

- You have approximately 110 minutes.
- The exam is closed book, closed calculator, and closed notes except your one-page crib sheet.
- Mark your answers ON THE EXAM ITSELF. Provide a *brief* explanation if applicable.
- For multiple choice questions,
  - ☐ means mark **all options** that apply
  - ☐ means mark a **single choice**
  - When selecting an answer, please fill in the bubble or square **completely** (● and ■)

First name	
Last name	
SID	
Student to your right	
Student to your left	

**Your Discussion/Exam Prep\* TA(s) (fill all that apply):**

- |                                  |                                  |                                  |                                   |                                   |
|----------------------------------|----------------------------------|----------------------------------|-----------------------------------|-----------------------------------|
| <input type="checkbox"/> Shizhan | <input type="checkbox"/> Cathy   | <input type="checkbox"/> Lindsay | <input type="checkbox"/> Andreea  | <input type="checkbox"/> Jinkyu   |
| <input type="checkbox"/> Carl    | <input type="checkbox"/> Peyrin* | <input type="checkbox"/> Gokul*  | <input type="checkbox"/> Chandan  |                                   |
| <input type="checkbox"/> Emma    | <input type="checkbox"/> Andy    | <input type="checkbox"/> Rachel  | <input type="checkbox"/> Sherman* | <input type="checkbox"/> Lawrence |
| <input type="checkbox"/> Mesut*  | <input type="checkbox"/> Wilson  | <input type="checkbox"/> Henry*  | <input type="checkbox"/> Mike     |                                   |
| <input type="checkbox"/> Jesse   | <input type="checkbox"/> Ryan    | <input type="checkbox"/> Alan    | <input type="checkbox"/> Danny*   | <input type="checkbox"/> Albert   |

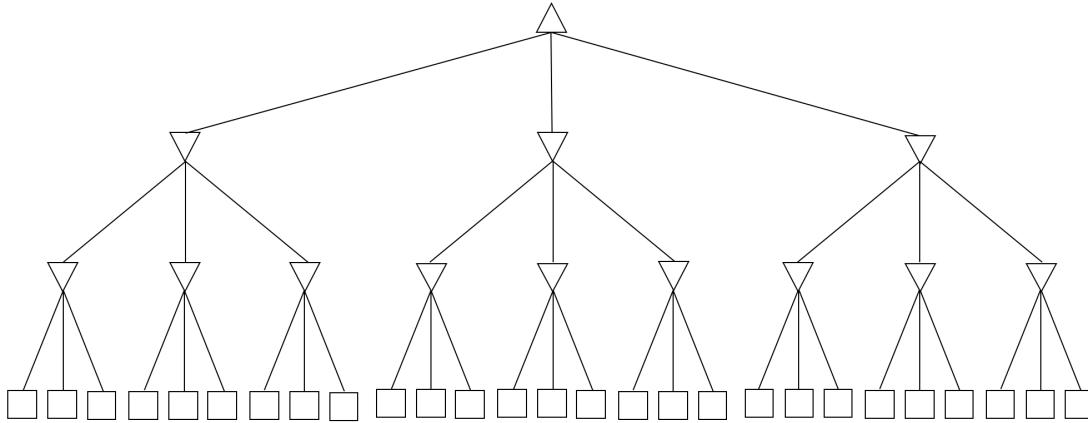
**For staff use only:**

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# Q1. [9 pts] Potpourri

(a) [3 pts] We are given the following game tree.



(i) [1 pt] What is the total number of **terminal nodes** (denoted by squares at bottom)? Your answer should be an integer.

Answer:

(ii) [2 pts] Suppose you use alpha-beta pruning to prune branches from game tree. What is the **maximum** total number of terminal nodes **whose value is never explored because an immediate or upstream branch is pruned** in one single set of assignments for the terminal nodes. Your answer should be an integer.

Answer:

(b) [2 pts] **Arc consistency**

$X \rightarrow Y$  is consistent if and only if \_\_\_\_\_(i)\_\_\_\_\_, \_\_\_\_\_(ii)\_\_\_\_\_

(i) [1 pt]

- ☐ for some value of  $x$  in  $X$
- ☐ for all values of  $x$  in  $X$
- ☐ for some value of  $y$  in  $Y$
- ☐ for all values of  $y$  in  $Y$

(ii) [1 pt]

- ☐ there exists some allowed  $x$  in  $X$
- ☐ all values of  $x$  in  $X$  are allowed
- ☐ there exists some allowed  $y$  in  $Y$
- ☐ all values of  $y$  in  $Y$  are allowed

(c) [4 pts] Which of the following algorithms is always guaranteed to provide the optimal solution for the corresponding problems they solve? We define "corresponding problem" as one of the following: uninformed/informed search, CSPs, MDPs, game trees, reinforcement learning.

