CS 188
Spring 2020

Introduction to
Artificial Intelligence

# Written HW 3

**Due:** Wednesday 04/08/2020 at 11:59pm (submit via Gradescope).

**Policy:** Can be solved in groups (acknowledge collaborators) but must be written up individually

**Submission:** Your submission should be a PDF that matches this template. Each page of the PDF should align with the corresponding page of the template (page 1 has name/collaborators, question 1 begins on page 2, etc.). **Do not reorder, split, combine, or add extra pages.** The intention is that you print out the template, write on the page in pen/pencil, and then scan or take pictures of the pages to make your submission. You may also fill out this template digitally (e.g. using a tablet.)

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**For staff use only:**

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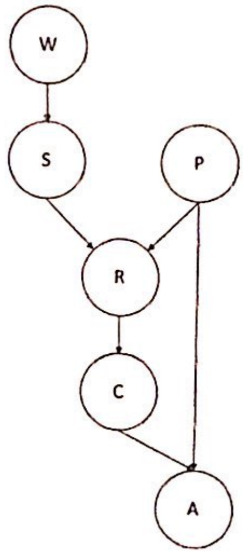
|---|---|---|
| Q1. | Quadcopter: Spectator | /30 |
| Q2. | Quadcopter: Data Analyst | /40 |
| Q3. | Quadcopter: Pilot | /30 |
| | Total | /100 |

# Q1. [30 pts] Quadcopter: Spectator

Flying a quadcopter can be modeled using a Bayes Net with the following variables:

- $W$ (weather) $\in$ {clear, cloudy, rainy}
- $S$ (signal strength) $\in$ {strong, medium, weak}
- $P$ (true position) $= (x, y, z, \theta)$ where $x, y, z$ **each** can take on values $\in \{0, 1, 2, 3, 4\}$ and $\theta$ can take on values $\in \{0^\circ, 90^\circ, 180^\circ, 270^\circ\}$
- $R$ (reading of the position) $= (x, y, z, \theta)$ where $x, y, z$ **each** can take on values $\in \{0, 1, 2, 3, 4\}$ and $\theta$ can take on values $\in \{0^\circ, 90^\circ, 180^\circ, 270^\circ\}$
- $C$ (control from the pilot) $\in$ {forward, backward, rotate left, rotate right, ascend, descend} (6 controls in total)
- $A$ (smart alarm to warn pilot if that control could cause a collision) $\in$ {bad, good}




**(a)** Representation

**(i)** [2 pts] What is $N_p$, where $N_p$ is the domain size of the variable $P$? Please explain your answer.

Answer: $N_p$ = 500

Explanation:
5 possible values for x, y, z
4 possible values for θ

**(ii)** [2 pts] Please list **all** of the Conditional Probability Tables that are needed in order to represent the Bayes Net above.

P(W), P(S|W), P(P); P(R|S,P), P(C|R), P(A|C,P)

**(iii)** [1 pt] What is the size of the Conditional Probability Table for $R$? You may use $N_p$ in your answer.

$3N_p^2$

$(R|S,P)$

Now, assume that we look at this setup from the perspective of Spencer – a spectator who can observe A and W. Spencer observes $A$=bad and $W$=clear, and he now wants to infer the signal strength. In BN terminology, he wants to calculate $P(S|A = \text{bad}, W = \text{clear})$.

**(b)** [5 pts] Inference by Enumeration

If Spencer chooses to solve for this quantity using inference by enumeration, what is the biggest **factor** that must be calculated along the way, and what is its **size**? You may use $N_p$ in your answer. Please show your work.

**Biggest factor:**

$P(w)\,P(S|w)\,P(P)\,P(R|P,S)\,P(C|R)\,P(a|C,P)$

Size of factor:

$3 \cdot N_p \cdot N_p \cdot 6 = 18N_p^2$

**(c)** [5 pts] Inference by Variable Elimination Order 1

Spencer chooses to solve for this quantity by performing variable elimination in the order of $R-P-C$. Answer the following prompts to work your way through this procedure.

