ASSIGNMENT-5

Q.1. Compare the following:-

I) IIR and FIR filters.

Ans I)=

CHARACTERISTIC	IIR	FIR
No. of necessary multiplications	least	Most
Stability	Depends upon system design	Guaranteed
Linear Phase	NO	Guaranteed
Required hardware Memory	least	Most
Supportive adaptive filtering	Yes	Yes
Availability of design software	Good	Very Good

II) Impulse Invariance method and bilinear tranformation

IMPULSE INVARIANCE METHOD	BILINEAR TRANSFORMATION
It is a technique for designing discrete-time infinite-impulse-response (IIR) filters from continuous-time filters in which the impulse response of the continuous-time system is sampled to produce the	The bilinear tranformation method is an alternative to impulse invariance that uses a different mapping that maps the continuous-time system's frequency response, out to infinite

impulse response of the discrete-time system. The frequency response of the discrete time system will be a sum of shifted copies of the frequency response of the continuous-time system.	frequency, into the range of frequencies up to the Nyquist frequency in the discrete time case, as opposed to mapping frequencies linearly with circular overlap as impulse invariance does.

(Q.2) Describe Window method for FIR filter design.

Ans2) Windows Method for FIR Filter Design.

The window method for digital filter design is fast, convenient, and robust but generally suboptimal. It is easily understood in terms of the convolution theorem for fourier transforms, making it instructive to study after the fourier theorems and windows for spectrum analysis.

The window method consists of simply "windowing" a theoretically ideal filter impulse response $h(n)$ by some suitable chosen window function $w(n)$, yielding

$$h_w(n) = w(n) \cdot h(n) \quad n \in Z.$$

$$W_R(n) = \begin{cases} 1 & \text{for } n = 0, 1, 2, \dots M-1 \\ 0 & \text{otherwise} \end{cases}$$

$$h(n) = h_d(n) \cdot W_R(n).$$

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$$W_R(\omega) = \sum_{n=0}^{M-1} 1 \cdot e^{-j\omega n}$$

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$$W_R(n) = u(n) - u(n-M).$$

$$F.T. \text{ of } u(n) = \sum_{n=0}^{\infty} 1 \cdot e^{-j\omega n} = \sum_{n=0}^{\infty} (e^{-j\omega})^n = \frac{1}{1-e^{-j\omega}}$$

$$F.T. \text{ of } u(n-M) \leftrightarrow e^{-j\omega M} F\{u(n)\}. = e^{-j\omega M} \cdot \frac{1}{1-e^{-j\omega}} = \frac{e^{-j\omega M}}{1-e^{-j\omega}}$$

$$W_R(\omega) = \frac{1}{1-e^{-j\omega}} - \frac{e^{-j\omega M}}{1-e^{-j\omega}}$$

$$= \frac{1-e^{-j\omega M}}{1-e^{-j\omega}} = \frac{\left|\sin\left(\frac{\omega M}{2}\right)\right|}{\left|\sin(\omega/2)\right|}$$

Q.3) State and explain following concepts:-

(i) Gibb's Phenomenon:- For a periodic signal with discontinuities, if the signal is reconstructed by adding the Fourier series, then overshoots appear around the edges. These overshoots decay outwards in a damped oscillatory manner away from the edges. This is known as GIBBS phenomenon.

(ii) Frequency Warping:- Frequency warping is a one types of transformation process where one spectral representation on a certain frequency scale measured in Hz. The amplitude response of digital

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IIR filter is expanded at lower frequencies and compressed at higher frequencies in comparison to the analog filter.

(iii) Butterworth filter Approximation:-

This type of response is called as butterworth response when the passband is maximally flat. That means there are no variations (ripples) in the passband.

The magnitude response of low pass butterworth filter is given by,

$$|H(\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega}{\Omega_c}\right)^{2N}}.$$

Q.4.) Design 2nd order Butterworth filter with cutt off frequency 1KHz and sampling frequency of 10^4 samples/sec by Bilinear Transformation.

