1 a) False ; if we re-arrange to

$$\sum_{i=1}^{n} a_i x_i = \left(\sum_{i=1}^{n} a_i\right)\left(\sum_{i=1}^{n} x_i\right)$$

Consider the case of $n=2$. If we expand the summations

$$a_1 x_1 + a_2 x_2 = (a_1 + a_2)(x_1 + x_2)$$

$$= a_1 x_1 + a_1 x_2 + a_2 x_1 + a_2 x_2$$

Which is only true if $a_1 x_2 + a_2 x_1 = 0$. Hence false.

Counter example: $n=2$, $a_1 = x_1 = 0$, $a_2 = x_2 = 1$

b) True; substituting for $\bar{x}$ we have,

$$n a_3 \bar{x} = n a_3 \left(\frac{1}{n} \sum_{i=1}^{n} x_i\right)$$

$$= a_3 \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} a_3 x_i$$

c) False ; substituting for $\bar{a}$ and $\bar{x}$ we have,

$$n \bar{a} \bar{x} = n \left(\frac{1}{n} \sum_{i=1}^{n} a_i\right)\left(\frac{1}{n} \sum_{i=1}^{n} x_i\right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} a_i \sum_{i=1}^{n} x_i$$

Similarly to part a) if we use $n=2$

then $$\sum_{i=1}^{n} a_i x_i = \frac{1}{n} \sum_{i=1}^{n} a_i \sum_{i=1}^{n} x_i$$

$$a_1 x_1 + a_2 x_2 = \frac{1}{2}(a_1 + a_2)(x_1 + x_2)$$

which is only true if

$$(a_1 x_1 + a_2 x_2) - a_1 x_2 - a_2 x_1 = 0$$

Counter example:

$n=2$, $a_1 = x_1 = 0$, $a_2 = x_2 = 1$

Scanned with CamScanner

2 a) $\sigma(-x) = \frac{1}{1+e^{x}} \cdot \frac{e^{-x}}{e^{-x}} = \frac{e^{-x}}{e^{-x}+1}$

$$= \frac{e^{-x}+1-1}{e^{-x}+1}$$

$$= \frac{1+e^{-x}}{1+e^{-x}} - \frac{1}{1+e^{-x}}$$

$$= 1 - \sigma(x)$$

b) using quotient rule

$$\frac{d}{dx}\sigma(x) = \frac{0 - \frac{d}{dx}(1+e^{-x})}{(1+e^{-x})^2}$$

$$= \frac{-e^{-x}}{(1+e^{-x})^2} = \frac{e^{-x}}{(1+e^{-x})} \cdot \frac{1}{(1+e^{-x})}$$

$$= \left(\frac{e^{x}}{e^{x}}\right) \cdot \frac{e^{-x}}{(1+e^{-x})} \cdot \frac{1}{(1+e^{-x})}$$

$$= \left(\frac{1}{1+e^{x}}\right)\left(\frac{1}{1+e^{-x}}\right)$$

$$= (1-\sigma(x))\,\sigma(x)$$

3. $f'(c) = \frac{1}{n} \sum_{i=1}^{n} \frac{d}{dc}\left((x_i - c)^2\right)$

$= \frac{1}{n} \sum_{i=1}^{n} -2(x_i - c)$

$= \frac{-2}{n}\left(-nc + \sum_{i=1}^{n} x_i\right) = 0$

$\Rightarrow \sum_{i=1}^{n} x_i = cn \Rightarrow \frac{1}{n}\sum_{i=1}^{n} x_i = c$

$\Rightarrow c = \bar{x}$

To justify that this value is a minimum we take the second derivative at $c = \bar{x}$

$f''(c) = \frac{d}{dc}\left(\frac{-2}{n}\left(-nc + \sum_{i=1}^{n} x_i\right)\right)$

$= \frac{-2}{n}\left(\frac{d}{dc}(-nc)\right) = \frac{-2}{n} \cdot (-n) = 2$

since $f''(c = \bar{x}) = 2$ which is > 0

the graph is concave upward at $c = \bar{x}$

meaning it is a minimum

4 a) 2. cannot be found with information in the article

while we know the percent of surveyed adults who have a great trust in religious leaders & scientists respectively, we don't know the intersection of the 2 groups. In other words; we don't know the percentage of the adults who greatly trust scientists, that trust religious leaders as well and vice versa by just knowing the information presented in the article

b) Toyota; Since the car was from Berkeley (Alameda County) and we know that the most common make of car in Alameda is Toyota, we guess Toyota. Also, since we know nothing about the relative frequencies of specific makes of cars getting speeding violations, the fact that the car was speeding gives us no additional information to assume the make of the car.

5 Let $A =$ the probability a 40 yr old women has breast cancer

$B =$ the probability a women tests positive for breast cancer

Using Bayes Rule:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

Note $P(\bar{A}) = 1 - P(A)$

Using total probability rule:

$$= \frac{P(B|A)\,P(A)}{P(B|A)\,P(A) + P(B|\bar{A})\,P(\bar{A})}$$

$$= \frac{(.80)\,(0.01)}{(.80)(.01) + (.096)(1-0.01)}$$

$$= \frac{.008}{.008 + .09504} = \frac{.008}{.10304} = 0.078$$

6 B The graph is approximately bell shaped and symmetric about the mean, so we can assume a normal probability distribution.

In a normal distribution about 68% of the data falls within 1 standard deviation of the mean.

About 68% of the data seems to be in the range 150 ± 6.1