- |  |   |
|--|---|
| <input type="checkbox"/> DFS                 | <input type="checkbox"/> Policy extracted from policy iteration   |
| <input type="checkbox"/> Greedy search       | <input type="checkbox"/> Policy extracted from value iteration  |
| <input type="checkbox"/> Hill climbing       | <input type="checkbox"/> Minimax search with $\alpha$ - $\beta$ pruning (assuming both players are playing optimally) |
| <input type="checkbox"/> Simulated annealing |   |
| <input type="radio"/> None of the above      |   |

## Q2. [17 pts] CSP: The Picnic

Six CS 188 TAs secretly scheduled a Picnic behind the back of other TAs! In order for the Picnic to be successful, Andy (A), Cathy (Ca), Chandan (Ch), Lindsay (L), Mesut (Me), and Mike (Mi) will each be assigned 1 of the 5 tasks. Please note that one task requires 2 TAs. In this question, the TAs are the variables, and the domains are the tasks  $\{1, \dots, 5\}$ . Your mission is to use your knowledge of CSPs to find an assignment that meets the following constraints:

- Only Andy (A) is capable of task 4.
- Cathy (Ca) and Mesut (Me) must be assigned adjacent (difference in number is 1) tasks. (e.g. task 4 and task 5 are adjacent, but task 1 and task 5 are not)
- Chandan (Ch) can only do odd-numbered tasks.
- Lindsay (L) can only do tasks with number  $\leq 2$ .
- Mesut (Me) can only do either task 2 or task 5.
- Mike (Mi) has to take a task with a bigger number than Lindsay (L)'s.
- Task 2 needs exactly 2 TAs.
- Every task other than task 2 needs exactly 1 TA.
- The 2 TAs doing task 2 needs to have the same initial letter.

(a) [8 pts] Let's start with looking into the constraints

(i) [1 pt] What type of constraint is *Mike (Mi) has to take a task with a bigger number than Lindsay (L)'s*?

- ☐ Unary Constraint ☐ Binary Constraint ☐ Higher Order Constraint

(ii) [1 pt] What type of constraint is *Only Andy (A) is capable of task 4*?

- ☐ Unary Constraint ☐ Binary Constraint ☐ Higher Order Constraint

(iii) [1 pt] What type of constraint is *Task 2 needs exactly 2 TAs*?

- ☐ Unary Constraint ☐ Binary Constraint ☐ Higher Order Constraint

(iv) [3 pts] Please select the elements in the domains that will be **crossed out** after enforcing unary constraints.

A	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5
Ca	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5
Ch	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5
L	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5
Me	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5
Mi	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5

(v) [2 pts] Please select the elements in the domains that will be **crossed out** after enforcing unary constraints and binary constraints through **arc-consistency**. Ignore high order constraints. (Note: this part will be graded **independently**, which means no partial credit if your answer is incorrect due to errors in previous parts. So please double check the correctness of previous parts.)

A	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5
Ca	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5
Ch	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5
L	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5
Me	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5
Mi	<input type="checkbox"/> 1	<input type="checkbox"/> 2	<input type="checkbox"/> 3	<input type="checkbox"/> 4	<input type="checkbox"/> 5

- (b) [2 pts] Heuristics in CSPs could help you find a satisfying assignment more quickly. Which of the heuristics fits each of the given descriptions?
- (i) [1 pt] Prioritize assigning a variable that has the minimum number of remaining values in its domain  
☐ MRV   ☐ LCV   ☐ Both   ☐ Neither
- (ii) [1 pt] Prioritize assigning a variable that is involved in the least number of constraints  
☐ MRV   ☐ LCV   ☐ Both   ☐ Neither
- (c) [2 pts] Suppose you can't use either of those helpful heuristics because you ran out of time implementing them [sad reacts only :( ] and decide to run the backtracking search algorithm manually. Fortunately, Mike told you an extra constraint: he (Mi) has to be assigned to task 2 at the picnic.
- (i) [1 pt] Using this additional information, could you find at least one assignment that satisfies all the constraints?  
☐ Yes   ☐ No
- (ii) [1 pt] If yes, which task(s) can Cathy possibly get assigned to?  
☐ 1   ☐ 2   ☐ 3   ☐ 4   ☐ 5   ☐ Not applicable
- (d) [5 pts] Now suppose all  $N$  CS 188 students and staff are going on the trip together. There are  $D$  tasks that need exactly 2 people, while all other tasks need exactly 1 person. In addition, there are  $b > 0$  binary constraints, and  $h > 0$  higher-order constraints. There exists a satisfying assignment where everyone gets one and only one task, and all tasks are filled to the required number.
- (i) [1 pt] Without further assumptions, which of the following is the tightest upper bound for the runtime for finding a satisfying assignment?  
☐  $O(D^N)$    ☐  $O(N^D)$    ☐  $O((N - D)^N)$    ☐  $O(N^{(N-D)})$    ☐  $O(ND^2)$   
☐  $O(N(N - D)^2)$    ☐  $O(N^2D)$    ☐  $O(N^2(N - D))$    ☐  $O(N^2D^3)$    ☐  $O(N^2(N - D)^3)$
- (ii) [2 pts] We **remove all the higher-order constraints**. Without further assumptions, which of the following is the tightest upper bound for the runtime for the AC-3 Algorithm in this setting?  
☐  $O(D^N)$    ☐  $O(N^D)$    ☐  $O((N - D)^N)$    ☐  $O(N^{(N-D)})$    ☐  $O(ND^2)$   
☐  $O(N(N - D)^2)$    ☐  $O(N^2D)$    ☐  $O(N^2(N - D))$    ☐  $O(N^2D^3)$    ☐  $O(N^2(N - D)^3)$
- (iii) [2 pts] Suppose the binary constraints turn out to be very sparse ( $b \ll N^2$ ). After **removing all the higher-order constraints**, without further assumptions, which of the following is the tightest upper bound for the runtime for the AC-3 Algorithm in this setting now?  
☐  $O(D\sqrt{b})$    ☐  $O(\sqrt{b}^D)$    ☐  $O((N - D)\sqrt{b})$    ☐  $O(\sqrt{b}^{(N-D)})$    ☐  $O(\sqrt{b}D^2)$   
☐  $O(\sqrt{b}(N - D)^2)$    ☐  $O(bD)$    ☐  $O(b(N - D))$    ☐  $O(bD^3)$    ☐  $O(b(N - D)^3)$

