to share your code generously on Teams to get feedback, or ask others for help.

```
boost_learn <- function(model, dataset, outcome, n=100, rate=0.1) {</pre>
    # get data frame of only predictor variable
    predictors <- dataset[, colnames(dataset)!= outcome]</pre>
    res <- dataset[, colnames(dataset) == outcome]</pre>
    models <- list()</pre>
    for (i in 1:n) {
        this_model <- update(model, data = cbind(charges=res, predictors))</pre>
        # update residual with learning rate
        res <- res - rate*predict(this model)</pre>
        # model storage
        models[[i]] <- this_model</pre>
    list(models=models, rate=rate)
boost predict <- function(boosted learning, new data) {</pre>
  boosted models <- boosted learning$models</pre>
  rate <- boosted_learning$rate</pre>
  # get predictions of new data from each model
  predictions <- lapply(1:length(boosted_models), \(i)</pre>
    rate*predict((boosted models[[i]]), new data))
  pred frame <- as.data.frame(predictions) |> unname()
  # apply a sum over the columns of predictions, weighted by learning rate
```

(See Next Page)

b. What is the RMSEoos for the boosted OLS regression?

```
set.seed(110078509)
model <- boost_learn(lm_trained,train,"charges", n=100, rate=0.1)

pred <- model|> boost_predict(test)
RMSEoos(charges_actual , pred )
```

```
## [1] 6222.843
```

#### The RMSEoos for the boosted OLS regression is 6222.843

c. What is the RMSEoos for the boosted decision tree?

```
boost_prediction <- boost_learn(tree_model,train,"charges", n=100, rate=0.1) |>
boost_predict(test)

RMSEoos(charges_actual , boost_prediction )
```

```
## [1] 4677.864
```

The RMSEoos for the boosted decision tree is 4677,864

Question 4) Let's engineer the best predictive decision trees. Let's repeat the bagging and boosting decision tree several times to see what kind of base tree helps us learn the fastest. Report the RMSEoos at each step.

a. Repeat the bagging of the decision tree, using a base tree of maximum depth 1, 2, ... n while the RMSEoos keeps dropping; stop when the RMSEoos has started increasing again.

```
ls <- c()
for (i in 1:5){
  old_tree_stump <- rpart(charges~., train, cp = 0, control = list( maxdepth = i)
  set.seed(110078509)
  preds <- bagged_learn(model = old_tree_stump,dataset = train,model_type = 'DT' , b
  = 100) |> bagged_predict(test)
  RMSE_this <- RMSEoos(charges_actual, preds)

ls <- append(ls,RMSE_this)
}
names <- c(multi_items(" Depth-", 1:5))
  cat(names, '\n' ,ls )

## Depth-1 Depth-2 Depth-3 Depth-4 Depth-5
## 8022.451 5080.104 4632.197 4554.376 4588.228</pre>
```

Ans: Should stop as the depth of the tree equal to 4

b. Repeat the boosting of the decision tree, using a base tree of maximum depth 1, 2, ... n while the RMSEoos keeps dropping; stop when the RMSEoos has started increasing again.

```
ls <- c()
for (i in 1:5){
    old_tree_stump <- rpart(charges~., train, cp = 0, control = list( maxdepth = i))
    set.seed(110078509)
    preds_i <- boost_learn(old_tree_stump, train, 'charges', n=100, rate=0.1) |>
    boost_predict(test)
    RMSE_this <- RMSEoos(charges_actual, preds_i)
    ls <- append(ls,RMSE_this)
    }
    names <- c(multi_items(" Depth-", 1:5))
    cat(names, '\n' ,ls )

### Depth-1 Depth-2 Depth-3 Depth-4 Depth-5
## 6207.541 4575.485 4646.622 4750.675 4852.06</pre>
```

Ans: Should stop as the depth of the tree equal to 2