As a member of the UC Berkeley community, I act with honesty, integrity, and respect for others. I will neither give nor receive assistance while taking this exam. I understand that I must write the exam on paper

Thursday 7:00 - 8:45 PDT (concurrent exam)

1 a) $E[X_2] = 10\,(2/6) = 10/3$

b) $E[X_1 + X_2 + X_3] = 10$ $\qquad 10 \times \left(\frac{1}{6} + \frac{2}{6} + \frac{3}{6}\right)$

c) No since $P[X_1 = x_1 \cap X_2 = x_2]$ is not always equal to $P[X_1 = x_1] \cdot P[X_2 = x_2]$

Consider the counter example $x_1 = 10$ $x_2 = 10$

$P[X_1 = 10 \cap X_2 = 10] = 0$ but $P[X_1 = 10]\, P[X_2 = 10] = \left(\frac{1}{6}\right)^{10} \left(\frac{2}{6}\right)^{10}$

d) $\binom{10}{2}\binom{8}{3}\left(\frac{1}{6}\right)^2\left(\frac{2}{6}\right)^3\left(\frac{3}{6}\right)^5$

2 a) $\hat{\theta}_0 = \bar{y} - \hat{\theta}_1 \bar{x} = 7.5 - (0.35)(20) = 0.5$

$\hat{\theta}_1 = r \frac{\sigma_y}{\sigma_x} = (0.7)\left(\frac{1}{2}\right) = 0.35$

b) $\hat{y} = (0.5) + (0.35)(30) = 11$

$\boxed{\text{residual} = y - \hat{y} = 19 - 11 = 8}$

c) $R^2 = r^2 = \frac{\sigma^2_{\hat{y}}}{\sigma^2_y}$

$\Rightarrow \sigma^2_{\hat{y}} = r^2 \sigma^2_y$

$= (0.7)^2 (1)^2 = 0.49$

d) $\sigma^2_y + \sigma^2_{\hat{y}} = 1 + .49 = 1.49$

3) a) $M = \frac{1}{n} S^2$

b) 0

4 a) 2

b) 6

c) 7

5 a)

$$L(\theta) = \hat{y}_i = \theta_0 (0.5) + \theta_0 \theta_1 x_1 + \sin(\theta_1) x_2$$

$$\nabla_\theta L(\theta) = \begin{bmatrix} \frac{\partial L(\theta)}{\partial \theta_0} \\ \frac{\partial L(\theta)}{\partial \theta_1} \end{bmatrix} = \begin{bmatrix} 0.5 + \theta_1 x_1 \\ \theta_0 x_1 + \cos(\theta_1) x_2 \end{bmatrix}$$

$$A = 0.5 + \theta_1^{(t)} x_1$$

$$B = \theta_0^{(t)} x_1 + \cos(\theta_1^{(t)}) x_2$$

b)

$$\begin{bmatrix} \theta_0^{(t+1)} \\ \theta_1^{(t+1)} \end{bmatrix} = \begin{bmatrix} \theta_0^{(t)} \\ \theta_1^{(t)} \end{bmatrix} - \begin{bmatrix} 0.5 + \theta_1^{(t)} x_1 \\ \theta_0^{(t)} x_1 + \cos(\theta_1^{(t)}) x_2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0.5 \\ x_2 \end{bmatrix} = \begin{bmatrix} -0.5 \\ -2 \end{bmatrix}$$

c) $\theta_0^{(t)}$ approaches 8

Scanned with CamScanner

6 a)

$$x^T\theta = -1.3 + (0.08)(3) - 0.08(1) + 0.001(50)$$

$$=$$

$$\boxed{\hat{y} = \sigma(-1.09)}$$

b)

$$-\log(1-\hat{y}) = -\log(1-\sigma(-1.09))$$

c)

The log odds ratio is given by $x^T\theta$

so a 1 unit increase in age increases

the log odds by 0.001

d)

$$x^T\theta = -1.3 + 0.08(3) - 0.08(1) + 0.001(age)$$
$$= 0.001(age) - 1.14$$

$$\hat{y} = \sigma(x^T\theta) > 0.5$$

$$= \quad 0.001(age) - 1.14 > \sigma^{-1}(0.5)$$

$$= 0.001(age) - 1.14 > 1$$

age = 2140

so the minimum age is 2141

Scanned with CamScanner

e) $\sigma(-1{,}3 + 0{,}08\,(\text{education}) - 0{,}08\,(\text{marriage}) + 0{,}001\,(\text{age})) = 0{,}8$

$= -1{,}3 + 0{,}08\,(\text{education}) - 0{,}08\,(\text{marriage}) + 0{,}001\,(\text{age}) - \sigma^{-1}(0{,}8) = 0$

$A = -1{,}3 - \sigma^{-1}(0{,}8) = -1{,}3 - \log\left(\frac{0{,}8}{(1-0{,}8)}\right) = -1{,}3 - \log(4)$

$B = 0{,}08$

$C = -0{,}08$

$D = 0{,}001$

Scanned with CamScanner

f) Yes, then the line from part e separates

the data so it is linearly separable.

g) I would choose the Random Forrest model since the area under its ROC curve is greater (closer to 1) h)

$$\hat{\theta} = [-1.30 \quad 0.02 \quad -0.08 \quad 0.002]^T$$

7 a) 4

b) 16/51 $\frac{4^2}{51}$

c) k=2 $\frac{9^2+4^2}{9^2+4^2+2^2+1} = \frac{97}{102} > .95$

d) 0.3

first row of x

$X_1 v_+[2] = \sigma_3 \; u[0,2]$

e) 2.1 $\frac{1}{2}(3(0.8)+3(0.6)+12(0)+5(0)+1(0))$

$= 2.1$

f) 0

g) Plot c

8 a) $E[\hat{\theta} - \theta] = E[\hat{\theta}] - \theta$ $\qquad E[\hat{y} - y]$

$$= n\left(\frac{8}{20}\right) - 8$$

$$= \frac{2n}{5} - 8$$

b) $n = 20$

$$\frac{2n}{5} - 8 = 0$$

$$n = 20$$

c) $\frac{6}{25} n$

$$\cancel{E[\hat{\theta}^2] - E^2[\hat{\theta}]}$$

$$= n\left(\frac{2}{5}\right)\left(\frac{3}{5}\right) \cancel{n\left(\frac{2}{5}\right)}$$

$$\cancel{n\frac{2n}{5} - \left(-\frac{2}{5}\right)}$$