(1a) First, we need to eliminate $R$. Which factors (from the 6 CPTs above) are involved?

$f(R|S,P)\ ,\ f(C|R)$

(1b) Show the multiplication of those factors and eliminate the variable of interest. What conditional probability **factor** results from this step?

$\sum_r P(R|S,P)\cdot P(C|R) = \sum_r f(C,R|S,P) = f(C|S,P)$

(2a) Second, we need to eliminate $P$. Which factors are involved?

$f(P),\ f(a|C,P)\ ,\ f(C|S,P)$

(2b) Show the multiplication of those factors and eliminate the variable of interest. What conditional probability **factor** results from this step?

$\sum_p P(P)\cdot P(a|C,P)\cdot P(C|S,P) = \sum_p f(a,P,C|S) = f(a,C|S)$

(3a) Third, we need to eliminate $C$. Which factor/s are involved?

$f(a, C|, S)$

(3b) Show the multiplication of those factors and eliminate the variable of interest. What conditional probability **factor** results from this step?

$\sum_c P(a, c|S) = f(a|S)$

(4) List the 3 conditional probability factors that you calculated as a result of the 3 elimination steps above, along with their domain sizes. Which factor is the biggest? Is this bigger or smaller than the biggest factor from the "inference by enumeration" approach?

$P(C|S, P) = 18 N_p$

$P(a|C, S) = 18$

$P(a|S) = 3$

$P(C|S,P)$ is the biggest factor, it is smaller than inference by enumeration.

(5) List the **1** unused conditional probability factor from the 3 that you calculated above, and also list the **2** unused conditional probability factors from the 6 original CPTs.

unused: $f(a|S)$

original: $f(w)$, $f(S|w)$

(6) Finally, let's solve for the original quantity of interest: $P(S|A = \text{bad}, W = \text{clear})$. After writing the equations to show how to use the factors from (5) in order to solve for $f(S|A = bad, W = clear)$, don't forget to write how to turn that into a probability $P(S|A = bad, W = clear)$.
Hint: use Bayes Rule, and use the 3 unused factors that you listed in the previous question.

$$f(S|a, w) = \frac{f(a, w, S)}{P(a, w)} = \frac{f(a, w, S)}{\sum_s f(a, s, w)}$$

$$= \frac{f(w)\, f(S|w)\, f(a|S)}{\sum_s f(w)\, P(s|w)\, P(a|s)}$$

to turn it into a probability we can normalize it by dividing by the probability of each entry by the sum of entries in the final table

**(d)** [5 pts] Inference by Variable Elimination: Order 2

Spencer chooses to solve for this quantity by performing variable elimination, but this time, he wants to do elimination in the order of $P - C - R$. Answer the following prompts to work your way through this procedure.

(1a) First, we need to eliminate $P$. Which factors (from the 6 CPTs above) are involved?

f(P) , f(a | c,P) , P(R | s,P)

(1b) Show the multiplication of those factors and eliminate the variable of interest. What conditional probability **factor** results from this step?

(2a) Second, we need to eliminate $C$. Which factors are involved? Recall that you might want to use the factor that resulted from the previous step, but you should not reuse factors that you already used.

(2b) Show the multiplication of those factors and eliminate the variable of interest. What conditional probability **factor** results from this step?

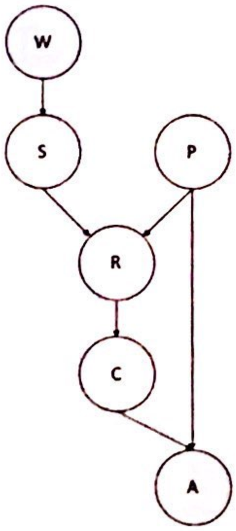
(3a) Third, we need to eliminate $R$. Which factor/s are involved? Recall that you might want to use the factor that resulted from the previous step, but you should not reuse factors that you already used.

(3b) Show the multiplication of those factors and eliminate the variable of interest. What conditional probability **factor** results from this step?

