

## Hypothesis Testing:

Description: Say you flip a coin 10 times and observe only one heads. What would be your null hypothesis and p-value for testing whether the coin is fair or not?

Solution:

First, let's talk about what we can't do:

- We can't set up a 2-sided hypothesis test
- We can't use central limit theorem due to a small n

$H_0 : p_0=0.5$

$H_1 : p_0<0.5$

$p_0$  here is simply the probability of observing heads on a fair coin: 50%.

The null hypothesis is that the coin is fair, and the alternative hypothesis is that the coin is biased: *biased towards tails (note that this a one-sided test)*:

$H_0 : p_0=0.5$

$H_1 : p_0<0.5$

Since the sample size here is 10, you cannot apply the Central Limit Theorem and so cannot approximate a binomial using a normal distribution.

The p-value here is the probability of observing the results obtained given that the null hypothesis is true, i.e., under the assumption that the coin is fair. For 10 flips of a coin, there are  $2^{10} = 1024$  possible outcomes, only 10 of which yield 9 tails and one heads.

Hence, the exact probability of the given result is the p-value, which is  $10/1024=0.0098102410=0.0098$ . Therefore, we can reject the null hypothesis at a 0.05 significance level.