2b) Mapping from $(0,\infty) \to (0,1)$.

We take: $x_i = c \tan\left(\frac{\pi y_i}{2}\right)$

$$\Rightarrow w_{x_i} = c\,\frac{\pi}{2}\,\sec^2\frac{\pi y_i}{2}\,w_{y_i}$$

$$= c\,\frac{\pi}{2}\,\frac{w_{y_i}}{\cos^2\left(\frac{\pi y_i}{2}\right)}$$

Where c is a constant.

I have taken $c = 5$ in following problem taking into account of the gaussian distribution of integration around given point.

2c) Gauss-Laguerre integration.

$$I = \int_0^\infty e^{-x} x^\alpha g(x)\,dx = \int_0^\infty \frac{x^3}{e^x - 1}\,dx$$

$$\Rightarrow \alpha = 3,\; g(x) = \frac{e^x}{e^x - 1}$$

(3) $\quad J = P\int_0^{\infty} \frac{f(x)}{x^2-a^2}$ $\leftarrow$ has Singularity around the point $x=a$.

$$= \underset{\varepsilon\to 0}{\mathrm{Lt}} \int_0^{a-\varepsilon} \frac{f(x)}{x^2-a^2} + \int_{a+\varepsilon}^{\infty} \frac{f(x)}{x^2-a^2}$$

$$P\int_0^{\infty} \underbrace{\frac{f(x)-f(a)}{x^2-a^2}}_{\text{I}} + \underbrace{P\int_0^{\infty} \frac{f(a)}{x^2-a^2}\,dx}_{\text{II}}$$

(II) $\quad P\int_0^{\infty} \frac{f(a)}{x^2-a^2}\,dx = \underset{\varepsilon\to 0}{\mathrm{Lt}} \int_0^{a-\varepsilon} \frac{f(a)}{x^2-a^2}\,dx + \underset{\varepsilon\to 0}{\mathrm{Lt}} \int_{a+\varepsilon}^{\infty} \frac{f(a)}{x^2-a^2}\,dx$

using Partial derivative technique:

$$\frac{f(a)}{x^2-a^2} = \frac{f(a)}{2a}\left\{\frac{1}{x-a} - \frac{1}{x+a}\right\}$$

$$\Rightarrow \underset{\varepsilon\to 0}{\mathrm{Lt}} \int_0^{a-\varepsilon} \frac{f(a)\,dx}{x^2-a^2} = \frac{f(a)}{2a}\left\{ \underset{\varepsilon\to 0}{\mathrm{Lt}} \left[\ln(x-a)\Big|_0^{a-\varepsilon} \right] - \log(x+a)\Big|_{a+\varepsilon}^{\infty} \right.$$

$$= 0.$$

$$\Rightarrow \quad P\int_0^{\infty} \frac{f(x)}{x^2-a^2} = P\int_0^{\infty} \frac{f(x)-f(a)}{x^2-a^2}$$

For $x=a$, we apply L'Hôpital rule.