(4) List the 3 conditional probability factors that you calculated as a result of the 3 elimination steps above, along with their domain sizes. Which factor is the biggest? Is this bigger or smaller than the biggest factor from the "inference by enumeration" approach? Is this bigger or smaller than the biggest factor from R-P-C elimination order?

**(e) D-Separation**

**(i)** [5 pts] Which variable (in addition to $W$ and $A$), if observed, would make $S \perp P$? Please shade in the variable that fits the criteria and run the D-separation algorithm on the resulting graph to support your answer. If no such variable exists, please provide an explanation for why each candidate fails.



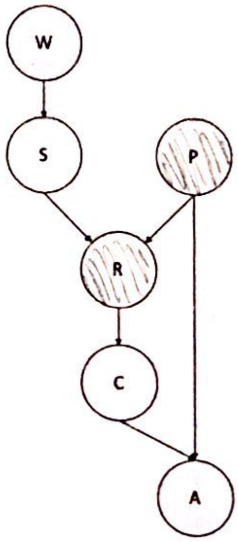

No such variable;

to make $S \to R \leftarrow$ inactive, R must not be evidence, but then the path $S \to R \to C \to A \leftarrow P$ has only active triples

**(ii)** [5 pts] Ivan claims that there exist two variables (which are NOT directly connected) such that if you know the value of all **other** variables, then those two variables are guaranteed to be independent. Is Ivan right?

● Yes, and I will shade in all but two variables in the graph below, and run D-separation algorithm to prove that those two variables are guaranteed to be independent conditioned on all other variables in the graph.

○ No, there is no such two variables, and I will provide a proof below.




$S \to R \to C$ is inactive

$S \to R \leftarrow R \to A \leftarrow C$

can be decomposed into $S \to R \leftarrow P$ inactive

So the whole path is inactive

Only inactive paths between the nodes

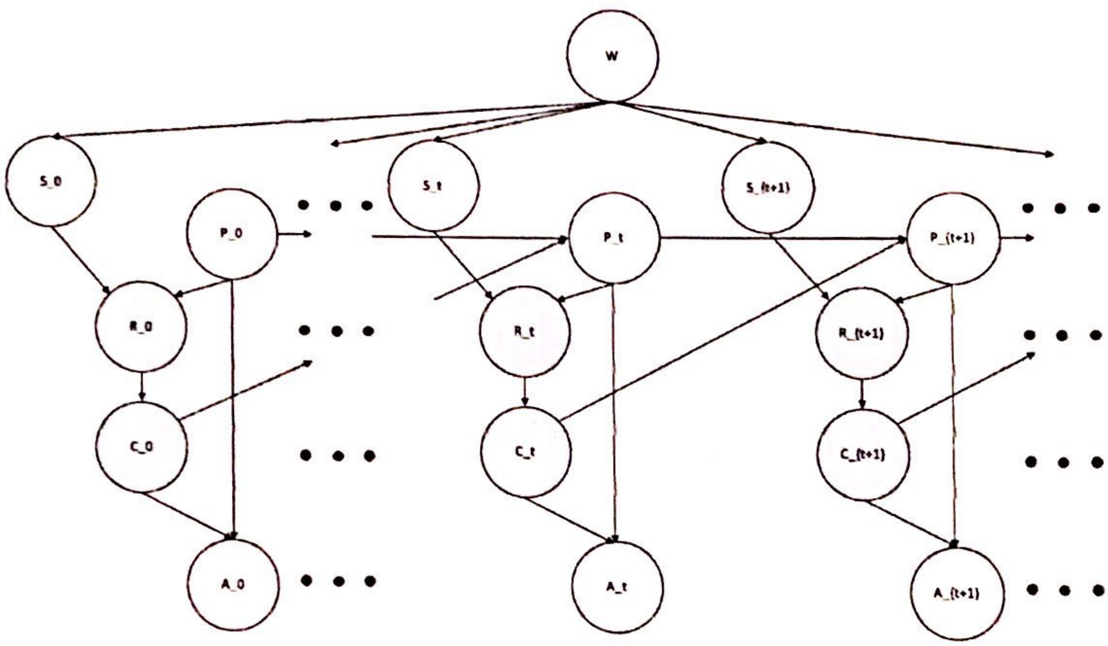
So $S \perp\!\!\!\perp C \mid \{W, P, R, A\}$