### Q3. [12 pts] MDP: Blackjack

There's a new gambling game popping up in Vegas! It's similar to blackjack, but it's played with a single die. CS188 staff is interested in winning a small fortune, so we've hired you to take a look at the game!

We will treat the game as an MDP. The game has states  $0, 1, \dots, 8$ , corresponding to dollar amounts, and a *Done* state where the game ends. The player starts with \$2, i.e. at state 2. The player has two actions: Stop and Roll, and is forced to take the Stop action at states 0, 1, and 8.

When the player takes the Stop action, they transition to the *Done* state and receive reward equal to the amount of dollars of the state they transitioned from: e.g. taking the stop action at state 3 gives the player \$3. The game ends when the player transitions to *Done*.

The Roll action is available from states 2-7. The player rolls a **biased** 6-sided die that will land on 1, 2, 3, or 4 with  $\frac{1}{8}$  probability each and 5 or 6 with probability  $\frac{1}{4}$  each.

If the player Rolls from state  $s$  and the die lands on outcome  $o$ , the player transitions to state  $s + o - 2$ , as long as  $s + o - 2 \leq 8$  ( $s$  is the amount of dollars of the current state,  $o$  is the amount rolled, and the negative 2 is the price to roll). If  $s + o - 2 > 8$ , the player busts, i.e. transitions to Done and does NOT receive reward.

- (a) [4 pts] In solving this problem, you consider using policy iteration. Your initial policy  $\pi^a$  is in the table below. Evaluate the policy at each state, with  $\gamma = 1$ . Note that the action at state 0, 1, 8 is fixed into the rule, so we will not consider those states in the update. (*Hint: how does the bias in the die affect this?*)

State	2	3	4	5	6	7
$\pi^a(s)$	Roll	Roll	Stop	Stop	Stop	Stop
$V^{\pi^a(s)}$						

- (b) [4 pts] Deciding against the previous policy, you come up with a simpler policy  $\pi^{(0)}$ , as shown below, to start with. Perform one iteration of Policy Iteration (i.e. policy evaluation followed by policy improvement) to find the new policy  $\pi^{(1)}$ . In this part as well, we have  $\gamma = 1$ .

In the table below, R stands for *Roll* and S stands for *Stop*. Select both R and S if both actions are equally preferred.

State	2	3	4	5	6	7
$\pi^{(0)}(s)$	Stop	Stop	Stop	Stop	Stop	Stop
$\pi^{(1)}(s)$	<input type="checkbox"/> R <input type="checkbox"/> S	<input type="checkbox"/> R <input type="checkbox"/> S	<input type="checkbox"/> R <input type="checkbox"/> S	<input type="checkbox"/> R <input type="checkbox"/> S	<input type="checkbox"/> R <input type="checkbox"/> S	<input type="checkbox"/> R <input type="checkbox"/> S

- (c) [2 pts] Suppose you start with a initial policy  $\pi_0$  that is the **opposite** of the optimal policy (which means if  $\pi^*(s) = \text{Roll}$ ,  $\pi_0(s) = \text{Stop}$ , and vice versa). Your friend Alice claims that the Policy Iteration Algorithm can still find the optimal policy in this specific scenario. Is Alice right?

- ☐ Alice is right, because Policy Iteration can find the optimal policy regardless of initial policy.
- ☐ Alice is right, but not for the reason above.
- ☐ Alice is wrong, because a very bad initial policy can block the algorithm from exploring the optimal actions.
- ☐ Alice is wrong, but not for the reason above.

