Write-up for Homework3

September 19, 2016

1 Problem-1

The machine precision of my device is of the order 10^{-7} for single precision and 10^{-16} for double precision. Hereby, for reducing the step size of h to below machine precision, I have taken the h values in my code from 10^0 to 10^{-10} for single precision and 10^0 to 10^{-20} for double precision.
For comparision with the analytical value, I've taken the analytical values of ***cos(x)*** and ***sqrt(x)*** at the values x = 0.1, 1, 30 from Wolfrum Alpha, and thus computed the relative error.

My codes and plots for problem-1 are included in the folder ***prob1***. This folder has two sub-folders namely: ***double*** - for double precision computation and ***single*** - for single precision computation. Each of above folders then includes 3 sub-folders: ***central*** -for computation of derivative via central differentiation method, ***extrapolated*** - for computation of derivative via extrapolated differentiation method and ***forward*** -for computation of derivative via forward differentiation method.
Furthermore, above folders include two sub-folders: ***cos*** -for derivative of cosine function and ***sqrt(x)*** -for derivative of square root function. I have then computed the derivatives at points x=0.1,1 and 30 and have included my results and plots in the corresponding ***x_0.1***, ***x_1***, ***x_30*** sub-folders.

About the truncation and round-off errors: When we plot the first derivative of a function, we observe that the value of $log_{10}(relative\_error)$ decreases with decrease in the value of $log_{10}(h)$(where h refers the step size) until when step-size(h) reaches to the value of machine precision. This part of plot indicates the *truncation error*. However when the value of h exceeds machine precision, $log_{10}(relative\_error)$ increases in value again. This part of plot indicates the *Round off error*.
I've observed the similar trend in plots for almost all of my plots for first order derivative of *cos(x)* and *sqrt(x)*. However, I'd like to share that I somehow found floating point errors for the differentiation of *sqrt(x)* at point x=0.1 via both central and extrapolated differentiation method, and couldn't compile it to observe the nature of plots.

2 Problem-2

As the problem indicates, I have derived the three and five point formulas for second order derivative and I have included the scanned copy of it in the *write-up* folder under the name *derivation.pdf.* I evaluated the second order derivative of *cos(x)* at points x=0.1,1 and 30 via both three-point and five-point formula. My fortran codes for problem-2 are included in the folder ***prob2***. This folder is subdivided into two sub-folders: ***fivepoint*** - for derivative via five-point formula and ***threepoint*** - for derivative via three-point formula. Each of above folder contains threee sub-folders: ***x_0.1***, ***x_1***, ***x_30***, which condain my fortran codes and plots of second order derivative of *cos(x)* at points x=0.1, 1 and 30 respectively.

In my codes I have started value of h at $\pi/10$ and then iterated it in the loop for 100 turns with value of h decreasing to 0.85h after each loop. The upper-limit of loop i.e 100 and decrement in h ie h*.85 were taken with an idea that value of h decreases to less than machine precision value (10^{-7}-for single precision) by the end of loop. The results I obtained were very close to the corresponding analytical values, however to my surprise I found some lower values in the iteration process have very unusual larger values, and I'm not sure how these values were generated in the looping process (as can be seen in the .dat file). It's most likely due to these unusually high values, my plots for this problem are way off, and I'm not able to make any sense out of them.

3 Problem-3

For this problem I have worked on the exact and numerical solution of euler equation provided, and the codes are saved in the file *cool.f90*. Data file of the codes that shows the difference change in the abs(exact solution-numerical solution) w.r.t new timestep (i.e time step provided Δt = 0.1, 0.05, 0.025 decreased by a factor of two) is saved in the file *cool.it* and the corresponding plot of difference as a function of new Δt is saved in the file *diff.agr*.