# Q2. [40 pts] Quadcopter: Data Analyst

In this question, we look at the setup from the previous problem, but we consider the quadcopter flight over time. Here, flight can be considered in discrete time-steps: $t \in 0, 1, 2, ...., N-1$ with, for example, $P_t$ representing the true position $P$ at discrete time-step $t$. **Suppose the weather (W) does not change throughout the quadcopter flight.**

One key thing to note here is that there are edges going between time $t$ and time $t+1$: The true position at time $t+1$ depends on the true position at time $t$ as well as the control input from time $t$.

Let's look at this setup from the perspective of Diana, a data analyst who can only **observe** the output from a data-logger, which stores **R (reading of position) and C (control from the pilot).**




**(a)** Hidden Markov Model

**(i)** [4 pts] List all the hidden variables and observed variables in this setup. In a few sentences, how is this setup different from the vanilla Hidden Markov Model you saw in lecture? You should identify at least 2 major differences.

Hidden variables:

A, S, W, P

Observed variables:

R, C

Differences:

1) Since W is not absened, there is an active triple between state $P_t$ and all other future states meaning $P_{i+1}$ is not conditionally independent of other states $P_{i-1}$ given $P_i$

2) The $P_i$ variables are not conditionally independent of past $R_i$ variables given $P_i$.

3) Each $P_i$ depends on current evidence, as well as past evidence, $C_{i-1}$

**(ii)** [3 pts] As a data analyst, Diana's responsibility is to infer the true positions of the quadcopter throughout its flight. In other words, she wants to find a list of true positions $p_0, p_1, p_2, \ldots, p_N$ that are the most likely to have happened, given the recorded readings $r_0, r_1, r_2, \ldots, r_N$ and controls $c_0, c_1, c_2, \ldots, c_N$.

**Write down the probability that Diana tries to maximize in terms of a joint probability, and interpret** the meaning of that probability. Note that the objective that you write below is such that Diana is solving the following problem: $\max_{p_0, p_1, \ldots, p_N}$ (maximization objective).

Maximization objective:

$$\max_{p_0 : p_N} P(p_0, \ldots p_N, r_0, \ldots, r_N, c_0, \ldots, c_N)$$

Explanation:

This is the probability of the evidence & maximizing position set occurring

**(iii)** [3 pts] Morris, a colleague of Diana's, points out that maximizing the joint probability is the same as maximizing a **conditional probability** where all evidence ($r_0, r_1, r_2, \ldots$ and $c_0, c_1, c_2, \ldots$) are moved to the right of the conditional bar. Is Morris right?

- ◉ Yes, and I will provide a proof/explanation below.
- ○ No, and I will provide a counter example below.

$$P[p_{0:N}, r_{0:N}, c_{0:N}] = P[p_{0:N} \mid r_{0:N}, c_{0:N}] \, P[r_{0:N}, c_{0:N}]$$

$$\propto P[p_{0:N} \mid r_{0:N}, c_{0:N}]$$

thus both distributions are maximized under the same assignment of $p_{0:N}$

**(b)** The Markov Property

**(i)** [5 pts] In this setup, conditioned on all observed evidence, does $P_0, P_2, ...., P_N$ follow the Markov property? Please justify your answer.

No; consider path $P_{t-1} \to R_{t-1} \leftarrow S_{t-1} \leftarrow W \to S_{t+1} \to R_{t+1} \leftarrow P_{t+1}$

This path only has active triples so $P_{t+1}$ is not conditionally independent of all P given $P_t$, which violates the Markov Property

**(ii)** [5 pts] In this setup, conditioned on all observed evidence, does $S_0, S_2, ...., S_N$ follow the Markov property? Please justify your answer.