- (d) [2 pts] Suppose you want to try a different approach, and implement a **value iteration** program to find the optimal policy for this new game. Your friend Bob claims that  $V_k(s)$  has to converge to  $V^*(s)$  for all states before the program declares it has found the optimal policy. Is Bob right?
- ☐ Bob is right, because  $V_k(s)$  always converge to  $V^*(s)$  for all states when the optimal policy  $\pi_*$  is found.
  - ☐ Bob is right, but not for the reason above.
  - ☐ Bob is wrong, because we cannot use value iteration to find the optimal policy.
  - ☐ Bob is wrong, but not for the reason above

## Q4. [17 pts] RL: Blackjack, Redux

After playing the Blackjack game in Q3 a few times with the optimal policy you found in the previous problem, you find that you're doing worse than expected! (Hint: you may want to do Q3 before attempting this problem.) In fact, you are beginning to suspect that the Casino was not honest about the probabilities of dice's outcome. Seeing no better option, you decided to do some good old fashioned reinforcement learning (RL).

(a) First, you need to decide what RL algorithm to use.

(i) [2 pts] Suppose you had a policy  $\pi$  and wanted to find the value  $V^\pi$  of each of the states under this policy. Which algorithms are appropriate for performing this calculation? Note that we **do not** know the transition probabilities, and we don't have sufficient samples to approximate them.

☐ Value Iteration ☐ Policy Iteration ☐ Q-learning ☐ Direct Evaluation ☐ Temporal difference learning

(ii) [2 pts] Being prudent with your money, you decide to begin with observing what happens when other people randomly play the blackjack game. Which of the following algorithms can recover the optimal policy given this play data?

☐ Value Iteration ☐ Policy Iteration ☐ Q-learning ☐ Direct Evaluation ☐ Temporal difference learning

(b) You decide to use Q-learning to play this game.

(i) [2 pts] Suppose your initial policy is  $\pi_0$ . Which of the following is the update performed by Q-learning with learning rate  $\alpha$ , upon getting reward  $R(s, a, s')$  and transitioning to state  $s'$  after taking action  $a$  in state  $s$ ?

- ☐  $Q_{k+1}(s, a) = (1 - \alpha)Q_k(s, a) + \alpha(R(s, a, s') + \gamma \max_{a'} Q_k(s', a'))$   
☐  $Q_{k+1}(s, a) = (1 - \alpha)Q_k(s, a) + \alpha(R(s, a, s') + \gamma Q_k(s', \pi_0(s')))$   
☐  $V_{k+1}(s) = (1 - \alpha)V_k(s) + \alpha(R(s, a, s') + \gamma \max_{s''} V_k(s''))$   
☐  $V_{k+1} = (1 - \alpha)V_k + \alpha(R(s, a, s') + \gamma V_k(s'))$

(ii) [2 pts] As with the previous problem, denote a policy at any time-step  $k$  as  $\pi_k$  (and  $\pi_k(a|s)$  means the **probability** of taking action  $a$  at state), and the Q values at that timestep as  $Q_k$ . In the limit of infinite episodes, which of these policies will always do each action in each state an infinite amount of times?

- ☐  $\pi_k(Roll|s) = \pi_k(Stop|s) = \frac{1}{2}$   
☐  $\pi_k(a|s) = 1 - \frac{\epsilon}{2}$  if  $a == \arg \max_a Q_k(s, a)$  else  $\frac{\epsilon}{2}$   
☐  $\pi_k(Roll|s) = 1, \pi_k(Stop|s) = 0$   
☐  $\pi_k(Roll|s) = \frac{1}{3}, \pi_k(Stop|s) = \frac{2}{3}$   
☐ None of the above

(iii) [3 pts] Suppose you decide to use an exploration function  $f(s', a')$ , used in-place of  $Q(s', a')$  in the Q-learning update. Which of the following choices of an exploration functions encourage you to take actions you haven't taken much before? (Recall that  $N(s, a)$  is the number of times the q-state  $(s, a)$  has been visited, assuming every  $(s, a)$  has been visited at least once.)

- ☐  $f(s, a) = Q(s, a)$   
☐  $f(s, a) = Q(s, a) + N(s, a)$   
☐  $f(s, a) = \max_{a'} Q(s, a')$   
☐  $f(s, a) = Q(s, a) + \frac{k}{N(s, a)}$ , where  $k > 0$   
☐  $f(s, a) = Q(s, a) + \sqrt{\frac{\log(\sum_{a'} N(s, a'))}{N(s, a)}}$   
☐  $f(s, a) = \frac{1}{N(s, a)^2}$   
☐ None of the above



(iv) [3 pts] Suppose you start with the following Q-value table:

State	2	3	4	5	6	7
Q(State, Roll)	0	0	5	3	4	2
Q(State, Stop)	2	3	4	5	6	7

After you observe the trajectory

$$(s = 2, a = \text{Roll}, s' = 4, r = 0), (s = 4, a = \text{Roll}, s' = 7, r = 0), (s = 7, a = \text{Stop}, s' = \text{Done}, r = 7)$$

What are the resulting Q-values after running one pass of Q-learning over the given trajectory? Suppose discount rate  $\gamma = 1$ , and learning rate  $\alpha = 0.5$ .