Ans 4):- Order of filter = N = 2
Cut off frequency, f_c = 1KHz = 1000Hz
Sampling frequency, $f_s = 10^4$

Part A: Calculation of the required design specification of digital filter:

We have equation to convert continuous frequency (F) into discrete frequency (f).

$$f = \frac{F}{F_s} \qquad \therefore f_c = \frac{F_c}{F_s}.$$

$$= \frac{1000}{10,000} = 0.1 \text{ cycles/sample.}$$

$$\omega_c = 2\pi f_c = 2\pi \times 0.1 = 0.2\pi$$

Part B Calculation of specifications of analog filter for butterworth approximations.

Step 1: $\Omega = \frac{2}{T_s} \tan(\omega/2)$ $\therefore \Omega_c = \frac{2}{T_s} \tan\left(\frac{\omega_c}{2}\right)$

$$T_s = \text{Sampling time} = \frac{1}{f_s} = \frac{1}{10,000}$$

$$\Omega_c = (2 \times 10,000) \tan\left(\frac{0.2\pi}{2}\right)$$

$$\Omega_c = 6498.39 \text{ (cut-off frequency)}$$

Step 2: calculate poles,

$$P_K = \pm \Omega_c e^{j(N+2K+1)\pi/2N}$$

Here, $N=2$. Thus, $k=0$ to $N-1$ means $k=0$ and $k=1$

for $k=0$, $P_0 = \pm 6498.39\left[\cos\left(\frac{3\pi}{4}\right) + j\sin\left(\frac{3\pi}{4}\right)\right]$

$$= \pm 6498.39[-0.707 + j0.707]$$

$$= -4595.05 + j4595.05 \text{ and } 4595.05 - j4595.05$$

for $k=1$, $P_1 = \pm 6498.39[-0.707 - 0.707]$

$$P_1 = -4595.05 - j4595.05 \text{ and } 4595.05 + j4595.05.$$

$S_1 = -4595.05 + j4595.05$
$S_1^* = -4595.05 - j4595.05$

Step.3 The transfer function of analog filter is obtained by using the equation.

$$H(s) = \frac{\Omega_c^N}{(s-s_1)(s-s_1^*)}$$

$$H(s) = \frac{(6498.39)^2}{(s+4595.05-j4595.05)(s+4595.05+j4595.05)}$$

$$H(s) = \frac{(6498.39)^2}{s^2+9190.1s+42.22\times10^6}$$

Part c: Designing digital filter using bilinear transformation:

$$s = \frac{2}{T_s}\left[\frac{z-1}{z+1}\right]$$

$$T_s = \frac{1}{10,000}$$

$$s = 2\times10^{+4}\left[\frac{z-1}{z+1}\right]$$

$$\therefore H(z) = \frac{(6498.39)^2}{\left[2\times10^4\left(\frac{z-1}{z+1}\right)\right]^2 + 183.802\times10^6\left(\frac{z-1}{z+1}\right) + 42.22\times10^6}$$

Req^rd transfer function for digital filter.

Q.5.) Compare various windowing functions.

Ans 5):-

WINDOW	FUNCTION
1. Triangular Bartlett	$W_T(n) = 1 - \frac{2\|n\|}{M-1}$ for $\|n\| \le M-1$
2. Hanning	$W_{hn} = \frac{1}{2}\left[\cos\left(\frac{2\pi n}{M-1}\right)\right]$
3. Hamming	$W_{hm}(n) = \begin{cases} 0.54 + 0.46 \cos\frac{2\pi n}{M-1} \\ 0 & \text{, elsewhere} \end{cases}$
4. Blackman	$W_B(n) = 0.42 + 0.5\cos\frac{2\pi n}{M-1} + 0.08\cos\frac{4\pi n}{M-1}$
5. Kaiser	$W_K(n) = \begin{cases} I_0(\beta)/I_0(\alpha) & \|n\| \le \varphi \\ 0 & \text{elsewhere} \end{cases}$ OR
	$W_K(n) = \begin{cases} I_0\left(\alpha\sqrt{1-(n/\varphi)^2}\right)/I_0(\alpha) & \|n\| \le \varphi \\ 0 & \text{elsewhere} \end{cases}$

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