Yes; this blocks the paths to W by creating an inactive triple which d-separates the P's meaning that $P_{t+1}$ is only conditionally dependent on $P_t$

**(c)** Forward Algorithm Proxy

Conner, a colleague of Diana's, would like to use this model (with the $R_t$ and $C_t$ observations) to perform something analogous to the forward algorithm for HMMs to infer the true positions. Let's analyze below the effects that certain decisions can have on the outcome of running the forward algorithm.

Note that when we say to **not include** some variable in the algorithm, we mean that we marginalize/sum out that variable. For example, if we do not want to include $W$ in the algorithm, then we replace $P(S_t|W)$ everywhere with $P(S_t)$, where $P(S_t) = \sum_W P(S_t|W)P(W)$.

**(i)** [4 pts] He argues that since W (weather) does not depend on time, and is not something he is directly interested in, he does not need to include it in the forward algorithm. What effect does not including W in the forward algorithm have on (a) the accuracy of the resulting belief state calculations, and on (b) the efficiency of calculations? Please justify your answer.

Accuracy:

Lower; because we exclude W the model will follow the Markov Property but that is not the case in the original model, as P depends on all prior states. This reduces the accuracy

Efficiency:

More efficient, we no longer have to eliminate W (hidden variable) when inferring $P[P_t | r_t, c_t]$

**(ii)** [3 pts] He also argues that he does not need to include hidden state A (smart alarm warning) in the forward algorithm. What effect does not including A in the forward algorithm have on (a) the accuracy of the resulting belief state calculations, and on (b) the efficiency of calculations? Please justify your answer.

Accuracy:

Same; all conditional dependencies are the same when not including A.

Efficiency: More efficient; we no longer have to join the factor $P[A_t \mid C_t, P_t]$ when eliminating hidden variable $A$.

**(iii)** [3 pts] Last but not least, Conner recalls that for the forward algorithm, one should calculate the belief at time-step $t$ by conditioning on evidence up to $t-1$, instead of conditioning on evidence from the entire trajectory (up to $N$). Let's assume that some other algorithm allows us to use evidence from the full trajectory ($t = 0$ to $t = N$) in order to infer each belief state. What is an example of a situation (in this setup, with the quadcopter variables) that illustrates that incorporating evidence from the full trajectory can result in better belief states than incorporating evidence only from the prior steps?

Suppose W takes on {cloudy, clear, rainy} w/ equal probability & P is highly dependent on S given R.
Since there is an active path between each previous P, W, and $P_t$, conditioning on all states gives a better inference for W and therefore $P_t$.

**(d)** Policy Reconstruction

**(i)** [2 pts] Denise states that the probabilistic model for the pilot's **policy** is entirely captured in one Conditional Probability Table from the Bayes Net Representation. Which table do you think this is, and explain why this table captures the pilot's policy.

Table:

$P[C \mid R]$

Explanation: The pilot will take an action dependent only on the reading of position.

**(ii)** [4 pts] Denise argues that if we were given a lot of data from the data logger, we could reconstruct the probabilistic model for the pilot's policy. Is she right?

- ● Yes, and I will provide an overview of how to reconstruct the pilot's policy from the data.
- ○ No, and I will provide a list of reasons for why we cannot reconstruct the policy.

We can fill in the values of the logger into the model. Then we can approximate the value of the missing C values using inference on each time step in increasing order

**(iii)** [4 pts] Supposed Diana (from earlier) now wants to use particle filtering as a way to estimate the hidden states. In order to do this, assume that she has the $R$ and $C$ observations (as above), along with the necessary transition models, sensor models, etc.

Let's assume that Diana has multiple logs of data, where each log contains many flights from a specific human operator. Diana reasons that the transition model $P(P_{t+1}|P_t, C_t)$ is determined by the dynamics of the quadcopter itself (so it doesn't change across human operators), and other models such as $P(R_t|S_t, P_t)$ also are pre-defined for this problem setting (and don't change across human operators). However, the factor discovered by Denise (above) is actually one that is slightly different per human operator.

(a) If Diana uses Denise's approach of reconstructing a policy from the observed data (from a particular human operator's log), would this be a more or less accurate model for representing that operator's control choices, as compared to some generic policy model that summarizes overall/average human behavior? Why?