State	2	3	4	5	6	7
Q(State, Roll)						
Q(State, Stop)						

(v) [1 pt] One of the other gamblers looks over your shoulder as you perform Q-learning, and tells you that you're learning too slowly. "You should use a learning rate of  $\alpha = 1$ ", they suggest.

If you use constant  $\alpha = 1$ , is Q-learning guaranteed to eventually converge to the optimal policy, assuming you observe every state, action pair an infinite amount of times? ☐ Yes ☐ No

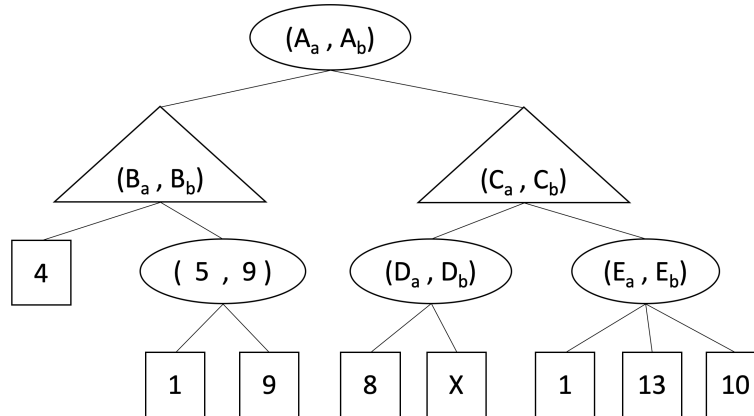
(vi) [2 pts] If you continue with constant  $\alpha = 0.5$ , is Q-learning guaranteed to eventually converge to the optimal policy, assuming you observe every state, action pair an infinite amount of times? ☐ Yes ☐ No

## Q5. [15 pts] Games

Alice is playing a two-player game with Bob, in which they move alternately. Alice is a maximizer. Although Bob is also a maximizer, Alice believes Bob is a minimizer with probability 0.5, and a maximizer with probability 0.5. Bob is aware of Alice's assumption.

In the game tree below, square nodes are the outcomes, triangular nodes are Alice's moves, and round nodes are Bob's moves. Each node for Alice/Bob contains a tuple, the left value being Alice's expectation of the outcome, and the right value being Bob's expectation of the outcome.

Tie-breaking: choose the left branch.



(a) [2 pts] In the blanks below, fill in the tuple values for tuples  $(B_a, B_b)$  and  $(E_a, E_b)$  from the above game tree.

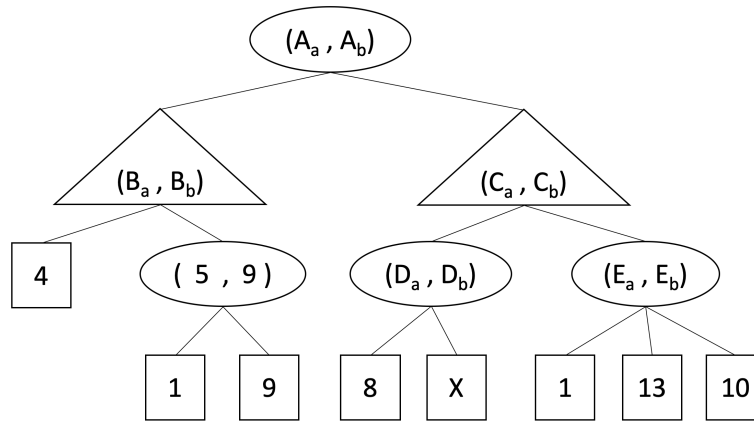
$(B_a, B_b) = ( \quad , \quad )$

$(E_a, E_b) = ( \quad , \quad )$

(b) [2 pts] In this part, we will determine the values for tuple  $(D_a, D_b)$ .

(i) [1 pt]  $D_a =$  ☐ 8 ☐ X ☐ 8+X ☐ 4+0.5X ☐ min(8,X) ☐ max(8,X)

(ii) [1 pt]  $D_b =$  ☐ 8 ☐ X ☐ 8+X ☐ 4+0.5X ☐ min(8,X) ☐ max(8,X)



(The graph of the tree is copied for your convenience. You may do problem e on this graph. )

- (c) [6 pts] Fill in the values for tuple  $(C_a, C_b)$  below. For the bounds of X, you may write scalars,  $\infty$  or  $-\infty$ .  
If your answer contains a fraction, please write down the corresponding **simplified decimal value** in its place. (i.e., 4 instead of  $\frac{8}{2}$ , and 0.5 instead of  $\frac{1}{2}$ ).

1. If  $-\infty < X < \boxed{\phantom{000}}$ ,  $(C_a, C_b) = (\boxed{\phantom{000}}, \boxed{\phantom{000}})$
2. Else,  $(C_a, C_b) = (\boxed{\phantom{000}}, \max(\boxed{\phantom{000}}, \boxed{\phantom{000}}))$

- (d) [4 pts] Fill in the values for tuple  $(A_a, A_b)$  below. For the bounds of X, you may write scalars,  $\infty$  or  $-\infty$ .  
If your answer contains a fraction, please write down the corresponding **simplified decimal value** in its place. (i.e., 4 instead of  $\frac{8}{2}$ , and 0.5 instead of  $\frac{1}{2}$ ).

1. If  $-\infty < X < \boxed{\phantom{000}}$ ,  $(A_a, A_b) = (\boxed{\phantom{000}}, \boxed{\phantom{000}})$
2. Else,  $(A_a, A_b) = (\boxed{\phantom{000}}, \max(\boxed{\phantom{000}}, \boxed{\phantom{000}}))$

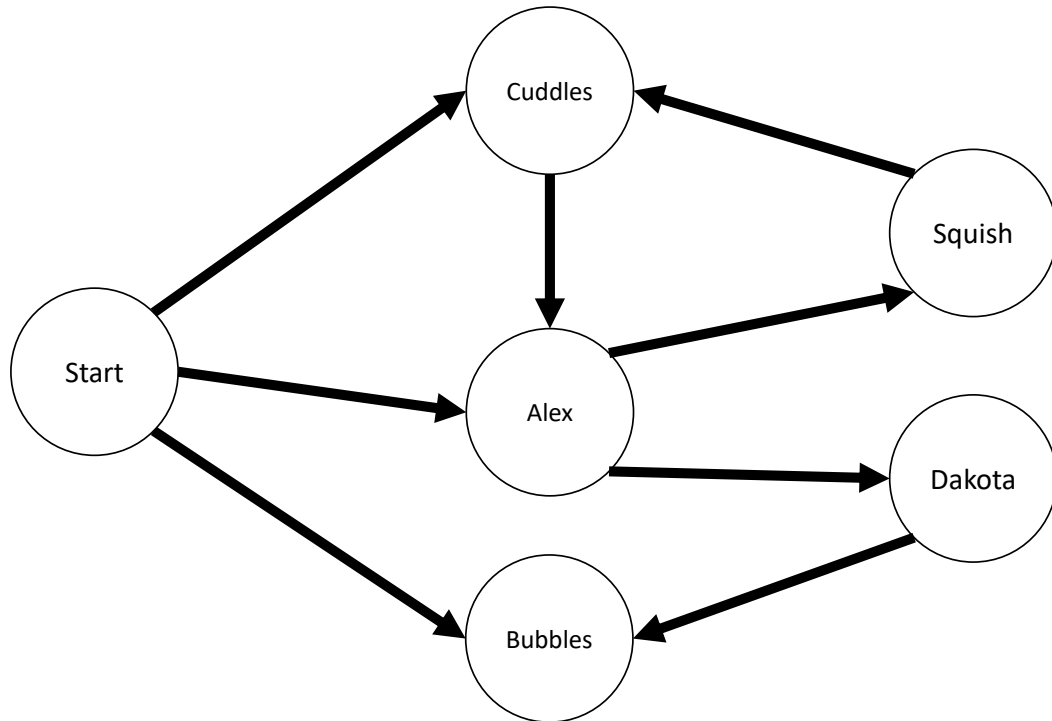
- (e) [1 pt] When Alice computes the left values in the tree, some branches can be pruned and do not need to be explored. In the game tree graph on this page, put an 'X' on these branches. If no branches can be pruned, mark the "Not possible" choice below.

Assume that the children of a node are visited in left-to-right order and that you should not prune on equality.

☐ Not possible

## Q6. [14 pts] Search: Snail search for love

Scorpborg the snail is looking for a mate. It can visit different potential mates based on a trail of ooze to nearby snails, and then test them for chemistry, as represented in the below graph, where each node represents a snail. In all cases, nodes with equal priority should be visited in alphabetical order.



### (a) [5 pts] Simple search

In this part, assume that the only match for Scorpborg is Squish (i.e. Squish is the goal state). Which of the following are true **when searching the above graph**?