More accurate; since the factor discovered is slightly different for each human operator, the inference would be different an less accurate than the data from an individual to determine his own factor.

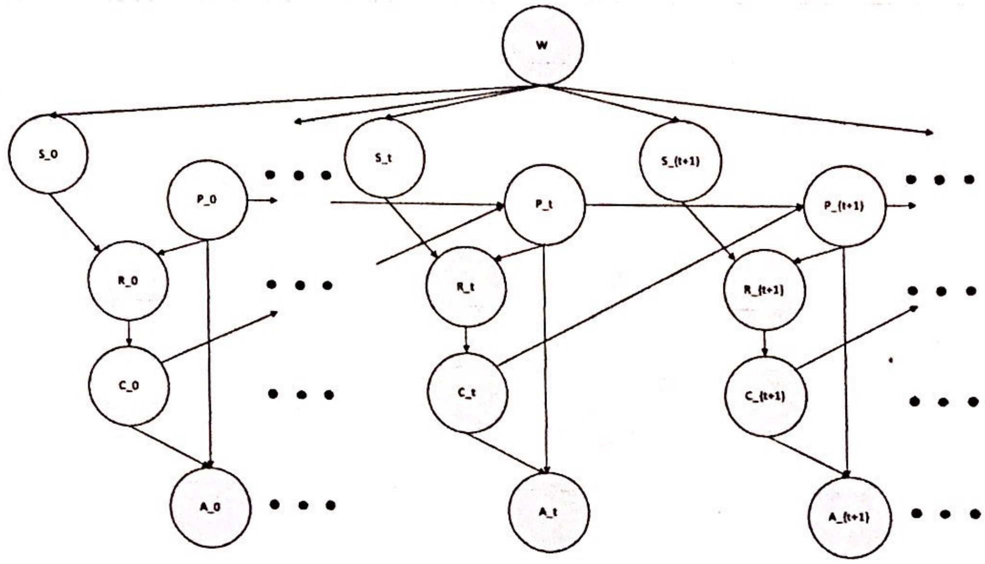
(b) How exactly would this new model be used in the particle filtering process? Would it lead to improvements in the results of particle filtering? Why?

This would be used in the observation update phase of particle filtering. The new sensor model will help us when resampling, since the sensor models won't change across human operators.

This helps us weight the distributions the same way and yeild a more accurate belief distribution

# Q3. [30 pts] Quadcopter: Pilot

In this question, we look at same setup from Question 2, but we now look at it from the perspective of Paul, a quadcopter pilot who can **observe W (weather), R (reading of position), C (control from the pilot), and A (smart alarm warning)**. As before, suppose weather (W) does not change throughout the quadcopter's flight.




**(a)** Forward Algorithm: The real deal

**(i)** [2 pts] Now that the only hidden states are $S_t$ and $P_t$, is this graph a well-behaving HMM (where $E_{t+1} \perp\!\!\!\perp E_t \mid X_{t+1}$ and $X_{t+1} \perp\!\!\!\perp E_t \mid X_t$, recall that $X$ is the hidden variable and $E$ is the evidence variable)? Please explain your reasoning. A subset of your responses from Q2 might apply here.

Yes; all paths between $P_{t-1}$ and $P_{t+1}$ given $P_t$ are blocked and only have inactive triples

all hidden variables are blocked by $P_t$ and $W_t$ so the properties hold

**(ii)** [4 pts] What is the **time-elapsed update** from time-step $t$ to time-step $t+1$? Be sure to include all hidden states and observed states, and show how to assemble the update from the conditional probability tables corresponding to the graph.

$$B'(w_{t+1}) = \sum_{w_t} P(w_{t+1} \mid w_t) B(w_t)$$

$$P[P_{t+1} \mid \{R_t, C_t, A_t, \ldots, W\}] = \sum_{P_t} P[P_{t+1} \mid p_t, c_t] P[p_t \mid \{R_t, C_t, A_t, \ldots W\}]$$

**(iii)** [4 pts] What is the **observation update** at time-step $t+1$? Be sure to include all hidden states and observed states, and show how to assemble the update from the conditional probability tables corresponding to the graph.

$$B(W_{t+1}) \propto P(L_{t+1} \mid W_{t+1}) \underbrace{\sum_{w_i} P(W_{t+1} \mid w_t) B(w_t)}_{\text{part ii}}$$

$$\downarrow$$

$$P[S_{t+1} \mid P_{t+1}] = \frac{P[S_{t+1} \mid R_{t+1}, P_{t+1}]\, P[R_{t+1}, P_{t+1}]}{P[S_{t+1}, P_{t+1}]}$$

$$B(P_{t+1}) \propto \frac{P[S_{t+1} \mid R_{t+1}, P_{t+1}]}{\sum_{S_{t+1}} P[S_{t+1} \mid P_{t+1}]} P[P_{t+1} \mid R_{t+1}, P_t] \sum_{P_t} P[P_{t+1} \mid p_t, C_t] P[p_t \mid \{R_t, C_t, A_t, \ldots W\}]$$

**(b)** Consider a simpler scenario where we only track the 2D position $(x, y)$ of the quadcopter, where $x, y$ **each** can take on values $\in \{0, 1, 2\}$. Consequently, we now only have four controls: forward, backward, left, and right. Paul, the pilot, wants to infer the quadcopter's true position $P$ as accurately as possible. In this problem, consider an additional variable $E_R$, which represents Paul's estimate of the current position, and this variable depends on the reading $R$. Consider a penalty that is equal to the L1 norm of the difference between the estimate of current position $E_R$ and the actual position $P$. Thus, the utility is defined to be $U(P, E_R) = U_{max} - ||P - E_R||_1$, in dollars.

Suppose for the rest of this question that your **reading R is** $(1, 0)$ and that the weather is always cloudy. Given this cloudy weather, the signal strength can take on 3 values with equal probability: weak, medium, and strong. These signal strengths correspond to the following errors in readings:

- Weak: The reading R returns a random number (for each position element) sampled uniformly from the domain of possible positions.
- Medium: The reading R has an error of at most 1, and that error occurs in only one of the position elements.
- Strong: The reading R is identical to the true position.

Additionally, suppose that when the signal strength is unknown, the reading R is assumed to have a 50% chance of being correct and 50% chance of being incorrect (where, if the reading is incorrect, it takes on a uniformly random value from the list of possible incorrect positions).

**(i)** [2 pts] Which variable should intuitively have the greatest VPI? Explain your answer. You should not do any calculations for this part.

Signal strength? ; it allows us to identify the position which has the most value

**(ii)** [8 pts] Suppose Paul's coworker offers to tell him the signal strength (S) in exchange for some cash. What is the most Paul should pay to know the value of S?
Hint: Recall the current reading R.

$$VPI = (E' \mid e) = MEU(e, E') - MEU(e)$$

$$= \frac{1}{3}[u_{max} - 2 + u_{max} - 1 + u_{max}] - \left[\frac{1}{2} \cdot \frac{1}{3}[(u_{max} - 2) + (u_{max} - 1) + u_{max}] + \frac{1}{2}(u_{max} - 2)\right]$$

$$= \frac{1}{3}(3u_{max} - 3) - \frac{1}{2}(u_{max} - 3) + (u_{max} - 2)$$

$$= \frac{1}{2}u_{max} - \frac{1}{2}$$

(c) (i) [5 pts] Suppose your coworker only tells you the signal strength with probability $p$, and with probability $1-p$, they don't tell you the signal strength even after payment. How much would you be willing to pay in this scenario?

Let $Y = 3u_{max} - 3$

$= \frac{1}{3} p Y + \frac{1}{3}(1-p)(u_{ymax} - 2) - \frac{1}{6} Y - \frac{1}{4}(u_{max} - 2)$

Assuming $p > \frac{1}{2}$ otherwise not buying

(ii) [5 pts] How much would you pay to know the true position (P)?

3 things part ii

or $\frac{3}{2} u_{max} - \frac{3}{2}$