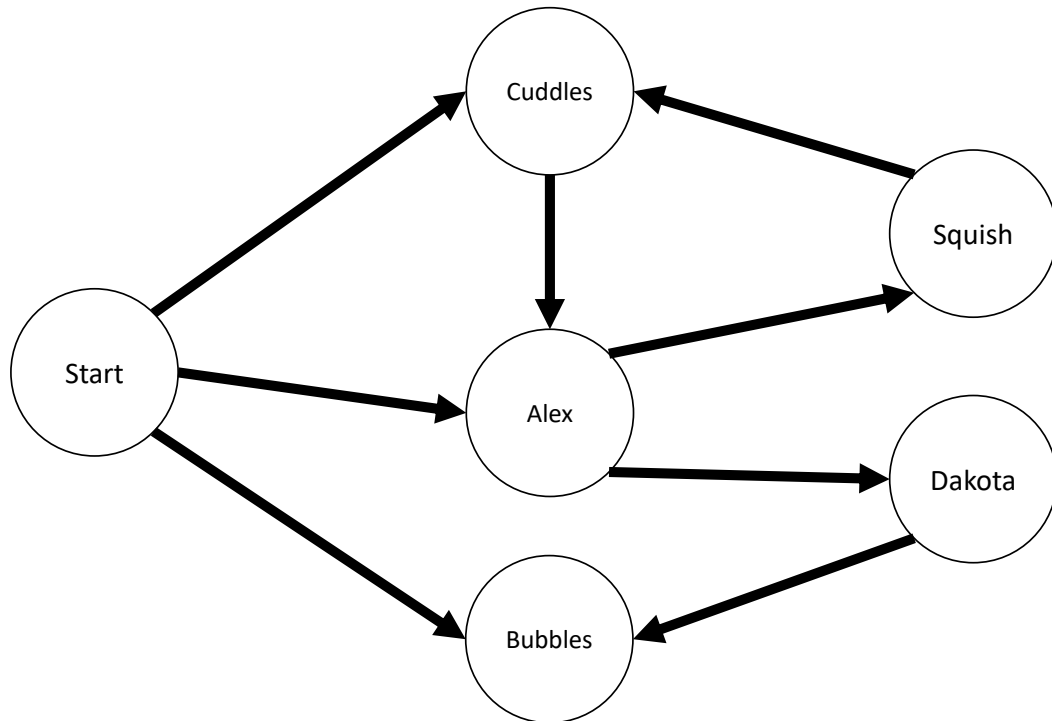
- (i) [1 pt] BFS Tree Search expands more nodes than DFS Tree Search ☐ True ☐ False
- (ii) [1 pt] DFS Tree Search finds a path to the goal for this graph ☐ True ☐ False
- (iii) [1 pt] DFS Graph Search finds the shortest path to the goal for this graph ☐ True ☐ False
- (iv) [2 pts] If we remove the connection from Cuddles → Alex, can DFS Graph Search find a path to the goal for the altered graph? ☐ Yes ☐ No

### (b) [5 pts] Third Time's A Charm

Now we assume that Scorpborg's mate preferences have changed. The new criteria she is looking for in a mate is that she has **visited the mate twice before** (i.e. when she visits any state for the third time, she has found a path to the goal).

- (i) [3 pts] What should the most simple yet sufficient new state space representation include?

- ☐ The current location of Scorpborg
- ☐ The total number of edges travelled so far
- ☐ An array of booleans indicating whether each snail has been visited so far
- ☐ An array of numbers indicating how many times each snail has been visited so far
- ☐ The number of distinct snails visited so far



(The graph is copied for your convenience)

- (ii) [1 pt] DFS Tree Search finds a path to the goal for this graph ☐ True ☐ False
- (iii) [1 pt] BFS Graph Search finds a path to the goal for this graph ☐ True ☐ False
- (iv) [2 pts] If we remove the connection from Cuddles → Alex, can DFS Graph Search find a path to the goal for the altered graph? ☐ Yes ☐ No

We continue as in part (b) where the goal is still to find a mate who is visited for the third time.

(c) [4 pts] **Costs for visiting snails**

Assume we are using Uniform cost search and we can now add costs to the actions in the graph.

- (i) [2 pts] Can one assign (non-negative) costs to the actions in the graph such that the goal state returned by UCS (Tree-search) changes? ☐ Yes ☐ No
- (ii) [2 pts] Can one assign (potentially negative) costs to the actions in the graph such that UCS (Tree-search) will never find a goal state? ☐ Yes ☐ No

## Q7. [16 pts] Searching with Heuristics

Consider the A\* searching process on the connected undirected graph, with starting node S and the goal node G. Suppose the cost for each connection edge is **always positive**. We define  $h^*(X)$  as the shortest (optimal) distance to G from a node X.

Answer Questions (a), (b) and (c). You may want to solve Questions (a) and (b) at the same time.

- (a) [6 pts] Suppose  $h$  is an **admissible** heuristic, and we conduct A\* **tree search** using heuristic  $h'$  and finally find a solution. Let  $C$  be the cost of the found path (directed by  $h'$ , defined in part (a)) from S to G

- (i) [4 pts] Choose **one best** answer for each condition below.

1. If  $h'(X) = \frac{1}{2}h(X)$  for all Node  $X$ , then ☐  $C = h^*(S)$  ☐  $C > h^*(S)$  ☐  $C \geq h^*(S)$
2. If  $h'(X) = \frac{h(X)+h^*(X)}{2}$  for all Node  $X$ , then ☐  $C = h^*(S)$  ☐  $C > h^*(S)$  ☐  $C \geq h^*(S)$
3. If  $h'(X) = h(X) + h^*(X)$  for all Node  $X$ , then ☐  $C = h^*(S)$  ☐  $C > h^*(S)$  ☐  $C \geq h^*(S)$
4. If we define the set  $K(X)$  for a node  $X$  as all its neighbor nodes  $Y$  satisfying  $h^*(X) > h^*(Y)$ , and the following always holds

$$h'(X) \leq \begin{cases} \min_{Y \in K(X)} h'(Y) - h(Y) + h(X) & \text{if } K(X) \neq \emptyset \\ h(X) & \text{if } K(X) = \emptyset \end{cases}$$

then,

- ☐  $C = h^*(S)$  ☐  $C > h^*(S)$  ☐  $C \geq h^*(S)$

5. If  $K$  is the same as above, we have

$$h'(X) = \begin{cases} \min_{Y \in K(X)} h(Y) + \text{cost}(X, Y) & \text{if } K(X) \neq \emptyset \\ h(X) & \text{if } K(X) = \emptyset \end{cases}$$

where  $\text{cost}(X, Y)$  is the cost of the edge connecting  $X$  and  $Y$ ,

then,

- ☐  $C = h^*(S)$  ☐  $C > h^*(S)$  ☐  $C \geq h^*(S)$

6. If  $h'(X) = \min_{Y \in K(X) \cup \{X\}} h(Y)$  ( $K$  is the same as above), ☐  $C = h^*(S)$  ☐  $C > h^*(S)$  ☐  $C \geq h^*(S)$

- (ii) [2 pts] In which of the conditions above,  $h'$  is still **admissible** and for sure to dominate  $h$ ? Check all that apply. Remember we say  $h_1$  dominates  $h_2$  when  $h_1(X) \geq h_2(X)$  holds for all  $X$ . ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

- (b) [7 pts] Suppose  $h$  is a **consistent** heuristic, and we conduct A\* **graph search** using heuristic  $h'$  and finally find a solution.

- (i) [5 pts] Answer exactly the same questions for each conditions in Question (a)(i).

1. ☐  $C = h^*(S)$  ☐  $C > h^*(S)$  ☐  $C \geq h^*(S)$
2. ☐  $C = h^*(S)$  ☐  $C > h^*(S)$  ☐  $C \geq h^*(S)$
3. ☐  $C = h^*(S)$  ☐  $C > h^*(S)$  ☐  $C \geq h^*(S)$
4. ☐  $C = h^*(S)$  ☐  $C > h^*(S)$  ☐  $C \geq h^*(S)$
5. ☐  $C = h^*(S)$  ☐  $C > h^*(S)$  ☐  $C \geq h^*(S)$
6. ☐  $C = h^*(S)$  ☐  $C > h^*(S)$  ☐  $C \geq h^*(S)$

- (ii) [2 pts] In which of the conditions above,  $h'$  is still **consistent** and for sure to dominate  $h$ ? Check all that apply.

- ☐ 1 ☐ 2 ☐ 3 ☐ 4 ☐ 5 ☐ 6

- (c) [3 pts] Suppose  $h$  is an **admissible** heuristic, and we conduct A\* **tree search** using heuristic  $h'$  and finally find a solution. If  $\epsilon > 0$ , and  $X_0$  is a node in the graph, and  $h'$  is a heuristic such that

$$h'(X) = \begin{cases} h(X) & \text{if } X = X_0 \\ h(X) + \epsilon & \text{otherwise} \end{cases}$$

- Alice claims  $h'$  can be inadmissible, and hence  $C = h^*(S)$  does not always hold.
- Bob instead thinks the node expansion order directed by  $h'$  is the same as the heuristic  $h''$ , where

$$h''(X) = \begin{cases} h(X) - \epsilon & \text{if } X = X_0 \\ h(X) & \text{if otherwise} \end{cases}$$

Since  $h''$  is admissible and will lead to  $C = h^*(S)$ , and so does  $h'$ . Hence,  $C = h^*(S)$  always holds.

The two conclusions (underlined) apparently contradict with each other, and **only exactly one of them are correct and the other is wrong**. Choose the **best** explanation from below - which student's conclusion is wrong, and why are they wrong?

- ☐ Alice's conclusion is wrong, because the heuristic  $h'$  is always admissible.
- ☐ Alice's conclusion is wrong, because an inadmissible heuristics does not necessarily always lead to the failure of the optimality when conducting A\* tree search.
- ☐ Alice's conclusion is wrong, because of another reason that is not listed above.
- ☐ Bob's conclusion is wrong, because the node visiting expansion ordering of  $h''$  during searching might not be the same as  $h'$ .
- ☐ Bob's conclusion is wrong, because the heuristic  $h''$  might lead to an incomplete search, regardless of its optimally property.
- ☐ Bob's conclusion is wrong, because of another reason that is not listed above.